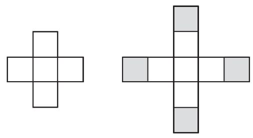
Sequences

Sequences can – deceptively – appear to be straightforward. Consequently the early stages of teaching the topic may be rushed so that pupils do not get the chance to compose the 'big picture' for themselves. They need to develop a full understanding of the relationship between the context of a sequence and the ways in which it can be expressed, first in words and later in algebraic terms. It is essential for pupils to understand the term-to-term and position-to-term structures and the relationship between them. Use pupils' knowledge of variables and functions. T(n) and n are not 'shorthand' for words, they are symbols representing variables that can take on specific values (in most examples these are positive integer values). To describe a sequence, we can use either a function that links one term T(n + 1) with the previous term T(n) or a function that links a term T(n) with its position in the sequence n.

Linear sequences: find and describe in words and symbols the rule for the next term and the nth term of a sequence, for example:



It is important for pupils to use physical patterns, rather than just tables of number sequences, to justify their algebraic forms. Furthermore, exploring different but equivalent forms will help pupils to understand and relate equivalent algebraic expressions and functions.

Pupils will believe that the algebra must be equivalent by relating to the structure but will also be motivated to manipulate the expressions or functions to confirm this equivalence. Be explicit about the use of symbols to represent variables and avoid the temptation to treat the algebra as an abbreviation of the words. Emphasise the meaning of the item they are constructing: for example, a function that relates the number of objects in the shape to its position in the sequence, say T(n) = 7 + 3n.

5 , 9 , 13 , 17	Use the context of a sequence to generate the related numerical terms
It goes up in 4s because you add 1 block to each arm of the cross. n 1 2 3 4 T(n) 5 9 13 17 add 4 T(n + 1) = T(n) + 4	Notice and describe how the sequence is growing term by term and relate this to the context
For the nth shape there is one block in the middle and then four arms of n blocks. T(n) is the total number of blocks in a shape and n is the position of the shape in the sequence. T(n) = 1 + 4n	Notice and describe a general term in the sequence and relate this to the context
There is a horizontal row of blocks, which is one more than twice the term number, then a top leg with n blocks and a bottom leg with n blocks. T(n) = (2n + 1) + n + n	Appreciate different forms for the general term and relate each to the context
	Generate different forms for the general term and relate each to the context

Quadratic sequences: find and describe in symbols the rule for the next term and the nth term of a sequence

Explore practical contexts for simple cases. Consider using ICT for generating sequences. Use the simple examples linked to difference tables and generalise from these examples.

For examples see the *Framework for teaching mathematics: Years 7, 8 and 9*, pages 151 and 153	Repeat the progression described on page 16

Recognise and describe types of sequence: for example, arithmetical sequences and multiples, triangular numbers, square numbers

Explore familiar spatial patterns that generate multiples, triangular numbers and square numbers. Encourage pupils to consider how these familiar sequences could appear in 'slight disguise' and how they could develop strategies to recognise them.

The *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 144 to 159, provides contexts in which pupils could develop mental processes in sequences.

For examples see the *Framework for teaching mathematics: Years 7, 8 and 9*, pages 155 and 159	Use knowledge of related geometrical patterns Use differences to test for types of sequence

Building a sequence, using tiles or linking cubes, gives pupils opportunities to think about the construction of each term.

Colour-coding can help to illustrate the structure of the start point and show increments from term to term. Use of colour can also help pupils find the words to describe what they see.

Encourage them to describe the stages of their thinking by suggesting sentence stems, for example:

- I started by thinking of this shape as …
- To get the next shape you …
- The hundredth shape will have …
- The *n*th shape will have ….

Model this process, thinking aloud as you work. Write the rules in words first and then move to symbols. Ask a pupil to give a commentary as you build a sequence of shapes. Illustrate the fact that apparently different algebraic forms can be used to describe the same sequence, depending on how you see each term built up. This equivalence may not be obvious to pupils, so it needs to be discussed explicitly.

This gives pupils opportunities to appreciate that the algebraic forms are equivalent by relating the algebra to the structure of the shape rather than manipulating expressions.

Classifying cards helps pupils to build a mental picture of the different types of sequence. For example, they might choose to sort the cards according to whether the sequences are ascending or descending, and by equal or unequal steps.

A set of cards could include:

- sequences of geometrical patterns;
- names of sequences (square numbers, triangular numbers, odd numbers, even numbers, multiples);
- functions relating the term to its position in the sequence (expressed in different ways);
- a list of numbers in a sequence;
- a table showing position and terms in a sequence.

For an illustration of the types of card, see the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 146 and 147.

For **Find the function**, pupils use ICT to compose and work with formulae, linking a term to its position in the sequence, for example, $T(n) = 5 + 4n$.

This can be done on a spreadsheet or a graphing calculator. Pupils work in groups of two pairs. Each pair uses ICT to compose and test a formula. They then give the other pair a short chain of terms in the sequence (these may not be the first terms of the sequence), challenging them to find the function. The guessing pair use ICT to test their attempts at finding the function and finally check their result with the opposite pair.

Discuss the strategies that pairs could use to find a function from a short sequence of terms. Consider exploring differences between consecutive terms, noting the links between these differences and the nature of the function. For an illustration of strategies and ICT applications, see the *Framework for teaching mathematics: Years 7, 8 and 9*, supplement of examples, pages 149 to 151.

Snapshots of a sequence can be used to allow pupils to speculate about preceding and subsequent terms. For example, use blocks to show one shape and tell pupils that this is the third in the sequence. Ask what the second and fourth shapes could look like. Share solutions and choose one or two to consider together (see the example below, showing the second, third and fourth terms for two possible sequences).

Generate the rule, in words, for the step from one term to the next. Write the function in symbols for the general term for both sequences. Illustrate both sequences in diagrams and numbers and show that the function gives the same number of blocks for the third term but generates a different sequence.