The Employment Equation:

Why our young people need more maths for today's jobs

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THE SUTTON TRUST

Improving social mobility through education

Contents

Foreword

Mathematics is becoming ever more important to our lives. It is at the heart of everyday technology from our smartphones and tablets to the increased automation in daily tasks from driving to shopping. The assumption is too often made that the increased sophistication of computers and the ubiquity of calculators means that most of us have little use for mathematics beyond the classroom.

In Britain, few would proudly proclaim their illiteracy. Yet many happily say they are no good with numbers. The education system reinforces this attitude. For most young people, mathematics finishes with GCSEs. Unlike most other developed countries where the majority study maths to 18, only 13% of our young people study maths at an advanced level beyond 16.

Maths matters too much to cut it off after sixteen. For young people from less affluent backgrounds, in particular, their ability to benefit fully from higher education and play a productive role in the workforce will depend on their mathematical competence.

Professor Jeremy Hodgen and Dr Rachel Marks, in this review of the literature on maths in the workplace for the Sutton Trust, explain not only the relevance of the subject in today's workplace, but also how important it is that people have the skills they need to apply what they learn in school to their everyday tasks at work.

The fascinating examples they cite showing the importance of estimation and measurement, for example, give the lie to the idea that everything can be done on a computer. Not knowing such concepts means not being able to spot errors that in some walks of life such as medicine could prove fatal. At the very least, lack of mathematical competence means an inability to function fully at work.

So, while GCSE maths may equip young people with a basic understanding of key concepts, it will not necessarily provide them with the capacity to apply those concepts in practical situations. Such skills are vital in the workplace, as this study shows, but also in a host of university subjects that are not traditionally seen as mathematical, where statistical and chronological skills are vital.

The Government has rightly accepted Alison Wolf's recommendation that those without a good GCSE in maths should be expected to study the subject in school or college through to the age of 18.

We think they should go further, and require continued study for all students, though, except for some, not at A-level standard. We are not alone in this view. We welcome the decision of the Opposition to support the requirement that students take maths and English through to 18. We are also pleased that so many young people themselves, in our polling with Ipsos MORI, see the good sense in being able to continue studying these two vital subjects beyond GCSE.

However, this research suggests that their ability to do so will also require the development of an advanced qualification that equips young people for the practical application of what they learned for GCSE, as well as reinforcing their knowledge of the key concepts.

I have argued in the past for a full Baccalaureate, which virtually all other OECD countries have, where students study eight or so subjects to 18 including English and Maths. But ensuring that everyone studies maths alongside A levels or vocational qualifications would be an important step in the right direction.

I am very grateful to Prof Hodgen and Dr Marks for their work on this report, and I hope it will help policymakers develop the right curriculum for all our young people.

Sir Peter Lampl
Chairman
The Sutton Trust and Education Endowment Foundation

Recommendations

1. It is vital that all students have a solid understanding of basic mathematics. For those not taking A-level mathematics, a new pathway should be developed for students covering fluency, modelling and statistics for those who already have at least a Grade C in GCSE maths, based on the mathematical needs for employment rather than on covering more advanced topics.

2. Changes in workplace practices – particularly an increased focus on efficiency measures – have resulted in mathematical application and understanding becoming an essential skill for all people in the workplace, even in relatively unskilled jobs. People in the workplace need to be able to make sense of the mathematics they are using if they are to avoid making mistakes in the workplace.

3. All young people should continue to study maths until the age of 18. The Government has introduced this expectation only for young people who don't gain a GCSE Grade C at 16. Continued study of maths should focus on its application, so that it is relevant to university courses and a growing number of modern jobs.

4. It is critical that alternatives to the traditional GCSE mathematics pathway are developed that are rigorous, engaging for students, provide sufficient breadth and are valued by employers. Mathematics is a critical skill for all, including to those who have not achieved a Grade C at GCSE by age 16.[1]

5. In general, students with at least a grade C at GCSE have already covered the critical mathematical techniques and concepts, but they do need to understand what they already know better. Any specialist mathematical techniques can be learnt in the workplace, provided students understand and can apply GCSE mathematics. The curriculum should also include more "simple maths in complex settings", by providing students with problem-solving opportunities involving "messy" contexts that do not have straightforward solutions. Students should have many more opportunities to collaborate and discuss, working together to understand, interpret and communicate the mathematics they are involved in.

6. To allow students to more easily transfer their mathematical skills into the workplace they should use computers extensively, particularly spreadsheets and computer-generated graphs, to apply and learn mathematics. Competence in these skills matters in the workplace.

Summary

1. This report reviews over 50 research studies to consider the level and type of mathematical skills needed by employers in today's economy. It considers five key questions:

   - What mathematics (level and content) is required in the workplace today?
   - How and why have the mathematical needs of the workplace changed over time?
   - In what ways is mathematics used in today's workplace?
   - To what extent do specific workplaces have specific mathematical demands?
   - What are the implications of mathematics use in the workplace for post-16 education?

2. This report looks in detail at the application of mathematics by those without numerate degrees in six key sectors: Health (predominantly nursing); Engineering; Construction and Manufacturing; Transportation; Retail; Finance.

3. Mathematics participation levels in England are recognised internationally to be low. While over half of young people gain at least a C grade at GCSE maths, only 20% continue to study any maths post-16, whereas across the OECD, the majority of young people in all other developed countries outside the UK continue to study maths until age 18.[2]

4. The level of mathematics used by people in the workplace and required by employers for all but the most highly numerate and technical jobs is "simple mathematics in complex settings". The academic level of the mathematics required lies almost wholly within the GCSE curriculum.

5. However, although the mathematical content may be at GCSE level, it is embedded within complex settings and the transfer of mathematical skill to the workplace is not always straightforward. Many workplace settings require the sophisticated use of these basic mathematical skills, particularly when people in the workplace are faced with modelling scenarios. Increasingly, and particularly in combination with the use of technology such as Computer Aided Design and modelling software, employees work in collaboration to reach joint understandings.

6. All the evidence suggests that workplaces are now technology-rich environments. Many people in the workplace are engaged in ICT, particularly in using spreadsheets and graphical outputs. However, this study finds many examples of people in the workplace using a 'black-box' approach to some mathematical techniques, where they lack the mathematical knowledge to understand fully the techniques they are using, to control the technology, and to understand and use the outputs.

Introduction, Context and Research Questions

Mathematics education in the UK is now acknowledged as internationally unusual in the low participation rates at upper secondary, as illustrated in the comparison table below.[3]

What are the participation rates in upper secondary (post-16) mathematics education?		
	Studying any mathematics	**Studying advanced mathematics**
England	20%-26%	13%
Germany (Rhineland-Palatinate)	>90%	8%-14%
Hong Kong	>95%	22-23%
New Zealand	71% (Y12), 44% (Y13)	66% (Y12), 40% (Y13)
Scotland	48% (S5), 21% (S6)	27%
Singapore	66%	39%
USA (Massachusetts)	>84%	>16%

Note: The base for the participation rates is the number of students of upper secondary age who are in education, employment or training.
England: The upper limit of those studying any mathematics includes the 6% who retake GCSE.
New Zealand: Figures for both Y12 and Y13 are given.
Scotland: Figures for S5 and S6 are given for studying any mathematics. For advanced mathematics, an aggregated figure is given that indicates the proportion of student completing advanced mathematics at some point during upper secondary education.

As a result, a consensus is developing around an urgent need to raise participation rates very substantially,[4] as reflected in much of the policy literature. To date, the focus of this work has been on supply side issues such as students' subject choices,[5] on understanding policy,[6] or on the needs of higher education.[7] Little attention has been given to the needs of employers or industry. In this study, we seek to address this imbalance through a focused literature review.

Whilst there have been studies into the mathematical needs of the non-technical workforce,[8] this literature is fairly limited and quite fragmentary. In this study we identify these key publications (from both academic and policy literature in the UK and internationally), and review and synthesise these in order to outline the mathematics needed in the modern workplace, situating this within the current, complex, policy context of a rising school-leaving age and debate around the mathematics pathways provided.

Research Questions

- What mathematics (level and content) is required in the workplace today?
- How and why have the mathematical needs of the workplace changed over time?
- In what ways is mathematics used in today's workplace?
- To what extent do specific workplaces have specific mathematical demands?
- What are the implications of mathematics use in the workplace for post-16 education?

Workplace Contexts: Employment and Industry Sectors

In line with other reports,[9] we looked across major employment and industry sectors in order to examine the mathematics used, and required, in the modern workplace:

- Health (predominantly nursing)

- Engineering
- Construction and manufacturing
- Transportation
- Retail
- Finance

Examples from this literature are used to illustrate the key themes covered within this report.

Workplace Contexts: The Workforce and Mathematical Need

Mathematical needs and usage in the workplace fall into two categories: the high-end sophisticated use of mathematics by those with degrees including a substantial mathematical element in specialised workplaces, and the use of mathematics by non-specialist people in the workplace. The focus of this report is on this latter group, those working in what we call the non-technical sector - roles not requiring a numerate degree or mathematical specialism. In our view, understanding the mathematical demands on this workforce is crucial to understanding what mathematics pathways and qualifications should be offered to those students who do not currently take mathematics through to age 18 in England.

"Simple Mathematics in Complex Settings"

One of our important findings is that the level of mathematics used by people in the workplace and required by employers for all but the most highly numerate and technical jobs is "simple mathematics in complex settings".[10] We use the terms "simple" and "complex" here with care; they are relative terms. The academic level of the mathematics required is almost wholly within the GCSE curriculum, covering the core areas of:

- Number, particularly mental maths, approximation, estimation and proportional reasoning
- Using and interpreting calculators and spreadsheets
- Statistics and probability, including data collection, interpretation and representation
- Algebra, particularly graphical representation and diagrams
- Geometry and measures, including 2D and 3D representation

However, the application of this 'simple' mathematics is not straightforward. Workplaces are often complex settings and the application of mathematics is no different. Workplace mathematics is carried out in ways that are very different to how mathematics is taught in schools. It often involves the use of technology and is more often than not used in collaboration with others. People in the workplace need to choose the appropriate mathematics, sometimes under pressure. They then need to apply it, and make informed judgements using the results. All this is central to the type of mathematics required today in employment and industry and it is recognised to be a significant deficit in the skills of today's people in the workplace.[11]

Diana Coben's[12] widely cited definition of numerate behaviour is relevant here:

> To be numerate means to be competent, confident, and comfortable with one's judgements on **whether** to use mathematics in a particular situation and if so, **what** mathematics to use, **how** to do it, what **degree of accuracy** is appropriate, and what the answer means in relation to the context.

