The National Strategies
Secondary

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Renewing the Framework for secondary mathematics

Spring 2008 subject leader development meeting: Sessions 2, 3 and 4

department for
children, schools and families

National Curriculum

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DSI COLOURWORKS 02-2008

A timeline for change

Follow the progress of each cohort by means of a diagonal line starting from the year group as they begin the academic year 2008–09. For example, the line for the cohort comprising Year 7 in 2008–09 (top row) runs diagonally down, from left to right, to end as Year 11 in 2013 (bottom row). The year groups most affected by the new programmes of study are shown in dark blue. Other, lighter shades could be used in evolving plans as the department works through a phased implementation.

	2007/08	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14
Year 7	old KS3	**new KS3**					
Year 8	old KS3	old or new KS3	**new KS3**				
Year 9	old KS3	old or new KS3	old or new KS3	**new KS3**			
Year 10	2 tier UAM in exam	2 tier UAM in exam	2 tier UAM in exam	**Functional skills GCSE 1 GCSE 2**	**Functional skills GCSE 1 GCSE 2**		
Year 11	2 tier with cwk	2 tier UAM in exam	2 tier UAM in exam	2 tier UAM in exam	**Functional skills GCSE 1 GCSE 2**	**Functional skills GCSE 1 GCSE 2**	

NB: The relationship of functional skills to GCSE will be informed by the pilot; however, to achieve grade C or above pupils will require functional skills level 2.

Mathematical processes and applications

In the new programmes of study at Key Stages 3 and 4 there is much greater emphasis on the key processes and attainment target 1 is different, now entitled 'Mathematical processes and applications'. This reflects the importance given to key processes in the 2008 curriculum. It also parallels the emphasis on key processes in other subjects and on aspects of process that reach across the curriculum, enshrined in the personal, learning and thinking skills (PLTS). A key aim of the curriculum is that pupils should see themselves more explicitly as learners and become aware of their developing skills, central to their work in school and to all aspects of their lives.

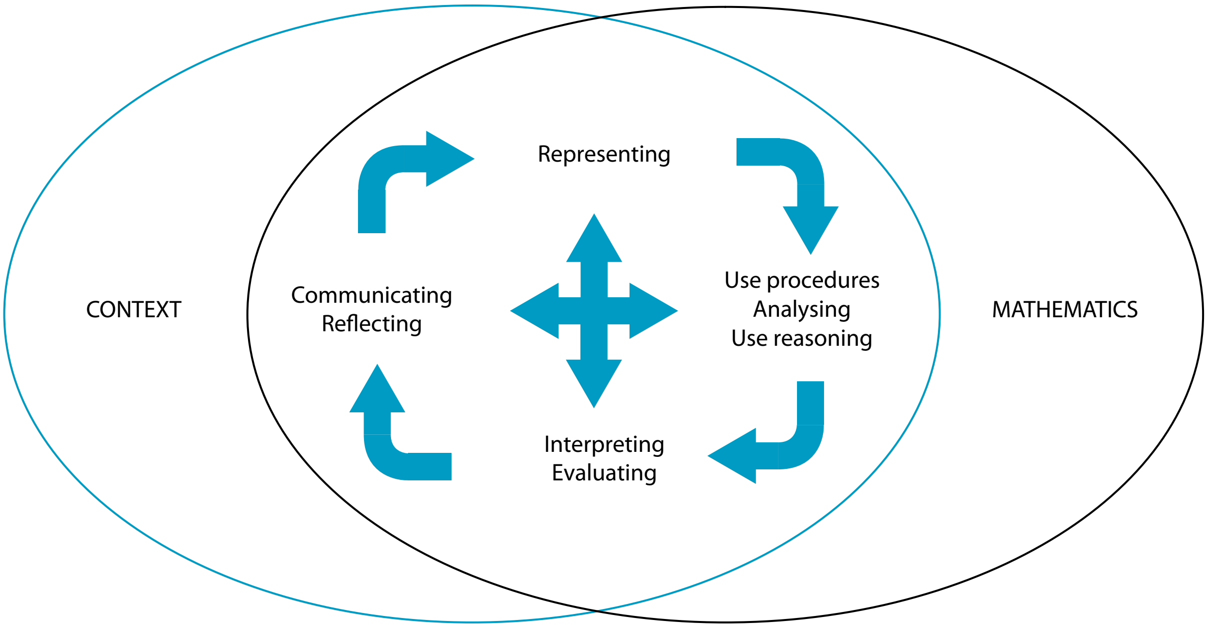
'Using and applying mathematics' was previously broadly described under the sub-headings of 'problem-solving', 'communicating' and 'reasoning'. Problem-solving lies at the heart of mathematics and involves a cycle of processes. These are elaborated in the key processes of the curriculum. By the inclusion of mathematical procedures, well-defined routines and algorithms, a more complete description of process is achieved:

- Representing
- Analysing
  - Use mathematical reasoning
  - Use appropriate mathematical procedures
- Interpreting and evaluating
- Communicating and reflecting

The process skills help pupils both to learn mathematics and to apply their mathematical subject knowledge to deal with situations from life and the world of work. To ensure that they make progress in developing these skills and can function mathematically, pupils need to experience a rich 'diet' of applications that includes:

- increasingly **complex** applications, including non-routine or multi-step problems and extended enquiries, that require them to analyse a situation and sustain their thinking
- situations that are **unfamiliar** (in the sense that they are different from the context where the mathematics was developed), including applications to other subjects or aspects of their lives, that requires them to make connections and transfer their skills, sometimes in creative ways
- situations or problems that increase the **technical demand** of the mathematics required to solve them, including the application of more advanced concepts, more difficult procedures, or more rigorous argument and proof
- opportunities to develop greater **independence** and autonomy in problem-solving skills, so that they can select and apply a higher level of mathematics for themselves.

In summary, it is the context, and the mathematics to be applied to it, that determines the nature of the processing skills that pupils need and the level of challenge they face. It is helpful to think of a 'problem-solving cycle' but, as the diagram overleaf shows, many contexts require movement in and out of the cycle. For example, the 'representing' phase of a more complex problem may require some 'analysing', 'interpreting' or 'communicating' in order to set up the model.



Mathematical processes should be embedded within the everyday teaching of the strands of number, algebra, geometry and measures, and statistics and in all cross-strand work. The related documents listed below give illustrative examples for each of the strands, including some observations of how ICT can be used to engage pupils with the key processes.

- *Key processes in number*
- *Key processes in algebra*
- *Key processes in geometry and measures*
- *Key processes in statistics*
- *ICT and the key processes*

Launching the new programme of study in mathematics

Introduction

These notes are designed to help you launch the new mathematics programme of study at Key Stage 3 with your department and to set out initial thoughts for a longer-term plan. The new secondary curriculum (phased in from September 2008) is based on some over arching principles, including:

Greater flexibility and coherence

The curriculum is focused on the key concepts and processes that underlie each subject, with less detailed prescription of content. This makes it easier to see links between subjects and increase the coherence for pupils across the curriculum and school activities.

New focus on aims and skills

The curriculum includes a framework for personal, learning and thinking skills (and functional skills for English, mathematics and ICT), embedded in the programmes of study. The key processes highlight the essential skills that learners need in order to make progress and achieve in each subject.

Emphasis on assessment for learning

Greater flexibility in the curriculum will give teachers more time to focus on assessment for learning strategies and to provide more targeted assessments to meet individual learners' needs.

In order to appreciate the scope for flexibility, it is essential first to recognise the impact that the key concepts and processes can have on pupils' learning in mathematics. It is also important to appreciate how engaging pupils explicitly in the key processes will strengthen their skills in solving problems and applying their mathematical knowledge, much more effectively than over-emphasis on coverage of content.

So the first requirement is for your department to become familiar with the key processes in mathematics and how they expand on 'using and applying mathematics'. The first five tasks **NC1–5** will help you to launch the new curriculum with your department. They are best completed in short sequence, preferably in a departmental half-day or several shorter meetings not too far apart.

The tasks have specific, practical outcomes and should leave you in a good position to assess your current practice, particularly how you address the key processes in your teaching. You can then plan for evolutionary change in your Key Stage 3 scheme of work, over the period 2008–2011. Do not be tempted to go for a superficial rewrite of your existing scheme of work. More effectively, establish a collaborative approach to planning with a measured and sustained programme of revising and updating units of work.

To support your phased implementation of the new programmes of study further, a Renewed *Framework for secondary mathematics* will be available from summer 2008. The summer term subject leader development meetings will support the use of the renewed Framework by introducing a

Secondary mathematics planning toolkit, modelled on the *Mathematics planning toolkit: Key Stage 4*, which has been in use in many departments since summer 2007.

The documents needed for the launch tasks are included in a folder **Launching the new KS3 curriculum (2008)**, on the CD-ROM. It includes a briefing note for an administrative assistant or technician to help with preparation of resources. One of the launch tasks involves reviewing and revising an early algebra unit and it would be helpful to bear this in mind in your preparatory work.

Suggested pre-reading for you and preferably all members of the mathematics department is:

- ***NC programme of study for Key Stage 3 (2008)***
- Extracts from guidance in the renewed *Framework for secondary mathematics* (to be available from summer 2008):
  - ***Mathematical processes and applications***
  - ***Key processes in algebra***
  - ***Teaching and learning approaches***

You will need copies of the above for each colleague at your launch meeting.

Launching the new programme of study

First, complete a classifying task designed to begin to familiarise everyone with the key processes in mathematics. You will need to print copies of the document ***Key processes classifying task***. Cut up sheets of the key process headings and statements so that each statement is on a separate slip of paper, one set per two teachers. You will also need, for each colleague:

- ***Mathematical processes and applications***

Note: This and the following four tasks are suitable for a half-day departmental meeting, or an equivalent sequence of shorter meetings.

Task NC1 Introducing the key mathematical processes

Use your pre-reading, the notes above and your knowledge of your department, to explain to colleagues:

- the aims of the new national Curriculum programmes of study;
- how you are proposing to respond as a first step.

Introduce the task, a card sort designed to familiarise everyone with the key processes in mathematics. Working in pairs, lay the five key process headings on the table:

Representing

Communicating and reflecting

Analysing – using mathematical reasoning

Analysing – using appropriate procedures

Interpreting and evaluating

Ask pairs to discuss each of the process statements in turn and assign them to one of the process headings. Expect a healthy debate, there is no right answer! It is a first step in becoming familiar with the new curriculum.

Allow time for classifying, then pose a couple of questions to the group:

- Which cards were difficult to place? Why?
- Can you find a set of three linked cards and explain the link?

Point out that the National Curriculum programme of study offers a categorisation, as a helpful prompt to thinking. However, there will always be room for debate about any description of processes.

The key processes are important when considering how pupils should engage with mathematics. You will consider this next, in the context of a particular example. Round off by asking colleagues to read (or re-read) the document ***Mathematical processes and applications***, drawn from the guidance in the renewed *Framework for secondary mathematics* (available from summer 2008).

To get to grips properly with the key processes it helps to reflect on a mathematical task that is sufficiently rich and open. Exploring patterns and relationships on a hundred square, familiar to many, is an accessible context for algebraic generalisation and problems can be posed in many ways. You might find the ***'Hundred square' prompt sheet*** useful when setting the task to colleagues. For personal preparation in leading the task you might also find it helpful to read ***Case study 'matchstick shapes'*** before the meeting. This gives an example of how a group of teachers built up a simple process map for themselves. For the meeting you will need the following as paper copies

For each colleague:

- ***Key processes in algebra, highlighter pens***

For pairs of colleagues:

- ***100 number grid*** (for 'hundred square' task),
- ***Key processes adaptable template***

For the whole group:

- ***Key processes blank template*** – a large hand-drawn version or the adaptable software version of the map to use on an interactive white board or projector.

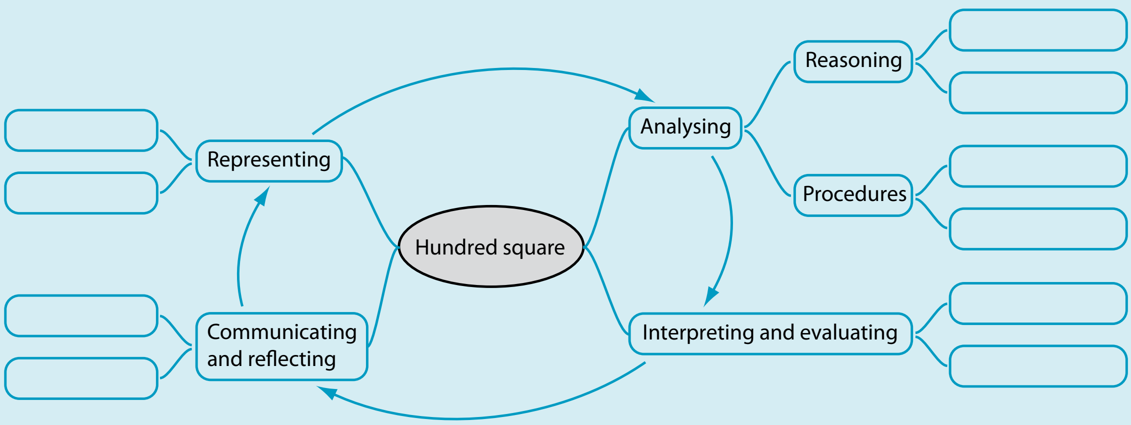
Task NC2 Exploring the key processes

Provide pairs with a hundred square and give them 10 minutes to explore the problem posed. The context may be familiar, but it is helpful to explore the mathematics in order to identify the potential for engaging pupils with the key processes. Provide direction, if needed.

Following their previous reading, remind everyone that the key processes describe how pupils should engage with their learning, at all levels of mathematics. Allow time for individuals to read the document ***Key processes in algebra***, having in mind particularly the task they have just been doing. It may be helpful to highlight particular sentences or sections which relate closely to the way pupils could be learning through this task.

