AS/A level subject criteria for mathematics: QCA consultation draft

For first teaching from September 2012

Contents

1 Introduction

1.1 These subject criteria set out the knowledge, understanding, skills and assessment objectives common to all advanced subsidiary (AS) and advanced (A) level specifications in mathematics. They provide the framework within which the awarding body creates the detail of the specification.

Subject criteria are intended to:

- help ensure consistent and comparable standards in the same subject across the awarding bodies

- define the relationship between the AS and A level specifications, with the AS as a subset of the A level

- ensure that the rigour of A level is maintained

- help higher education institutions and employers know what has been studied and assessed.

Any specification that contains significant elements of the subject mathematics must be consistent with the relevant parts of these subject criteria.

2 Aims and learning outcomes

Aims

2.1 AS and A level specifications in mathematics should encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment

- develop an understanding of coherence and progression in mathematics

- use technology as a tool for exploring mathematics, modelling with mathematics and solving mathematical problems

- develop awareness of the historical and cultural roots of mathematics

- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general

- take increasing responsibility for their own learning and the evaluation of their own mathematical development

- be well prepared for progression to further study and to employment.

Learning outcomes

2.2 AS and A level specifications in mathematics should enable students to:

- reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs

- apply a range of mathematical skills and techniques to both structured and unstructured problems

- develop their understanding of how different areas of mathematics are connected

- recognise that a situation may be represented mathematically by a mathematical model and understand the model's relationship with the corresponding 'real world' problem

- refine and improve mathematical models

- communicate mathematical ideas effectively and use mathematics as an effective means of communication

- read and comprehend mathematical arguments and information concerning mathematics and its applications

- use technology and recognise when such use may be inappropriate, and be aware of limitations.

3 Subject content

3.1 Mathematics is a rigorous and coherent discipline with a rich historical and cultural heritage. While there is a progression of material through all levels at which the subject is studied, there is also the possibility to develop breadth and depth within many aspects. The criteria build on the knowledge, understanding and skills studied in higher tier GCSE mathematics. The core content for AS is a subset of the core content for A level. Progression in the subject may extend in a number of ways from AS and A level, into the complementary study of further mathematics or onto related courses in higher education.

Knowledge, understanding and skills

3.2   AS and A level specifications in mathematics should require:

- construction and presentation of mathematical arguments, including proofs, through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language

- correct understanding and use of mathematical language, notation and grammar – these are listed in the Appendix.

These requirements should pervade the core content material set out below.

3.3   Core content material for AS and A level examinations in mathematics is listed below. AS core content is listed in the second column, with A2 core content in the right-hand column.

3.3.1 Algebra and functions

	AS core content	A2 core content
(a)	Laws of indices for all rational exponents.	
(b)	Use surds and pi in exact calculations.	
(c)	Quadratic functions and their graphs. The discriminant of a quadratic function. Completing the square. Solution of quadratic equations.	
(d)	Simultaneous equations: analytical solution by substitution, eg of one linear and one quadratic equation.	
(e)	Solution of linear and quadratic inequalities.	
(f)	Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem.	Simplification of rational expressions including factorising and cancelling, and algebraic division.

(g)	Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.	
(h)		Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.
(i)		The modulus function.
(j)	The effect of simple transformations on the graph of $y = \mathrm{f}\,(x)$ as represented by $y = a\mathrm{f}\,(x)$, $y = \mathrm{f}\,(x) + a$, $y = \mathrm{f}\,(x + a)$, $y = \mathrm{f}\,(ax)$.	Combinations of these transformations.
(k)		Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).

3.3.2 Coordinate geometry in the (*x, y*) plane

	AS core content	A2 core content
(a)	Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$. Conditions for two straight lines to be parallel or perpendicular to each other. Coordinates of a mid-point.	
(b)	Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$, and including use	

	of the following circle properties: • the angle in a semicircle is a right angle • the perpendicular from the centre to a chord bisects the chord • the perpendicularity of radius and tangent.	
(c)		Parametric equations of curves and conversion between Cartesian and parametric forms.

3.3.3 Sequences and series

	AS core content	A2 core content
(a)	Sequences, including arithmetic and geometric sequences, those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.	
(b)	Arithmetic series, including the use of Σ notation.	
(c)	Geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$ and the use of Σ notation.	
(d)	Binomial expansion of $(1+x)^n$ for positive integer n.	
(e)		Binomial series for any rational n.