Overview of Findings

Here we provide a tabulated overview of our key findings. These are then each discussed in detail within section 3: “Simple Mathematics in Complex Settings”, and inform our recommendations**.**

“Simple mathematics”	The mathematics used by the majority of people in the workplace is at a level no higher than that contained within the current GCSE syllabus.
Mathematical content	The mathematical content used within, and appropriate to, the workplaces of today includes: Number, Statistics and Probability, Algebra and Geometry and Measures. Of particular importance are mental proportional reasoning, approximation and estimation, the interpretation of graphs, and the use of spreadsheets and calculators.
“Complex settings”	Although the mathematical content may be at GCSE level, it is embedded within complex settings. These settings require the sophisticated use of mathematics particularly when people in the workplace are faced with modelling scenarios.
Technology rich environments	Evidence across the literature base suggests that workplaces are technology rich environments with much of the mathematics people in the workplace are engaged in embedded within ICT, particularly in the use of spreadsheets and graphical outputs. This has implications for how mathematics in taught in schools.
Collaborative working environments	Increasingly, and particularly in combination with the use of technology such as Computer Aided Design and modelling software, employees work in collaboration reaching joint understandings. This should be reflected in approaches to post-16 mathematics provision.

Our literature review indicates that, aside from those with highly numerate degrees, the majority of people in the workplace only use relatively simple mathematics,[13] the content of which is covered in the current GCSE curriculum.

However, this simple mathematics is used in situations – both environments and tasks – which appear complex to the outsider.[14] The mathematics often involves idiosyncratic procedures, graphs and other tools that are often specific to particular sectors or even individual workplaces that are very different to those used in school mathematics. Some people are extremely competent in applying mathematics to solve problems in sophisticated ways. Many others encounter difficulties. Some, but not all, of these difficulties are due to poor mathematical understanding. In many cases, it is the application of mathematics in the workplace that causes difficulties.

Competence matters in the workplace. The incorrect application or interpretation of mathematics can have significant economic or safety implications. Serious errors appear to be relatively infrequent, but when they do happen, the consequences can be serious. The literature gives examples across sectors from agriculture, health care, banking and finance,[15] to chemical spraying,[16] health care,[17] and nursing, retail, finance and defence.[18] It can also have serious consequences for individuals’ decisions involving health or financial risks.[19] For example, in Gail FitzSimons’[20] study of chemical handling and spraying, they report on the need for employees to carefully calibrate spraying equipment taking into account variables such as the tractor used and the gradient of the field. All calculations are conducted as teams and double checked because any mistakes ‘may threaten public safety and the livelihoods of the workers and their managers.’ Robin Riall and David Burghes cite the example of a Post Office employee who incorrectly “rounded up” a number by adding an extra zero, thus launching a major internal inquiry into increased mail levels.[21] The consequences of mathematical error were also strongly highlighted in the incorrect use of statistics and subsequent serious miscarriage of justice in the Sally Clark case.[22]

In this section, we look in detail at how competence is developed. We draw on in-depth studies to provide examples of people in the workplace with a strong understanding of workplace mathematics and also look at where this can go wrong, highlighting the role of technological change leading to invisibility and ‘black-boxing’.

Workplace Competence

In this section, we discuss and give examples of mathematical competence in the workplace. Workplace mathematics looks different to the mathematics of the school classroom. Workplace mathematics is “situated”[23] and can often only be understood in context. Indeed, because they involve idiosyncratic conventions and sometimes break the “rules” of mathematics, outsiders often struggle to make sense of how mathematics is carried out, which we believe has considerable implications for how mathematics is taught post-16 (see Section 4). Approximation and estimation are often, but not always, more important than an “accurate” result as calculation is very often carried out with calculators, spreadsheets or other electronic tools.

Let us first consider accountancy. Margaret Dawes’[24] study of accountants was unusual in looking at a group of skilled professionals who are highly numerate, yet not required to have a mathematics, engineering or similar numerate degree. The accountants in her study were highly adept numerically, although they make very considerable use of calculation aids. Dawes’ draws on theories of distributed,[25] and situated,[26] cognition to show how the accountants used mathematics to develop shared judgments about the extent to which large bank loans were secure. Whilst the accountants’ mathematics has features peculiar to accountancy, many aspects of their practices are common in other workplaces.

Tom, a partner in a global accountancy firm, was extremely competent mathematically. Indeed, he was the most mathematically competent accountant in Dawes' study. Although he used no mathematics beyond that contained in the GCSE curriculum, he applied this mathematics with considerable sophistication, and with fluency in that it was carried out without a pause or break in his analysis (see Box 1).

Box 1: Assessing the safety of loans in accountancy

In Dawes' study (pp. 106-116) of the bank audit team in a large international accountancy firm, she presents the case of Tom, the partner responsible for UK bank audits. Tom was observed individually reviewing his team's cash flow model to assess the debt of a company (Kookaburra) and to decide whether it was reasonable to assume that a bank's loans to Kookaburra were safe. Given the size of the debt, this was a significant decision for the bank, for the company and perhaps even for the UK economy. Tom began by commenting on the report, 'unhelpfully, this doesn't tell me what the total debt is', then started to make his own calculation using figures from the report.

Tom was typical of other accountants in Dawes' study in that he did relatively little calculation either mentally or on paper. We note that his team had produced their models using spreadsheets. He did perform some relatively simple addition and subtraction mentally [7.4 – 6.3=1.1 (billion)] but for anything less straightforward, such as multiplication and division, he used a calculator. For example, having rounded a debt of £6.3 billion up to £6.5 billon, he used a calculator to estimate the annual interest due on the debt, 6.5 × 0.05, reading the display [0.325] as 'approximately 350 million to 400 million interest every year'.

To estimate the debt write off, he constructed the following algebraic equation,[27] which he recorded in his notebook:

x /90 + x*0.05 = 350

Tom did not solve this equation algebraically but used trial and error. He began by evaluating his expectation that the company should write off 50% of the debt. To do this, he substituted his anticipated write off, 3.25 billion, for *x*, carrying out the calculation, 3.25/90 + 3.25 × 0.05 [with the answer read as 199], on a calculator. Tom was surprised that this was around 150 billion less than the 350 billion cash flow, commenting, 'I thought they needed to halve the debt to make it.' As a result, he changed his judgment of the appropriate write off to 25%. He did not check this using the equation. Instead, he estimated that a quarter of the available cash flow would be available for cash repayments, calculated this using a calculator [350 × 1/4], rounded the result [87.5] to 85 and multiplied this by 83, the unexpired term of the company's concession, again using a calculator. He concluded from the resulting £7033 million that the debt could be repaid, 'So we might have to write off 25%. Maybe prices aren't as far away as I thought'.

Tom used two distinct models of debt repayment in his analysis. The algebraic equation, a standard repayment model, was used as a framework to structure Tom's thinking and to set out the logic of his argument rather than as a formal mathematical problem to be solved. Since he considered that he 'knew' the answer (the company should write off 50% of the debt), he tested this using the formula. Surprised when his expectation was disproved, he revised his hypothesis to a write off of 25% of the debt, then used a different model to test this: could the company repay the debt over the remaining term of the concession at an annual rate of £85 million. Dawes notes that implicit but unstated in this second model was an assumption that, if 25% of the debt is written off, then 25% of the cash flow would be released for repayments.

The mathematics used by Tom is not easy for an outsider to understand. In reconstructing and extending the work that his team had done, he used numbers and relationships between numbers to construct an argument about the company's debt and the necessary write off. His competence was not so much in an ability to do formal calculation since most calculations were carried out using a calculator. Using a calculator reduced the cognitive effort he expended on calculation, allowing him space for analysis.[28] Calculators can speed up the calculation process,[29] as Tom recognised. We note also that Tom did not manipulate the algebraic equation. Rather, Dawes suggests that the algebra enabled him to work quickly with mathematical models, to round and approximate numbers and to retain the situated or commercial meanings of the numbers and the relationships.

Tom's story shows that competence is not just about being able to do basic mathematics but about knowing which mathematics to use and being able to construct appropriate models and interpret and communicate the results. Although Tom's mathematics was right in these circumstances, it might well be judged "incorrect" by a school teacher. His rounding and approximation procedures certainly did not follow the normal mathematical rules. Yet in the context of debt assessment, rounding 0.325 billion to 350 million to 400 million, given the magnitude of the figures he is working with, was appropriate. It was important in this case to have some flexibility in the application of basic mathematical ideas.

Dawes' looked at a highly qualified graduate profession where high levels of numeracy are required, but a numerate degree is not required. Whilst Tom's mathematical fluency is impressive, competence in mathematics is not exclusive to graduate professions. Celia Hoyles and colleagues give the example of Jim, a technician who left school aged 16.[30] By interpreting extremely complex graphs relating to key performance indicators, Jim could successfully identify the causes of abnormalities in the manufacture of specialist packaging films. Yet despite such technical proficiency, Jim was unable to describe this in mathematical terms. As a result, his skills were not shared with other people in the workplace, some of whom described him as practising "black arts". Hence, whilst Jim's team was very much more effective than other teams, there was no mechanism by which the other teams could learn his mathematical understandings to improve their own effectiveness.

Dawes' accountants were extremely numerate. Nevertheless, many aspects of the mathematics they used are similar to those used in other workplaces. Celia Hoyles and colleagues demonstrated that it is crucial for safety of dosages that the nurses keep the meaning of the numbers in mind as they calculate (see Box 2).[31] These nurses' calculation methods were "fit for purpose", although the nurses were working with a different level of precision and used more grounded calculation methods. Informal methods such as these are a feature of workplace mathematics. For example, Edwin Hutchins found that experienced navigators could calculate the speed of a ship using the three minute rule: how many hundred yards a ship travels in three minutes as its speed in nautical miles per hour.[32] Using this rule, experienced navigators could read a distance measure simply as a measure of speed with little calculation. Such use of mathematics reflects the essence of Arthur Bakker and others' analysis of the use of measurement in the workplace by employees without numerate degrees.[33] They provide an example of lab technicians using a spectrophotometer to ascertain the concentration of a particular chemical. These employees, by understanding their work environment, dilute the chemicals before the analysis by a simple calculation allowing the technology to be used appropriately, effectively and efficiently.