Next draw everyone's attention to the map ***Key processes adaptable template*** and allow a couple of minutes for pairs to peruse it. Perhaps refer back to the classification task and note differences and similarities. Emphasise that there is no perfect map or classification but that you and your colleagues need to have a shared vocabulary if you are to discuss the processes productively.

Explain that the adaptable map provides a graphic way of detailing general aspects of the processes. Mapping can provide a mechanism for you to get to know the processes by constructing your own version, based on a specific context. It is possible to start from the adaptable template however, your map will be considerably smaller and for this reason it is probably easier to begin from the blank template and create a few simple branches. Use a flip chart, board or interactive white board, putting the title 'Hundred square' in the centre of the map.



- Working together as a department, add some key processes that could be developed through this task, drawing on ideas from the classifying task, from exploring the mathematics and from reading.
- Begin to discuss and note the range of opportunities that could emerge for introducing algebra through using this as an extended task across a number of lessons in Year 7.

The next task links very closely to **NC2** and is best completed at the same time or very soon afterwards. You are beginning to consider the potential of this task as part of an early algebra unit in Year 7. Working with your department, the aim is to design one or two objectives relating to mathematical processes and application. The renewed *Framework for secondary mathematics* includes objectives adapted from the previous 'using and applying' objectives better to reflect the focus of the new programmes of study. Keeping the focus of the discussion on one specific example (the hundred square), the aim of task **NC3** is to provide a 'light touch' introduction to ways of working with these objectives. Process is crucially affected by the context and task so you will need to tailor the objectives with the learning in mind. A possible objective could be:

Pupils should learn to:

represent 'hundred square' patterns, using symbols and expressions, and work logically to produce and explain generalisations; compare different approaches and recognise where they are equivalent.

Task **NC3** sets out the stages of thinking to help your colleagues design a similar objective based on your earlier work with the key processes. Don't be tempted to short-cut this thinking or align too closely to the example above.

You will need, for pairs of colleagues:

- ***Key processes map 'hundred square' (your agreed version from NC2)***
- ***Year 7 mathematical processes and applications objectives***
- ***Year 7 algebra objectives***

Task NC3 Tailoring the mathematical process and application objectives

Remind colleagues about the last task. In particular mention the map of the cycle of key processes which you produced after exploring the algebra of the hundred square and the interrelated elements of the map which you noted as possible foci for an algebra unit plan in Year 7.

Introduce the table of 'mathematical process and application' (MPA) objectives and explain that these have been adapted from the 'using and applying' objectives to reflect the focus of the new programmes of study better. Say that you will work in more detail on these objectives at a later stage, for now you are simply 'getting to know them'.

- Ask pairs of colleagues to highlight phrases in the ***Year 7 MPA objectives*** which seem appropriate to the hundred square task. They may find that working from your **Key process map** (from NC2) helps them to do this.
- Discuss the highlighted phrases, reach some consensus and design one or two composite objectives that are simple enough to describe the learning opportunity presented by the hundred square problem.

It is important to see the MPA objectives as part of a collection of objectives in a unit. To complete this picture you need to consider which 'range and content' objectives you would select if you were to include the 'hundred square' as a major part of an algebra unit in Year 7.

- Working as a group, consider the ***Year 7 algebra objectives*** and select a small collection which would be suitable, alongside the MPA objective, for a Year 7 algebra unit plan.

Summarise tasks NC2 and NC3 and emphasise that an extended and rich task of this kind can help to ensure that pupils learn through the key processes and understand the range and content in a more connected way.

The fourth task prepares you for a 'stock-take' on your approach to algebra in Year 7. For departments whose approach is well-aligned with the new curriculum, the main outcomes will be to introduce the language of key processes and to refine existing units. Departments that need to make a greater level of change might start by revising one or two units in Year 7, before setting out a manageable plan for the longer term. Think about how to draw in all members of the department. Enlarged or projected documents may help to do this. You will need:

- **copies of one or more algebra units from early in Year 7, including any resources and textbooks that you use.**
- **a unit planning template, either the one you use in school, or chosen from the three provided in the folder on the CD-ROM.**

Each colleague will also need a copy of:

- **Teaching and learning approaches**

Task NC4 Revising a Year 7 algebra unit

Explain that the purpose of this task is to review an early algebra unit in Year 7. The ***Teaching and learning approaches*** document synthesises and interprets the aims, key concepts, key processes and curriculum opportunities in the new curriculum. It can help you reach a consensus on priorities for planning and teaching.

Ask everyone to read just the six sub-sections of 'Some principles for effective learning' (first main section of the document only). They should have in mind the context of teaching algebra to pupils in the first term of Year 7 and highlight text that they think is important when reviewing early algebra units.

Discuss individual suggestions and agree two or three priorities.

Now introduce the main task, which is to review and revise a Year 7 algebra unit. In order to address priorities in the new curriculum, your plan is likely to include a rich task developed over several lessons to:

- engage pupils in particular aspects of the key processes
- develop other aspects of effective learning.

Your unit might include a new task (e.g. 'hundred square') or an adaptation of an existing task. You might choose to:

Either adapt an existing unit,

Or drop notes into a unit planning template, if you want to start afresh.

Spend most of your time exploring how the task should be developed and incorporated in the unit, including:

- how you might present the task to pupils who had not encountered algebraic representation or used algebraic procedures before
- ways in which pupils could develop or extend the problem and become more autonomous in using the key processes.

Finally, identify what needs to be completed beyond this meeting in order for colleagues to prepare for teaching the unit. This will include new objectives adapted from the mathematical process and application strand and renewed objectives drawn from the algebra strand.

The final task in this sequence looks ahead to when you teach the unit. It will be essential to evaluate the unit, how you have adapted your teaching and the impact on pupils' learning. It would be helpful if you could project the templates or work on enlarged paper versions of:

- ***Teaching and learning review template: lessons/unit***
- ***Teaching and learning review template: pupils' views***

Task NC5 Preparing to review teaching and learning

Explain that the final task is to give further consideration as to how you will teach the revised Year 7 unit and to identify points to note, to help you review the impact of the changes made.

From the ***Teaching and learning approaches*** guidance, ask colleagues to read 'Some principles for effective teaching'. Individually, highlight points in the text particularly relevant to teaching this unit (you are not trying to cover everything!). Then discuss and agree priorities for the department identifying important aspects to develop, without being over-ambitious at this stage.

Together, adapt the ***Teaching and learning review template: lessons/unit*** so that it is suitable as an observation or reflection sheet to help you review the unit later. On the basis of your agreed priorities, decide which sections of the template are relevant to copy and adapt as part of the agreed review prompt. Without being overambitious, your template should include:

- the particular key processes with which you expect pupils to engage
- other aspects of pupil learning you are seeking to develop
- the particular teaching principles you are seeking to improve.

Having designed this review template you may wish to select matching prompts for gathering pupils' views through small-group discussions based on the same priorities. To do this, use the adaptable template ***Teaching and learning review template: pupils' views***. Copy, paste and adapt the suggested questions for your chosen developments and agree when and how pupils' views will be gathered.

Discuss how the adapted teaching and learning review sheet and the results of pupil discussions can be used in preparation for a departmental review meeting:

- by all teachers as self-reflection on their lessons
- for any lesson observations that may be possible
- to inform discussions with small groups of pupils about their experiences in the lessons.

Agree when the unit will be taught and set a date for a review meeting.

Conclude by noting that starting on a small scale to establish the key processes in selected units of work will help all staff to move forward with the new curriculum and will inform the department's long-term development plan.

Drafting a plan for the longer term

It would be appropriate to allow time to reflect on the outcomes of your launch meeting before setting out a longer-term plan. The timing of this next task will depend on such factors as whole-school planning for the new curriculum and whether you feel ready to set out a plan, or whether you would prefer to allow time for some trialling of one or two units of work, say in Year 7, before thinking ahead to the longer term.

For this task you will need:

- ***A timeline for change***

Task NC6 Drafting a development plan

This task is for the subject leader working with a colleague, such as the second in department or teacher with responsibility for Key Stage 3.

Reflect on and discuss:

- the 'big picture' in the school and plans or points for consideration from the senior leadership team as they seek to implement the new curriculum;
- issues arising from the launch with your colleagues (and any subsequent classroom trialling), related to implementing the new mathematics curriculum, particularly the incorporation of key processes.

Your discussion should help you to address questions about priorities and phasing. Use the chart ***A timeline for change*** to consider which year groups are most affected by the changes to the curriculum. In the light of this consider whether you will:

- *work on the planning and teaching for a particular year group or the whole key stage;*
- *review the whole curriculum by working on critical units in all strands or review a larger number of units in a selected strand.*

When you have formulated your thoughts, it would be appropriate to discuss plans with a senior leader. This provides an opportunity to set out what you see as the challenges for your department and possible ways forward. Further work may be needed to arrive at an agreed plan that can lead to sustained phased development towards a scheme of work which fully reflects the new programmes of study.

Having launched the new programme of study with your department and considered some of the implications for your teaching, set out early thoughts on a development plan and discussed them with a school senior leader and the department, you should be ready to clarify your plans, identify priorities and start working on them.

In summer 2008 the renewed *Framework for secondary mathematics* will be available along with a *Secondary mathematics planning toolkit*, based on the structure of the *Key Stage 4 planning toolkit* which has been in use in many departments since summer 2007.

The toolkit will include a *Key Stage 3 planning handbook* which will offer guidance in the same format as this document to support the ongoing process of collaborative planning to implement the new curriculum at Key Stage 3. The handbook should enable you to step back and look more broadly at the issues of developing your scheme of work and help you to prioritise and firm up your plans for a phased implementation of the new programme of study. It will also help you to become familiar with the renewed *Framework for secondary mathematics*.

Hundred square grid

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Key processes in algebra

Algebra in Key Stages 3 and 4 is based on the generalisations of relationships familiar from basic number. It is developed to include the use of equations, formulae and identities, and sequences, functions and graphs. Algebra is purposeful when pupils encounter sufficiently complex situations where objects or relationships require representation in symbolic or graphical form. These occur frequently when describing generalisations underlying particular relationships. To use and make sense of algebra pupils need opportunities to relate it to their knowledge of the arithmetical operations. Suitable contexts for algebraic representation may come from within mathematics (for example, exploring number patterns and puzzles or finding areas of shapes), by linking with other subjects or from real-life applications. It should include use of ICT, such as graph-plotting and spreadsheet software to explore functions.

Representing

Representing a situation places it into a mathematical form that enables it to be worked on. In algebra this might mean trying out and choosing between different diagrammatic, graphical and symbolic forms arising from looking at the problem or situation from different points of view. Aspects of representing within algebra include:

- identifying assumptions, variables and relationships in order to create a mathematical model
- developing understanding of algebraic conventions, for example, conventions of writing terms and expressions, coordinate points and equations of lines, vectors and magnitude of vectors
- constructing algebraic expressions, equations, formulae and identities, for example, understanding and using signs such as $=$, $<$ and $>$ to represent relationships between variables
- choosing appropriate algebraic representation of such relationships, using knowledge of equivalence forms, for example, of tables, functions and graphs so that the context can be analysed and the solution communicated
- choosing the tools most appropriate to represent the mathematics drawn from the situation, for example, a graphical calculator or a spreadsheet.

As well as giving point to the subject, experience of algebraic representation is crucial if pupils are to understand and use precise algebraic language. Giving explicit attention to this helps them to understand the conventions for using letter symbols and constructing algebraic expressions. It can also give pupils insights into algebraic structure and order of operations, needed when transforming or interpreting symbolic and graphical representations.

Analysing – use mathematical reasoning

Algebra as a tool lies at the heart of much mathematical reasoning. Pupils need opportunities to experience the power of algebra in expressing generality. This includes:

- identifying and describing numerical patterns and relationships, both symbolically and graphically
- making connections with arithmetical operations and with equivalent algebraic forms when transforming expressions and equations
- making connections between sequences, functions and graphs and exploring the effects of varying values
- making generalisations, explaining and proving, relating results to the context of the problem.

Analysing – use appropriate mathematical procedures

Using appropriate procedures involves manipulating expressions, equations and graphs, using and applying techniques and accurate notation and monitoring the accuracy of methods and solutions. Appropriate procedures in algebra include:

- generating equivalent expressions and equations including a simplified form
- factorising and expanding expressions and equations
- solving equations exactly and approximately
- manipulating formulae, including changing the subject of the formula
- substituting values into equations and formulae, for example, evaluating a formula to convert temperature in degrees C to degrees F.

Algebra at Key Stages 3 and 4 is generalised arithmetic. It requires understanding of the commutative, associative and distributive laws as they apply to the number operations, and of relationships between operations, including inverses. Pupils can be supported to generalise the rules with letters in place of numbers, for example, $ab = c$ implies:

$ba = c$ $b = c/a$ $a = c/b$ $2ab = 2c$ $2ab + 1 = 2c + 1\ldots$

Taking an exploratory approach to transforming algebraic expressions and equations, where pupils are regularly asked to write expressions in different ways ('find as many ways as you can'), builds their algebraic skills. They:

- gain confidence in manipulating expressions into different equivalent forms
- gain insights into which of a range of possible transformations will be both valid and efficient as a next step, for example, in solving an equation or rearranging a formula
- develop increasing fluency with algebraic manipulation without being rule-bound and, when the steps in a procedure are not obvious, are able to resolve difficulties for themselves.

Interpreting and evaluating

Aspects of interpreting and evaluating in algebra include:

- relating numerical results, such as the solution of an equation, to the context under consideration
- interpreting general statements or conclusions expressed in algebraic form (e.g. an expression or formula) and considering their significance
- recognising the difference between numerical evidence and algebraic proof
- interpreting graphs and graphical features such as points of intersection, gradients and the general shape of a graph
- evaluating different approaches, for example, where someone else has represented the problem or approached its solution in a different way.

