Finnish pupils' success in mathematics

Factors that contribute to Finnish pupils' success in mathematics

This report focuses on the factors that contribute positively to Finnish pupils' success in mathematics and considers the implications for policy and practice in England.

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Background

International comparisons have highlighted Finnish children's success in mathematics. In 2000, 2003 and 2006, they were ranked very highly in mathematics, science and literacy on the Programme for International Student Assessment (PISA) tests organised by the Organisation for Economic Cooperation and Development (OECD). In 1999, they were also ranked highly on the Trends in Mathematics and Science Study (TIMSS) tests.[1]

Even though Finland's highest attainers were not the highest attainers overall, on average its lowest attainers did much better than the lowest attainers internationally. The gap between the higher and lower attainers was narrower than in other countries, including England.

A substantial body of literature has investigated the reasons for this performance, motivated in part by the fact that the Finns did not expect it in 2000 and 2003. Researchers emphasise that there is no clear, single explanation for the success. Some factors identified in the literature and most relevant to this study are:

- Finnish people's high regard for education, schools and teachers
- high levels of literacy
- research-based teacher education leading to reflective practitioners
- education of teachers to Master's level
- professional trust in teachers
- the emphasis on problem-solving in the curriculum since 1985
- support for low-attaining pupils
- low levels of pupils' anxiety about mathematics (although their interest in mathematics is also low)
- promotion of equity at all levels in education
- decentralisation with local autonomy at school level and for teachers
- small class sizes
- cultural and social homogeneity
- the range of in-service education
- high levels of care for pupils in school.

[1] Annex B gives examples of items from the PISA and TIMSS tests.

The stability of the country's curriculum has enabled the roots of the success to be traced back over time.

Methodology

In September 2009, two of Her Majesty's Inspectors visited three schools and a university from each of three towns in Finland to gather evidence about the factors that contributed to Finnish pupils' success in mathematics and which have the potential to aid improvement in England. This report draws on the first-hand evidence gathered during the visit from the various groups involved in mathematics education in the country, tested against what was observed in lessons. Much of what the inspectors saw and discussed in the schools visited reflected the wider-reaching accounts of the researchers, teacher trainers and national representatives with whom inspectors held discussions. In observing lessons in the schools visited, the inspectors could see that the schools and the teachers used the flexibility in the national guidelines and expectations to ensure that what was provided responded to local circumstances as well as to individual pupils.

Key contributory factors

The key factors that contribute to success fall into six areas. In some cases, these areas are interrelated.

- Ethos (and special education): identifying children's needs as soon as they arise and following them up immediately enable pupils to overcome difficulties and keep up with their peers.
- Problem solving: pupils learn from the outset to solve problems posed in realistic or meaningful contexts, aided by their high levels of literacy. They demonstrate fluency in functional mathematics.
- Text books: textbooks underpin all children's development of problem solving, thinking and rigour in applying mathematical techniques. Although the content is not specifically structured or presented with the aim of developing conceptual understanding for every topic, the text books provide effective support for progression through the mathematics curriculum. They create a fairly consistent pedagogical approach to teaching mathematics as they are adhered to closely in many lessons.
- Teaching: teachers ensure that all pupils are able to carry out the methods needed in the topic being taught, including in the complex cases.
- Teaching practice during initial teacher education: the depth and quality of teaching practice, which is shorter than placements in England, develops reflective teachers and leads to effective links between theory and practice. These links quickly become embedded in trainees' teaching methods. From the outset, trainees are supported with an inclusive ethos similar to that from which school pupils benefit to ensure that they do not fall behind.

- Mathematics pedagogy in primary teacher education: the mathematics education and subject-specific pedagogical training of primary teachers contributes to the quality of their teaching and the confidence they convey during lessons in mathematics and to pupils' ability to learn it.

Although the visit did not set out to explore any links between pupils' capabilities in mathematics and their literacy, it quickly became apparent that pupils' good skills in literacy enabled them to concentrate on the mathematics in written problems without being hampered by difficulties in reading and making sense of the context. A high proportion of the time in the first two years of schooling is spent on developing pupils' literacy skills.

Implications for policy and practice in England

Each of these factors is explored in greater detail in Part A of this report. Part A also considers the possible implications of the above factors for developing policy and practice to help raise mathematics standards in English schools. In doing so, the report recognises that:

- factors that contribute to successful teaching in one subject cannot be isolated from overall educational philosophy which, in turn, reflects wider beliefs and culture
- aspects of the current Finnish approach might not be transferred easily to the current English context.

Part A: Key findings and implications

The report concentrates on those aspects considered to have a positive influence on Finnish pupils' success in mathematics. It therefore does not aim to paint a full picture of mathematics education in Finland. Some weaknesses or 'downsides' were observed but these are not discussed in the report, except where they have implications for policy and practice in England.

Ethos and special education

Key contributory factors

- Teachers expect each pupil to succeed and support those who need it in lessons; this is facilitated by small class sizes. They act quickly if a pupil has difficulties, or returns from absence, and provide extra support sessions after school, for which schools have a budget, or a brief spell with a small special class.
- Each school has 'special teachers'[2] who work closely with class and subject teachers to assess and support pupils in mathematics, the home languages of

[2] A common Finnish term for 'special teachers' is 'erityisopettajat'.

Finnish or Swedish, and English. They meet pupils who need support before they transfer to another school. Initial teacher education is available for special teachers.

- The special teachers assess all pupils on entry to Grade 1 (at the age of seven) and plan appropriate support with the class teacher that may include attendance at special class small-group lessons with the special teacher during mathematics teaching time.
- Special classes use textbooks that correspond to those used by the whole class, with simpler presentation yet covering the same concepts and using the same problem-solving approach. Consequently, pupils do not miss any topics and are able to rejoin lessons with their classmates when their progress permits it.
- Individual schools and local clusters devise the support provided by teachers, special teachers and teaching assistants flexibly to meet the changing needs of pupils each year.
- Teachers in their pastoral and subject roles communicate directly with parents if a pupil has difficulties with her or his work or behaviour. Also, many parents readily contact class teachers if they have concerns or if they want extra support for their child on any topic.
- Teachers often remain with the same class for a number of years and so come to know each pupil's needs well and work hard to meet them. Teachers know pupils' progression in the mathematics curriculum. Staff turnover is lower than in England.
- Pupils, particularly in the lower secondary schools, help each other in lessons and are often given the option of answering harder or easier questions on the same concepts. They expect to experience challenge and to overcome it. No stigma is attached to asking the teacher or peers for help, selecting the easier questions, or attending the special class.

Downsides observed

- The most able pupils were not always challenged, either because the work set for them was too easy or because they had already grasped what the teacher had to explain further to the rest of the class.
- Although primary pupils generally took up the after-school support offered, lower secondary pupils did not always do so.

Considerations for policy

- Exploring the most effective ways of supporting teachers to identify all pupils' difficulties in key mathematical concepts that form the building blocks for the future; and providing timely resources to help them to overcome them so that pupils do not fall behind.
- Reviewing the provision of initial teacher education for special needs teachers.