3.3.4 Trigonometry

	AS core content	A2 core content
(a)	The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$.	
(b)	Radian measure, including use for arc length and area of sector.	
(c)	Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.	
(d)		Knowledge of secant, cosecant and cotangent, and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.
(e)	Knowledge and use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.	Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\csc^2\theta = 1 + \cot^2\theta$.
(f)		Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$), $\cos(A \pm B)$ and $\tan(A \pm B)$, and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$.
(g)	Solution of simple trigonometric equations in a given interval.	Solution of trigonometric equations in a given interval.

3.3.5 Exponentials and logarithms

	AS core content	A2 core content

(a)	$y = a^x$, $a > 0$, and its graph.	The exponential function (e) and its graph.
(b)	Laws of logarithms: ($a > 0$): $\log_a x + \log_a y \equiv \log_a(xy)$, $\log_a x - \log_a y \equiv \log_a\left(\dfrac{x}{y}\right)$, $k\log_a x \equiv \log_a(x^k)$.	The logarithmic function ($\ln$) and its graph; logarithmic and exponential functions as the inverse of one another.
(c)	The solution of equations of the form $a^x = b$ ($a, b > 0$).	
(d)		Exponential growth and decay.

3.3.6 Differentiation

	AS core content	A2 core content
(a)	The tangent to the graph of $y = f(x)$ as a limit of chords; the gradient of the tangent as a limit. The terminology of derivatives and their interpretation as a rate of change; second-order derivatives. Differentiation of simple functions from first principles.	
(b)	Differentiation of x^n, and related sums and differences.	Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.
(c)	Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions.	

| (d) | | Differentiation using the product rule, the quotient rule, the chain rule |
| (e) | | Differentiation of simple functions defined implicitly or parametrically. |

3.3.7 Integration

	AS core content	A2 core content
(a)	Indefinite integration and differentiation as inverse processes.	
(b) (c)	Integration of x^n, $n \neq -1$. Simple approximation of area under a curve. Interpretation of the definite integral as the area under a curve. Evaluation of definite integrals.	Integration of e^x, $\dfrac{1}{x}$, $\sin x$, $\cos x$.
(d)		Evaluation of volumes of revolution.
(e)		Simple cases of integration by substitution and integration by parts. These methods as the inverse processes of the chain and product rules, respectively.
(f)		Simple cases of integration using partial fractions.
(g)		Formation and analytical solution of simple first-order differential equations with separable variables.

3.3.8 Numerical methods

	AS core content	A2 core content
(a)		Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$, in an interval of x in which $f(x)$ is continuous.
(b)		Approximate solution of equations using iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.
(c)		Numerical integration of functions.

3.3.9 Vectors

	AS core content	A2 core content
(a)		Vectors in two and three dimensions.
(b)		Magnitude of a vector.
(c)		Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.
(d)		Position vectors. The distance between two points. Vector equations of lines.
(e)		The scalar product. Its use for calculating the angle between two lines.

3.3.10 Applications

The AS applications develop from the pure mathematics content. The references given below are to the related pure mathematics sections from which the applications can be developed.

	AS core content – basic ideas of mathematical modelling, including business, mechanical and statistical contexts	A2 core content – further development of mathematical modelling
(a)	Optimisation: linear programming Simple algorithmic procedures 3.3.6, 3.3.1e, 3.3.2a	Graphs and networks Primm's and Kruskal's algorithms, minimum spanning tree, Dijkstra's algorithm
(b)	Kinematics: speed–, distance–, acceleration–, velocity–time graphs Constant acceleration, including vertical motion under gravity Variable acceleration (polynomial functions of *time*) 3.3.1g, 3.3.2a, 3.3.6, 3.3.7	Newton's laws Friction Tension and thrust – connected Particles (not compound pulleys) Projectiles as an example of motion in 2-D where displacement is a function of time
(c)	Forces: force as a vector (magnitude, direction), resolution of forces in 2-D, drawing force diagrams, triangle of forces, inclined plane, resistance, equilibrium/dynamic movement (limited to predicting whether the object will move and, if so, in which direction) 3.3.4	
(d)	Working within the statistical problem-solving process: interpret, explain and evaluate data using appropriate diagrams and summary statistics (including standard deviation and use of Σ notation) 3.3.3	Simple continuous random variables Normal distribution Estimation of population parameters Distribution of the sample mean

(e)	Probability: sample space, combined events, conditional probabilities, random variable Modelling with probability distributions (geometric and binomial) 3.3.3c,d	Central limit theorem for the sample mean Inference

4 Key skills

4.1   AS and A level specifications in mathematics should provide opportunities for developing and generating evidence for assessing relevant key skills from the list below. Where appropriate, these opportunities should be directly cross-referenced, at specified level(s), to the key skills standards, which may be found on the QCA website (www.qca.org.uk).