Communicating and reflecting

Aspects of communicating and reflecting in algebra include:

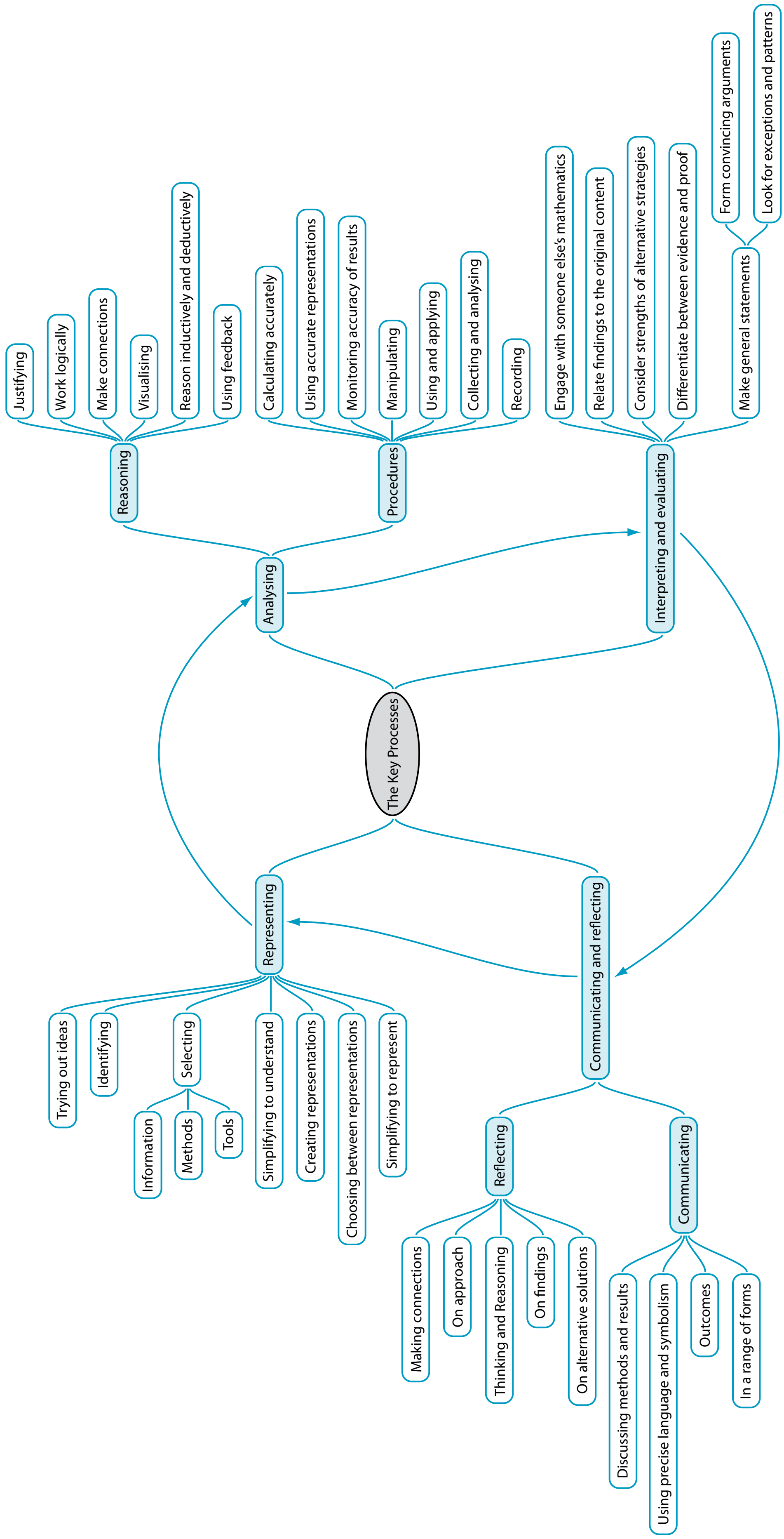
- recognising and using the fact that algebraic language (symbolic and graphical) is a powerful form of communication for expressing the steps in an argument or conclusions of an enquiry
- considering alternative approaches, for example, comparing algebraic, graphical and numerical approaches to tackling a problem
- making links to related problems or to different problems with a similar structure.

Resources for algebra

A range of resources to support the development of key processes in algebra are included in the 'Ideas for rich tasks' folder in the *Secondary mathematics planning toolkit*.

- *Interacting with mathematics in Key Stage 3 – algebra:*
  - *Constructing and solving linear equations* Year 7, Year 8 and Year 9 booklets
- *Teaching mental mathematics from level 5:*
  - *Algebra*
  - *Measures and mensuration in algebra*
- Standards Unit *Improving learning in mathematics:*
  - *Mostly algebra*, sessions A1 – A14

Key processes adaptable template



Case study: 'Matchstick shapes'

Mapping the key processes: building on 'using and applying mathematics' in the current scheme of work

The context

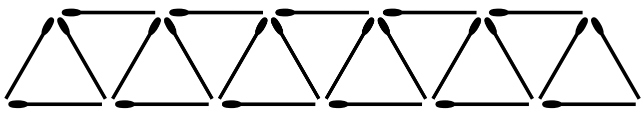
Department Y decided they would look at some of the rich tasks they have been using in Year 7 to cover 'using and applying' elements of the 2001 Framework to see how well they mapped to the new 'key processes'. They wanted to get a sense of what they needed to do in order to begin to meet the demands of the new curriculum. They decided that each member of the Key Stage 3 team would take turns to bring a problem they liked, and that they thought had potential, to their regular meetings and look at how it could be adapted to address the key processes.

The story

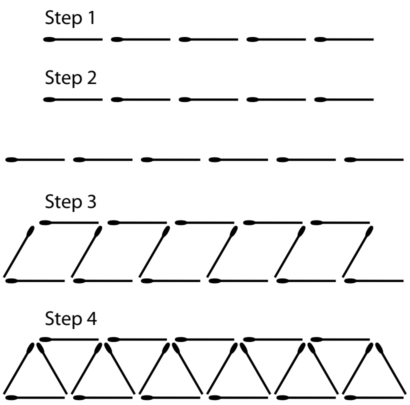
M, one of the most experienced members of the department, started by sharing a problem she used with her Year 7 in the summer term every year. It was 'Matchstick shapes' (p.32 of the *Supplement of Examples*).

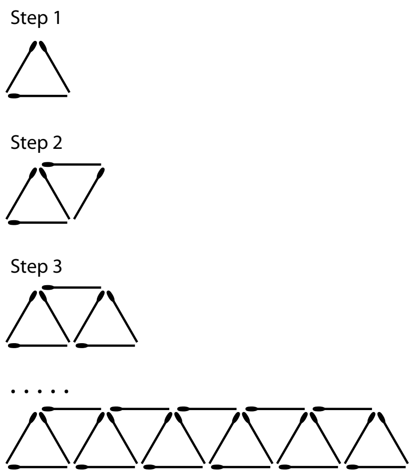
M described briefly how she currently used and introduced the task. She had an introduction based on the image from page 32 of the *Supplement of Examples*, where she discussed how the class might tackle the problem. Together (normally with some gentle nudging from her) they would decide to start with one triangle and then build up the triangles one at a time and draw up a table on the board. They could then identify a pattern and from that generate a general rule. She knew that more was possible and she felt that the problem had lots of potential. She had recently been to a local network group meeting with teachers from other schools and one of them had described how they introduced the task:

Pupils arrive in the room and a line of triangles is already on the board.



Pupils are asked to work in pairs – one of the pair draws the match design and the other watches how they do it. After a few minutes several pupils are asked to describe how their partner did it. This is modelled by the teacher who writes the associated arithmetic on the board (two examples below):





Despite the different methods they all end up with the same answer. How about if there were 20 triangles? 45 triangles? 100 triangles? *n* triangles?

The pupils work on answers to each of these in turn – using their own methods and sharing them. The general case yields different algebraic expressions all of which can be simplified to show their equivalence (see below). Pupils then look back to make sense of each other's expressions and see how they reflect the way they had drawn the image and that they are all equivalent.

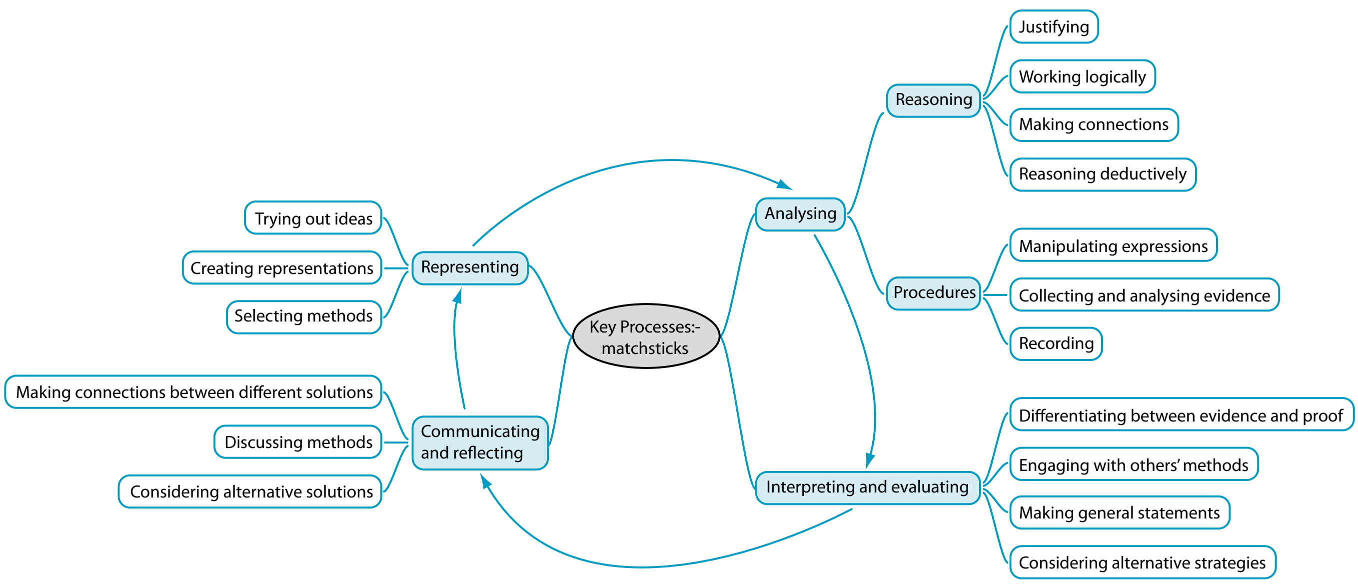
11 triangles (5+6)+(6+6) = 11 + 12 = 23	**11 triangles** 3+2+2+2+2+2+2+2+2+2+2= 3+10x2 = 23
20 triangles (10+10)+(11+10) = 20 + 21= 41	**20 triangles** 3+19x2 = 41
***n* triangles** $(n)+(n+1) = 2n+1$	***n* triangles** $3 + 2(n-1) = 2n+1$

M felt the power of this was in the lack of reliance on a table of results and a focus on the structure of the mathematics and how it really does reflect what it is representing.

There was lots of discussion about how the task could be presented and extended – either with more triangles or different situations (rows of squares) or other growing shapes.

The department sat down and produced the following mapping of the 'Matchstick shapes' task against the processes. It was pretty obvious it covered an enormous range of opportunities but they decided that when they next used the tasks they would focus on three inter-related elements:

- discussing methods
- considering alternative solutions
- engaging with others' methods.



Year 7, 8 and 9 MPA objectives

Mathematical processes and applications

Solve problems, explore and investigate in a range of contexts

Increase the **challenge** and build **progression** across the key stage, and for groups of pupils by:

- increasing the **complexity** of the application, for example, non-routine, multi-step problems, extended enquiries
- reducing the **familiarity** of the context, for example, new context in mathematics, context drawn from other subjects, other aspects of pupils' lives
- increasing the **technical demand** of the mathematics required, for example, more advanced concepts, more difficult procedures
- increasing the degree of **independence** and autonomy in problem solving and investigation

	Year 7	Year 8	Year 9
Representing	Identify the necessary information to understand or simplify a context or problem; represent problems, making correct use of symbols, words, diagrams, tables and graphs; use appropriate procedures and tools, including ICT	Identify the mathematical features of a context or problem; try out and compare mathematical representations; select appropriate procedures and tools, including ICT	Break down substantial tasks to make them more manageable; represent problems and synthesise information in algebraic, geometrical or graphical form; move from one form to another to gain a different perspective on the problem
Analysing – use mathematical reasoning	Classify and visualise properties and patterns; generalise in simple cases by working logically; draw simple conclusions and explain reasoning; understand the significance of a counter-example; take account of feedback and learn from mistakes	Visualise and manipulate dynamic images; conjecture and generalise; move between the general and the particular to test the logic of an argument; identify exceptional cases or counter-examples; make connections with related contexts	Use connections with related contexts to improve the analysis of a situation or problem; pose questions and make convincing arguments to justify generalisations or solutions; recognise the impact of constraints or assumptions
Analysing – use appropriate mathematical procedures	Within the appropriate range and content: make accurate mathematical diagrams, graphs and constructions on paper and on screen; calculate accurately, selecting mental methods or calculating devices as appropriate; manipulate numbers, algebraic expressions and equations, and apply routine algorithms; use accurate notation, including correct syntax when using ICT; record methods, solutions and conclusions; estimate, approximate and check working		
Interpreting and evaluating	Interpret information from a mathematical representation or context; relate findings to the original context; check the accuracy of the solution; explain and justify methods and conclusions; compare and evaluate approaches	Use logical argument to interpret the mathematics in a given context or to establish the truth of a statement; give accurate solutions appropriate to the context or problem; evaluate the efficiency of alternative strategies and approaches	Justify the mathematical features drawn from a context and the choice of approach; generate fuller solutions by presenting a concise, reasoned argument using symbols, diagrams, graphs and related explanations
Communicating and reflecting	Communicate own findings effectively, orally and in writing, and discuss and compare approaches and results with others; recognise equivalent approaches	Refine own findings and approaches on the basis of discussions with others; recognise efficiency in an approach; relate the current problem and structure to previous situations	Review and refine own findings and approaches on the basis of discussions with others; look for and reflect on other approaches and build on previous experience of similar situations and outcomes