Considerations for practice

- Exploring ways of enabling teachers to remain with pupils for longer than one year in order to give them more responsibility for pupils' mathematical progression; for example, throughout a key stage.
- Seeking to inform and involve parents more.

Potential limitations in the English context

- Much secondary mathematics and some primary mathematics teaching takes place in class sets based on previous mathematics attainment. It is common for pupils to be transferred between mathematics sets if their performance improves or declines slightly. This moves the responsibility from one teacher to another and sometimes causes gaps in pupils' progression through mathematical topics.
- Staff turnover is greater in England than Finland.
- English class sizes are larger and resources for support are deployed differently.
- Special needs teachers in England have to train initially as primary or secondary teachers before specialisation.

Problem solving

Key contributory factors

- There is a clear emphasis in curriculum guidelines for Grades 1 to 9 on interpreting information and solving problems. Mathematics is also a subject field within the one year of pre-primary education, which is attended by almost all children.
- When problem solving was introduced into the Finnish curriculum in 1985, the National Board of Education provided in-service training for teacher trainers and textbook writers and, subsequently, for teachers. Thus, a consistent interpretation of problem solving developed.
- Contexts for problems used in textbooks and by teachers are chosen well for realism and appeal, and require pupils from an early age to formulate in symbols the mathematics needed to express and solve a problem, then interpret their results.
- Textbooks from Grade 1 upwards, including those for lower attaining pupils, contain multi-step and logical problems using visual cues, then gradually more complex language so that reading is not a barrier to the mathematics.
- All pupils spend a high proportion of their working time in lessons solving problems, whatever their attainment level, and persist with them as they expect to be able to solve them. This expectation is built from Grade 1 through the fact that pupils are given problems with an appropriate level of demand and experience success in solving them.

- High literacy levels that help to give access to some realistic problems are achieved through substantial teaching time devoted to literacy in Grades 1 and 2, support for those who have difficulties, and the cultural emphasis on reading.

Considerations for policy

- ensuring that national assessments truly test functional mathematics and the solving of problems in non-routine as well as familiar contexts
- influencing writers of textbooks for primary pupils upwards to raise the profile of problem solving and reduce the use of repetitive exercises

Considerations for practice

- Raising teachers' awareness of the central importance of problem solving and its use, including with lower attainers and younger pupils.
- Guiding teachers, through initial teacher education and in-service training, into using realistic and meaningful problem solving.

Potential limitations in the English context

- The narrowness of approaches to problem solving through routine decoding of simple word problems is widespread, particularly in primary schools.
- There is a lack of meaningful, realistic and challenging problems in textbooks and other published resources.
- Teachers often set repetitive questions from exercises rather than selecting any questions that might involve problem solving, leaving any such questions to the end or setting them only for the higher attainers or fastest workers.
- There is a strong link between national assessment items, particularly at GCSE level, and the content of textbooks: the focus of many textbooks (often written by GCSE examiners) is on passing examinations rather than teaching mathematics.
- The interpretation of 'functional mathematics' varies within the mathematics education community.

Textbooks

Key contributory factors

- Progression is enhanced by the coherent revision of the curriculum roughly every 10 years; all grades are revised at the same time, starting with pre-school.
- The National Board of Education took a strategic approach to the introduction of problem solving within the curriculum in 1985, drawing on a wide range of subject expertise. It provided the first wave of in-service training for teacher trainers and textbook writers so that they could work together on developing

textbook series. When state control was later relaxed, the approach to problem solving was sufficiently embedded to remain in textbooks.

- The involvement of publishers, practitioners and researchers in curriculum development ensures that textbooks convey consistent messages not only about mathematical content and correctness, but also about pedagogy, such as the use of mental work, meaningful contexts and homework.
- Textbooks are accompanied by teachers' handbooks that give teachers guidance on activities and pedagogy for complete lessons.
- All pupils in compulsory education from Grades 1 to 9 are entitled to free textbooks which they can retain. In primary school, these are mostly of write-on format so that pupils have questions and answers clearly recorded.
- A corresponding textbook produced for pupils in special classes includes the same concepts and pedagogical style.
- The first section of each textbook revises the key concepts from the previous year that underpin the coming year's work. Elsewhere in the textbook there is little repetition.
- Worked examples are chosen carefully to demonstrate solutions to both straightforward and complex questions. Exercises are demanding and introduce harder questions and problems early on. Because they do not divide exercises by type or include repetitive questions, pupils have to think carefully about how to answer each question.

Downsides observed

- Where there was too close an adherence to textbooks, the pupils were engaged in little open-ended investigatory work.

Considerations for policy

- Conducting an independent review of textbooks and mathematics resources, including e-resources, and the feasibility of providing a textbook to every pupil each year.
- Providing guidelines and requirements for textbooks, with possibly a system of approval or kite-marking.
- Ensuring that conceptual development is underpinned with a good-quality teachers' handbook accompanying all textbook series.
- Requiring the Department for Education's delivery partners to work with publishers and writers at the early stages of designing textbooks to ensure a match with curriculum and pedagogical priorities.

Considerations for practice

- Providing guidance for teachers on how to use current textbooks to challenge pupils' thinking by avoiding piecemeal approaches and repetitive exercises.

Potential limitations in the English context

- English textbooks usually provide a large quantity of similar questions and, typically, leave any problem solving to the end of a long exercise or chapter.
- Pupils are not required to think about how to apply an algorithm across the breadth of cases. This is because consecutive exercises require them to make progress in only very small, separate steps and so they do not succeed in grasping the bigger picture.
- Textbooks do not provide clear mathematical pathways from Year 1 to Year 11. For example, textbooks for Key Stages 3 and 4 overlap in their mathematical content.
- Providing textbooks for each pupil, especially small numbers of textbooks for lower attaining pupils, would be costly.
- Few textbooks pay sufficient attention to building pupils' understanding of concepts and why methods work in addition to how they work.

Teaching

Key contributory factors

- Through careful exposition and questioning, teachers develop methods slowly, using only one or two examples, and then reiterate all the main points.
- They choose examples that span the breadth of the topic and include a complex case, sometimes starting with it. They then set work that covers the breadth and complexity of the topic, and that includes problem solving. This gives pupils a holistic overview of each topic.
- Teachers use mathematical language and symbols precisely and demonstrate mathematically correct presentation. They expect similarly high standards of the pupils; exercise books with squared paper aid accurate mathematical presentation.
- Teachers support individuals and spend long enough on each topic to ensure that everyone gains the required skills rather than rushing on to the next topic. In the primary and lower secondary schools visited, several teachers explained that they can spend this time because they do not feel the pressure of covering material in preparation for an external examination.
- Pupils are determined to succeed in the problems that they attempt, rather than rushing on to other ones. They answer questions using a pencil, retrying until they are successful.

Considerations for policy

- Improving teachers' insight, through initial teacher education and in-service training, into how to choose examples carefully to build a more holistic approach to each topic and then set wide-ranging and challenging questions.

Considerations for practice

- Making a clearer distinction in teaching between informal written methods and the necessary precision of formal methods, supported by working in exercise books with squared paper.