Box 2: Calculating drug dosages in paediatric nursing

In an ethnographic study of practicing paediatric nurses, Celia Hoyles found that practitioners' mathematical skills are considerably better in the workplace than in formal pencil and paper tests.[34] Typically, drugs are prescribed by weight, but produced as solutions in standard packages. Hence, nurses need to calculate how much solution is to be administered to the patient. So, in training, student nurses are taught - and tested on - a formula that is intended to retain this meaning:

$$\frac{What\ you\ want}{What\ you've\ got} \times The\ volume\ it\ comes\ in$$

OR

$$\frac{Dose\ prescribed\ (mg)}{Amount\ per\ package\ (mg)} \times Volume\ of\ each\ package\ (ml)$$

Although this formula is widely known, it is used less frequently than more informal 'grounded' methods. So, for example, one nurse, Belinda, described how she calculated the volume of liquid (2.4ml) required for 120mg dose of amakacine: 'I knew the doses … I know that that one is two point four … two point four mils. With the amakacine, whatever the dose is, if you just double the dose, it's what the mil is. Don't ask me how it works, but it does' (p.19). Amakacine was available in 2ml vials each containing 100mg of the drug. Belinda's method of 'doubling' works because of the particular way in which the drug is package and can be expressed in terms of the nursing formula above:

$$\frac{120\ mg}{100\ mg} \times 2ml = 1.2 \times 2ml = 2.4ml$$

In 'doubling', Belinda has 'seen' 120mg as 1.2 and simply doubled to get the dose. This method was fast, itself an advantage in a hospital ward. The method was also 'grounded' in the packaging of the particular drug and thus safe, because the method retained meaning (at least for as long as the drug was packaged in this way).

Estimation and approximation are critical in workplace mathematics. The pilots in Richard Noss and colleagues' study used estimation rather than exact calculation when landing in windy conditions where cross wind speed is a critical factor.[35] This could be calculated very accurately using trigonometry, but this would be slow and would prevent the pilots from carrying out other important tasks. So, rather than exact calculations, they instead estimated cross winds using a linear approximation for the sine function, which was quicker and carried sufficient accuracy for their purposes. Further examples are seen in Arthur Bakker's analysis where they argue that, where successful, the purpose of the measurements informs the degree of accuracy with which they are taken.[36] This emphasis on linear estimation is further illustrated within the a 2011 report by the Advisory Committee on Mathematics Education where estimation is shown being used in a range of workplaces from large corporations (a graduate trainee in a bank estimating ratios within a mortgage model) to a catering company (estimating the cost of producing sandwiches).[37]

The accountants are not unusual in their use of spreadsheets; they are used in many workplaces. Spreadsheets have considerable advantages over pencil and paper in that they can provide an ongoing record and enable the structuring of problems, both of which were crucial to the work of Margaret Dawes' accountants. Similarly, Julian Williams and Geoff Wake[38] give the example of an engineer, who had an engineering degree, estimating his plant's energy use with a spreadsheet.[39] Dan constructed a spreadsheet to predict the amount of gas that would be required overnight. He then inputted readings by other people in the workplace (0600 INTEGRATING READING is the meter reading taken by a worker at 6am):

[[[[["2nd INTEGRATING READING"-"0600 INTEGRATING READING"] + [[["2nd INTEGRATING READING"] – ["1st INTEGRATING READING"]]/ T2]*TIME4]]/100000] /3.6*CALCV*1000000/29.3071]

At first sight this function appears complex and difficult to understand, partly due to the multiple nested brackets (which in turn reflect how the function was constructed). Indeed, the employees who took the various readings did not understand how these were used. Yet, this function is simply a linear estimation that takes the gas used already that day and adds an estimate of the gas that will be used for the rest of the day, based on the current rate.[40] Moreover, although it is expressed algebraically, Dan has chosen to construct the model using a spreadsheet rather than formal symbolic algebra.[41] Dan's spreadsheet function also illustrates the idiosyncratic meanings and processes that can develop in the workplace, with the mathematics embedded in ways that may be difficult for even some knowledgeable insiders to understand.[42]

In other cases, idiosyncratic meanings and embedded mathematical processes are understood only by a small number of "experts", such as Dan, the engineer in Julian Williams and Geoff Wake's study[43] or Jim, the technician in Celia Hoyles' study[44] whose colleagues regarded his successful and effective interpretation of graphs and key performance indicators as "black arts". Difficulties in some employees' interpretation and understanding of graphical representations and other mathematical outputs, particularly within statistical process control, are also highlighted in the ACME case studies where managers and senior employees ranging from clinical governance in health to engineers within the railway industry make critical decisions based on these outputs. Yet, these procedures can have very significant benefits. Edwin Hutchins study notes that seamen reading measurements under pressure or in conditions of poor visibility could make errors by transposing or misreading digits.[45] As a result, procedures that may seem opaque or odd to an outsider have been developed to avoid such errors.[46]

Margaret Dawes' study highlights a further common aspect of the workplace. The accountants' work – and the mathematics involved – was carried out with others. Whilst Tom worked individually, his work was in a social context. He made sense of the interim reports prepared by his team – the mathematical work of others – so that he could make a judgment about debt write off. This judgment would form the basis of the next steps in his team's investigations and help frame a final collective decision. The accountants presented data to each other in numeric form. They argued over the data and, in doing so, they corrected their own and each other's errors. In the end, they reached collective judgments on the basis of the mathematics. Their numeracy was at a very high level. Indeed, most expert accountants, such as Tom, are not just highly *competent* but highly *fluent* in that their application of mathematics appeared to be seamless. But accountancy is not unique. Workplace mathematics is social and collaborative. This can involve a collaborative social setting where people in the workplace – as with the accountants - very deliberately work together on a common task,[47] or it can involve the same tasks being carried out by different nurses at different times – as in Hoyles' study[48] – resulting in seemingly idiosyncratic procedures becoming part of a shared mathematical wisdom.

Finally, the type of mathematics involved obviously varies across sectors. Perhaps surprisingly, Dawes' accountants make very little use of graphs or of statistical approaches, although Dawes suggests that this is due to the particular nature of these accountants' work (loan loss reviews) and statistics is fundamental to many financial and other workplaces.[49] But, despite this variation, the mathematical content is taught within GCSE mathematics: multiplicative reasoning, approximation and estimation, mental maths, simple statistics, using and interpreting calculators and spreadsheets, interpreting graphs, and measurement. This type of mathematical content reflects the mathematical content required for quantitative literacy, a skill set, which, across studies, is generally agreed as being essential.[50]

In this section we have explored the extent to which mathematics in the workplace is deeply embedded and specific to particular workplaces, leading to the need for a sophisticated application of mathematics, even where the mathematical principles may be covered by the GCSE curriculum. In these contexts, employees spend time, both individually and collaboratively, coming to understand the meaning of the mathematics they use and applying this to their own jobs. So, what are the barriers to such mathematical competence?

Barriers to Mathematical Competence

There are many situations where employees do not appear to have sufficient mathematical competence. In this section, we consider the causes of such a lack of competence. Several studies indicate that, in familiar and routine settings, people in the workplace can use basic mathematics confidently and generally successfully,[51] often because procedures have been developed that support and enable these routine calculations. In the studies that we reviewed, many of the respondents considered that they did little more than "basic" arithmetic.[52] The mathematics becomes partially invisible because the employees can do it without thought, making it appear trivial and non-mathematical to them and their employers. Unlike classroom mathematics, where practices and artefacts are clearly labelled (such as a mathematics class, a mathematics teacher, a mathematics textbook) workplace mathematics is embedded in specific roles, tasks and goals.[53]

A number of other studies raise concerns that people in the workplace lack this basic competence,[54] resulting in significant skills gaps.[55] This lack of competence reflects findings from a recent study of English secondary school pupils who demonstrated poor understanding of decimal number, ratio and multiplicative reasoning generally.[56] Whilst these levels of misunderstanding are extremely serious, the problem is not simply one of being able to "do", or calculate with, mathematics but of being able to understand its practical application in the workplace.

In Celia Hoyles' study[57] of mortgage advisers, employees used a mortgage illustration tool to model repayments for a current account mortgage (CAM)[58] based on financial information supplied by the client. On the basis of this information, a computerised mortgage calculator produced a graphical illustration showing the repayments the client would make and their savings compared with a standard repayment mortgage (see Figure 1).

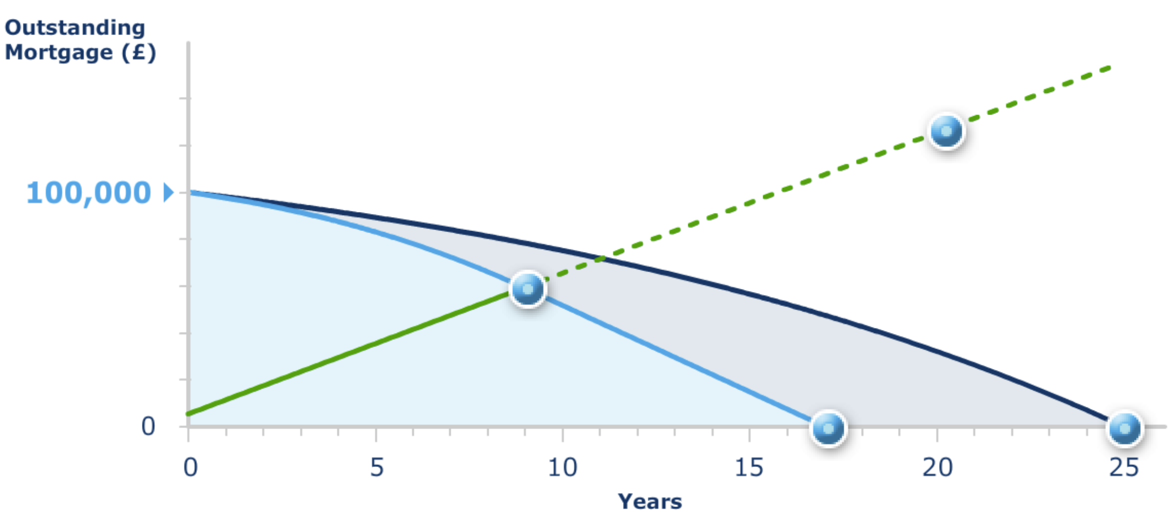

Figure 1: A mortgage calculator similar to that used by the mortgage advisers in Hoyles' study. The mortgage adviser was also presented with information about the predicted savings: 'This is what you will save: £29,504.99 interest saved. Term reduction: Three years and seven months.'