Year 7, 8 and 9 algebra objectives

Algebra	Year 7	Year 8	Year 9
Equations, formulae, expressions and identities	Use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation* Understand that algebraic operations follow the rules of arithmetic Simplify linear algebraic expressions by collecting like terms; multiply a single term over a bracket (integer coefficients)	Recognise that letter symbols play different roles in equations, formulae and functions; know the meanings of the words *formula* and *function* Understand that algebraic operations, including the use of brackets, follow the rules of arithmetic; use index notation for small positive integer powers Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket	Distinguish the different roles played by letter symbols in equations, identities, formulae and functions Use index notation for integer powers and simple instances of the index laws Simplify or transform algebraic expressions by taking out single-term common factors; add simple algebraic fractions
	Construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations)	Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in same way) Use graphs and set up equations to solve simple problems involving direct proportion	Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution) Use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^2 + x = 20$ Use algebraic methods to solve problems involving direct proportion; relate algebraic solutions to graphs of the equations; use ICT as appropriate *Explore ways of constructing models of real-life situations by drawing graphs and constructing algebraic equations and inequalities*
	Use simple formulae from mathematics and other subjects; substitute positive integers into linear expressions and formulae and, in simple cases, derive a formula	Use formulae from mathematics and other subjects; substitute integers into simple formulae, including examples that lead to an equation to solve; substitute positive integers into expressions involving small powers, e.g. $3x^2 + 4$ or $2x^3$; derive simple formulae	Use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject

Teaching and learning approaches

Guidance on teaching and learning approaches is presented in three sections:

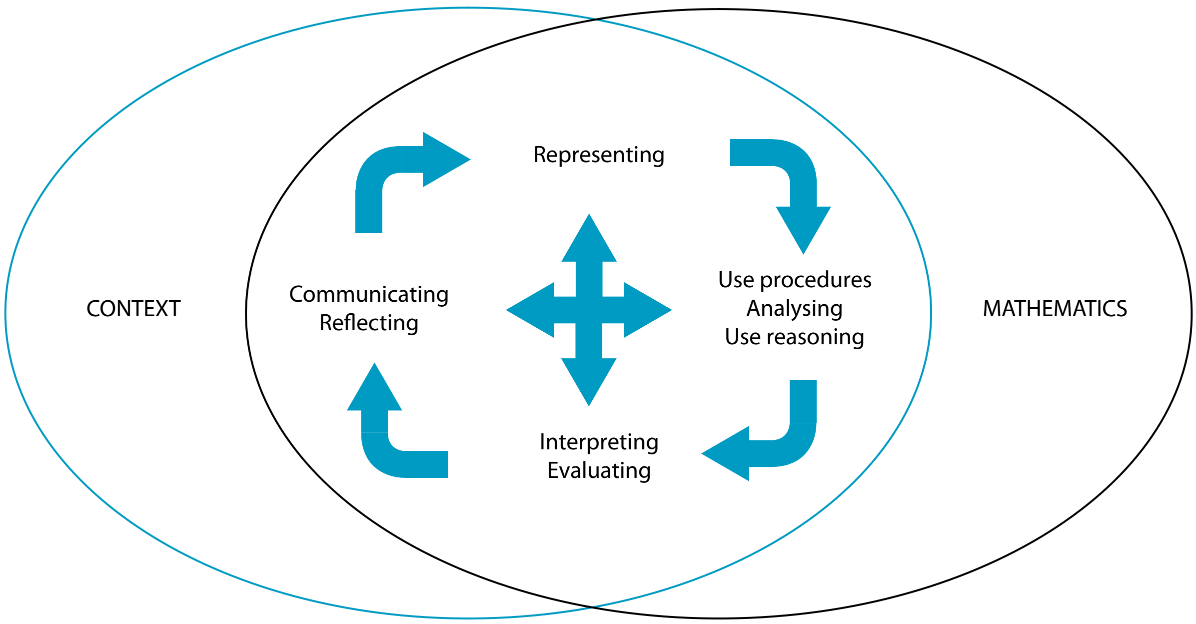
- 'Some principles for effective learning', based on the 2008 programme of study in mathematics
- 'Some principles for effective teaching', based on research over many years into the teaching of mathematics
- 'Further support to develop pedagogy and practice', which references existing Strategy guidance on lesson design, teaching repertoire, etc.

Some principles for effective learning

This section is informed by the curriculum aims of the 2008 programme of study. By synthesising and interpreting the aims, key processes, key concepts and curriculum opportunities the intention is to provide a supportive reference paper which the whole department can use to reflect on priorities for development in teaching and learning and so phase the implementation of the new programme of study.

Pupils learn about and learn through the key mathematical processes

Key processes need to be experienced as components of a whole cycle and this can be reflected within a single lesson as well as through a unit of work. Investigative and problem-solving opportunities should be designed so that pupils cycle through the processes several times and also move backwards and forwards between the stages as ideas mature, modify and change. In this way the notion of a cycle provides a helpful structure but does not become restrictive.



The diagram represents the dual nature of mathematics, both as a tool for solving problems in a wide range of contexts and as a discipline with a distinctive and rigorous structure. So pupils become successful learners by developing competence in applying mathematics effectively in a range of contexts, including those from within mathematics itself. There are two ways of thinking about pupils' experience of the key mathematical processes that lie at the heart of the revised programme of study.

They need opportunities to **learn about** the mathematical processes and to reflect on how they are improving in these skills. This could include designing lessons or units where there is no new content and the focus is on improving the process skills.	They need opportunities to **learn through** using the mathematical processes. As pupils gain confidence in the skills of applying these processes they can use them to develop their understanding of topics within the range and content of the curriculum.

Pupils work collaboratively and engage in mathematical talk

It is through paired and group work that pupils gain confidence in their ability to communicate mathematics effectively. Choosing a rich task will usually provide pupils with the chance to explain and justify, question and disagree. Over time the level of dialogue in the classroom becomes more mathematically rich as pupils pose questions to each other and develop more convincing arguments orally. As this kind of dialogue becomes a regular part of their work on mathematics pupils are forced to think in this way, preparing questions for one another and rehearsing arguments. We could describe this as developing a habit of 'self-talk'; that is they are naturally developing the thinking which will support more independent work.

Pupils work on sequences of tasks

Within the planning and teaching of units of work there need to be sequences of lessons which do not move too quickly from one topic to another or from one task to another. Instead, pupils need to be provided with a sequence of learning which is planned to become more challenging within a phase of a unit. One way of doing this is to select a task or sequence of related tasks which develop over a number of lessons. This has the advantage of reducing the burden of producing and introducing different tasks in each lesson. It means that more of a lesson is dedicated to pupils actively doing mathematics rather than listening to instructions for new and different topics and tasks. Most importantly a sequence of tasks, involving the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts makes mathematical progression more explicit to the pupils. In this way pupils develop the competence to apply suitable mathematics accurately within the classroom and beyond.

Pupils select the mathematics to use

Pupils can begin to see the power and purpose of their mathematical learning when they are given the opportunity to make decisions about the mathematical tools (including ICT) to help them to solve a problem or investigate from a given stimulus. As pupils use existing mathematical knowledge to investigate or create solutions to unfamiliar problems their confidence increases and they come to see that doing mathematics is an interesting and enjoyable activity. They are then more likely to apply mathematical skills in life effectively, in their wider studies and ultimately in employment. Unit plans adapted to meet the new curriculum should build in a variety of open and closed tasks, ensuring that the contexts for some task are real and others are abstract. In this way pupils come to appreciate mathematics for itself as well as understanding that it is used as a tool in a wide range of contexts.

Pupils tackle relevant contexts beyond the mathematics classroom

In order for pupils to be functional in mathematics and motivated to take their learning further they need to hone their knowledge, skills and understanding by applying suitable mathematics accurately within the classroom and beyond. This means planning units where pupils are not learning new content but are working on problems that arise in other subjects and in contexts beyond the school, such as architecture or engineering. In many cases a solution will involve using mathematics as a model to interpret or represent situations. Applications involving modelling changes in society and the environment or managing risk (for example, insurance and investments) could be used to stimulate discussion about important issues such as financial capability or environmental dilemmas. The assumptions and simplifications involved in the process of modelling a real context should be made explicit so that pupils come to realise that mathematics itself is essentially abstract and that a model or representation has limitations to its scope.

Pupils are exposed to the historical and cultural roots of mathematics

If they are given the chance to learn about problems from the past that led to the development of particular areas of mathematics, pupils can begin to appreciate that people of all cultures use mathematics to make sense of the world around them. They may be fascinated to find out that pure mathematical findings sometimes precede practical applications, and their curiosity may be aroused to think that mathematics continues to develop and evolve. This will engage and motivate pupils to become more aware of the nature of mathematics and of the mathematics around them.

Some principles for effective teaching

Research shows that the following principles underlie effective teaching. They are based on those included in *Improving learning in mathematics: challenges and strategies*, by Malcolm Swan University of Nottingham, in the 'Standards Unit box' (Improving learning in mathematics, The Standards Unit, DfES1599-2005DOC-EN). The list is provided to support evaluation of current teaching approaches and to stimulate departmental discussions about improving the effectiveness of current teaching.

Build on the knowledge pupils bring to a sequence of lessons

Design activities which uncover prior learning and offer pupils opportunities to express their understanding. For example:

- pose a problem to the whole class to stimulate paired discussion and to set the agenda for the next few lessons
- set up pairs or groups to draw and share a concept map or equivalent diagram showing the interconnections of existing understanding. Revisit the 'maps' and add to them as the learning emerges throughout the unit.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach (Rich tasks folder):

- *Leading in learning* (KS3 and KS4 training materials and exemplification in mathematics)
- *Bridging plans: from KS3 to KS4*
- Interacting with mathematics in Key Stage 3 – proportional reasoning
  - *Year 7 Fractions and ratio* minipack and resources, especially the key lesson

Expose and discuss common misconceptions

Pupils make mistakes for a variety of reasons. Some are due to lapses in concentration, hasty reasoning, memory overload or a failure to notice important features of a problem. Others, however, are symptoms of more profound mathematical difficulties. Where mistakes are the result of consistent, alternative interpretations of mathematical ideas we refer to them as misconceptions. These should not be dismissed as 'wrong thinking' as they may be necessary stages of conceptual development. Design activities so that misconceptions are systematically exposed by allowing time in the lessons for pupils to reflect and discuss these conflicts. For example:

- ask pupils to complete a task, using more than one method, and then to resolve conflicting answers
- present statements to be classified and justified as always true, sometimes true or never true.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach:

- *Misconceptions in mathematics* (Pedagogy folder, Improving subject knowledge sub-folder))
- *Teaching mental mathematics from level 5*
  - *Shape and space* (Rich tasks folder)

You might also find useful, as a separately available CD-ROM including video:

- *Mathematics: developing dialogue and reasoning* (DfES 00243-2006CDO-EN)

Develop effective questioning

Aim to invite a range of responses to your questions by asking more open and probing questions which promote higher-level reflective thinking. Allow time for pupils to think before offering help or moving on to ask someone else and allow time for yourself so that you think about your response. For example:

- establish a routine through which pupils share their answers in pairs before you take whole-class feedback. This 'pair/share' not only builds confidence, it also increases the number of pupils who feel that their response has been heard.
- be explicit about types of questions you use and encourage the pupils to use the same types of question. A display of question stems can be helpful for you and the pupils:

  What if…?

  Why do you think…?

  When would it not work…?

  How do you know…?

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach (Pedagogy folder):

- Standards Unit *Improving learning in mathematics* (Pedagogy folder)
- *Pedagogy and practice: Teaching and learning in secondary schools*
  - *Unit 7 questioning* (Pedagogy folder)

Also, as separately available CD-ROMs:

- *Assessing pupils' progress in mathematics at Key Stage 3*, probing questions (DfES 00007-2007CDO-EN)
- *Mathematics: developing dialogue and reasoning* (DfES 00243-2006CDO-EN)

Use cooperative small-group work

Ensure that everyone is confident and benefits from participating in discussions by designing tasks which require collaboration in pairs or small groups and establish this as a regular feature of mathematics lessons. For example:

- use classification activities with only one set of objects per pair or group so that joint decisions have to be made
- ask pupils to create a spider diagram of connections with one large sheet of paper for three pupils.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach:

- *Pedagogy and practice: Teaching and learning in secondary schools*
  - *Unit 10 group work* (Pedagogy folder)
- *Interacting with mathematics in Key Stage 3 – handling data*
  - *Y8 handling data minipack* (Rich tasks folder)

Emphasise methods rather than answers

Focus on pupils developing their repertoire of appropriate methods rather than on obtaining correct answers to a long list of similar problems. This is likely to involve aiming to work on fewer problems than would appear in a typical textbook exercise. For example:

- direct pupils to tackle the same problem, using more than one method, and work in pairs to compare solutions and evaluate their efficiency
- ask pupils to redesign a problem so that it is more challenging or simpler and give it to their group to solve.

The following Strategy resources in the *Secondary mathematics planning toolkit* may help with this approach.

- *Interacting with mathematics in Key Stage 3* (Rich tasks folder)
  - *Handling data* (Wise words and other tasks)
  - *Proportional reasoning* (Year 8 multiplicative relationships, Year 9 proportional reasoning, Enhancing PR in Year 8 and Year 9)

Use rich collaborative tasks

Think about how to design tasks that motivate a need to learn and encourage the pupils to be creative, curious and reflective. Pupils' mathematical thinking will be improved if they have to make decisions and ask questions. The learning is made memorable when pupils enjoy the tasks and are surprised by outcomes. Richer tasks allow all learners to find something challenging and at an appropriate level to work on. Examples of accessible and extendable tasks can be developed from 'routine' tasks by changing the initial stimulus and the questions asked. For example:

- help pupils to consolidate their understanding of algebraic factorisation, expansion and simplification by working with jigsaw or domino cards showing matching expressions. Extend this to include their own design of such cards

- ask two groups of pupils to debate opposing arguments which support or refute a hypothesis where data is supplied in a spreadsheet.