Potential limitations in the English context

- The common approach of teaching one small part of a topic at a time and giving pupils many similar questions does not build an overview of the topic as each part is introduced incrementally and separately. Also, the quantity of questions diverts time from thinking about the important differences between cases.
- Pupils' written work using formal methods often contains incorrect mathematical syntax such as wrong use of the = sign. This often goes unchallenged by teachers and is sometimes incorrectly demonstrated by them. This contributes to pupils' errors in solving problems and can impede the development of reasoning.

Teaching practice during initial teacher education

Key contributory factors

- The provision of all initial teacher education in universities, with the early placements of all trainees in a nearby practice school, enables very close collaboration between lecturers and teachers on training and observing trainees.
- A focus on developing quality in teaching rather than expertise through quantity of practice is delivered through trainees teaching only a small number of lessons, for each of which supervising teachers discuss planning and evaluation at some length.
- Teachers' self-evaluation is gradually enhanced through the style of interaction with supervisors; required logs within portfolios; and evaluating each other's lessons within the group of mathematics or primary trainees at the same practice school.

Consideration for policy

- Arranging initial placement of substantial groups of trainees at selected good-practice schools.

Ofsted

Consideration for practice

- Developing in-depth approaches at the initial stage of teaching practice.

Potential limitations in the English context

- Providing supervising teachers in practice schools with sufficient time to work closely with trainees has resource implications.
- The presence in all lessons of supervisors and often some observing trainees affects the development of behaviour management skills.
- The practicalities of managing large cohorts of trainees and the physical space required for them would be a challenge for many schools, although there is already some good practice in this area.

Mathematics pedagogy in primary teacher education

Key contributory factors

- Trainees enter university having studied mathematics in upper secondary school (equivalent to Years 12 to 14), so their experience of mathematics is more recent and at a higher level than most of their English counterparts.
- All trainees build on this by studying a substantial amount of mathematics pedagogy as part of the five-year Master's degree teaching qualification. Roughly 10% of trainees take additional optional mathematics courses.

Considerations for policy

- Incorporating more mathematics pedagogy in primary initial teacher education at BEd and PGCE level (as recommended in Ofsted's report *Mathematics: understanding the score*).[3]
- Incorporating a substantial proportion of mathematics-specific pedagogy in developing initial primary teacher education to Masters level, including current new courses leading to the deferred Masters in Teaching and Learning (MTL) studied by recently qualified teachers.
- Introducing mathematics enhancement studies for primary teacher trainees who do not have a higher qualification than a grade C in GCSE mathematics.

Potential limitations in the English context

- Limited time is available for mathematics-specific pedagogy during one-year PGCE courses.

[3] *Mathematics: understanding the score* (070063), Ofsted, 2008; www.ofsted.gov.uk/publications/070063

- BEd courses offer greater potential for increasing the time spent on mathematics pedagogy but the proportion of primary trainees studying BEd courses is decreasing.

Part B: Evidence and evaluation

Background information

International mathematics assessments

1. Finnish pupils aged 15 scored very highly in the OECD's PISA tests in 2000, 2003 and 2006 in mathematics, science and literacy. While all three subjects are tested each time, one is assessed in particular depth: literacy in 2000, mathematics in 2003, and science in 2006. Finnish pupils also scored highly in the TIMSS tests in 1999. (Finland did not participate in TIMSS in 2003 and 2007.) Even though Finland's pupils were not the highest-attaining overall, on average its lowest attaining pupils did much better than the lowest attainers internationally. The gap between the higher and lower attainers was narrower than in other countries, including England.

2. The PISA 'mathematical literacy' items are realistic problems, presented in writing with diagrams, which pupils need to interpret and synthesise. The items require the pupil to make sense of the problem, and then to decide on the mathematics to use and the method of solving them. They test 'functional mathematics', the crucial mathematical skills that sit within 'using and applying mathematics' and the 'key process skills' of England's National Curriculum.

3. By contrast, the items in the TIMSS test are more routine and often only require a single step to solve them. Many are context-free; others have contexts which add little to the mathematical demand. Since 1985, the objectives of the Finnish mathematics curriculum have emphasised formulating and solving problems that occur in everyday situations and can be solved with the aid of mathematical thinking. This has led to the use of a wide range of problems in textbooks and by teachers. While these are generally briefer than the PISA items, they build the necessary reasoning skills. Annex B gives examples of items from the PISA and TIMSS tests.

Finnish culture and attitudes to education

4. Education has been greatly valued in Finland since the Second World War when the Finnish people decided that it was key to their subsequent economic development. Teaching is a highly respected profession. Finns' avid reading of books and newspapers contributes to their high literacy levels. Finland has two official languages, Finnish and Swedish, both of which are used in some areas of the country. Most education is offered in Finnish: approximately 6% of pupils are taught in Swedish. Local authorities are required to provide education in the Sami language in parts of Lapland. The development of concentration is one of the aims of early primary mathematics education; 'sisu' is the word Finns use to

describe characteristics that include perseverance, determination and community spirit.[4]

Finnish schools and the mathematics curriculum

5. Pupils attend pre-school at the age of six and begin primary school at seven. Finns describe compulsory education as basic education. It is free and provided in primary school from Grades 1 to 6 (equivalent to Years 3 to 8 in England) and in lower secondary school from Grades 7 to 9 (equivalent to Years 9 to 11). The schools are comprehensive and pupils are taught in small mixed-ability form classes.

6. After completing Grade 9 (the end of compulsory education), if pupils wish to remain in education, they must apply for entry and demonstrate appropriate attainment in the teacher assessments that are conducted in each subject. The vast majority of pupils go on either to upper secondary school or vocational school to study generally for three years. At this stage there may be some specialism, such as the upper secondary school visited by the inspectors that specialised in the arts. Mathematics remains a compulsory subject at this stage, although not necessarily for the whole three years. In upper secondary school, mathematics may be studied at an advanced level or as a more basic course. Students may choose whether or not to include mathematics from either of these courses within the range of subjects they take in their matriculation examination. Students who, subsequently, wish to continue into higher education take the matriculation examination and, often, university entrance examinations at the end of upper secondary and occasionally vocational school.

7. The school year starts in the middle of August. It is commonly split into five periods of around seven weeks. The number of hours in school each week increases with pupils' age. The time allocated to mathematics lessons each week is determined locally within national guidelines for different age groups.

8. The management structure of Finnish schools is flat. There is a principal and deputy principal(s): in one of the primary schools visited, the post of principal rotated among staff on a two-yearly cycle. There are no subject leaders although, in practice, some enthusiastic teachers voluntarily take on some form of leadership role, with collaborative support from colleagues. An ethos of professional trust about the quality of teachers' work exists and there is no school-based system for evaluating teachers' performance through observing lessons or analysing pupils' results. One upper secondary school visited was in the early stages of establishing a system of professional development through mentoring and lesson observations.