The mortgage adviser was given a script to present these savings to the client:

> Mr Smith, on the basis of the information you have given me, with a current account mortgage you could pay off your borrowing three years and seven months early, and save yourself £29,504.99 in interest. Is that alright for you? [59]

It was crucial to the mortgage company that they should meet their legal requirement to identify clients who would not benefit from a CAM mortgage and to ensure that clients understood the advantages - and disadvantages - of a CAM mortgage over a standard repayment mortgage. However, when faced with queries or atypical clients, the mortgage advisers were unable to explain where the apparent savings of approximately £30,000 arose and they could not use the graphs to explain the reduced term. In part, the mortgage advisers had very limited experience of interpreting graphs.[60] It is even more surprising that the advisers saw little connection between the mortgage and credit card debt, a relationship fundamental to the CAM mortgage.[61]

In understanding the extent to which these mortgage advisers did not display mathematical competence, it is useful to compare their case with the competence demonstrated by Margaret Dawes' accountants:[62]

> Participants were very competent in setting up and performing calculations. They were skilled and knowledgeable users of arithmetic. Generally they chose to perform an appropriate calculation and set up that calculation without too much thought or effort. They made few mistakes when calculating and these were almost always picked up and corrected, either almost immediately or during the review process. For written work electronic means of calculation were used for all but the simplest calculations and calculations were checked for completeness and accuracy where they were deemed important. Mental calculations were used in oral discussions observed: these were generated from information in written texts, set up, described and performed in conversation. Also, for critical calculations, time was spent ensuring that there was a common understanding of and agreement as to what was to be calculated, the result and/or its meaning. Generally, all calculation participants kept commercial meanings attached to the input and output numbers as they calculated. They or other team members were interested in the answers produced and their commercial meanings; routinely making sense of the calculations. These practices taken together explain the participants' general high level of competence.

The accountants certainly were more numerically competent than the mortgage advisers. But, unlike the mortgage advisers, the accountants ensured that they understood what the calculations they were doing meant and they were interested in the "commercial meanings" of the results. For the mortgage advisers, the mortgage calculator was what Bruno Latour calls a black-box 'about which they need to know nothing but its input and output'.[63] In the workplace, black boxes can often be seen where spreadsheets have been constructed by others. Although the mortgage calculator was designed to help the adviser, it has not made the meaning of the calculations any more transparent.

Mathematical competence is also challenged when people in the workplace are faced with unexpected problems – or "breakdown situations" – and are forced into confronting mathematics with which they would not usually engage on a day-to-day basis.[64] When people in the workplace are taken outside their comfort zone and asked to explain processes usually hidden by a black-box, weaknesses in their mathematical competence quickly become apparent. For example, Richard Noss and colleagues asked nurses to interpret a hospital memo about a new drug administration procedure.[65] They were asked to judge whether the proportion of a dose in a catheter was safe to inject. Getting this wrong mattered, because it could result in a serious drug overdose. Although the nurses had been able to calculate familiar drug dosages successfully,[66] they struggled with this unfamiliar, but real, problem and several required help to do so.

This literature review indicates that there are significant barriers to mathematical competence. It is important to emphasise that simply being able to carry out the relevant mathematical procedures is not enough. Employees need both to understand the mathematical content (though only at GCSE level) and have the capacity to understand it within their workplace. This understanding needs to include making sense of a problem, interpretation and communication. Neither the mathematical understanding nor the application capacities are sufficient on their own. Studies such as those by Julian Williams and Geoff Wake,[67] and by Celia Hoyles,[68] indicate that the relevant mathematical understanding is best developed through the use of problem solving techniques and a consideration of mathematics in context.

Technology and Efficiency

Recent changes in today's workplaces – particularly the growth in technology – have significant implications for the mathematics needed in the workplace. Technological advancement, alongside globalisation and demographics, is a major driver of workplace change.[69] Much of this technological advancement has been in the field of information and communications technologies (ICT) although it is predicted that other technologies – such as biotechnology or nanotechnology – will result in workplace changes in the future. The use of ICT in the workplace has continually increased: in 1986, 40.3% of all employees used computers or computerised equipment, rising to 73.7% in 2001 and 77.4% in 2006.[70] Alan Felstead's report, from which these figures are extracted, suggests that this final figure represents a saturation point, yet others predict technological progress will continue or accelerate.[71]

This increase in technology has changed the mathematical practices in which people in the workplace are required to engage,[72] although few studies have examined these changes over time. For some semi-skilled employees, technology has resulted in deskilling. Luxury boat builders cited in Robyn Zevenbergen's study,[73] now use templates rather than measuring, while the electricians cited in Wolff-Michael Roth's study used conduit benders set to predetermined values rather than trigonometry in bending tubing to the correct angle.[74] In some cases, it is surprising how little technology is used. For example, the delicatessen-counter employees in Kim Hastwell's study of a large suburban supermarket in New Zealand relied almost exclusively on paper-based record keeping.[75]

For other people in the workplace, technology has increased the mathematical demands. Alain Mercier's study of agricultural technicians found the mathematics in calculating the dietary requirement of cattle feed to have increased.[76] Whereas these technicians previously used a specific procedure called the 'diagonal cross' to calculate weighted ratios, together with ready reckoner tables, the introduction of computers means that the technicians needed to interpret non-linear graphs instead of doing traditional calculations.

Statistical analysis shows an increasing use of numeracy across managers, professionals and trade workers between 1996 and 2006,[77] and a rise in the need for mathematical skills – such as the interpretation of non-linear graphs – not used by previous generations.[78] Staff are now required to be 'more mathematically competent … [and are] … required to undertake higher cognitive tasks such as interpreting the meaning of the computer generated results of calculations',[79] and understand the significance of the numerical output,[80] as calculations move from manuals and charts to spreadsheets and calculators. The mathematics has become more embedded in working life.[81] However, the impacts of technological change may be dependent on the skill level of employees. For example, semi-skilled manufacturing employees initially tended to make very little use of ICT or used it as a supporting tool rather than to change traditional ways of working.[82] Now, the introduction of computer aided design (CAD) has substantially increased technology within manufacturing, reducing development costs and wastage, and changing the required types of mathematical engagement.

A significant increase in ICT is visible in the production and use of spreadsheets across many sectors. Mathematical understanding of the principles underlying the spreadsheet is essential in order to make sense of the output. However, a mathematical understanding is not sufficient. For example, Celia Hoyles quotes the owner/manager of a family-run farm bakery, who describes how employees need to use mathematics differently as a result of the introduction of spreadsheets:[83]

> One of the things we're struggling with at the moment is updating the cost per unit of the products ... You've got the transfer of information from recipes, a lot of which are historic and written in imperial units, going into a metric formula, because a lot of the raw ingredients — not all though, it's still a mix are in metric units. So you've got to watch carefully, for example a can of golden syrup is sold as '4 pounds', but the spreadsheet has a metric volume. So there are little things, not necessarily mathematical to the eye, which they've got to be watching all the time. **Once the spreadsheet is in place it should be working itself out. The staff have got to be looking for anomalies.** We have found that suddenly the price of a cake based on the raw ingredients will go from £1 to £1.50 in a six month period, and so I've had to say, go back and look because there's obviously a problem there, the raw materials haven't increased by 50%. And when they go back they'll find that often a raw ingredient has been entered in imperial instead of metric or vice versa, or one of the cooks has changed the recipe and that information hasn't been fed through. They might have changed from 4 ounces of ground almonds to 5 or 6. Although there's a mathematical problem there, it's a matter of attention to detail in the workings.

By recording production data over time, a well-designed spreadsheet can enable efficiency savings that would be almost impossible otherwise. But spreadsheets cannot themselves think nor solve problems. The bakery employees need to be able to use the spreadsheet to identify anomalies and anticipate potential problems. The mathematical concepts involved are well within GCSE mathematics: the effect of a 50% gain and the difference between metric and imperial units. However, this "attention to detail" requires staff to use mathematics to notice anomalies such as an unbalanced increase in the cost of production.

Spreadsheets are commonly used for modelling across industries, particularly in the financial sector. In finance, they give people in the workplace greater control – and more responsibility – over whole products, such as providing advice on choosing pensions, investments and mortgage products. But that greater responsibility brings its own problems if people in the workplace don't understand the outputs of the models, as with the workings of compound interest cited earlier.[84] The result can be incorrect advice to clients.

Technological change – particularly the use of models or similar procedures – is also ingrained in wider industry processes. Process Improvement and Statistical Process Control are good examples. These procedures are often used on production lines to identify bottlenecks and increase overall efficiency through monitoring of outputs at various stages and modelling to ascertain the potential impacts of changes. At Master Cake, a large automated bakery producing sponge cakes observed in Celia Hoyles' study,[85] Process Improvement is shown through the use of new technology to increase production line efficiency, but problems occur when only a few people in the workplace can engage with and fully interpret the outputs. This is similar to what Jim the technician was involved in as he engaged with key performance indicators.

Summary

In this section we have discussed the need for mathematical competence in the workplace. Changes in workplace practices – particularly an increased focus on efficiency measures – have resulted in mathematical application / understanding becoming an essential skill for all people in the workplace, even in relatively unskilled jobs. People in the workplace need to be able to make sense of the mathematics they are using (individually and in collaborative situations), interpreting inputs and outputs rather than entering data blindly in a 'black-box'. Additionally, many people in the workplace need the capacity to spot anomalies and to respond to these in appropriate ways. It is also essential that all people in the workplace are able to communicate outputs coherently, both internally and to external clients.