The following resources in the Secondary mathematics planning toolkit may help with this approach (Rich tasks folder):

- *Teaching mental mathematics from level 5*
- Standards Unit *Improving learning in mathematics*

Create connections between mathematical topics

Design activities for existing units which make explicit connections within and across mathematical topics. For example:

- matching tasks which require pupils to recognise different representations of the same mathematical idea.

You may also plan to include more cross-strand units to develop stronger connections. For example:

- a functional mathematics unit presenting a real context requiring exploration or investigation. In such units pupils could work on a range of mathematical connections needed to reach a resolution.

The following resources in the *Secondary mathematics planning toolkit* may help with this approach (Rich tasks folder):

- *Standards Unit Improving learning in mathematics*
- *Teaching mental mathematics from level 5*
  - *Measures and mensuration booklets*
- *Interacting with mathematics in KS3 – proportional reasoning*
  - *Year 9 proportional reasoning*

Use technology in appropriate ways

Present mathematical concepts in dynamic, visually exciting ways that engage and motivate learners. Introduce, explore and represent concepts, structures and processes in new and revealing ways. Often dynamic images will permit insights and understandings which are difficult to convey in other ways. For example:

- Display an equation of the form $y = mx + c$ on the same screen as the associated table and graph in order to explore the relationship between them
- Explore a dynamic diagram showing how the angle formed between two straight lines changes as the lines move. Extend to parallel lines and an intercept.

The following resources from the *Secondary mathematics planning toolkit* may help with this approach (Rich tasks folder):

- *Interacting with mathematics in Key Stage 3 – proportional reasoning*
  - *Year 7 fractions and ratio*, interactive teaching programmes
- *ICT in mathematics*, ICT lesson plans

Further support to develop pedagogy and practice

The 2001 *Framework for teaching mathematics: Years 7, 8 and 9* established several principles which teachers found useful in guiding their planning. They included dimensions of a teaching repertoire such as modelling, questioning and explaining and aspects of lesson design such as structuring learning and starters and plenaries.

As further support for these developments the Pedagogy and Practice materials were published a few years later.

Pedagogy and Practice: Teaching and learning in secondary schools is often referred to as 'the ped pack'. It is a suite of study guides created to support the professional development of teachers across all subjects in secondary schools. They provide guidance on the relationship between pedagogic approaches (teaching models), teaching strategies, techniques and methods of creating the conditions for learning in order to inform lesson design. The techniques suggested in each study guide are tried and tested and draw on both academic research and the experience of practising teachers.

Many teachers, who began to work on new teaching strategies from the initial, brief guidance in the Framework, moved on to more detailed developments through the 'ped pack' guidance.
For example, the structuring learning booklet elaborates the original Framework guidance on structured lessons. It describes dividing lessons into a series of episodes, choosing from a range of strategies and techniques to motivate pupils and examines three pedagogic approaches – direct interactive, inductive and exploratory – to show how they can help pupils develop tools for learning, such as inductive thinking or enquiry skills.

The full list of booklets is given below, but you are unlikely to require the entire set at any one time. Instead, think about what support you need and consider downloading one or two booklets; most are only 24 pages and can be accessed at www.standards.dcsf.gov.uk/secondary/keystage3/all/respub/sec ppt10.

Designing lessons

Unit 1 Structuring learning

Unit 2 Teaching models

Unit 3 Lesson design for lower attainers

Unit 4 Lesson design for inclusion

Unit 5 Starters and plenaries

Teaching repertoire

Unit 6 Modelling

Unit 7 Questioning

Unit 8 Explaining

Unit 9 Guided learning

Unit 10 Group work

Unit 11 Active engagement techniques

Creating effective learners

Unit 12 Assessment for learning

Unit 13 Developing reading

Unit 14 Developing writing

Unit 15 Using ICT to enhance learning

Unit 16 Developing effective learners

Creating conditions for learning

Unit 17 Improving the climate for learning

Unit 18 Learning styles

Unit 19 Classroom management

Teaching and learning review template: lessons/unit

This is an adaptable template. It should be used to support your implementation of the new programme of study in mathematics and help you work together as a department on your chosen priorities.

It is intended to be used to inform discussions in departmental meetings as you review teaching and learning approaches. This can be done by using the template as a notepad to capture:

- personal reflections on a lesson or sequence of lessons
- reflections made during or after a focused lesson observation by pairs of colleagues
- reflections made during or after a focused lesson observation by a subject or senior leader

The first step is to use the three 'copy and paste' sheets to drop (and adapt) priorities for review into the template. There are only two cells beside each priority to encourage you to focus closely. You may wish to adjust this number bearing in mind that reflecting on a smaller number of development points can have a more significant impact.

Teaching and learning review template: lessons/unit

In this lesson/unit we are looking for…	…development of these chosen priorities	Notes from personal reflections or observations including next steps
…pupils working on these aspects of key processes		
…these aspects of learning		
…these aspects of teaching		

Representing	Analysing – using mathematical reasoning	Analysing – using appropriate mathematical procedures	Interpreting and evaluating	Communicating and reflecting
Identifying mathematical aspects of a problem and trying out ideas	Making and using connections within mathematics and between problems	Using accurate graphs, charts, constructions and diagrams (including with ICT)	Engaging with someone else's mathematical reasoning or modelling	Discussing methods and results
Creating representations, including with ICT	Visualising and working with dynamic images	Using and applying procedures, using accurate notation (including with ICT)	Relating findings to the original context, identifying whether they support or refute conjectures	Communicating outcomes effectively, in a range of forms and for different audiences
Choosing between representations	Working logically, recognising impact of assumptions and constraints	Calculating accurately, selecting mental methods or calculating devices	Considering assumptions made and the appropriateness and accuracy of results	Reflecting on the elegance, efficiency and equivalence of alternative solutions
Simplifying a problem in order to understand it and to represent it mathematically	Justifying, explaining, convincing and proving	Recording methods, solutions and conclusions (including with ICT)	Making general statements and forming convincing arguments	Reflecting on the approach, thinking and findings
Selecting mathematical information	Reasoning inductively and deductively, considering covariance and invariance	Manipulating – using numbers, algebra, graphs and geometric images (including routine algorithms)	Looking for patterns and exceptions	Making connections between different outcomes and with problems having a similar structure
Selecting mathematical methods and tools	Identifying and classifying patterns, specialising and generalising	Monitoring accuracy of results by estimating, approximating and checking	Considering the strengths of alternative strategies	
	Making conjectures and using counter-examples	Collecting and analysing data, evidence and information (including with ICT)	Evaluating evidence (including taking account of bias), differentiating between evidence and proof	
	Using feedback from the mathematical context and from discussion			

Some aspects of effective learning	Summary explanation – for more detail see 'Teaching and learning approaches'
Pupils learning about the key processes	Learning **about** the mathematical processes and reflecting on how they are improving in these skills. Engaging in lessons or units with no new content and focusing on improving the process skills.
Pupils learning through the key processes	Learning **through** using the mathematical processes. Gaining confidence in the skills of applying these processes and using them to develop their understanding of topics within the range and content of the curriculum.
Pupils working collaboratively and engaging in mathematical talk	Working in pairs or groups on a rich task, explaining and justifying, questioning and disagreeing, developing the thinking which will support more independent work.
Pupils working on sequences of tasks	Working on a sequence of learning through an extended task or sequence of closely related tasks over a few lessons, either involving the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts.
Pupils selecting the mathematics to use	Making decisions about the mathematical tools (including ICT), using existing mathematical knowledge to investigate or create solutions to unfamiliar problems, enjoying mathematics from real and abstract contexts.
Pupils tackling relevant contexts from beyond the mathematics classroom	Applying suitable mathematics accurately within the classroom and beyond, working on problems that arise in other subjects and in contexts beyond the school, using mathematics as a model to interpret or represent situations.
Pupils engaging with the historical and cultural roots of mathematics	Finding out about the mathematics of other cultures, learning about problems from the past, finding out about the ways that mathematics continues to develop.

Some aspects of effective teaching	**Summary explanation** – for more detail see 'Teaching and learning approaches'
Building on the knowledge pupils bring to a sequence of lessons	Lessons and units including activities which uncover prior learning and offer pupils opportunities to express their understanding.
Exposing and discussing common misconceptions	Lessons or units including activities so that misconceptions are systematically exposed, allowing for time in the lessons for pupils to reflect and discuss these conflicts.
Developing effective questioning	Teachers inviting a range of responses to open and/or probing questions, promoting higher-level reflective thinking, allowing time for pupils to think before offering help or moving on and allowing time for themselves to think about their response.
Using cooperative small-group work	Lessons or units involving tasks which require collaboration in pairs or small groups, pupils showing confidence and benefiting from participating in discussions.
Emphasising methods rather than answers	Pupils working on their repertoire of appropriate methods rather than on obtaining correct answers to a long list of similar problems. This is likely to involve working in depth on fewer problems in each lesson, aiming to develop more powerful and generalisable methods.
Using rich collaborative tasks	Pupils working on enjoyable and extendable tasks which encourage mathematical talk and thinking. There is something appropriately challenging for all pupils, involving them in more complex tasks which motivate a need to learn.
Creating connections between mathematical topics	Pupils working on tasks which explicitly use knowledge and connect closely related concepts and notations. These ideas may be abstract and explored within mathematics or brought together in application, solving a problem in a real context.
Using technology in appropriate ways	Mathematical concepts presented in dynamic, visually exciting ways, introducing, exploring and representing concepts, structures and processes in new and revealing ways. Using dynamic images to gain insights and understandings.

Teaching and learning review template: pupils' views

This is an adaptable template. It should be used to support your implementation of the new programme of study and help you work together as a department on your chosen priorities.

It is intended to be used to prompt focused discussions with small groups of pupils, perhaps five or six. The discussions should be used as a major source of evidence that agreed developments in teaching and learning are having an impact on the pupils.

The first step is to complete the template by using the three 'copy and paste' sheets to drop (and adapt) questions which relate to the priorities for review. To keep the discussion accessible, general questions have been suggested for the 'key process' section. You may wish to elaborate on these questions with particular examples of aspects of key processes which have been a recent focus for development.

There are only two cells beside each priority to encourage you to focus closely. You may wish to adjust this number bearing in mind that reflecting on a smaller number of development points can have a more significant impact.

Prompt sheet for discussion with small groups of pupils

In this discussion we are trying to find out …	…about the development of these chosen priorities	Notes – responses
…pupils' awareness of the **key processes**	In your recent lessons on (insert topic and task) you were working on (insert specific key process) – can you describe this? Use examples of what you did in the lessons. Tell me about where you have worked on (insert specific key process) before? Did the lessons and tasks help you get better at (topic)? Did you get better at (insert specific key process)?	
…how often pupils experience these **aspects of learning**		
…and if it is helping them to make progress		
…if pupils are aware of an increased focus on these **aspects of teaching**		
…and if it is helping them to make progress		

Some aspects of effective learning	Summary explanation – for more detail see 'Teaching and learning approaches'	Adapt these questions/prompts to suit the stage of implementation and the groups you are convening
Pupils learning about the key processes	Learning **about** the mathematical processes and reflecting on how they are improving in these skills. Engaging in lessons or units with no new content and focusing on improving the process skills.	Tell me about how your recent mathematics lessons have helped you to get better at (insert specific key process). When and where do you think you will use this skill? In mathematics, in other subjects, in daily life?
Pupils learning through the key processes	Learning **through** using the mathematical processes. Gaining confidence in the skills of applying these processes and using them to develop their understanding of topics within the range and content of the curriculum.	We have recently been working on (insert specific key process), tell me about how this has helped you to make progress in (insert topic).
Pupils working collaboratively and engaging in mathematical talk	Working in pairs or groups on a rich task, explaining and justifying, questioning and disagreeing, developing the thinking which will support more independent work.	Tell me about some mathematical tasks which meant you needed to work together in pairs or groups. What was it about the task which made you work in a group or pair? When you work as part of a group or in a pair: • How do you work on the task together, e.g. do you each do different parts? • What types of decisions do you make? • Are you able to learn from others in the group. How? Give an example. Does the chance to talk and think together help you to learn? Give examples.
Pupils working on sequences of tasks	Working on a sequence of learning through an extended task or sequence of closely-related tasks over a few lessons, either involving the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts.	Tell me about a set of lessons which were linked in some way, e.g. they were on the same topic or you were working on a task which extended across a few lessons. • How did each lesson progress from the one before? • How did the sequence of lessons end? • Do you know when you might use what you have learned through the sequence of lessons?
Pupils selecting the mathematics to use	Making decisions about the mathematical tools (including ICT), using existing mathematical knowledge to investigate or create solutions to unfamiliar problems, enjoying mathematics from real and abstract contexts.	Tell me about a task where you had to make decisions about how to start it. Tell me about an occasion when you had to choose the methods or tools to use. Give an example of choosing to use ICT for a mathematical task. Tell me about a time when you were set a challenge that meant you had to pull together knowledge from different parts of mathematics.
Pupils tackling relevant contexts from beyond the mathematics classroom	Applying suitable mathematics accurately within the classroom and beyond, working on problems that arise in other subjects and in contexts beyond the school, using mathematics as a model to interpret or represent situations.	Tell me about tasks/activities that you have done recently where you used mathematics in other subjects or at home, e.g. in clubs, in your hobby, as part of a job. Have you used your understanding of mathematics to explain something to anyone recently? Give me an example of when you understood something outside your mathematics lesson because of your learning in mathematics.
Pupils engaging with the historical and cultural roots of mathematics	Finding out about the mathematics of other cultures, learning about problems from the past, finding out about the ways that mathematics continues to develop.	Do you know where any of the mathematics you have learnt came from or why it was developed? Can you name any famous mathematicians from the present or the past? Why are they famous? What do you know about mathematics from different cultures? – Who? Which? What? Do you think mathematics is still developing? How? Why?