[4] Discussion of PISA 2006 by the Centre for Educational Assessment at the University of Helsinki, http://www.pisa2006.helsinki.fi/finland/sisu/sisu.htm

9. There is not an exact match between the secondary mathematics curriculum at Key Stage 4 in England and Grades 7 to 9 in Finland. For instance, the Finnish mathematics curriculum places greater emphasis on functions, while the English curriculum includes more statistics. Higher-attaining Key Stage 4 pupils in England study more advanced material, such as the solution of quadratic equations by factorising and the formula, the sine and cosine rules, and conditional probability. However, the description of good performance (a score of 8) in Finland aligns fairly well with that for GCSE grade C in England.

10. While the National Board of Education determines the core mathematics curriculum, including objectives and assessment criteria, the detail is determined locally by schools and/or municipalities. Teachers have autonomy over the choice of teaching methods and resources. Although parents are represented on local curriculum boards, little evidence was seen during the visit of their direct influence on the mathematics curriculum at a local or individual school level.

Ethos

11. The nine Finnish schools visited were typified by their strong dedication to identifying children's needs as soon as they arose and by following them up immediately. There was close attention to pupils' care and well-being, including through counselling, and an ethos of mutual respect among all parties involved in the school. At the same time, there was an onus on each pupil to work to the best of her or his ability and to make the most of the activities and support offered.

12. The teachers seen during the survey, in their pastoral and subject roles, communicated directly with parents, especially if a pupil was having difficulties with work or behaviour. The teachers said that parents readily contacted class teachers if they had concerns or if they wanted extra support for their child on any topic. The textbooks make it clear what is needed so parents can easily see if their children are behind in their work. Parents' meetings are usually held twice a year to inform parents about the curriculum, pastoral issues, and how they can help their child. The teachers told inspectors that attendance at these meetings was high. In addition, there is generally one meeting each year when each family has half an hour to discuss their child's progress with the form teacher. This is always provided in Grades 1 to 7 and as needed in Grades 8 and 9.

13. Pupils generally remain in the same mixed-ability class throughout primary school and again throughout lower secondary school. Mathematics is taught to classes of fewer than 20 pupils in primary schools and approximately 20 in lower secondary schools. Teachers often stay with pupils through consecutive years in primary school, for example Grades 1 to 3 or 4 to 6, and occasionally right the way through Grades 1 to 6; and Grades 7 to 9 in lower secondary school. Consequently, they know pupils very well. In the schools visited,

relationships were friendly and informal: there was no school uniform and pupils called teachers by their first names. There was a noticeable family atmosphere with teachers caring about and supporting the progress of every pupil. Teachers sat with their class during lunch time, when a free hot lunch was provided for all pupils. The strong sense of mutual respect between teacher and pupils contributed to relaxed, positive behaviour in lessons and during the 15-minute break that many of the schools provided after each 45-minute lesson.

14. There was an unspoken assumption that all pupils would meet the full range of mathematical concepts and techniques, and be able to answer questions about them, albeit at different levels of demand. This was achieved through a combination of teachers' expectations and support and pupils' perseverance, which, as noted above, is consciously developed from Grade 1.

Learning support and special education

Identification of needs

15. Pupils' needs are identified immediately on entry to school and they are allocated support accordingly. At the start of Grade 1, information from pre-school and assessments of all pupils in their home language and mathematics are evaluated by the special teacher.

16. Special teachers are involved in assessing pupils' mathematical needs. In one of the primary schools visited, for instance, the special teacher marked tests set for all pupils at the start of Grades 1 to 6. In a lower secondary school, the special teacher set tests for pupils who had previously been in special classes or if the teachers requested them.

17. Special teachers in lower secondary schools visit the partner primary schools to discuss Grade 6 pupils' needs. Special teachers regularly discuss all pupils in special classes with their class teacher in primary school or their mathematics teacher in lower secondary school and with the educational psychologist, who works for a number of days in the school each week. When class teachers or mathematics teachers find that a pupil needs extra help, they first discuss with the special teacher the most effective ways of providing this.

Types of support

18. Finnish schools provide multiple levels of support, which are designed flexibly to meet their pupils' needs.

Support in class

19. The teachers expect that all pupils will succeed and help them to do so during the lesson. Pupils are happy to help each other and are aware that it helps them learn. They do not belittle each other and are not judgemental about

other pupils being slower or faster in their work; there is no peer pressure against working hard. The combination of small classes and teachers helping all pupils when they are stuck means that pupils experience success. This raises their self-esteem and so pupils expect to be able to succeed. While inspectors saw that most pupils persevered diligently, sometimes referring to their theory book or notes, others asked for assistance too readily from teachers.

20. In addition, there are support workers in class lessons for pupils who have individual education plans but who do not need the nurturing environment or simpler approach of the special class. These support staff also help the rest of the pupils in the class. Much of this provision is part-time.

Short-term support to help overcome a specific difficulty or catch up after absence

21. If a pupil is stuck on a topic or does badly in a test, teachers offer quick provision of focused, short-term support lessons to ensure that the pupil can overcome her or his difficulty and build on the work in the ensuing topic. These support lessons take place after school and are usually provided by the pupil's teacher. Pupils are entitled to such support to help them catch up or overcome difficulties, and parents can request it. Primary school pupils take up these offers eagerly but some lower secondary school pupils choose not to attend.

22. When pupils have been absent from lessons, they may be offered after-school support or a brief period in the special class where the teacher can quickly go through with them the work that they missed.

Various forms of medium-term or part-time support

23. Individual schools have considerable flexibility about how they arrange the school day and they may opt for different start and finish times on different days. Each school devises mechanisms for support to meet its pupils' needs, for example, in one of the schools visited, a class teacher who was not teaching provided support once a week for pupils from another class who had difficulty concentrating. In another school visited, the principal arranged for some lessons in Grade 2 classes to be provided to half the class at a time so that harder topics and practical work could be taught in smaller groups. One half attended school from 8.00am to 12 noon and started the day with mathematics. The other group started at 9.00am and ended the day at 1.00pm, with mathematics last.

Special classes

24. Pupils attend special classes for mathematics, Finnish/Swedish and English as needed or as indicated on their individual education plan. In general, one special teacher works with the pupils for all of these subjects and gets to know the pupils well. However, in the lower secondary school, special classes are sometimes taught mathematics by subject teachers. Mostly, pupils attend the

special class for all of their mathematics lessons but, occasionally, they attend for one or two of the week's lessons. Some special classes include mixed ages, for example, some primary schools have special classes for Grades 1 to 2, 3 to 4 and 5 to 6. In some municipalities, one primary school is designated to provide the special class for certain year groups, so all pupils who need to attend it join the roll at that school. Needs are reviewed regularly and a few pupils in special classes reintegrate into the class during the year.

Special schools

25. Two percent of pupils attend special schools. They are entitled to two additional years of free schooling.

Teaching, learning and assessment

Emphasis on problem solving, logical reasoning and literacy

26. The emphasis on problem solving has been in the Finnish national curriculum since 1985 and was defined more explicitly in 1994. There has been much in-service training on the subject. Everyday problems are used in mathematics lessons. Also, the out-of-school environment includes more numeracy. For example, there is a substantial amount of data in house advertisements and tourist information posters, so that interpreting mathematical information is a natural part of day-to-day life. A museum in Jyväskylä, visited by all school pupils as part of the curriculum, has a large number of thought-provoking mathematical activities in context for pupils to tackle.