However, developing mathematical competence is not straightforward, and learning cannot simply be transferred from a classroom context to the workplace. Mathematics in the workplace is highly contextual (to the extent of becoming invisible), embedded in workplace practices through idiosyncratic procedures and processes, and very different from school mathematics.[86] Much mathematics carried out in the workplace is now carried out using ICT (although we note the persistence of paper based work in some low-skilled sectors), with spreadsheet usage being particularly prevalent. In general, technological advances appear to have increased, not decreased, the mathematical demands of many staff.

People in today's workplaces certainly do need mathematics skills. This curriculum *content* is predominantly covered sufficiently in the current GCSE curriculum. However, it is the *application* of this content to the workplace that is not straightforward. Therefore, in the next section we consider how competence in the application of this content can best be developed in post-16 mathematics education.

Implications for Mathematics Post-16

What does the literature on workplace mathematics tell us about the implications for teaching and learning after GCSE and the needs of those students who do not currently study any mathematics beyond the age of 16 in England?

This is a diverse group of students. A-level mathematics participation is heavily skewed towards the highest attaining group of students.[87] There are three groups of students essentially. First, there are the 13% of 17 and 18 year-olds who take a maths AS or A level course and are, thus, already catered for. Second, there are the students who get at least a grade C at GCSE aged 16 but do not take a maths AS or A level course (just under half of the cohort). More than half of those who get A or A* grades at GCSE don't continue studying maths, in addition to the vast majority of those who get a B or C grade. It is critical that a new pathway is created targeted at the needs of this group. Finally, there are those students who don't gain a grade C at GCSE (just under half of the cohort of all 16 year-olds). As a result of the Wolf Report,[88] this latter group will continue to study towards GCSE mathematics if they are on a school or college course. For this group, it is important to identify ways to tailor the traditional GCSE mathematics pathway to the needs of this group that are rigorous, engaging for students, provide sufficient breadth and are valued by employers.[89]

Our review of the literature indicates that the transfer of mathematical skills from school to the workplace is not straightforward. Mathematical understanding is important and the literature highlights the importance of particular mathematical topics. However, it is not on its own sufficient for students to successfully develop mathematical competence in the workplace. In part, this is because the ways in which mathematics is applied at work are very different to school mathematics. We have termed this as "simple mathematics in complex settings" to emphasise that the problem is more about *how* mathematics is taught and learnt at school rather than the content of the curriculum.

Within this study, we found the literature on workplace mathematics to broadly fall into four categories.

- There are a number of policy focused commentaries which introduce and debate terms such as quantitative and mathematical literacy.[90]
- There are several broad studies into workplace mathematical skills which examine practices across sectors and are often based on large-scale surveys or case-studies.[91]
- There are studies which specifically look at learning within the workplace which we considered relevant to a discussion of school-based interventions/implications in that they serve to identify very specific focal areas, particularly in the use of spreadsheets, models and graphs, in the mathematics needed in the workplace.[92]
- There are also a few studies which examine how we can better match classroom learning and workplace needs.[93]

Broadly the first group argues for an approach reflecting the use of mathematics in the workplace, particularly when engaged in Process Improvement activities or working with numerical outputs in modelling financial scenarios. However, as Geoff Wake and Julian Williams argue, workplace mathematics is so 'complex and messy' that it 'demands a critical, inquiring disposition'.[94] On this basis, we concur with Celia Hoyles that the 'skills deficit' is less about mathematical and calculation skills narrowly and more about how mathematics is applied and interpreted.[95] For example, our own ICCAMS study shows that students' difficulties relate to knowing *when* contexts involve multiplicative and proportional relationships.[96]

Although the mathematical content required differs from workplace to workplace, this content is largely covered in the GCSE curriculum. Whilst many GCSE topics are important, we note the particular importance of proportional (or multiplicative) reasoning, approximation and estimation, the interpretation of graphs, and the use of spreadsheets.[97] However, this content looks different in the workplace. For example, although relatively little symbolic algebra was apparent in these studies, people in the workplace are often required to understand relationships between variables and to use spreadsheet algebra.[98] Similarly, rather than following mathematical rules and conventions to the letter, approximation and estimation is used in ways that are "fit for purpose", as we saw with Tom. However, this use of approximation and estimation needs to be grounded on a good understanding of measurement and error. Measurement is key to many workplaces – it is important, for example, that people in the workplace understand error is cumulative.[99] Yet, measurement receives relatively little attention in the curriculum.

Statistical reasoning is used in some workplaces, but it is very different to the formal procedures that students encounter in the classroom. For the workplace, exploratory and descriptive techniques appear to be often more important than the use of formal statistical tests.[100] Of course, statisticians do need to understand a range of sophisticated techniques, but for others a thorough understanding of measures of centre and spread, together with a feel for statistical relationships and simple probability, is sufficient. Even in degree subjects where more advanced statistics may be needed, such as the biosciences, these skills can be taught later provided students have a thorough grounding in simple GCSE level statistics.[101] Similarly, for the workplace, using a computer to interpret statistical relationships is very much more important than competence with pencil and paper procedures.[102] Recent work by Dave Pratt and colleagues on using simulation to better understand these relationships provides one model for this.[103]

The literature reviewed suggests significant features of workplace mathematics not generally reflected in school mathematics. For instance, much workplace mathematics is not a solitary activity but one involving collaboration at different levels, either with people in the workplace coming together as a team or where employees with responsibility for specific processes bring these together to create whole products. Collaboration in the form of teamwork is particularly important in that it enables participants to build a strong shared understanding of the mathematics they are engaged in and this approach should underlie post-16 mathematics provision. Such approaches have previously been seen in mathematics pathways such as the Free Standing Mathematics Qualifications (FSMQs) and the AS-Level Use of Maths.[104] This review also suggests a need to address the computational methods in use; whilst workplaces tend to use calculators and computers for anything beyond the most basic of calculations, school mathematics remains a largely pencil and paper world.[105] Furthermore, the application of mathematics learnt in the classroom to the workplace is not straightforward.[106]

Further Research

Elsewhere we have recommended that a new advanced mathematics pathway should be introduced with a focus on fluency, modelling and statistics, but that research was needed about the mathematical needs for employment to guide the design of this pathway and associated qualification.[107] This literature review has begun to address this question. However, there are still research needs. We note, in particular, that more research is needed about the mathematics of professionals and about how workplace mathematics is changing over time.

We also noted that, in implementing the Wolf Report recommendation that GCSE Mathematics should remain compulsory until students have attained a Grade C, it is important to avoid a remedial focus. In a subsequent report, we recommended identifying alternatives to the traditional GCSE mathematics pathway that are rigorous, engaging for students, provide sufficient breadth and are valued by employers. Students should have opportunities to develop their understanding of "simple mathematics in complex settings". Providing such learning opportunities in school is far from straightforward. Further research is needed into the design and effectiveness of these.

A Note on Methodology

An Assessment of the Literature

The literature on workplace mathematics is partial. We found that many of the studies sourced – particularly those that were industry specific – were skewed towards apprentices or other work placements often linked with the researchers' institutions, rather than long-term employees. This may have implications for the types of mathematical practices reported. Further, few of the journal papers (in comparison with studies published as books) tackled the problems or provided in-depth examples and as such many longer illustrations are taken from a limited literature base. Several of the in-depth ethnographic studies are very good indeed and we rely on these heavily in this review.[108] There were no longitudinal studies, a somewhat surprising finding given the scale of technological change in the workplace.

Literature Sourcing

Literature was drawn from two sources: expert knowledge of the field and online/database searches. For the online/database searches, a list of search strings – including 10 industry specific searches – was used. Full-text and title only searches were conducted across nine databases and Google Scholar with a limit of articles published from 2000 onwards. Databases were selected from the researchers' knowledge of the field. The total number of articles sourced was tabulated in a matrix for each string and database. Potentially relevant documents from database searches and the first 250 results from Google Scholar searches were tabulated and imported into an Endnote database.[109] Further limited literature sourced through researcher contacts and following up of references was added to this database. After cleaning, the database contained 311 entries. These entries were coded as relevant to, or beyond the limits of, a scoping study. Examples of exclusion criteria included being concerned with subject matter outside of mathematics or educational stages (such as primary schooling) irrelevant at this stage. Coding was discussed as a research team and references put through a process of snowballing, resulting in 133 articles retained as relevant for our scoping study. Further classification resulted in 54 core articles and 79 background articles. A further 25 policy documents were identified as important.

Literature Categorisation, Synthesis and Report-Writing

The 54 core articles were categorised as general literature (31 articles) or industry specific e.g. manufacturing, retail (29 articles). There was a small degree of double-categorisation with some articles covering multiple industries (6 articles).[110] The key themes of the general literature and policy documents were noted. Industry specific literature was read in industry groups and their key findings were recorded on a template in Excel. Key issues were then synthesised from the general literature, policy and industry-specific tables/templates to evidence and illustrate the key themes. The table below provides an overview of the literature sourced, categorised and drawn on.

Initial full-text search results (excluding Google Scholar)	127946
Initial results considered from Google Scholar	3500
Literature included in database following cleaning	311
Literature relevant to this scoping study	133
Literature classified as core	54
Literature classified as background (but within the scope of the study)	79
Relevant policy documents	25

Core Studies Included in Review

The 54 items of literature classified as core and included within this review are as follows:

Advisory Committee on Mathematics Education. (2011). *Mathematical needs: Mathematics in the workplace and in Higher Education*. London: The Royal Society.

Autor, D., Levy, F., & Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly Journal of Economics, 118*(4), 1279-1333.

Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. *ZDM, 43*(5), 737-746. doi: 10.1007/s11858-011-0328-3

Bessot, A., & Ridgway, J. (Eds.). (2000). *Education for mathematics in the workplace*. Dordrecht: Kluwer.

Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). Adult numeracy: review of research and related literature. London: National Research and Development Centre for Adult Literacy and Numeracy.

Dawes, M. (2007). *Workplace mathematics : a study of mathematics in use in the UK assurance division of an international accounting firm.* PhD, King's College London.

Earle, D. (2011). *Literacy and Numeracy at Work: Skill Education and Job Tasks*. Wellington: Ministry of Education.

Felstead, A., Gallie, D., Green, F., & Zhou, Y. (2007). Skills at Work, 1986 to 2006. Oxford and Cardiff: ESRC Centre on Skills, Knowledge and Organisational Performance based at the Universities of Oxford and Cardiff.