Some aspects of effective teaching	Summary explanation – for more detail see 'Teaching and learning approaches'	Adapt these questions to suit the stage of implementation and the groups you are convening
Building on the knowledge pupils bring to a sequence of lessons	Lessons and units including activities which uncover prior learning and offer pupils opportunities to express their understanding.	Are your mathematics lessons linked to something you've done before? – Describe an example. When and how do you get the opportunity to show what you know about a topic? How could you show someone what you already know about the mathematics you are learning? Does this happen in your lessons?
Exposing and discussing common misconceptions	Lessons or units including activities so that misconceptions are systematically exposed, allowing time in the lessons for pupils to reflect and discuss these conflicts.	What do you do when you notice that you, or the person you are working with, has misunderstood the topic/problem or has the wrong idea? Do you get opportunities to explain why a piece of mathematics is wrong in lessons? Why is this important? Tell me about a time when you thought you understood something but then you worked on a task or had a discussion that exposed a gap or flaw in your understanding.
Developing effective questioning	Teachers inviting a range of responses to open and probing questions, promoting higher-level reflective thinking, allowing time for pupils to think before offering help or moving on and allowing time for themselves to think about their response.	Tell me about an interesting question you have been asked in a mathematics lesson. Tell me about the kinds of question that make you think a lot. Can you remember an example? When a question is difficult, what do you do? What happens in the class when someone is finding a question hard? How long do you get to think about your answers? Does this vary? Does just one person give an answer to a question or are a range of answers taken?
Using cooperative small-group work	Lessons or units involving tasks which require collaboration in pairs or small groups, pupils showing confidence and benefiting from participating in discussions.	Tell me about some mathematical tasks which meant you needed to work together in pairs or groups? What was it about the task which made you work in a group or pair? When you work as part of a group or in a pair: • How do you work on the task together, e.g. do you each do different parts? • What types of decisions do you make? • Are you able to learn from others in the group – how/give an example? Does the chance to talk and think together help you to learn? Give examples
Emphasising methods rather than answers	Pupils working on their repertoire of appropriate methods rather than on obtaining correct answers to a long list of similar problems. This is likely to involve working in depth on fewer problems in each lesson, aiming to develop more powerful and generalisable methods.	Do you get opportunities to hear how others in your class worked out a problem or reached a solution? How does this help you if you've already got the problem right? If you get an answer to a mathematics question wrong, do you get the chance to show your teacher or partner how you worked it out? What is more important, the method or the answer?

Some aspects of effective teaching	**Summary explanation** – for more detail see 'Teaching and learning approaches'	**Adapt these questions to suit the stage of implementation and the groups you are convening**
Using rich collaborative tasks	Pupils working on enjoyable and extendable tasks which encourage mathematical talk and thinking. There is something appropriately challenging for all pupils involving them in more complex tasks which motivate a need to learn.	Tell me about a task which you worked on that was really interesting. • Why did you enjoy working on it? • What was it about the task that was interesting? • Did this task make you want to learn? Do you find it helps to talk in a group when you are faced with a really hard mathematics problem? Does it help you learn more?
Creating connections between mathematical topics	Pupils working on tasks which explicitly use knowledge and connect closely-related concepts and notations. These ideas may be abstract and explored within mathematics or brought together in application, solving a problem in a real context.	Give me an example of how different topics in mathematics link together. Tell me about a problem or investigation where you had to connect your knowledge of different parts of mathematics in order to reach a solution.
Using technology in appropriate ways	Mathematical concepts presented in dynamic, visually exciting ways, introducing, exploring and representing concepts, structures and processes in new and revealing ways. Using dynamic images to gain insights and understandings.	Tell me about the last time you used ICT in a mathematics lesson. Was it your choice? Give an example where using ICT in mathematics really helped you understand the topic or reach a solution. Can you explain why?

Key processes classifying task

Representing	
Analysing – use mathematical reasoning	**Analysing – use appropriate mathematical procedures**
Interpreting and evaluating	**Communicating and reflecting**

Process statements

identify the mathematical aspects of a situation or problem	manipulate numbers, algebraic expressions and equations and apply routine algorithms
choose between representations	use accurate notation, including correct syntax when using ICT
simplify the situation or problem in order to represent it mathematically, using appropriate variables, symbols, diagrams and models	record methods, solutions and conclusions
select mathematical information, methods and tools to use	estimate, approximate and check working

make connections within mathematics	**form convincing arguments based on findings and make general statements**
use knowledge of related problems	**consider the assumptions made and the appropriateness and accuracy of results and conclusions**
visualise and work with dynamic images	**be aware of the strength of empirical evidence and appreciate the difference between evidence and proof**
identify and classify patterns	**look at data to find patterns and exceptions**
make and begin to justify conjectures and generalisations, considering special cases and counter-examples	**relate findings to the original context, identifying whether they support or refute conjectures**
explore the effects of varying values and look for invariance and covariance	**engage with someone else's mathematical reasoning in the context of a problem or particular situation**
take account of feedback and learn from mistakes	**consider the effectiveness of alternative strategies**
work logically towards results and solutions, recognising the impact of constraints and assumptions	**communicate findings effectively**
appreciate that there are a number of different techniques that can be used to analyse a situation	**engage in mathematical discussion of results**

reason inductively and deduce	**consider the elegance and efficiency of alternative solutions**
make accurate mathematical diagrams, graphs and constructions on paper and on screen	**look for equivalence in relation to both the different approaches to the problem and different problems with similar structures**
calculate accurately, selecting mental methods or calculating devices as appropriate	**make connections between the current situation and outcomes, and situations and outcomes they have already encountered**

‘Hundred square’ prompt sheet

This is a rich task which could be used as an early experience of algebra so that pupils can see how it helps them to generalise and explain. Think about a Year 7 class, perhaps starting along the following lines.

Tell me what you know about patterns of numbers in the 100 square…

	24	
33	34	35
	44	

The cross-shaped ‘window’ can be moved to different positions on the grid, to reveal different sets of five numbers. Place the window anywhere you like.

Add the left- and right-hand numbers in the window and compare with the middle number.
Do the same for the top and bottom numbers. What do you notice?

Try different positions of the window. What do you notice?

Will it always work? Can you explain why? (Pupils are encouraged to give verbal explanations.)

Let’s see if we can use algebra to make the explanation simpler and clearer…

You told me that lots of numbers could be in the middle position. We could give the middle number a symbol, let’s call it n.

If we have chosen this symbol what can we say about the left-hand number? … and the right-hand number? How can we write this in symbols? (Pupils may use words and then try symbols.)

So the sum can be written as…

$(n - 1) + (n + 1) = n + n - 1 + 1 = n + n = n \times 2 = 2n$

(Explain the notation as necessary.)

Can you write down an expression for the sum of the top and bottom numbers?

$(n - 10) + (n + 10) = 2n$

Can this help us to explain why the two totals are the same for any position of the window?

…

By extending the task, pupils could work more independently to try other ideas such as starting by giving a symbol to the top number or exploring sums and differences of various numbers in the window. The shape of the window could also be changed.

Lines of progression in mathematics

Overview of strands

Strands	Sub-strands
1 Mathematical processes and applications	
	1.1 Representing
	1.2 Analysing – use reasoning
	1.3 Analysing – use procedures
	1.4 Interpreting and evaluating
	1.5 Communicating and reflecting
2 Number	
	2.1 Place value, ordering and rounding
	2.2 Integers, powers and roots
	2.3 Fractions, decimals, percentages, ratio and proportion
	2.4 Number operations
	2.5 Mental calculation methods
	2.6 Written calculation methods
	2.7 Calculator methods
	2.8 Checking results

3 Algebra	
	3.1 Equations, formulae, expressions and identities
	3.2 Sequences, functions and graphs
4 Geometry and measures	
	4.1 Geometrical reasoning
	4.2 Transformations and coordinates
	4.3 Construction and loci
	4.4 Measures and mensuration
5 Statistics	
	5.1 Specifying a problem, planning and collecting data
	5.2 Processing and representing data
	5.3 Interpreting and discussing results
	5.4 Probability

Learning objectives

1 Mathematical processes and applications

Solve problems, explore and investigate in a range of contexts

Increase the **challenge** and build **progression** across the key stage, and for groups of pupils by:

- increasing the **complexity** of the application, e.g. non-routine, multi-step problems, extended enquiries
- reducing the **familiarity** of the context, e.g. new contexts in mathematics, contexts drawn from other subjects, other aspects of pupils' lives
- increasing the **technical demand** of the mathematics required, e.g. more advanced concepts, more difficult procedures
- increasing the degree of **independence** and autonomy in problem-solving and investigation

1.1 Representing

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• identify the necessary information to understand or simplify a context or problem; represent problems, making correct use of symbols, words, diagrams, tables and graphs; use appropriate procedures and tools, including ICT	• identify the mathematical features of a context or problem; try out and compare mathematical representations; select appropriate procedures and tools, including ICT	• break down substantial tasks to make them more manageable; represent problems and synthesise information in algebraic, geometrical or graphical form; move from one form to another to gain a different perspective on the problem	• compare and evaluate representations; explain the features selected and justify the choice of representation in relation to the context	• choose and combine representations from a range of perspectives; introduce and use a range of mathematical techniques, the most efficient for analysis and most effective for communication	• systematically model contexts or problems through precise and consistent use of symbols and representations, and sustain this throughout the work

1.2 Analysing – use reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• classify and visualise properties and patterns; generalise in simple cases by working logically; draw simple conclusions and explain reasoning; understand the significance of a counter-example; take account of feedback and learn from mistakes	• visualise and manipulate dynamic images; conjecture and generalise; move between the general and the particular to test the logic of an argument; identify exceptional cases or counter-examples; make connections with related contexts	• use connections with related contexts to improve the analysis of a situation or problem; pose questions and make convincing arguments to justify generalisations or solutions; recognise the impact of constraints or assumptions	• identify a range of strategies and appreciate that more than one approach may be necessary; explore the effects of varying values and look for invariance and covariance in models and representations; examine and refine arguments, conclusions and generalisations; produce simple proofs	• make progress by exploring mathematical tasks, developing and following alternative approaches; examine and extend generalisations; support assumptions by clear argument and follow through a sustained chain of reasoning, including proof	• present rigorous and sustained arguments; reason inductively, deduce and prove; explain and justify assumptions and constraints

1.3 Analysing – use procedures

Within the appropriate range and content:

make accurate mathematical diagrams, graphs and constructions on paper and on screen; calculate accurately, selecting mental methods or calculating devices as appropriate; manipulate numbers, algebraic expressions and equations, and apply routine algorithms; use accurate notation, including correct syntax when using ICT; record methods, solutions and conclusions; estimate, approximate and check working

1.4 Interpreting and evaluating

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• interpret information from a mathematical representation or context; relate findings to the original context; check the accuracy of the solution; explain and justify methods and conclusions; compare and evaluate approaches	• use logical argument to interpret the mathematics in a given context or to establish the truth of a statement; give accurate solutions appropriate to the context or problem; evaluate the efficiency of alternative strategies and approaches	• justify the mathematical features drawn from a context and the choice of approach; generate fuller solutions by presenting a concise, reasoned argument using symbols, diagrams, graphs and related explanations	• make sense of, and judge the value of, own findings and those presented by others; judge the strength of empirical evidence and distinguish between evidence and proof; justify generalisations, arguments or solutions	• show insight into the mathematical connections in the context or problem; critically examine strategies adopted and arguments presented; consider the assumptions in the model and recognise limitations in the accuracy of results and conclusions	• justify and explain solutions to problems involving an unfamiliar context or a number of features or variables; comment constructively on reasoning, logic, process, results and conclusions

1.5 Communicating and reflecting

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• communicate own findings effectively, orally and in writing, and discuss and compare approaches and results with others; recognise equivalent approaches	• refine own findings and approaches on the basis of discussions with others; recognise efficiency in an approach; relate the current problem and structure to previous situations	• review and refine own findings and approaches on the basis of discussions with others; look for and reflect on other approaches and build on previous experience of similar situations and outcomes	• use a range of forms to communicate findings effectively to different audiences; review findings and look for equivalence to different problems with similar structure	• routinely review and refine findings and approaches; identify how other contexts were different from, or similar to, the current situation and explain how and why the same or different strategies were used	• use mathematical language and symbols effectively in presenting convincing conclusions or findings; critically reflect on own lines of enquiry when exploring; search for and appreciate more elegant forms of communicating approaches and solutions; consider the efficiency of alternative lines of enquiry or procedures

2 Number

2.1 Place value, ordering and rounding

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use decimal notation and place value; multiply and divide integers and decimals by 10, 100, 1000, and explain the effect • compare and order decimals in different contexts; know that when comparing measurements the units must be the same	• read and write positive integer powers of 10; multiply and divide integers and decimals by 0.1, 0.01 • order decimals	• extend knowledge of integer powers of 10; recognise the equivalence of 0.1, $\frac{1}{10}$ and 10^{-1}; multiply and divide by any integer power of 10	• express numbers in standard index form, both in conventional notation and on a calculator display • convert between ordinary and standard index form representations	• use standard index form to make sensible estimates for calculations involving multiplication and/ or division	
• round positive whole numbers to the nearest 10, 100 or 1000, and decimals to the nearest whole number or one decimal place	• round positive numbers to any given power of 10; round decimals to the nearest whole number or to one or two decimal places	• use rounding to make estimates and to give solutions to problems to an appropriate degree of accuracy	• round to a given number of significant figures; use significant figures to approximate answers when multiplying or dividing large numbers	• understand how errors can be compounded in calculations	• understand upper and lower bounds

2.2 Integers, powers and roots

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand negative numbers as positions on a number line; order, add and subtract integers in context • recognise and use multiples, factors, primes (less than 100), common factors, highest common factors and lowest common multiples in simple cases; use simple tests of divisibility • recognise the first few triangular numbers; recognise the squares of numbers to at least 12 × 12 and the corresponding roots	• add, subtract, multiply and divide integers • use multiples, factors, common factors, highest common factors, lowest common multiples and primes; find the prime factor decomposition of a number, e.g. 8000 = $2^6 \times 5^3$ • use squares, positive and negative square roots, cubes and cube roots, and index notation for small positive integer powers	• use the prime factor decomposition of a number • use ICT to estimate square roots and cube roots • use index notation for integer powers; know and use the index laws for multiplication and division of positive integer powers	• use index notation with negative and fractional powers, recognising that the index laws can be applied to these as well • know that $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} =$ for any positive number n	• use inverse operations, understanding that the inverse operation of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$	• understand and use rational and irrational numbers