27. Pupils have good language skills: they can discuss and reason well, although this is not exploited greatly by teachers through oral work. They have high levels of literacy which help them to make sense of problems written in words. Finnish pupils' language and thinking skills enabled them to answer correctly the questions on probability in international assessments, even though they had not been taught probability. The research evidence speaks of pupils' interest in reading, including newspapers, which reflects a wider cultural interest in reading and the prevalence of public libraries. The development of pupils' literacy and oral skills is given great prominence in the first two years of compulsory schooling. Pupils learn about and through the Finnish or Swedish language for eight of the 21 school hours in the week. Subjects such as science and history are not studied until pupils are older. By the end of Grade 2, all are reading well enough for the written language of mathematical problems to be accessible, allowing the pupil to concentrate on the mathematics rather than on the words in which it is expressed.

Textbooks

28. Textbooks are the principal vehicle for delivering the mathematics curriculum. Researchers and teachers are involved in the writing teams, which helps to ensure that the focus is on teaching and learning mathematics. Currently, most

textbooks are produced by three main publishers, and all emphasise the solving of a variety of problems. For a number of years before 1992, state control of textbooks supported consistency in the emphasis on and interpretation of realistic problem solving, as did the in-service training that accompanied the introduction of the 1985 National Curriculum. This resulted in textbooks for Grades 1 to 9 having many questions set in contexts that seem realistic rather than contrived.

29. Each year, all pupils in primary and lower secondary schools (compulsory education) are entitled to be given a textbook to keep, so schools generally review and order new books annually. Textbooks usually start with a large revision section on the main points covered in the previous grade, the purpose being to ensure that key building blocks for new work are secure. This takes about four to six weeks of lessons. One primary teacher said that he felt this took the pressure off completion of the previous year's textbook because he knew important concepts would be covered again when he took the class the following year and that he would be aware of any small gaps in pupils' knowledge.

30. Most primary textbooks are write-in books and typically include:

- many illustrations
- spaces for answers to mental arithmetic
- worked examples
- a small number of questions of differing style and complexity
- extension activities
- homework for each lesson.

31. Resource sheets and practical aids are often provided within the books; for example a pocket at the back might contain a 100 square, a number line made of folded card and a metre rule.[5] Because pupils keep the books, they do not have to spend time copying worked examples.

32. The textbook page can appear rather crowded but this does not seem to worry pupils. Books for pupils in special classes have larger pages and the size of the print is larger. The language and the problems are simpler language but the same concepts are covered. Teachers' guides accompany the textbooks. The teachers' handbook for Grades 1 to 4 used at the primary school visited in Vantaa had a continuing storyline that involved the same fictional family and which set each mathematical topic in context. Some textbooks include traffic lights or self-assessment in the form of a 'Do you know this?' set of questions

[5] A 100 square is a grid in which the numbers 1 to 100 are arranged in rows of 10. A number line is a visual representation of a sequence of numbers at equal intervals such as 0, 1, 2, 3, 4 and so on, but can include negative numbers, decimals and fractions.

at the end of a section, for pupils to use in preparation for the end-of-chapter/section test. Some textbooks also include pages of puzzles.

33. Secondary textbooks are not generally write-on, but are again provided each year for pupils to keep. Some have harder and easier questions identified by a symbol against the question or sections of a chapter; pupils are often asked to select which questions to answer. The textbooks provide a variety of question styles within each exercise and topic. This contrasts with many English textbooks in which questions are often similar in style and include few believable problems posed in words. Finnish texts link coherently with the mathematics needed outside the classroom, such as that seen in newspapers and in public notices.

34. Primary textbooks include homework questions that are linked to each lesson, so homework is set in small amounts and often. Sometimes teachers ask pupils to 'finish off' the lesson's work. After Grade 4, answers are usually provided in the back of the textbook, which allows pupils to check their work. In two of the lessons seen, a minority of pupils had not attempted homework: the teachers did not make an issue of this during the lesson, and explained that they had regular communication with parents where there were concerns. In some of the secondary classes, pupils presented their solutions to their peers using a visualiser or by writing on the board. [6]

35. Mathematical symbols are used consistently; pupils learn to read and interpret many of them at an early age. Pupils also learn to use symbols for pre-algebra work and logic puzzles, for example, with items illustrated by different images such as a ball or a car. Thus abstraction is introduced early on through accessible images and problems that require pupils to formulate in symbols the mathematics needed to express and solve a problem.

Teaching and learning

36. Teachers are confident in their professional approach and expect learning in lessons and sequences of lessons to be successful but are not concerned to cover a fixed amount of content in a specified time. For example, this was illustrated in a lesson seen in an upper secondary school. The teacher introduced speed, velocity and acceleration through graphs and practical activity, but he told inspectors that he did not expect all the students to understand all of the concepts by the end of the first lesson and that they would be developed gradually during the sequence of lessons. A primary teacher seen preferred to spend a little longer on establishing the key concepts required to underpin the following year's work rather than completing all sections of the textbook; he taught the same pupils from Grades 1 to 6. Teachers did not show any insecurity in their knowledge of the mathematics

[6] A visualiser projects what is being written onto a board or screen.

being taught or that it might have been too difficult for the pupils to grasp, although they sometimes judged that to explain the underpinning mathematics might have been too complex for pupils.

37. Teachers have a long-term view of learning the whole sequence and leave secondary pupils to decide what to finish. They do not feel pressurised if pupils fall behind as they can carry on in another lesson. In most of the lessons the inspectors watched, the pupils recorded the methods demonstrated by the teacher, paying close attention to the precise use of symbols and the way solutions were set out. Upper secondary students have a lengthy formula book that they use across all their mathematics, physics and chemistry lessons. In the lessons seen, older pupils often chose to use a separate theory notebook.

38. There is a notable difference in the way Finnish teachers use examples in lessons. Whereas English teachers often introduce a simple example and build up incrementally, sometimes waiting until later lessons to cover the breadth of the topic, Finnish teachers introduce quite difficult examples early on. Teachers pay attention to mathematical notation and correctness in setting out the steps to a solution themselves and by the pupils. They also illustrate the algorithm and steps in thinking out loud, asking some pupils for answers and checking that they are following the method. Pupils then tackle only a few questions, which are not all similar in type. This means that they have to try hard questions without having seen an identical example, and therefore have to think it through for themselves.

39. The following three examples illustrate some of the distinctive characteristics of Finnish mathematics teaching. An important feature in these examples is teachers' careful but demanding use of examples.

> **Example 1** (upper school): At the end of a mechanics lesson introducing speed, velocity and acceleration, the teacher set students five graphs for homework, each of which represented a different physical situation. Through skilful questioning about the graphs, he drew out key points which prepared students well for thinking through their homework. In this discussion, he used a visualiser with the graphs worksheet to point to the graphs and write brief notes that the students copied down. Earlier in the lesson, students used a data logger to show the results of a vehicle rolling down an inclined plane. Before each roll, the teacher asked the students what the graph would be like. In that one lesson, he lectured on speed, time, velocity, and acceleration from a theoretical and practical perspective. Although immediately after the lesson not all the students were clear on the difference between speed and velocity, they could explain that the distance–time graph was steeper at faster speeds and that the vehicle accelerated.