- Application of number

- Communication

- Improving own learning and performance

- Information and communication technology

- Problem-solving

- Working with others

5 Assessment objectives

5.1   All candidates must be required to meet the following assessment objectives. The assessment objectives are to be weighted in all specifications as indicated in the following table.

Assessment objectives		Weighting	
		AS level	A2 level
AO1	Recall, select and use mathematical facts, concepts and techniques.	45–55	30–40
AO2	Select and apply mathematics to model real and abstract situations. Solve structured and unstructured problems in both familiar and less familiar contexts.	20–25	25–30
AO3	Interpret information and results, and evaluate the effectiveness of any model or strategy used.	10–15	15–20
AO4	Use precise statements, logical deduction and inference to construct and test the validity of mathematical arguments.	10–20	15–25

6   Scheme of assessment

6.1   A level specifications in mathematics will consist of two units at AS and two units at A2.

Synoptic assessment

6.2   Synoptic assessment in mathematics should take place across the A2 units and should encourage candidates to:

- demonstrate understanding of the connections between different elements of the subject

- solve problems that require candidates to bring different aspects of mathematics together

- solve substantial unstructured problems that require extended reasoning.

6.3   All AS and A level specifications in mathematics must explicitly refer to the importance of candidates using clear, precise and appropriate mathematical language. These references must draw attention to the relevant demands of assessment objective AO4.

6.4   AS and A level specifications in mathematics must:

- explicitly include all the material in the relevant 'Knowledge, understanding and skills' section of the criteria – specifications are permitted to elaborate on details in different ways, but all specifications must show a very high degree of consistency and comparability in addressing the material in sections 3.2 and 3.3; for both AS and A level, the 'Knowledge, understanding and skills' must attract total credit for the qualification

- permit the use of graphing calculators in all units

- indicate the mathematical notation that will be used – this is listed in the Appendix.

6.5   Applications should collectively have a weighting of 33–40 per cent.

Quality of written communication

6.6   AS and A level specifications will be required to assess the candidates' quality of written communication in accordance with the guidance document produced by QCA.

7   Performance descriptions

To be added

Appendix

1 Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{\, x_1,\, x_2,\, \ldots \}$	the set with elements $x_1,\, x_2,\, \ldots$
$\{x: \ldots\}$	the set of all x such that …
$n(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathcal{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{ \dfrac{p}{q} : p \in \mathbb{Z},\, q \in \mathbb{Z}^+ \right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
$(x,\, y)$	the ordered pair $x,\, y$
$A \times B$	the Cartesian product of sets A and B, i.e. $A \times B = \{(a,\, b) : a \in A,\, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection

$[a, b]$	the closed interval $\{x \in \mathrm{R} : a \le x \le b\}$
$[a, b)$	the interval $\{x \in \mathrm{R} : a \le x < b\}$
$(a, b]$	the interval $\{x \in \mathrm{R} : a < x \le b\}$
(a, b)	the open interval $\{x \in \mathrm{R} : a < x < b\}$
$y\,R\,x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous Symbols

$=$	is equal to
$\ne$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\le,\ \not>$	is less than or equal to, is not greater than
$>$	is greater than
$\ge,\ \not<$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b

$a - b$ a minus b

$a \times b$, ab, $a.b$ a multiplied by b

$a \div b$, $\dfrac{a}{b}$ a divided by b

$\displaystyle\sum_{i=1}^{n} a_i$ $a_1 + a_2 + \ldots + a_n$

$\displaystyle\prod_{i=1}^{n} a_i$ $a_1 \times a_2 \times \ldots \times a_n$

$\sqrt{a}$ the positive square root of a

$|a|$ the modulus of a

$n!$ n factorial

$\dbinom{n}{r}$ the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^{+}$

or $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

or $_{n}C_{r}$

4 Functions

$\mathrm{f}(x)$ the value of the function f at x

$\mathrm{f} : A \rightarrow B$ f is a function under which each element of set A has an image in set B

$\mathrm{f} : x \rightarrow y$ the function f maps the element x to the element y

f^{-1} the inverse function of the function f

gf the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$

or $\mathrm{g} \circ \mathrm{f}$

$\displaystyle\lim_{x \to a} \mathrm{f}(x)$ the limit of $\mathrm{f}(x)$ as x tends to a