2.3 Fractions, decimals, percentages, ratio and proportion

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• express a smaller whole number as a fraction of a larger one; simplify fractions by cancelling all common factors and identify equivalent fractions; convert terminating decimals to fractions, e.g. 0.23 $= \frac{23}{100}$; use diagrams to compare two or more simple fractions	• recognise that a recurring decimal is a fraction; use division to convert a fraction to a decimal; order fractions by writing them with a common denominator or by converting them to decimals	• understand the equivalence of simple algebraic fractions; know that a recurring decimal is an exact fraction	• distinguish between fractions with denominators that have only prime factors 2 or 5 (terminating decimals), and other fractions (recurring decimals)	• use an algebraic method to convert a recurring decimal to a fraction	
• add and subtract simple fractions and those with common denominators; calculate simple fractions of quantities and measurements (whole-number answers); multiply a fraction by an integer	• add and subtract fractions by writing them with a common denominator; calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction	• use efficient methods to add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse; cancel common factors before multiplying or dividing	• understand and apply efficient methods to add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse		

• understand percentage as the 'number of parts per 100'; calculate simple percentages and use percentages to compare simple proportions • recognise the equivalence of percentages, fractions and decimals • understand the relationship between ratio and proportion; use direct proportion in simple contexts; use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio; solve simple problems involving ratio and proportion using informal strategies	• interpret percentage as the operator 'so many hundredths of' and express one given number as a percentage of another; calculate percentages and find the outcome of a given percentage increase or decrease • use the equivalence of fractions, decimals and percentages to compare proportions • apply understanding of the relationship between ratio and proportion; simplify ratios, including those expressed in different units, recognising links with fraction notation; divide a quantity into two or more parts in a given ratio; use the unitary method to solve simple problems involving ratio and direct proportion	• recognise when fractions or percentages are needed to compare proportions; solve problems involving percentage changes • use proportional reasoning to solve problems, choosing the correct numbers to take as 100%, or as a whole; compare two ratios; interpret and use ratio in a range of contexts	• understand and use proportionality and calculate the result of any proportional change using multiplicative methods • calculate an original amount when given the transformed amount after a percentage change; use calculators for reverse percentage calculations by doing an appropriate division	• calculate an unknown quantity from quantities that vary in direct proportion using algebraic methods where appropriate	• understand and use direct and inverse proportion; solve problems involving inverse proportion (including inverse squares) using algebraic methods

2.4 Number operations

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use the rules of arithmetic and inverse operations in the context of positive integers and decimals • use the order of operations, including brackets	• understand and use the rules of arithmetic and inverse operations in the context of integers and fractions • use the order of operations, including brackets, with more complex calculations	• understand the effects of multiplying and dividing by numbers between 0 and 1; consolidate use of the rules of arithmetic and inverse operations • understand the order of precedence of operations, including powers	• recognise and use reciprocals; understand 'reciprocal' as a multiplicative inverse; know that any number multiplied by its reciprocal is 1, and that zero has no reciprocal because division by zero is not defined	• use a multiplier raised to a power to represent and solve problems involving repeated proportional change, e.g. compound interest	

2.5 Mental calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• recall number facts, including positive integer complements to 100 and multiplication facts to 10×10, and quickly derive associated division facts	• recall equivalent fractions, decimals and percentages; use known facts to derive unknown facts, including products involving numbers such as 0.7 and 6, and 0.03 and 8	• use known facts to derive unknown facts; extend mental methods of calculation, working with decimals, fractions, percentages, factors, powers and roots; solve problems mentally			• use surds and π in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
• strengthen and extend mental methods of calculation to include decimals, fractions and percentages, accompanied where appropriate by suitable jottings; solve simple problems mentally	• strengthen and extend mental methods of calculation, working with decimals, fractions, percentages, squares and square roots, and cubes and cube roots; solve problems mentally				
• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations	• make and justify estimates and approximations of calculations by rounding numbers to one significant figure and multiplying or dividing mentally		

2.6 Written calculation methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use efficient written methods to add and subtract whole numbers and decimals with up to two places • multiply and divide three-digit by two-digit whole numbers; extend to multiplying and dividing decimals with one or two places by single-digit whole numbers	• use efficient written methods to add and subtract integers and decimals of any size, including numbers with differing numbers of decimal places • use efficient written methods for multiplication and division of integers and decimals, including by decimals such as 0.6 or 0.06; understand where to position the decimal point by considering equivalent calculations	• use efficient written methods to add and subtract integers and decimals of any size; multiply by decimals; divide by decimals by transforming to division by an integer			

2.7 Calculator methods

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• carry out calculations with more than one step using brackets and the memory; use the square root and sign change keys • enter numbers and interpret the display in different contexts (decimals, percentages, money, metric measures)	• carry out more difficult calculations effectively and efficiently using the function keys for sign change, powers, roots and fractions; use brackets and the memory • enter numbers and interpret the display in different contexts (extend to negative numbers, fractions, time)	• use a calculator efficiently and appropriately to perform complex calculations with numbers of any size, knowing not to round during intermediate steps of a calculation; use the constant, π and sign change keys; use the function keys for powers, roots and fractions; use brackets and the memory	• use an extended range of function keys, including the reciprocal and trigonometric functions • use standard index form, expressed in conventional notation and on a calculator display; know how to enter numbers in standard index form	• use calculators to explore exponential growth and decay, using a multiplier and the power key • calculate with standard index form, using a calculator as appropriate	• use calculators, or written methods, to calculate the upper and lower bounds of calculations in a range of contexts, particularly when working with measurements

2.8 Checking results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• check results by considering whether they are of the right order of magnitude and by working problems backwards	• select from a range of checking methods, including estimating in context and using inverse operations	• check results using appropriate methods	• check results using appropriate methods	• check results using appropriate methods	• check results using appropriate methods

3 Algebra

3.1 Equations, formulae, expressions and identities

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use letter symbols to represent unknown numbers or variables; know the meanings of the words *term*, *expression* and *equation* • understand that algebraic operations follow the rules of arithmetic	• recognise that letter symbols play different roles in equations, formulae and functions; know the meanings of the words *formula* and *function* • understand that algebraic operations, including the use of brackets, follow the rules of arithmetic; use index notation for small positive integer powers	• distinguish the different roles played by letter symbols in equations, identities, formulae and functions • use index notation for integer powers and simple instances of the index laws	• know and use the index laws in generalised form for multiplication and division of integer powers • square a linear expression; expand the product of two linear expressions of the form $x \pm n$ and simplify the corresponding quadratic expression; establish identities such as $a^2 - b^2 = (a + b)(a - b)$	• factorise quadratic expressions, including the difference of two squares, e.g. $x^2 - 9 = (x + 3)(x - 3)$ cancel common factors in rational expressions, e.g. $\frac{2(x + 1)^2}{(x + 1)}$ • simplify simple algebraic fractions to produce linear expressions; use factorisation to simplify compound algebraic fractions	

• simplify linear algebraic expressions by collecting like terms; multiply a single term over a bracket (integer coefficients) • construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations)	• simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket • construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in same way) • use graphs and set up equations to solve simple problems involving direct proportion	• simplify or transform algebraic expressions by taking out single-term common factors; add simple algebraic fractions • construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution) • use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^2 + x = 20$	• solve linear equations in one unknown with integer and fractional coefficients; solve linear equations that require prior simplification of brackets, including those with negative signs anywhere in the equation • solve linear inequalities in one variable; represent the solution set on a number line • solve a pair of simultaneous linear equations by eliminating one variable; link a graph of an equation or a pair of equations to the algebraic solution; consider cases that have no solution or an infinite number of solutions	• solve equations involving algebraic fractions with compound expressions as the numerators and/or denominators • solve linear inequalities in one and two variables; find and represent the solution set • explore 'optimum' methods of solving simultaneous equations in different forms • solve quadratic equations by factorisation	• solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, where one is linear in each unknown and the other is linear in one unknown and quadratic in the other or of the form $x^2 + y^2 = r^2$ • solve quadratic equations by factorisation, completing the square and using the quadratic formula, including those in which the coefficient of the quadratic term is greater than 1

		• use algebraic methods to solve problems involving direct proportion; relate algebraic solutions to graphs of the equations; use ICT as appropriate **Y9 extension objective** • *explore ways of constructing models of real-life situations by drawing graphs and constructing algebraic equations and inequalities*			
• use simple formulae from mathematics and other subjects; substitute positive integers into linear expressions and formulae and, in simple cases, derive a formula	• use formulae from mathematics and other subjects; substitute integers into simple formulae, including examples that lead to an equation to solve; substitute positive integers into expressions involving small powers, e.g. $3x^2 + 4$ or ; $2x^3$ derive simple formulae	• use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject	• derive and use more complex formulae; change the subject of a formula, including cases where a power of the subject appears in the question or solution, e.g. find r given that $A = \pi r^2$	• derive and use more complex formulae; change the subject of a formula, including cases where the subject occurs twice	• derive relationships between different formulae that produce equal or related results

3.2 Sequences, functions and graphs

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• describe integer sequences; generate terms of a simple sequence, given a rule (e.g. finding a term from the previous term, finding a term given its position in the sequence) • generate sequences from patterns or practical contexts and describe the general term in simple cases	• generate terms of a linear sequence using term-to-term and position-to-term rules, on paper and using a spreadsheet or graphics calculator • use linear expressions to describe the *n*th term of a simple arithmetic sequence, justifying its form by referring to the activity or practical context from which it was generated	• generate terms of a sequence using term-to-term and position-to-term rules, on paper and using ICT • generate sequences from practical contexts and write and justify an expression to describe the *n*th term of an arithmetic sequence	• find the next term and the *n*th term of quadratic sequences and explore their properties; deduce properties of the sequences of triangular and square numbers from spatial patterns		

• express simple functions in words, then using symbols; represent them in mappings	• express simple functions algebraically and represent them in mappings or on a spreadsheet	• find the inverse of a linear function • generate points and plot graphs of linear functions, where y is given implicitly in terms of x (e.g. $ay + bx = 0$, $y + bx + c = 0$), on paper and using ICT; find the gradient of lines given by equations of the form , given values for m and c	• plot the graph of the inverse of a linear function • understand that equations in the form $y = mx + c$ represent a straight line and that m is the gradient and c is the value of the y-intercept; investigate the gradients of parallel lines and lines perpendicular to these lines	• identify the equations of straight-line graphs that are parallel; find the gradient and equation of a straight-line graph that is perpendicular to a given line • plot graphs of more complex quadratic and cubic functions; estimate values at specific points, including maxima and minima	• know and understand that the intersection points of the graphs of a linear and quadratic function are the approximate solutions to the corresponding simultaneous equations • construct the graphs of simple loci, including the circle $x^2 + y^2 = r^2$; find graphically the intersection points of a given straight line with this circle and know this represents the solution to the corresponding two simultaneous equations

• generate coordinate pairs that satisfy a simple linear rule; plot the graphs of simple linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise straight-line graphs parallel to the x -axis or y- axis • plot and interpret the graphs of simple linear functions arising from real-life situations, e.g. conversion graphs	• generate points in all four quadrants and plot the graphs of linear functions, where y is given explicitly in terms of x, on paper and using ICT; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs • construct linear functions arising from real-life problems and plot their corresponding graphs; discuss and interpret graphs arising from real situations, e.g. distance–time graphs	• construct functions arising from real-life problems and plot their corresponding graphs; interpret graphs arising from real situations, e.g. time series graphs	• explore simple properties of quadratic functions; plot graphs of simple quadratic and cubic functions, e.g. $y = x^2$, $y = 3x^2 + 4$, $y = x^3$ • understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines	• find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function • identify and sketch graphs of linear and simple quadratic and cubic functions; understand the effect on the graph of addition of (or multiplication by) a constant	• plot and recognise the characteristic shapes of graphs of simple cubic functions (e.g. $y = x^3$), reciprocal functions (e.g. $y = \frac{1}{x}$, $x \neq 0$), exponential functions ($y = kx$ for integer values of x and simple positive values of k) and trigonometric functions, on paper and using ICT • apply to the graph $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$ and $y = af(x)$ for linear, quadratic, sine and cosine functions

		Y9 extension objectives • *Use ICT to explore the graphical representation of algebraic equations and interpret how properties of the graph are related to features of the equation, e.g. parallel and perpendicular lines* • *interpret the meaning of various points and sections of straight-line graphs, including intercepts and intersection, e.g. solving simultaneous linear equations*			

4 Geometry and measures

4.1 Geometrical reasoning

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes • identify parallel and perpendicular lines; know the sum of angles at a point, on a straight line and in a triangle; recognise vertically opposite angles	• identify alternate angles and corresponding angles; understand a proof that: – the angle sum of a triangle is 180° and of a quadrilateral is 360° – the exterior angle of a triangle is equal to the sum of the two interior opposite angles	• distinguish between conventions, definitions and derived properties • explain how to find, calculate and use: – the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons – the interior and exterior angles of regular polygons	• distinguish between practical demonstration and proof in a geometrical context • solve multi-step problems using properties of angles, of parallel lines, and of triangles and other polygons, justifying inferences and explaining reasoning with diagrams and text	• show step-by-step deduction in solving more complex geometrical problems	• understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid

• identify and use angle, side and symmetry properties of triangles and quadrilaterals; explore geometrical problems involving these properties, explaining reasoning orally, using step-by-step deduction supported by diagrams	• solve geometrical problems using side and angle properties of equilateral, isosceles and right-angled triangles and special quadrilaterals, explaining reasoning with diagrams and text; classify quadrilaterals by their geometrical properties	• know the definition of a circle and the names of its parts; explain why inscribed regular polygons can be constructed by equal divisions of a circle • solve problems using properties of angles, of parallel and intersecting lines, and of triangles and other polygons, justifying inferences and explaining reasoning with diagrams and text	• know that the tangent at any point on a circle is perpendicular to the radius at that point; explain why the perpendicular from the centre to the chord bisects the chord	• prove and use the facts that: – the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference – the angle subtended at the circumference by a semicircle is a right angle – angles in the same segment are equal – opposite angles on a cyclic quadrilateral sum to 180°	• prove and use the alternate segment theorem