Example 2 (upper school): A lesson on quadratic equations started with students voluntarily writing out the solutions to homework questions on the board. The teacher went through their solutions, emphasising key points, for example, inserting a dot between bracketed linear expressions to remind students about the unseen multiplication. She connected a specific example of $x^2-1 = 0$ with the more general $ax^2+b = 0$, making a link with the new work that was to follow. She set out the methods carefully, as had the students, following the Finnish convention of showing with a vertical line the operation to be applied to both sides of the equation to obtain the next line of working. The teacher then introduced the quadratic formula without deriving it, responding to a student's question that she need only remember it and did not need to know where it came from. She chose a difficult first example, $-x^2+2x+3 = 0$, the negative x^2 coefficient causing debate among the students. She prompted them by comparison with x^2 where the coefficient is 1 so that they realised that $-x^2$ had a coefficient of -1. Then she set questions that varied greatly in difficulty, covering the three cases of two, one and no real roots, and involving negative coefficients and fractions. Where there were no roots, she explained that there was no number that could be put in place of x that would make the equation zero. However, she did not show that solutions could be checked by substitution. Students used the answers in the back of the book to check their answers. By the end of the lesson, she had covered all the cases (the two 'incomplete cases' formed by $b=0$ and $c=0$ and the three cases that stemmed from the use of the formula). This categorisation of incomplete and complete quadratic equations is a different approach from that commonly used in England, and so is the coverage of all the cases immediately on meeting the formula. Students were aware of the value of memorising the formula even though they had it in their formula books.

Example 3: In a Grade 5 lesson on multiplying a fraction by a whole number, the teacher demonstrated two examples: one that required the answer to be cancelled down and the other needing conversion to a mixed number. In his explanation, he used images of a rectangle, a number line, and repeated addition to ensure that pupils understood what was happening and could use the multiple representations of a fraction. He required pupils to write out the repeated addition steps when tackling the exercise.

40. There was an air of calm and quiet productivity about the lessons observed, often described as typical by the Finns who observed the lessons jointly with the inspectors. Most pupils took responsibility for their learning. The pupils in the secondary schools selected appropriate problems and could give reasons why, for example, when choosing between the A (easier) and B (harder) exercises in a textbook some preferred the challenge of B, others found B too hard or wanted to try A before doing B at home. However, some pupils did not

push themselves hard enough. Teachers did not spend time pursuing this during the lessons observed but explained that they had plenty of contact with the parents of pupils for whom they had concerns.

Assessment

41. Matriculation takes place at the end of upper secondary school (at the age of 19). There are no other national examinations. The National Board of Education monitors national performance at Grades 6 and 9 by a process of random sampling every two to three years. For those schools selected, the tests are compulsory. Lower-performing schools can request support but none is imposed. Data show that there has been an increase in the variation between schools in recent years.

42. Although there are no national assessments, teachers test pupils regularly, generally half-way through and at the end of a course (a period of seven weeks), as well as quick check-ups in between. From Grade 6, teachers assess pupils' performance on a scale of 4 (fail) to 10 (excellent) with 8 representing good. (In this report, we refer to these scores as levels 4 to 10.) At the end of Grade 9, teachers assess each pupil's final level in mathematics, and this affects entry to the local upper secondary school. The 'cut off' score varies from roughly 7 to around 9 in different areas of the country.

43. In general, higher levels are based on higher marks for identical tests rather than more demanding additional content. Pupils' levels are based mainly on their test marks but can be raised one level if their class work and contribution are better. One teacher, for example, kept records of marks for pupils' solutions presented on the board to their classmates. Test marks are not downgraded if class work is of a lower standard. Curriculum guidelines from the National Board of Education include descriptions of 'good performance' at the end of Grades 2, 5 and 9 (English Years 4, 7 and 11, respectively). Teachers felt confident that they could distinguish between performances at different levels despite the absence of a set of descriptors for each of the levels. They considered that they had developed over time an inbuilt sense of each level. This is a very different system from the assessment and moderation of English pupils' work against grade/level descriptions or by using the guidance for Assessing Pupils' Progress.[7]

44. Teachers test their classes on work taught, often writing tests themselves or using those from textbook chapters. Fewer than 10% of Finnish schools purchase tests from the mathematics teachers association. Teachers say that they do not feel pressure to rush through or narrow their teaching of the mathematical content as there are no external tests during the compulsory

[7] Further information on Assessing Pupils' Progress can be found at: http://nationalstrategies.standards.dcsf.gov.uk/primary/assessment/assessingpupilsprogressapp

schooling years. This differs somewhat in the upper secondary schools where teachers prepare students for the matriculation examination. A recent development in Jyväskylä has been the publication in each of the last two years of local schools' results in Grade 9 assessments in Finnish/Swedish and mathematics.

Teachers' training and professional development

Initial teacher education

45. The main routes to becoming a primary class teacher, a secondary mathematics teacher or a special teacher are different. There is a very high application rate for primary teacher training, with only a small minority of applicants being successful. Secondary mathematics teachers are harder to attract but courses are, nevertheless, oversubscribed.

46. Entry into a permanent teaching post is through a Master's degree, which normally takes five years to complete. It consists of many separate units which students study at different rates. Primary trainees generally embark on the five-year initial teacher education programme to become a class teacher, having studied at least basic level mathematics in upper secondary school. This is more mathematics, and studied for longer, than would be the case for a trainee in England, the majority of whom study no mathematics beyond grade C GCSE in Year 11. There are some compulsory mathematics courses as well as mathematics options for those who wish to specialise in the subject. About 10% of Finnish primary trainees choose to take an additional specialist course in mathematics, a proportion similar to that recommended by the review of mathematics teaching in England in 2008.[8] This proportion is much greater at the University of Joensuu, and data show higher boys' attainment in schools in that region than others. By choosing to specialise further in mathematics during the final two years of their course, primary teachers can become qualified to teach mathematics at lower secondary schools too, although not all choose to do so.

47. For secondary trainees, the majority of time is allocated to mathematics, some time to a subsidiary subject and some to education. Many complete a three-year undergraduate degree in mathematics before converting to secondary teaching and gaining their Master's degree. Some gain experience through temporary teaching before or during their studies.

[8] *Independent review of mathematics teaching in early years settings and primary schools*, DCSF, 2008; http://publications.education.gov.uk/default.aspx?PageFunction=productdetails&PageMode=publications&ProductId=DCSF-00433-2008.

48. A distinctive feature of the Finnish system is the initial teacher education course for special teachers. This contrasts with the English system where separate training in special educational needs follows qualification as a teacher.

49. All trainees are required to complete a thesis, but this is not often on mathematics pedagogy. Most primary trainees, other than those who choose to specialise in mathematics, write it on another subject or a general topic. Secondary trainees usually choose mathematics rather than mathematics education.

50. Initial teacher education is conducted at universities, each of which works with a neighbouring ‘practice school’. These schools play a significant role in trainees’ development of pedagogic practice. Trainees observe lessons, sometimes alongside their peers, and teach a few lessons. These are evaluated in depth at the planning stage and afterwards. Placements for teaching practice in the practice school ensure high-quality feedback on these few lessons.

