Education Scotland
Foghlam Alba

Scottish Survey of Literacy and Numeracy (SSLN) 2011

Professional Learning Resource

Transforming lives through learning

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Section 1 Introduction

> 'A strong focus on literacy and numeracy is essential: all children and young people require these skills to gain access to learning and to succeed in life. Confidence and competence in literacy and numeracy provide the foundations for lifelong learning.'
>
> *Building the Curriculum 3*

This professional learning resource is for all members of the learning community to work on together, in small groups or departments or as individuals. The resource offers a range of rich materials for in-service days and other professional development activities. It is designed to meet the needs of individual teachers, groups within schools, learning communities, and education authority staff with responsibility for numeracy. The resource lends itself well to promoting professional reflection, dialogue and debate about numeracy and how to improve it. It offers a reference point for teachers to evaluate the quality of the delivery of the numeracy aspect of the mathematics curriculum and numeracy across learning as experienced by learners across stages and at transition points.

Numeracy, alongside literacy and health and wellbeing, sits at the heart of Curriculum for Excellence, as the knowledge, skills and attributes which equip children and young people for learning, life and work.

This professional learning resource provides guidance and advice to help inform learning and teaching practices in line with the Scottish Survey of Literacy and Numeracy (SSLN) survey's main objectives. It provides practitioners with more detail on children and young people's strengths and areas for improvement in numeracy identified within the in-depth analysis of the SSLN numeracy survey data 2011.

The number of responses to the different test items within each stage of the SSLN survey allow exploration of children and young people's performance across the numeracy organisers. This professional resource aims to share with practitioners children and young people's performance in:

- measurement,
- fractions, decimal fractions and percentages (including ratio and proportion).

The resource aims to help you to use these findings to:

- reflect on your own practice in developing and promoting numeracy,
- consider how to enhance children and young people's numeracy skills, to support their learning across the curriculum,
- plan how to develop your practice to incorporate some new concepts and ideas, and
- share views on numeracy across learning.

The analysis of children and young people's performance within these numeracy organisers provide an opportunity to reflect on and explore planning for effective learning and teaching. Section 2 sets out in more detail features of effective learning and teaching which will lead to improvements in children and young people's achievements in numeracy.

This presentation incorporates the key ideas and messages within the resource. It will support professional learning activities focused on planning for effective learning and teaching.

Link to: PowerPoint presentation - SSLN Numeracy: Presentation on Professional Learning Resource

Further to this, the resource considers the views and experiences of both learners and teachers gathered from questionnaire responses. Primary teachers and secondary non-mathematics teachers expressed less confidence in delivering 'ideas of chance and uncertainty'. Section 3.3 provides further guidance and support in delivering this aspect of numeracy across learning.

The resource is designed to be used flexibly, for example, through exploring the survey findings either at a stage across each of the identified organisers or across the stages within an organiser. Section 3 sets out in more detail children and young people's strengths and areas for improvement based on the in-depth analysis of their responses to the SSLN survey tasks.

The resource will be updated to reflect the findings of the analysis of performance across other numeracy organisers within the SSLN 2011 survey and the SSLN 2013 survey.

Section 2 Numeracy – developing fluency, confidence and accuracy through a well planned curriculum

Section 2.1 The place of numeracy within the curriculum

Numeracy development progresses as part of mathematics and underpins much of what is taught in this curriculum area. Learning mathematics gives children and young people access to the wider curriculum and the opportunity to pursue further studies and interests. Numeracy across learning provides essential analytical, problem-solving and decision-making skills across the curriculum. Numeracy is not just a subset of mathematics, it represents important skills for life, learning and work. It is also a life skill which permeates and supports all areas of learning.

Within their curriculum, schools are likely to develop programmes and courses for mathematics. They might not necessarily have a separate programme for numeracy across learning. However, all schools, working with their partners, need to have strategies to ensure that all children and young people develop high levels of numeracy skills through their learning across the curriculum. To achieve this, all teachers need to be increasingly confident about the standards and expectations within the numeracy experiences and outcomes. This will enable them to ensure children and young people have appropriately challenging opportunities to develop and apply their skills in numeracy across learning.