Δx, δx an increment of x

$\dfrac{\mathrm{d}y}{\mathrm{d}x}$ the derivative of y with respect to x

$\dfrac{\mathrm{d}^{n}y}{\mathrm{d}x^{n}}$ the nth derivative of y with respect to x

$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, ..., $\mathrm{f}^{(n)}(x)$ the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x

$\int y \, dx$ the indefinite integral of y with respect to x

$\int_a^b y \, dx$ the definite integral of y with respect to x between the limits $x = a$ and $x = b$

$\dfrac{\partial V}{\partial x}$ the partial derivative of V with respect to x

$\dot{x}, \ddot{x}, \ldots$ the first, second, ... derivatives of x with respect to t

5 Exponential and Logarithmic Functions

e base of natural logarithms

$e^x, \quad \exp x$ exponential function of x

$\log_a x$ logarithm to the base a of x

$\ln x, \quad \log_e x$ natural logarithm of x

$\lg x, \quad \log_{10} x$ logarithm of x to base 10

6 Trigonometric and Hyperbolic Functions

sin, cos, tan
cosec, sec, cot   }   the trigonometric functions

$\sin^{-1}, \cos^{-1}, \tan^{-1}$
$\text{cosec}^{-1}, \sec^{-1}, \cot^{-1}$,
arcsin, acrcos,
arctan, arccosec,
arcsec, arccot   }   the inverse trigonometric functions

sinh, cosh, tanh
cosech, sech, coth   }   the hyperbolic functions

$\sinh^{-1}, \cosh^{-1}, \tanh^{-1}$
$\text{cosech}^{-1}, \text{sech}^{-1}$,
$\coth^{-1}$ }   the inverse hyperbolic functions

7 Complex Numbers

i square root of -1

z a complex number, $z = x + i\,y = r(\cos\theta + i\sin\theta)$

$\operatorname{Re} z$ the real part of z, $\operatorname{Re} z = x$

$\operatorname{Im} z$ the imaginary part of z, $\operatorname{Im} z = y$

$|z|$ the modulus of z, $|z| = \sqrt{x^2 + y^2}$

$\arg z$ the argument of z, $\arg z = \theta$, $-\pi < \theta \le \pi$

$z*$ the complex conjugate of z, $x - \mathrm{i}\,y$

8 Matrices

$\mathbf{M}$ a matrix $\mathbf{M}$

$\mathbf{M}^{-1}$ the inverse of the matrix $\mathbf{M}$

$\mathbf{M}^{\mathrm{T}}$ the transpose of the matrix $\mathbf{M}$

$\det \mathbf{M}$ or $|\mathbf{M}|$ the determinant of the square matrix $\mathbf{M}$

9 Vectors

$\mathbf{a}$ the vector $\mathbf{a}$

$\overrightarrow{AB}$ the vector represented in magnitude and direction by the directed line segment AB

$\hat{\mathbf{a}}$ a unit vector in the direction of $\mathbf{a}$

$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ unit vectors in the directions of the Cartesian coordinate axes

$|\mathbf{a}|$, a the magnitude of $\mathbf{a}$

$\left|\overrightarrow{AB}\right|$, AB the magnitude of $\overrightarrow{AB}$

$\mathbf{a.b}$ the scalar product of $\mathbf{a}$ and $\mathbf{b}$

$\mathbf{a} \times \mathbf{b}$ the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and Statistics

A, B, C, etc.   events

$A \cup B$ union of the events A and B

$A \cap B$ intersection of the events A and B

$\mathrm{P}(A)$ probability of the event A

A' complement of the event A

$\mathrm{P}(A \mid B)$ probability of the event A conditional on the event B

X, Y, R, etc.   random variables

x, y, r, etc.   values of the random variables X, Y, R etc

$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
p(x)	probability function $P(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
f(x), g(x), $\ldots$	the value of the probability density function of a continuous random variable X
F(x), G(x), $\ldots$	the value of the (cumulative) distribution function $P(X \leqslant x)$ of a continuous random variable X
E(X)	expectation of the random variable X
E(g(X))	expectation of g(X)
Var(X)	variance of the random variable X
G(t)	probability generating function for a random variable which takes the values 0, 1, 2, $\ldots$
B(n, p)	binomial distribution with parameters n and p
N(μ, σ^2)	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$, m	sample mean
s^2, $\hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution N(0, 1)
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
Cov(X, Y)	covariance of X and Y