51. School supervisors are trained in coaching and the psychology of how trainees learn. Optimum use is made of each lesson to develop the teacher’s reflection and skills. For each trainee, lessons include joint observation by training staff from the school and the university at different points in the placement, followed by discussion with the trainee. Joint observation with other trainees and peer evaluation also help to build up a thoughtful, critical and supportive learning environment. Part-way through a placement, a progress meeting between the school and university supervisors is commonly held to discuss each trainee’s strengths, weaknesses and ways to improve.

52. Earlier and later placements have different professional and pedagogical emphases, and reflect the goals of initial teacher education. These are to produce highly educated, pedagogical experts who are well-informed about education research, and committed to lifelong professional development. There are no national criteria for the assessment of trainees, so criteria vary across universities although there is some liaison about them. This differs from the English system where the professional standards set out what every newly qualified teacher must know, understand and be able to do.

53. Most teachers stay in the profession for life. The structure of teachers’ pay is very different from England. It is related to the number of hours they teach and length of teaching experience. Teachers receive payment for the occasional additional lessons that they provide after school. Each school sets a budget for this; for example, one of the primary schools visited budgeted for a total of three such lessons each week. Teachers in practice schools receive higher salaries as they are also required to support trainee teachers.

Continuing professional development

54. Each teacher is granted three days a year for continuing professional development, although the teachers told inspectors that it was possible to

spend longer than this in development if they wished. In discussion with the teachers, it was difficult to establish the extent of the opportunities for continuing professional development because they did not necessarily recognise some of their networks, meetings or other curricular activities as training or professional development. Instead, there appeared to be an unspoken assumption that teachers reflect on their practice in wanting to teach as well as they can and therefore naturally take up opportunities, read articles and discuss their work with their colleagues to help them achieve that objective.

55. Some initiatives appear to have had considerable impact on individual teachers or schools. In Vantaa, the Matiikkamaa project, with funding from the municipality, provides in-service demonstration lessons. These have helped teachers to use practical equipment to improve pupils' understanding of concepts such as fractions and negative numbers. The project, which has spread to a number of regions, was influenced by ideas from Hungary to improve mathematical thinking and understanding. The national LUMA programme, which ran from 1996 to 2002, aimed to increase the take-up of mathematics and science courses in upper secondary school and higher education. Through networks linking subjects, schools, universities and vocational institutions, it had a strong impact on developing and sharing ideas for those involved, but not more widely.

Notes

Two of Her Majesty's Inspectors visited a university and three schools in each of three towns in Finland: Helsinki/Vantaa, Jyväskylä, and Vaasa. The objective was to gather evidence of the factors that contributed to the country's success in mathematics. Evidence was collected through:

- observations of lessons and support sessions, and discussions with teachers, principals and pupils in three primary, four lower secondary and two upper secondary schools, including two schools used for teaching practice; lessons were observed jointly with educational researchers, advisers, principals and teachers, some of whom provided translation as well as evaluation of the lessons
- discussion of lessons, pupil support, teaching and training with teachers in each school
- observation of a trainee's lesson and its evaluation by the supervising teacher, trainee and other trainees
- discussion with mathematics education researchers and teacher trainers in three universities
- discussion with officers from the National Board of Education and members of the union of local government advisers
- analysis of research publications, articles, national and school documentation, and text books.

Acknowledgements

This report would not have been possible without the very helpful way in which visits were arranged and the openness with which people spoke to inspectors. Ofsted is grateful for the opportunities inspectors were given to observe teaching and for the time that teachers, principals, students, researchers, teacher trainers, advisers and officers of the National Board of Education made available to talk to us. In particular, Ofsted would like to thank Inspector Emeritus Martin Gripenberg, Professor Ole Björkqvist and Professor Pekka Kupari for their extensive help in organising the programme in the three towns visited.

Further information

Publications by Ofsted

The education of six-year-olds in England, Denmark and Finland (HMI 1660), Ofsted, 2003;

Mathematics: understanding the score (070063), Ofsted, 2008; www.ofsted.gov.uk/publications/070063

Other publications

Independent review of mathematics teaching in early years settings and primary schools, DCSF, 2008; http://publications.education.gov.uk/default.aspx?PageFunction=productdetails&PageMode=publications&ProductId=DCSF-00433-2008.

How Finns Learn Mathematics and Science, E Pehkonen, M Ahtee and J Lavonen (Eds), Sense Publishers, 2007.

Education in Finland, (ISBN 952-13-2829-0) 2010, Finnish National Board of Education; www.oph.fi/english/publications/2008/education_in_finland.

Websites

The programme for international student assessment (PISA) of the Organisation for Economic Cooperation and Development
www.pisa.oecd.org.

Trends in international mathematics and science study (TIMSS)
http://www.nces.ed.gov/timss/

Eurydice: Information on Education Systems and Policies in Europe;
www.eurydice.org.

Annex A: Schools and universities visited

Helsinki / Vantaa
Helsinki University
Uomarinteen Primary School
Kilterin Lower Secondary School
Helsingin Kuvataidelukio, Helsinki Upper Secondary School of Visual Arts

Vaasa
Abo Akademi University
Vikinga Primary School
Borgaregatans Lower Secondary School
Vasa Ovningsskola, Upper Secondary Practice School

Jyväskylä
Jyväskylä University
Kortepohja Primary School
Viitaniemi Lower Secondary School
Jyväskylän Normaalikoulu, Lower Secondary Practice School

Annex B: Examples of items from the PISA and TIMSS tests

Examples of PISA items

Note the realistic nature of most of the contexts and the demand on pupils' mathematical understanding and problem-solving skills.

Problem 1: reasoning and proportion

This is a challenging question that probes pupils' understanding of proportion. Pupils need to get all four statements right to earn the mark in Question 1.

People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.

For example, a man living in an apartment that occupies one fifth of the floor area of all apartments will pay one fifth of the total price of the building.

Question 1

Circle Correct or Incorrect for each of the following statements.

Statement	Correct / Incorrect
A person living in the largest apartment will pay more money for each square metre of his apartment than the person living in the smallest apartment.	Correct / Incorrect
If we know the areas of two apartments and the price of one of them we can calculate the price of the second.	Correct / Incorrect
If we know the price of the building and how much each owner will pay, then the total area of all apartments can be calculated.	Correct / Incorrect
If the total price of the building were reduced by 10%, each of the owners would pay 10% less.	Correct / Incorrect

Question 2

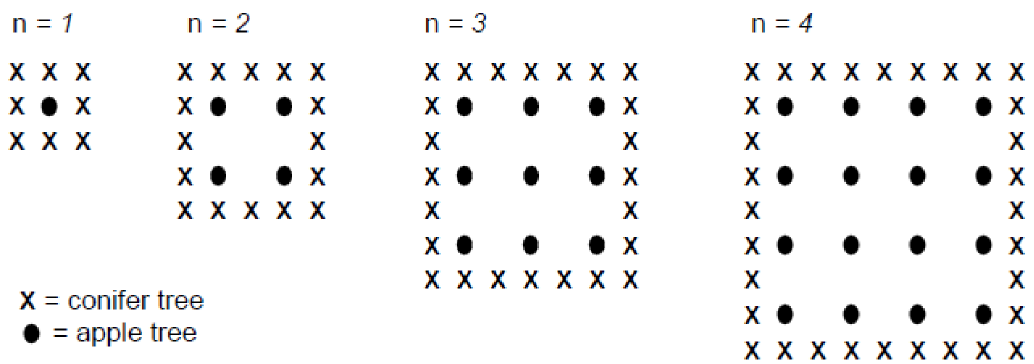
There are three apartments in the building. The largest, apartment 1, has a total area of $95m^2$. Apartments 2 and 3 have areas of $85m^2$ and $70m^2$, respectively. The selling price for the building is 300,000 zeds.