Reflective questions
How do we ensure a balance between the explicit teaching of numeracy skills within mathematics lessons and across learning?

How do we know when we have a balance?

Activity 1

Within your establishment, observe learning in action to evaluate the level of challenge expected of children and young people as they develop and apply their numeracy skills across their learning.

Is there a shared understanding of how to develop children and young people's numeracy skills across learning?

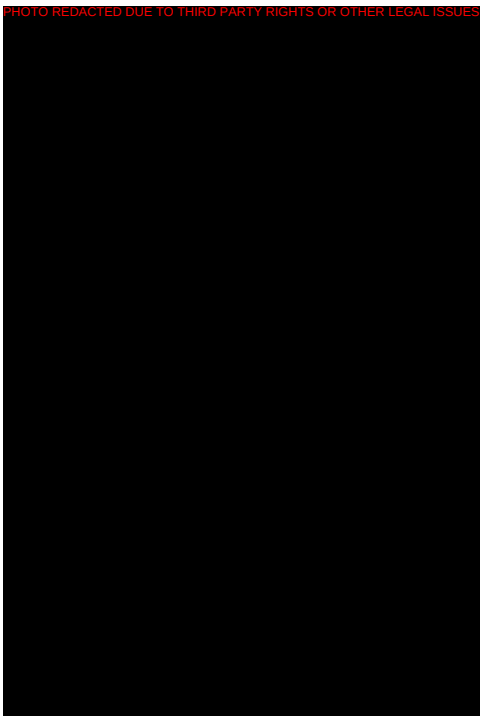
How do you know which numerical skills you should develop and that they are at an appropriate level?

You may find the following proforma helpful to organise your findings from observing numeracy across learning. Appendix 3

For example, when observing learning in action, you may wish to consider:

- whether learners estimate an answer before carrying out a calculation,
- learners' skills in mental agility including efficiency of strategies and how well they can explain the strategies used,
- the accuracy and presentation of children and young people's written calculations,
- whether learners consider the reasonableness of their answer,

- whether the specific numeracy skills required have not yet been learned in mathematics and/or in another subject area, and
- whether the level of numeracy is at an appropriate level of challenge for all learners.



Section 2.2 Discrete and contextualised learning

Reflective question
How do we ensure contexts for learning provide appropriate opportunities for learners to apply their numeracy skills?

Read

Within the SSLN 2011 survey, pupils completed tasks within two booklets as well as taking part in a pupil-teacher interactive assessment. The tasks involved pupils in answering:

- short individual questions usually set within a specific context,
- a series of questions linked to stimulus datasheets (**Appendix 4** contains an exemplar).

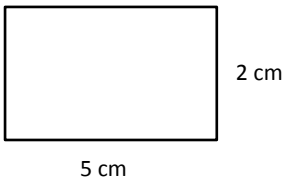
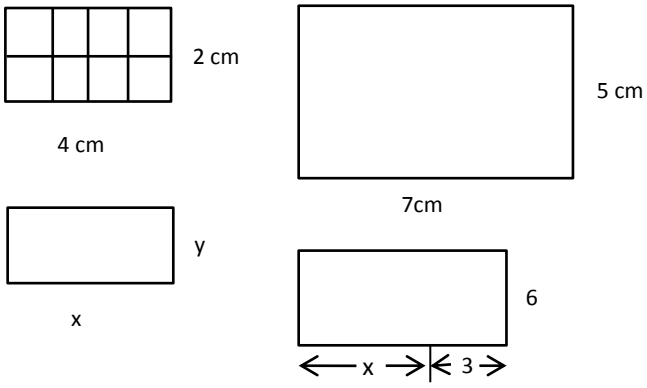
In all stages, pupils found the short individual questions the most challenging. Across the numeracy organisers, children and young people performed better with short atomistic questions such as 'What is 8 x 6?' than in questions which involved:

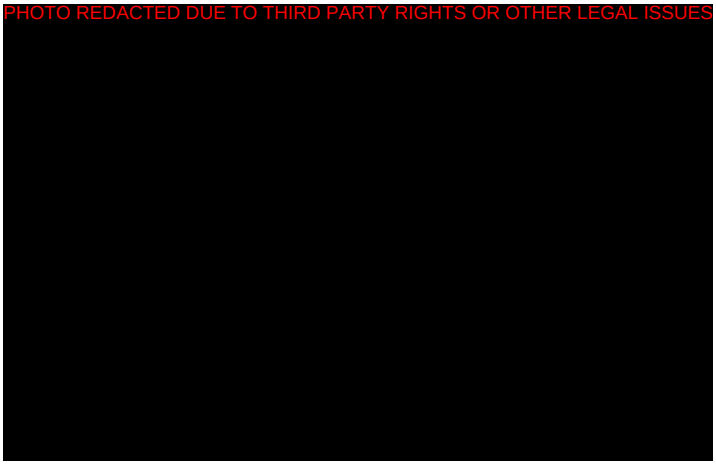
- a number of steps,
- a secondary organiser,
- the application of inverse operations, or
- the identification of a strategy in order to answer the problem.

Short questions can be constructed to include different degrees of challenge within and across the range of topics within the numeracy experiences and outcomes at each level.

The level of challenge can be increased by, for example, setting questions which require pupils to extract, assimilate and work with more than one piece of information from given tables, charts and diagrams.

For example, the following questions illustrate the different degrees of complexity within a short question:

How many millimetres are in a centimetre?	Tell me two lengths that together make a centimetre.
What is 8 x 6?	If 8 x 6 = 48, what else can you work out?
Four children out of a hundred on a school trip forgot to bring a packed lunch. What percentage is this?	The price of a coat is £60 after it has been reduced by 5%. How much did it cost before the reduction?
What is the area of the rectangle?  	What is the area of each of these rectangles? 



Contextualised tasks can contain basic stepped-out short response questions which do not require a significant level of reasoning, as well as more complex questions which require learners to extract, interpret and use information which integrates learning from across the experiences and outcomes.

Performance in the SSLN survey highlights the need for children and young people to experience a broader range of tasks and activities which strengthen their skills in:

- selecting and identifying appropriate strategies,
- reasoning and applying,
- comprehension and interpretation,
- extracting relevant information,
- using appropriate mathematical notation and vocabulary within their answers,
- the need for accurate calculations.

In planning such opportunities teachers should consider whether learners are 'using and applying' or learning new numeracy skills through another aspect of their learning.

Regular experience of working with simple to complex tasks develops the basic techniques required to tackle more challenging activities. Planned activities should support learners to think more flexibly and marshal these skills for more demanding multi-step problems. Multi-step problems require learners to break up the problem into manageable steps. This skill needs to be developed and teachers need to plan a range of tasks and activities to develop this skill. Learners need to be able to tackle multi-step problems without having these problems broken down into a succession of steps.

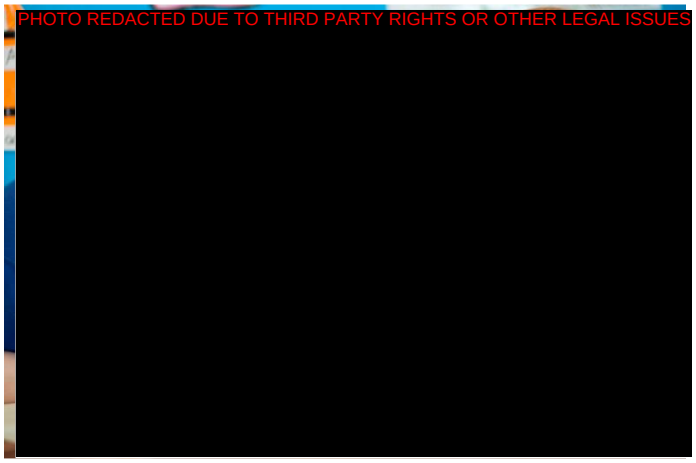

Activity

Activity 2

In planning appropriate tasks and activities how often do you consider:

- the number of operations required to reach an answer,
- the number of intermediate steps, stepped out or otherwise, to reach an answer,
- the application of inverse operations which require relevant direct facts to be known well,
- the integration of techniques rather than setting problems which require individual steps,

- an appropriate blend in the nature and frequency of examples selected to ensure appropriate challenge and development of key skills,
- connecting learning from across the numeracy experiences and organisers?