Fitzgerald, A. (1976). School mathematics and the requirements of industry. *Journal of Vocational Education and Training, 28*(70), 43-49.

FitzSimons, G., & Coben, D. (2009). Adult numeracy for work and life: Curriculum and teaching implications of recent research *International Handbook of Education for the Changing World of Work* (pp. 2731-2745). Dordrecht: Springer.

FitzSimons, G. E., Mlcek, S., Hull, O., & Wright, C. (2005). Learning Numeracy on the Job: A Case Study of Chemical Handling and Spraying. An Adult Literacy National Project Report: National Centre for Vocational Education Research.

Gainsburg, J. (2005). School mathematics in work and life: what we know and how we can learn more. *Technology in Society, 27*(1), 1-22.

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Hall, R., & Stevens, R. (1995). Making space: a comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The Cultures of Computing* (pp. 118-145). Oxford: Blackwell Publishers.

Harraway, J., & Barker, R. (2005). Statistics in the workplace: A survey of use by recent graduates with higher degrees. *Statistics Education Research Journal, 4*(2), 43-58.

Hastwell, K., Strauss, P., & Kell, C. (2013). "But pasta is pasta, it is all the same": the language, literacy and numeracy challenges of supermarket work. *Journal of education and work, 26*(1), 77-98.

Hoyles, C., Morgan, C., & Woodhouse, G. (1999). *Rethinking the mathematics curriculum* (Vol. 1). Abingdon, Oxon: Falmer Press.

Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. Abingdon, Oxon: Routledge.

Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education, 32*(1), 4-27.

Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council.

Hutchins, E. (1995). *Cognition in the Wild*. Cambridge, MA: Massachusetts Institute of Technology.

Joint Mathematics Council. (2011). Digital technologies and mathematics education: A report from a working group of the Joint Mathematical Council of the United Kingdom.

Lesh, R. (2000). Beyond constructivism : identifying mathematical abilities that are most needed for success beyond school in an age of information. *Mathematics Education Research Journal, 12*(3), 177-195.

Magajna, Z., & Monaghan, J. (2003). Advanced mathematical thinking in a technological workplace. *Educational Studies in Mathematics, 52*(2), 101-122.

Markes, I. (2006). A Review of Literature on Employability Skill Needs in Engineering. *European Journal of Engineering Education, 31*(6), 637-650.

Marr, B., & Hagston, J. (2007). Thinking beyond numbers: Learning numeracy for the future workplace. Adelaide, SA: National Centre for Vocational Education Research.

Masingila, J. O. (1994). Mathematics practice in carpet laying. *Anthropology & Education Quarterly, 25*(4), 430-462.

McClure, R., & Sircar, S. (2008). Quantitative literacy for undergraduate business students in the 21st century. *Journal of Education for Business, 83*(6), 369-374.

Michaels, G., Natraj, A., & Van Reenan, J. (2010). Has ICT Polarised Skill Demand? Evidence from Eleven Countries over 25 Years. CEP DP 987 LSE. Cambridge, MA: The National Bureau of Economic Research.

Millroy, W. L. (1992). An Ethnographic Study of the Mathematical Ideas of a Group of Carpenters. *Journal for Research in Mathematics Education. Monograph 5*, i+ix-x+1-210.

Moschkovich, J. (2002). Chapter 6: Bringing Together Workplace and Academic Mathematical Practices during Classroom Assessments. *Journal for Research in Mathematics Education. Monograph 11*, 93-110.

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Osmond, R. (2000). The importance of mathematics to employers. *Teaching Mathematics and its Applications, 19*(2), 50-55.

Parker, R., Kent, J., & Brown, T. (2001). The perceived importance of statistics in the logistics and transportation industry. *The Journal of Education for Business, 76*(4), 185-188.

Reyna, V. F., & Brainerd, C. J. (2007). The Importance of Mathematics in Health and Human Judgment: Numeracy, Risk Communication, and Medical Decision Making. *Learning and Individual Differences, 17*(2), 147-159.

Riall, R., & Burghes, D. (2000). Mathematical needs of young employees. *Teaching Mathematics and its Applications, 19*(3), 104-113.

Rice, S., Griffin, P. E., Care, E., & McPherson, J. (2012). Compulsory hurdle literacy and numeracy requirements for senior secondary students: what do the stakeholders think? *Research in Post-Compulsory Education, 17*(2), 143-161.

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Smith, E., & Gorard, S. (2011). Is there a shortage of scientists? A re-analysis of supply for the UK. *British Journal of Educational Studies, 59*(2), 159-177.

Smith, J. (2002). Chapter 7: Everyday Mathematical Activity in Automobile Production Work. *Journal for Research in Mathematics Education. Monograph 11*, 111-130.

Tikly, C., & Wolf, A. (Eds.). (2000). *The maths we need now: demands, deficits and remedies*. London: The Institute of Education, University of London.

Torpey, E. (2012). Math at Work: Using Numbers on the Job. *Occupational Outlook Quarterly, 56*(3), 2-13.

Trenerry, R. (2002). Talking About Training. Literacy and Numeracy Training Practices in Industry: A Comparative Study across Five Industries. Brisbane: Adult Literacy and Numeracy Australian Research Consortium.

van der Kooij, H. (2001). Mathematics and key skills for the workplace. In G. E. FitzSimons, J. O'Donoghue & D. Coben (Eds.), *Adult and Lifelong Education in Mathematics: Papers from Working Group for Action (WGA) 6 : 9th International Congress on Mathematics Education, ICME 9* (pp. 231-242): Adults Learning Mathematics.

Wake, G., & Williams, J. (2003). Using Workplace Practice to Inform Curriculum Design. In S. Lamon, W. Parker & S. Houston (Eds.), *Mathematical modelling a way of life* (pp. 189-200). Chichester, UK: Horwood Publishing.

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Endnotes

[1] Mathematics qualifications at any level have considerable value to individuals and the economy, but GCSE is the most valuable mathematics qualification. An A-level qualification in mathematics increases earnings by 7-10% (see Dolton, P. J., & Vignoles, A. (2002). The return on post–compulsory school mathematics study. *Economica,* 69(273), 113-142). GCSE Grade C / Level 2 qualification reduces the likelihood of unemployment by 7 percentage points and increases earnings by roughly 18% and the effect of numeracy is greater than the effect for literacy (see Layard, R., McIntosh, S., & Vignoles, A. (2002). Britain's record on skills. CEEDP, 23. London: Centre for the Economics of Education, London School of Economics and Political Science). This skills gap at GCSE accounts for around half of the 20% productivity gap between the UK and German economies, see also: Wolf, A. (2011). Review of Vocational Education: The Wolf Report. London: DfE.

[2] Hodgen, J., Pepper, D., Sturman, L., & Ruddock, G. (2010). Is the UK an outlier? An international comparison of upper secondary mathematics education. London: The Nuffield Foundation. In all developed countries outside the UK included in the survey most young people were found to continue with some study of mathematics until the age of 18. In England the post-16 figure may rise to 26% with the inclusion of GCSE retakes.

[3] Table extracted from p.23 of: Hodgen, J., Marks, R., & Pepper, D. (2013). Towards universal participation in post-16 mathematics: lessons from high performing countries. London: The Nuffield Foundation.

[4] Gove, M. (2011). Speech to the Royal Society, 29 June 2011.

[5] See for example Advisory Committee on Mathematics Education. (2012). Post-16 Mathematics: A strategy for improving provision and participation. London: Royal Society; and Harris, J. (2012). Rational Numbers: Investigating compulsion for mathematics study to 18. London: The Pearson Think Tank.

[6] Hodgen, J., Marks, R., & Pepper, D. (2013). Towards universal participation in post-16 mathematics: lessons from high performing countries. London: The Nuffield Foundation; and Norris, E. (2012). Solving the maths problem: international perspectives on mathematics education. London: The Royal Society.

[7] See for example Koenig, J. (2011). A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education. Leeds: UK Centre for Bioscience, Higher Education Academy.

[8] See for example Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society; Bessot, A., & Ridgway, J. (Eds.). (2000). *Education for mathematics in the workplace*. Dordrecht: Kluwer; Dawes, M. (2007). *Workplace mathematics : a study of mathematics in use in the UK assurance division of an international accounting firm*. PhD, King's College London.

[9] See for example Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society.

[10] Dawes, M. (2007). *Workplace mathematics : a study of mathematics in use in the UK assurance division of an international accounting firm*. PhD, King's College London.

[11] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. Abingdon, Oxon: Routledge.

[12] Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (pp. 33-50). Cresskill, NJ: Hampton Press. Cited in Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). Adult numeracy: review of research and related literature. London: National Research and Development Centre for Adult Literacy and Numeracy. (p. 10, emphasis in the original).

[13] In fact, those people in the workplace, who are required to have a numerate degree, often use largely simple mathematics. For example, ACME cites a graduate trainee in a bank working on modelling and competitor analysis who feels that a 'mathematics degree was not essential to get the job but has helped on several business management projects'; see: Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society (p.19).

[14] Gainsburg, J. (2005). School mathematics in work and life: what we know and how we can learn more. *Technology in Society,* 27(1), 1-22.

[15] Trenerry, R. (2001). Putting It All Together: Learning for Work and Learning About Work in the Horticulture Industry. Melbourne, Victoria: Literacy and Numeracy Australian Research Consortium.

[16] FitzSimons, G. E., Mlcek, S., Hull, O., & Wright, C. (2005). Learning Numeracy on the Job: A Case Study of Chemical Handling and Spraying. An Adult Literacy National Project Report: National Centre for Vocational Education Research.

[17] Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 4-27.

[18] Riall, R., & Burghes, D. (2000). Mathematical needs of young employees. *Teaching Mathematics and its Applications*, 19(3), 104-113.

[19] Gigerenzer, G. (2002). *Reckoning with risk.* London: Penguin.

[20] Op cit., p.15

[21] Op cit.

[22] http://en.wikipedia.org/wiki/Sally_Clark

[23] Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

[24] Op cit.

[25] Hutchins, E. (1995). *Cognition in the Wild.* Cambridge, MA: Massachusetts Institute of Technology.

[26] Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life.* Cambridge: Cambridge University Press.