How much should the owner of apartment 2 pay? Show your work.

Problem 2: sequences and generalisations

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:



x = conifer tree
● = apple tree

Question 1

Complete the table:

n	*Number of apple trees*	*Number of conifer trees*
1	**1**	**8**
2	**4**	
3		
4		
5		

Question 2

There are two formulae you can use to calculate the number of apple trees and the number of conifer trees for the pattern described above:

Number of apple trees = n^2

Number of conifer trees = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifer trees. Find the value of n and show your method of calculating this.

Question 3

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer.

Problem 3: understanding formulae

This problem requires pupils to understand the effect of changing the coefficients in a formula.

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:

3 points = Excellent
2 points = Good
1 point = Fair

Question 1

To calculate the total score for a car, the car magazine uses the following rule, which is a weighted sum of the individual score points:

$$\text{Total Score} = (3 \times S) + F + E + T$$

Calculate the total score for Car "Ca". Write your answer in the space below.

Question 2

The manufacturer of car "Ca" thought the rule for the total score was unfair.

Write down a rule for calculating the total score so that Car "Ca" will be the winner.

Your rule should include all four of the variables, and you should write down your rule by filling in positive numbers in the four spaces in the equation below.

Total score = ...× S + ...× F + ...× E + ...× T.

Problem 4: area, scales and estimation

Below is a map of Antarctica.

Estimate the area of Antarctica using the map scale.

Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation.)

Problem 5: estimation, area

In this problem, pupils are not given all the necessary information and need to bring their own knowledge of everyday measures to estimate how much space one person would occupy.

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A 2,000 D 50,000
B 5,000 E 100,000

C 20,000

Examples of TIMSS items that assess similar topics

Note that not all contexts are realistic or generate a sense of purpose. However, the abstract nature of some questions can place greater demands on pupils.

Problem 1: proportional reasoning within algebra

This question requires understanding that the second algebraic fraction is half of the first. A common incorrect answer would be 140. The multiple choice questions offer answers generated by dividing, subtracting, adding and multiplying by 2.

If $\frac{a}{b} = 70$, then $\frac{a}{2b} =$

(A) 35

(B) 68

(C) 72

(D) 140

Problem 6: fractions

A teacher and a doctor each have 45 books. If $\frac{4}{5}$ of the teacher's books and $\frac{2}{3}$ of the doctor's books are novels, how many more novels does the teacher have than the doctor?

(A) 2

(B) 3

(C) 6

(D) 30

(E) 36

Problem 3: forming (but not solving) simultaneous equations

At a market, 7 oranges and 4 lemons cost 43 zeds, and 11 oranges and 12 lemons cost 79 zeds. Using x to represent the cost of an orange and y to represent the cost of a lemon, write two equations that could be used to find the values of x and y.

Problems 4 and 5: solving equations

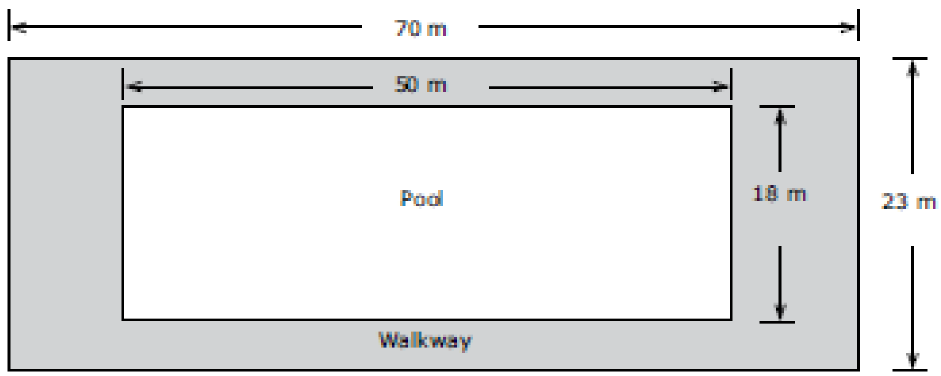
If $a + 2b = 5$ and $c = 3$, what is the value of $a + 2(b + c)$?

Find the value of x if $12x – 10 = 6x + 32$

Problem 6: area

This is a relatively routine problem.

A rectangular shaped swimming pool has a paved walkway around it as shown.



What is the area of the paved walkway?

Ⓐ $100\ m^2$

Ⓑ $161\ m^2$

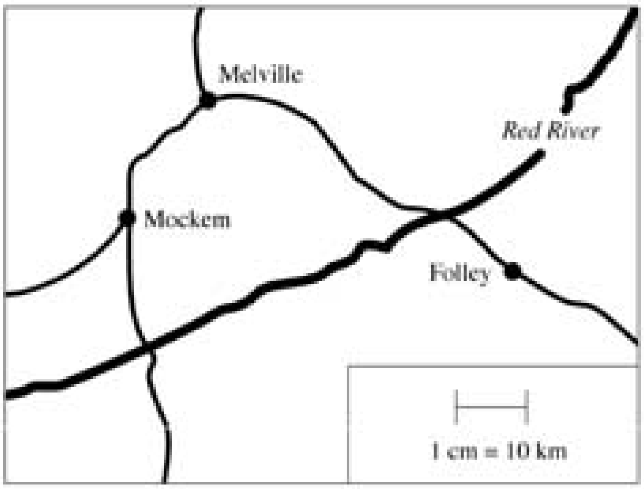
Ⓒ $710\ m^2$

Ⓓ $1,610\ m^2$

Problem 7: scales and estimation

This is a straightforward question with an easy scale for the map.

On the map, 1 cm represents 10 km on the land.



On the land, about how far apart are the towns Melville and Folley?

A. 5 km

B. 30 km

C. 40 km

D. 50 km

Problem 8: volume and estimation

The fact that all three dimensions are divisible by six reduces the demand of this question. Packing the second layer in the dips created by the first layer does not increase the number that may be packed, so pupils thinking more deeply do not gain additional credit.

Oranges are packed in boxes. The average diameter of the oranges is 6 cm, and the boxes are 60 cm long, 36 cm wide, and 24 cm deep.

Which of these is the BEST approximation of the number of oranges that can be packed in a box?

Ⓐ 30

Ⓑ 240

Ⓒ 360

Ⓓ 1,920