	• know that if two 2-D shapes are congruent, corresponding sides and angles are equal	• understand congruence and explore similarity	• know that if two 2-D shapes are similar, corresponding angles are equal and corresponding sides are in the same ratio; understand from this that any two circles and any two squares are mathematically similar while in general any two rectangles are not		• prove the congruence of triangles and verify standard ruler and compass constructions using formal arguments
		Y9 extension objective • *investigate Pythagoras' theorem, using a variety of media, through its historical and cultural roots including 'picture' proofs*			

• use 2-D representations to visualise 3-D shapes and deduce some of their properties	• visualise 3-D shapes from their nets; use geometric properties of cuboids and shapes made from cuboids; use simple plans and elevations	• visualise and use 2-D representations of 3-D objects; analyse 3-D shapes through 2-D projections, including plans and elevations	• understand and apply Pythagoras' theorem when solving problems in 2-D and simple problems in 3-D • understand and use trigonometric relationships in right-angled triangles, and use these to solve problems, including those involving bearings	• understand and use Pythagoras' theorem to solve 3-D problems • use trigonometric relationships in right-angled triangles to solve 3-D problems, including finding the angles between a line and a plane	• calculate the area of a triangle using the formula $\frac{1}{2}ab\sin C$ • draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scalings in either or both of the x and y directions • use the sine and cosine rules to solve 2-D and 3-D problems

4.2 Transformations and coordinates

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• understand and use the language and notation associated with reflections, translations and rotations	• identify all the symmetries of 2-D shapes	• identify reflection symmetry in 3-D shapes			
• recognise and visualise the symmetries of a 2-D shape • transform 2-D shapes by: – reflecting in given mirror lines – rotating about a given point – translating	• transform 2-D shapes by rotation, reflection and translation, on paper and using ICT	• recognise that translations, rotations and reflections preserve length and angle, and map objects on to congruent images	• transform 2-D shapes by combinations of translations, rotations and reflections, on paper and using ICT; use congruence to show that translations, rotations and reflections preserve length and angle		

• explore these transformations and symmetries using ICT	• try out mathematical representations of simple combinations of these transformations	• devise instructions for a computer to generate and transform shapes	• use any point as the centre of rotation; measure the angle of rotation, using fractions of a turn or degrees; understand that translations are specified by a vector		
		• explore and compare mathematical representations of combinations of translations, rotations and reflections of 2-D shapes, on paper and using ICT			

	• understand and use the language and notation associated with enlargement; enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor; explore enlargement using ICT	• enlarge 2-D shapes, given a centre of enlargement and a positive integer scale factor, on paper and using ICT; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; recognise that enlargements preserve angle but not length, and understand the implications of enlargement for perimeter	• enlarge 2-D shapes using positive, fractional and negative scale factors, on paper and using ICT; recognise the similarity of the resulting shapes; understand and use the effects of enlargement on perimeter	• understand and use the effects of enlargement on areas and volumes of shapes and solids	
	• make scale drawings	• use and interpret maps and scale drawings in the context of mathematics and other subjects			

• use conventions and notation for 2-D coordinates in all four quadrants; find coordinates of points determined by geometric information	• find the midpoint of the line segment AB, given the coordinates of points A and B	• use the coordinate grid to solve problems involving translations, rotations, reflections and enlargements	• find the points that divide a line in a given ratio, using the properties of similar triangles; calculate the length of AB, given the coordinates of points A and B	• understand and use vector notation to describe transformation of 2-D shapes by combinations of translations; calculate and represent graphically the sum of two vectors	• calculate and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors • understand and use the commutative and associative properties of vector addition • solve simple geometrical problems in 2-D using vectors

4.3 Construction and loci

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use a ruler and protractor to: – measure and draw lines to the nearest millimetre and angles, including reflex angles, to the nearest degree – construct a triangle, given two sides and the included angle (SAS) or two angles and the included side (ASA) • use ICT to explore constructions • use ruler and protractor to construct simple nets of 3-D shapes, e.g. cuboid, regular tetrahedron, square-based pyramid, triangular prism	• use straight edge and compasses to construct: – the midpoint and perpendicular bisector of a line segment – the bisector of an angle – the perpendicular from a point to a line – the perpendicular from a point on a line – a triangle, given three sides (SSS) • use ICT to explore these constructions • find simple loci, both by reasoning and by using ICT, to produce shapes and paths, e.g. an equilateral triangle	• use straight edge and compasses to construct triangles, given right angle, hypotenuse and side (RHS) • use ICT to explore constructions of triangles and other 2-D shapes • find the locus of a point that moves according to a simple rule, both by reasoning and by using ICT	• understand from experience of constructing them that triangles given SSS, SAS, ASA or RHS are unique, but that triangles given SSA or AAA are not • find the locus of a point that moves according to a more complex rule, both by reasoning and by using ICT		

4.4 Measures and mensuration

Year 7	Year 8	Year 9	Year10	Year 11	Extension
• choose and use units of measurement to measure, estimate, calculate and solve problems in everyday contexts; convert one metric unit to another, e.g. grams to kilograms; read and interpret scales on a range of measuring instruments • distinguish between and estimate the size of acute, obtuse and reflex angles	• choose and use units of measurement to measure, estimate, calculate and solve problems in a range of contexts; know rough metric equivalents of imperial measures in common use, such as miles, pounds (lb) and pints • use bearings to specify direction	• solve problems involving measurements in a variety of contexts; convert between area measures (e.g. mm^2 to cm^2, cm^2 to m^2, and vice versa) and between volume measures (e.g. mm^3 to cm^3, cm^3 to m^3, and vice versa)	• understand and use measures of speed (and other compound measures such as density or pressure); solve problems involving constant or average rates of change	• apply knowledge that measurements given to the nearest whole unit may be inaccurate by up to one half of the unit in either direction and use this to understand how errors can be compounded in calculations	• recognise limitations in the accuracy of measurements and judge the proportional effect on solutions
		Y9 extension objective • *interpret and explore combining measures into rates of change in everyday contexts (e.g. km per hour, pence per metre); use compound measures to compare in real-life contexts (e.g. travel graphs and value for money), using ICT as appropriate*			

• know and use the formula for the area of a rectangle; calculate the perimeter and area of shapes made from rectangles • calculate the surface area of cubes and cuboids	• derive and use formulae for the area of a triangle, parallelogram and trapezium; calculate areas of compound shapes • know and use the formula for the volume of a cuboid; calculate volumes and surface areas of cuboids and shapes made from cuboids	• know and use the formulae for the circumference and area of a circle • calculate the surface area and volume of right prisms	• solve problems involving lengths of circular arcs and areas of sectors • solve problems involving surface areas and volumes of cylinders	• solve problems involving surface areas and volumes of cylinders, pyramids, cones and spheres • understand and use the formulae for the length of a circular arc and area and perimeter of a sector • consider the dimensions of a formula and begin to recognise the difference between formulae for perimeter, area and volume in simple contexts	• solve problems involving more complex shapes and solids, including segments of circles and frustums of cones • understand the difference between formulae for perimeter, area and volume by considering dimensions

5 Statistics

5.1 Specifying a problem, planning and collecting data

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• suggest possible answers, given a question that can be addressed by statistical methods • decide which data would be relevant to an enquiry and possible sources	• discuss a problem that can be addressed by statistical methods and identify related questions to explore • decide which data to collect to answer a question, and the degree of accuracy needed; identify possible sources; consider appropriate sample size	• suggest a problem to explore using statistical methods, frame questions and raise conjectures • discuss how different sets of data relate to the problem; identify possible primary or secondary sources; determine the sample size and most appropriate degree of accuracy	• independently devise a suitable plan for a substantial statistical project and justify the decisions made • identify possible sources of bias and plan how to minimise it • break a task down into an appropriate series of key statements (hypotheses), and decide upon the best methods for testing these	• consider possible difficulties with planned approaches, including practical problems; adjust the project plan accordingly • deal with practical problems such as non-response or missing data • identify what extra information may be required to pursue a further line of enquiry	• select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling • understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn

• plan how to collect and organise small sets of data from surveys and experiments: – design data collection sheets or questionnaires to use in a simple survey – construct frequency tables for gathering discrete data, grouped where appropriate in equal class intervals • collect small sets of data from surveys and experiments, as planned	• plan how to collect the data; construct frequency tables with equal class intervals for gathering continuous data and two-way tables for recording discrete data • collect data using a suitable method (e.g. observation, controlled experiment, data logging using ICT)	• design a survey or experiment to capture the necessary data from one or more sources; design, trial and if necessary refine data collection sheets; construct tables for gathering large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables • gather data from specified secondary sources, including printed tables and lists, and ICT-based sources, including the internet	• gather data from primary and secondary sources, using ICT and other methods, including data from observation, controlled experiment, data logging, printed tables and lists		

5.2 Processing and representing data

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• calculate statistics for small sets of discrete data: – find the mode, median and range, and the modal class for grouped data – calculate the mean, including from a simple frequency table, using a calculator for a larger number of items	• calculate statistics for sets of discrete and continuous data, including with a calculator and spreadsheet; recognise when it is appropriate to use the range, mean, median and mode and, for grouped data, the modal class	• calculate statistics and select those most appropriate to the problem or which address the questions posed	• use an appropriate range of statistical methods to explore and summarise data; including estimating and finding the mean, median, quartiles and interquartile range for large data sets (by calculation or using a cumulative frequency diagram)	• use an appropriate range of statistical methods to explore and summarise data; including calculating an appropriate moving average for a time series • use a moving average to identify seasonality and trends in time series data, using them to make predictions	

• construct, on paper and using ICT, graphs and diagrams to represent data, including: – bar-line graphs – frequency diagrams for grouped discrete data – simple pie charts	• construct graphical representations, on paper and using ICT, and identify which are most useful in the context of the problem. Include: – pie charts for categorical data – bar charts and frequency diagrams for discrete and continuous data – simple line graphs for time series – simple scatter graphs – stem-and-leaf diagrams	• select, construct and modify, on paper and using ICT, suitable graphical representations to progress an enquiry and identify key features present in the data. Include: – line graphs for time series – scatter graphs to develop further understanding of correlation	• select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry and identify key features present in the data. Include: – cumulative frequency tables and diagrams – box plots – scatter graphs and lines of best fit (by eye)	• select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry, including histograms for grouped continuous data with equal class intervals	• construct histograms, including those with unequal class intervals
		Y9 extension objective • *work through the entire handling data cycle to explore relationships within bivariate data, including applications to global citizenship, e.g. how fair is our society?*			

5.3 Interpreting and discussing results

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• interpret diagrams and graphs (including pie charts), and draw simple conclusions based on the shape of graphs and simple statistics for a single distribution • compare two simple distributions using the range and one of the mode, median or mean • write a short report of a statistical enquiry, including appropriate diagrams, graphs and charts, using ICT as appropriate; justify the choice of presentation	• interpret tables, graphs and diagrams for discrete and continuous data, relating summary statistics and findings to the questions being explored • compare two distributions using the range and one or more of the mode, median and mean • write about and discuss the results of a statistical enquiry using ICT as appropriate; justify the methods used	• interpret graphs and diagrams and make inferences to support or cast doubt on initial conjectures; have a basic understanding of correlation • compare two or more distributions and make inferences, using the shape of the distributions and appropriate statistics • review interpretations and results of a statistical enquiry on the basis of discussions; communicate these interpretations and results using selected tables, graphs and diagrams	• analyse data to find patterns and exceptions, and try to explain anomalies; include social statistics such as index numbers, time series and survey data • appreciate that correlation is a measure of the strength of association between two variables; distinguish between positive, negative and zero correlation, using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship'	• interpret and use cumulative frequency diagrams to solve problems • recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis • compare two or more distributions and make inferences, using the shape of the distributions and measures of average and spread, including median and quartiles	• use, interpret and compare histograms, including those with unequal class intervals

			• examine critically the results of a statistical enquiry; justify choice of statistical representations and relate summarised data to the questions being explored		

5.4 Probability

Year 7	Year 8	Year 9	Year 10	Year 11	Extension
• use vocabulary and ideas of probability, drawing on experience • understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; identify all the possible mutually exclusive outcomes of a single event	• interpret the results of an experiment using the language of probability; appreciate that random processes are unpredictable • know that if the probability of an event occurring is p, then the probability of it not occurring is $1 - p$; use diagrams and tables to record in a systematic way all possible mutually exclusive outcomes for single events and for two successive events	• interpret results involving uncertainty and prediction • identify all the mutually exclusive outcomes of an experiment; know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems	• use tree diagrams to represent outcomes of two or more events and to calculate probabilities of combinations of independent events • know when to add or multiply two probabilities: if A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas if A and B are independent events, the probability of A and B occurring is P(A) × P(B)	• use tree diagrams to represent outcomes of compound events, recognising when events are independent and distinguishing between contexts involving selection both with and without replacement	• recognise when and how to work with probabilities associated with independent and mutually exclusive events when interpreting data

• estimate probabilities by collecting data from a simple experiment and recording it in a frequency table; compare experimental and theoretical probabilities in simple contexts	• compare estimated experimental probabilities with theoretical probabilities, recognising that: – if an experiment is repeated the outcome may, and usually will, be different – increasing the number of times an experiment is repeated generally leads to better estimates of probability	• compare experimental and theoretical probabilities in a range of contexts; appreciate the difference between mathematical explanation and experimental evidence	• understand relative frequency as an estimate of probability and use this to compare outcomes of experiments	• understand that if an experiment is repeated, the outcome may – and usually will – be different, and that increasing the sample size generally leads to better estimates of probability and population parameters	

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