[27] In the equation, x represents an estimate for the long-term debt that can be sustained subsequent to debt write-off. $x/90$ is an estimate for the annual repayments on the loan principal, where 90 is an approximation for 83 years, the unexpired term of the company's concession. $x*0.05$ is an estimate for the average annual interest charge over the entire concession. 350 is an estimate for the annual cash flow (rounded up from £334 million).

[28] We use cognitive effort loosely in Margaret Dawes' sense.

[29] Calculators can be slower than mental methods. So, for Tom, adding or subtracting two digit approximations appeared to be quicker mentally than with a calculator. It is, of course, crucial that people can identify key stroke errors through estimation

[30] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge.

[31] Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 4-27.

[32] Hutchins, E. (1995). *Cognition in the Wild.* Cambridge, MA: Massachusetts Institute of Technology.

[33] Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. *ZDM*, 43(5), 737-746.
[34] See also Hutton, M., Coben, D., Hall, C., Rowe, D., Sabin, M., Weeks, K., & Woolley, N. (2010). Numeracy for nursing, report of a pilot study to compare outcomes of two practical simulation tools–An online medication dosage assessment and practical assessment in the style of objective structured clinical examination. *Nurse Education Today*, 30(7), 608-614.
[35] Noss, R., Hoyles, C., & Pozzi, S. (2000). Working knowledge: Mathematics in use. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 17-25). Dordrecht: Kluwer.
[36] Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. *ZDM*, 43(5), 737-746.
[37] Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society.
[38] Williams, J., & Wake, G. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics,* 64(3), 345-372.
[39] We use this example to emphasise that even professionals with highly numerate degrees use a great deal of 'simple' GCSE mathematics rather than more complex functions or calculus. ACME (2011) found that complex models and functions were used by engineers and others, although linear models were used most commonly (Personal communication, Huw Kyffin, 20 May 2013).
[40] The gas used already that day is the difference of two readings, ["2nd INTEGRATING READING"-"0600 INTEGRATING READING"]. The estimate of the gas that will be used for the rest of the day is calculated by taking an estimate of the current rate of use, [[["2nd INTEGRATING READING"] – ["1st INTEGRATING READING"]]/T2], and multiplying by the time remaining, TIME4. The remaining elements are multipliers to calculate the payment due to the gas supplier. The spreadsheet has "too many" brackets, reflecting the process of its construction.
[41] The accuracy of this estimate was extremely important, because over and under estimates both carried financial penalties. Hence, Dan took various steps to evaluate and minimise this error using the spreadsheet, although as with the estimate itself, these were constructed for the context and differed from standard procedures in school mathematics.
[42] For a similar example in which structural engineers have to spend time understanding their own calculations in context, see: Hall, R., & Stevens, R. (1995). Making space: a comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The Cultures of Computing* (pp. 118-145). Oxford: Blackwell Publishers.
[43] Op cit
[44] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge.
[45] Hutchins, E. (1995). *Cognition in the Wild.* Cambridge, MA: Massachusetts Institute of Technology.
[46] Further discussion of these procedures – and the inherent difficulty in seeing the underlying mathematics – is provided in Section 3.2 where we explore in more depth the related concepts of invisibility and black-boxes.
[47] See for example Edwin Hutchins' (1995) study of US navigators, op cit.
[48] Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education,* 32(1), 4-27.
[49] See for example: Hahn, G., & Doganaksoy, N. (2011). *The role of statistics in business and industry.* Hoboken, NJ: Wiley; Harraway, J., & Barker, R. (2005). Statistics in the workplace: A survey of use by recent graduates with higher degrees. *Statistics Education Research Journal,* 4(2), 43-58; Khattree, R., & Rao, C. (2003). *Statistics in industry* (Vol. 22): Elsevier Publishing Company; Nair, V., Hansen, M., & Shi, J. (2002). Statistics in advanced manufacturing. In A. Raftery (Ed.), *Statistics in the 21st century* (pp. 264-271). Boca Raton, FL: Chapman and Hall/CRC; and Parker, R., Kent, J., & Brown, T. (2001). The perceived importance of statistics in the logistics and transportation industry. *The Journal of Education for Business,* 76(4), 185-188.
[50] See for example: Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council; Madison, B., & Steen, L. (Eds.). (2008). *Calculation vs. Context: Quantitative Literacy and Its Implications for Teacher Education*. Washington, DC: The Mathematical Association of America; McClure, R., & Sircar, S. (2008). Quantitative literacy for undergraduate business students in the 21st century. *Journal of Education for Business*, 83(6), 369-374; Rheinlander, K., & Wallace, D. (2011). Calculus, Biology and Medicine: A Case Study in Quantitative Literacy for Science Students. *Numeracy,* 4(1), 3; and Rosen, L., Weil, L., & von Zastrow, C. (2003). *Quantitative literacy in the workplace: Making it a reality.* Paper presented at the Quantitative Literacy: Why Numeracy Matters for Schools and Colleges - National Forum on Quantitative Literacy, National Academy of Sciences, Washington DC, December 1-2, 2001.
[51] See for example Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge; and Zevenbergen, R., & Zevenbergen, K. (2009). The Numeracies of Boatbuilding: New Numeracies Shaped by Workplace Technologies. *International Journal of Science and Mathematics Education*, 7(1), 183-206.
[52] See for example the results of the National Employers Skills Survey, Hudson, C. (2007). Working mathematics for the 21st century? A discussion paper on workplace numeracy and mathematics. England: National Research and Development Centre for Adult Literacy and Numeracy; the Australian NCVER report, Marr, B., & Hagston, J. (2007). Thinking beyond numbers: Learning numeracy for the future workplace. Adelaide, SA: National Centre for Vocational Education Research, which considers, amongst other issues, invisibility; and Riall, R., & Burghes, D. (2000). Mathematical needs of young employees. *Teaching Mathematics and its Applications*, 19(3), 104-113, who examined numeracy in use across multiple industries.
[53] Smith, J. (2002). Chapter 7: Everyday Mathematical Activity in Automobile Production Work. *Journal for Research in Mathematics Education.* Monograph 11, 111-130.
[54] See for example Fitzgerald, A. (1976). School mathematics and the requirements of industry. Journal of Vocational Education and Training, 28(70), 43-49; and Gainsburg, J. (2005). School mathematics in work and life: what we know and how we can learn more. *Technology in Society*, 27(1), 1-22.
[55] UKCES. (2010). Skills for Jobs: Today and Tomorrow - The National Strategic Skills Audit for England 2010 - Volume 2: The Evidence Report. Wath-upon-Dearne: UK Commission for employment and skills.
[56] Brown, M., Küchemann, D., & Hodgen, J. (2012). The struggle to achieve multiplicative reasoning 11-14. In M. Joubert & P. Andrews (Eds.), *Seventh British Congress of Mathematics Education (BCME7)* (Vol. 30, pp. 49-56): University of Manchester: BSRLM; and Hodgen, J., Küchemann, D., Brown, M., & Coe, R. (2010). Multiplicative reasoning, ratio and decimals: A 30 year comparison of lower secondary students' understandings. In M. F. Pinto & T. F. Kawaski (Eds.), *Proceedings of the 34th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 3, pp. 89-96). Belo Horizonte, Brazil.

[57] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge.
[58] A current account mortgage is also commonly referred to as an offset mortgage. It can be beneficial to self-employed people or those with a variable income.
[59] Hoyles, et al., (2010, p. 149), Op cit.
[60] Interpreting graphs is not straightforward. In a Roth and Bowen's (Roth & Bowen, 2001) study of academic physicists reading unfamiliar graphs about population growth, the physicists made similar errors to those made by children encoutering graphs fpr the first time. In order to make sense of the graphs, they drew on their sometimes incorrect) knowledge of population growth. See: Roth, W., & Bowen, G. (2001). Professionals read graphs: A semiotic analysis. *Journal for Research in Mathematics Education,* 32(2), 159-194.
[61] Moreover, these mortgage advisers, who were giving advice to clients on the basis of different interest rates, displayed a lack of awareness of percentages in relation to finance. For example, they did not realise that by paying, for example, 90% of the remaining debt off each month, clients would never repay the outstanding loan.
[62] Op cit. (p.194)
[63] Latour (1987, pp. 2-3), quoted Williams, J., & Wake, G. (2007a). Black boxes in workplace mathematics. *Educational Studies in Mathematics,* 64(3), 317-343, (p. 321).
[64] Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics*, 36(2), 105-122, (p.108)
[65] Noss, R., Hoyles, C., & Pozzi, S. (2002). Abstraction in expertise: A study of nurses' conceptions of concentration. *Journal for Research in Mathematics Education,* 33(3), 204-229.
[66] Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education,* 32(1), 4-27.
[67] Williams, J., & Wake, G. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics,* 64(3), 345-372
[68] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge.
[69] Wilson, R. (2009). The future of work and implications for education. Warwick: IER.
[70] Felstead, A., Gallie, D., Green, F., & Zhou, Y. (2007). Skills at Work, 1986 to 2006. Oxford and Cardiff: ESRC Centre on Skills, Knowledge and Organisational Performance based at the Universities of Oxford and Cardiff.
[71] See for example: Committee on Science Technology Engineering and Mathematics Workforce Needs for the U.S. Department of Defence and the U.S. Defence Industrial Base (CSTEM). (2012). Assuring the U.S. Department of Defence a Strong Science, Technology, Engineering, and Mathematics (STEM) Workforce Washington, DC: National Academies Press; and Wilson, R. (2009). The future of work and implications for education. Warwick: IER.
[72] Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). Adult numeracy: review of research and related literature. London: National Research and Development Centre for Adult Literacy and Numeracy.
[73] Zevenbergen, R., & Zevenbergen, K. (2009). The Numeracies of Boatbuilding: New Numeracies Shaped by Workplace Technologies. *International Journal of Science and Mathematics Education,* 7(1), 183-206.
[74] Roth, W-M. (2012). Rules of bending, bending the rules: the geometry of electrical conduit bending in college and workplace. *Educational Studies in Mathematics,* Online First. doi: 10.1007/s10649-011-9376-4
[75] Hastwell, K., Strauss, P., & Kell, C. (2013). "But pasta is pasta, it is all the same": the language, literacy and numeracy challenges of supermarket work. *Journal of education and work,* 26(1), 77-98.
[76] Mercier, A. (2002). Identification of some mathematical needs linked to the use of mathematics at work: An educational survey. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 199-208). Dordrecht: Kluwer.
[77] Ryan, C., & Sinning, M. (2009). Literacy and Numeracy Skills and Their Use by the Australian Workforce. Adelaide, SA: National Centre for Vocational Education Research.
[78] Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council.
[79] Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society, p.18.
[80] Marr, B., & Hagston, J. (2007). Thinking beyond numbers: Learning numeracy for the future workplace. Adelaide, SA: National Centre for Vocational Education Research.
[81] Gainsburg, J. (2005). School mathematics in work and life: what we know and how we can learn more. *Technology in Society,* 27(1), 1-22.
[82] Hall, R., & Stevens, R. (1995). Making space: a comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The Cultures of Computing* (pp. 118-145). Oxford: Blackwell Publishers.
[83] Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council, (p.31, our emphasis).
[84] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge.
[85] Op cit.
[86] See for instance the chart comparing school and workplace mathematics in: Wedege, T. (2002). Numeracy as a Basic Qualification in Semi-Skilled Jobs. *For the Learning of Mathematics,* 22(3), 23-28.
[87] Gill, T. (2010). *Uptake of GCE A-Levels 2011. Statistics Report Series No. 42.* Cambridge: Cambridge Assessment.
[88] Wolf, A. (2011). Review of Vocational Education: The Wolf Report. London: DfE.
[89] Kyffin, H., Hodgen, J., & Tomei, A. (2013). The Supply of Mathematics teachers in the Further Education Sector.
[90] See for example: British Academy. (2012). Society Counts: Quantitative Skills in the Social Sciences (A Position Paper). London: British Academy; Madison, B., & Steen, L. (Eds.). (2008). Calculation vs. Context: Quantitative Literacy and Its Implications for Teacher Education. Washington, DC: The Mathematical Association of America; National Workforce Literacy Project. (2010). National Workforce Literacy Project: report on employers views on workplace literacy and numeracy skills. North Sydney NSW: Australian Industry Group; OECD. (2012). Literacy, Numeracy and Problem Solving in Technology-Rich Environments: Framework for the OECD Survey of Adult Skills. Paris: OECD Publishing; Rosen, L., Weil, L., & von Zastrow, C. (2003). Quantitative literacy in the workplace: Making it a reality. *Paper presented at Quantitative Literacy: Why Numeracy Matters for Schools and Colleges - National Forum on Quantitative Literacy, National Academy of Sciences*, Washington DC, December 1-2, 2001; Ryan, C., &

Sinning, M. (2009). Literacy and Numeracy Skills and Their Use by the Australian Workforce. Adelaide, SA: National Centre for Vocational Education Research; and Steenken, W. (2007). Quantitative Literacy-Beyond Crossroads Gets It Right. *AMATYC Review*, 28(2), 23-25.

[91] See for example: Advisory Committee on Mathematics Education. (2011). Mathematical needs: Mathematics in the workplace and in Higher Education. London: The Royal Society; CBI. (2012). Learning to grow: what employers need from education and skills - Education and skills survey 2012. London: Pearson / CBI; DfES. (2003). The Skills for Life survey: a national needs and impact survey of literacy, numeracy and ICT skills. Norwich: Her Majesty's Stationery Office; Harraway, J., & Barker, R. (2005). Statistics in the workplace: A survey of use by recent graduates with higher degrees. *Statistics Education Research Journal,* 4(2), 43-58; Hillage, J., Regan, J., Dickson, J., & McLoughlin, K. (2002). *Employers Skill Survey 2002.* London: DfES Publications; Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council; Koenig, J. (2011). A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education. Leeds: UK Centre for Bioscience, Higher Education Academy.

[92] See for example: Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. Abingdon, Oxon: Routledge; Hudson, C. (2007). Working mathematics for the 21st century? A discussion paper on workplace numeracy and mathematics. England: National Research and Development Centre for Adult Literacy and Numeracy; Kent, P., Bakker, A., Hoyles, C., & Noss, R. (2011). Measurement in the workplace: the case of process improvement in manufacturing industry. ZDM, 43(5), 747-758; Roth, W. (2003). Competent workplace mathematics: How signs become transparent in use. International Journal of Computers for Mathematical Learning, 8(2), 161-189; Triantafillou, C., & Potari, D. (2010). Mathematical practices in a technological workplace: the role of tools. *Educational Studies in Mathematics,* 74(3), 275-294; and van der Kooij, H. (2001). Mathematics and key skills for the workplace. In G. E. FitzSimons, J. O'Donoghue & D. Coben (Eds.), *Adult and Lifelong Education in Mathematics: Papers from Working Group for Action (WGA) 6 : 9th International Congress on Mathematics Education, ICME 9* (pp. 231-242): Adults Learning Mathematics.

[93] See for example: Bessot, A., & Ridgway, J. (Eds.). (2000). *Education for mathematics in the workplace.* Dordrecht: Kluwer; Moschkovich, J. (2002). Chapter 6: Bringing Together Workplace and Academic Mathematical Practices during Classroom Assessments. *Journal for Research in Mathematics Education. Monograph 11*, 93-110; Office for Standards in Education. (2005). *Ofsted research into the relevance of education for the workplace.* London: OFSTED; Wake, G., & Williams, J. (undated). Using Workplace Practice to Inform Curriculum Change. Paper presented at the ICTMA; Wedege, T. (2002). Numeracy as a Basic Qualification in Semi-Skilled Jobs. *For the Learning of Mathematics,* 22(3), 23-28; Williams, J., & Wake, G. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics,* 64(3), 345-372; Williams, J., Wake, G., & Boreham, N. (2001). School or college mathematics and workplace practice: An activity theory perspective. *Research in Mathematics Education,* 3(1), 69-83.

[94] Wake, G., & Williams, J. (2003). Using Workplace Practice to Inform Curriculum Design. In S. Lamon, W. Parker & S. Houston (Eds.), *Mathematical modelling a way of life* (pp. 189-200). Chichester, UK: Horwood Publishing, (p.189).

[95] Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge, (p.168).

[96] Hodgen, J., Küchemann, D., Brown, M., & Coe, R. (2010). Multiplicative reasoning, ratio and decimals: A 30 year comparison of lower secondary students' understandings. In M. F. Pinto & T. F. Kawaski (Eds.), *Proceedings of the 34th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 3, pp. 89-96). Belo Horizonte, Brazil.

[97] See for example: Bakker, A., Wijers, M., Jonker, V., & Akkerman, S. (2011). The use, nature and purposes of measurement in intermediate-level occupations. *ZDM,* 43(5), 737-746; Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council; and Wedege, T. (2002). Numeracy as a Basic Qualification in Semi-Skilled Jobs. *For the Learning of Mathematics,* 22(3), 23-28.

[98] Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council.

[99] Bessot, A. (2000). Visibility of mathematical objects present in professional practice. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 225-238). Dordrecht: Kluwer.

[100] For example, one study found sampling, graphics and probability to be the most common uses of statistics, see: Parker, R., Kent, J., & Brown, T. (2001). The perceived importance of statistics in the logistics and transportation industry. *The Journal of Education for Business,* 76(4), 185-188.

[101] Koenig, J. (2011). A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education. Leeds: UK Centre for Bioscience, Higher Education Academy.

[102] See for example: Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies.* Abingdon, Oxon: Routledge; Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council; and Kent, P., Bakker, A., Hoyles, C., & Noss, R. (2005). Statistical reasoning in the workplace: techno-mathematical literacies and learning opportunities. *Proceedings of the British Society for Research into Learning Mathematics,* 25(2), 37-42. For a discussion of the important role of statistics in addressing many problems in business and industry see also: Hahn, G., & Doganaksoy, N. (2011). *The role of statistics in business and industry.* Hoboken, NJ: Wiley.

[103] Pratt, D., Ainley, J., Kent, P., Levinson, R., Yogui, C., & Kapadia, R. (2011). Role of Context in Risk-Based Reasoning. *Mathematical Thinking and Learning,* 13(4), 322-345.

[104] Wake, G. (In Press). Making sense of and with mathematics: the interface between academic mathematics and mathematics in practice. Educational Studies in Mathematics; Wake, G., & Williams, J. (2003). Using Workplace Practice to Inform Curriculum Design. In S. Lamon, W. Parker & S. Houston (Eds.), *Mathematical modelling a way of life* (pp. 189-200). Chichester, UK: Horwood Publishing.

[105] Joint Mathematics Council. (2011). Digital technologies and mathematics education: A report from a working group of the Joint Mathematical Council of the United Kingdom.

[106] See for example: Hoyles, C., Wolf, A., Molyneux-Hodgson, S., & Kent, P. (2002). Mathematical skills in the workplace: final report to the Science Technology and Mathematics Council; Jablonka, E. (1997). What Makes a Model Effective and Useful (or not)? In S. Houston, W. Blum, I. Huntley & N. Neill (Eds.), *Teaching and Learning Mathematical Modelling: Innovation, Investigation and Applications* (pp. 39-50). Chichester, UK: Albion Publishing; Wake, G., & Williams, J. (2003). Using Workplace Practice to Inform Curriculum Design. In S. Lamon, W. Parker & S. Houston (Eds.), *Mathematical modelling a way of life* (pp. 189-200). Chichester, UK: Horwood Publishing.

[107] Hodgen, J., Marks, R., & Pepper, D. (2013). Towards universal participation in post-16 mathematics: lessons from high performing countries. London: The Nuffield Foundation.

[108] For example: Dawes, M. (2007). *Workplace mathematics: a study of mathematics in use in the UK assurance division of an international accounting firm.* PhD, King's College London; Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for*

techno-mathematical literacies. Abingdon, Oxon: Routledge; and Wake, G., & Williams, J. (2001). Using College mathematics in understanding workplace practice: Summative Report of Research Project Funded by the Leverhulme Trust. Manchester: University of Manchester.

[109] The initial full-text search produced a very large number of results: 127946 results from the databases and a further 8997900 results from Google Scholar. 3500 of these were considered for inclusion in the review.

[110] For example, Wedege (2002, op cit.) explores the mathematics used in both the construction and transportation industries.