Section 3 contains illustrative questions to stimulate your thinking.

Reflective question
How are you incorporating these features into your practice?

Activity

Activity 3

Explore, over a period of a week, how you plan this range of experience for all learners.

You may wish to focus your activities by using the following prompts:

Are there particular tasks and activities which learners are confident with?
Are there particular tasks and activities which learners require more support with?
Do learners demonstrate the ability to think flexibly about how to apply their skills?
Can learners apply their skills accurately when working independently and with others, and can they then evaluate their solutions?
Do learners demonstrate the ability to be independent thinkers who can discuss and explain their reasoning?
Do learners demonstrate the ability to work as self-reliant individuals and stick to the task in hand?
Can learners explain their thinking to show their understanding of number processes and concepts?

In designing tasks and activities within familiar and unfamiliar contexts how often do we consider the following:

- starting with one or two more challenging questions,
- making some positive, informed choices about which questions to use, particularly from commercial resources (being scheme-assisted rather than scheme driven),
- using a greater range of problems involving different sorts of numbers (whole numbers/integers/fractions/decimal fractions/percentages),
- explore using similar numbers/different contexts and different numbers/same context,

- explore using fewer problems but in greater depth to support children and young people to think structurally,
- present word problems in ways that are not amenable to merely extracting key words and numbers.

Section 2.3 Word problems, real-life problems and problem solving

Read

Numeracy tasks can be planned to provide routine practice in a technique or combination of techniques. Tasks which involve the need to use an aspect of numeracy need to be well-designed to support learners' fluency in number as well as their capacity to handle a wider range of arithmetical and algebraic reasoning procedures. The SSLN numeracy survey included tasks with different degrees of challenge.

2.3.1 Word problems

Simple word problems require to be part of pupils' daily diet from the early stages. These could involve a few short sentences, given in words, where the learner is required to extract the meaning and any required information and then identify what needs to be done.

The development of the skill to extract and interpret information supports the development of mathematical reasoning and literacy skills which are important for learning across the curriculum as well as progression in learning. For example, in the 2011 SQA National Examinations, the following comments were made about areas which candidates found demanding:

- The lack of preparation for candidates to tackle **extended response questions** which assess **problem solving skills.**
- An apparent lack of understanding of the importance of **reading a question** carefully and being clear about what is being asked.

Standard Grade Mathematics

- **Drawing a conclusion** from information in a table.

Standard Grade Biology

- Centres should stress to candidates that they should **read the entire introduction** to a question before attempting the answer.

Intermediate 2 Chemistry

- Centres are encouraged to remind candidates to **read each question** very carefully and ensure that their response accurately answers the question asked.

Intermediate 2 Graphic Communication

- As in previous years, the main reasons why candidates forfeited marks included either **misreading instructions** in the stem of the question, particularly in relation to **'describe' or 'explain'.**

Standard Grade Geography

Mathematical literacy tasks provide contexts that support problem solving and in turn learners' achievements across all aspects of their learning.

Children and young people need to develop their numeracy skills and mathematical vocabulary through learning to read, write and discuss aspects of numeracy within a range of contexts. Mathematical vocabulary needs to be introduced in a planned way with real objects, mathematical instruments, pictures and diagrams to aid understanding.

Children and young people's mathematical literacy skills can be developed through opportunities to:

Listening and talking	• Listen to their teachers and peers using appropriate vocabulary • Use appropriate vocabulary within verbal responses and interactions • Describe, define and compare mathematical properties, techniques, patterns, relationships, rules • Discuss ways of tackling a problem • Hypothesise or make predictions • Present, explain and justify their methods and answers
Reading	• Reading instructions and explanations • Reading definitions in dictionaries • Reading texts with mathematical references
Writing	• To describe, compare, predict, interpret, explain, justify • Drawing and annotating diagrams, charts and tables • Write formulae, initially in words

Reflective questions

How are you planning to provide the range of experiences which develop children's and young people's skills in mathematical literacy?

Building on what you have learned, think about or discuss the following:

How often do we plan to develop children and young people's mathematical literacy skills through challenging learners to read about a new topic and develop their confidence in tackling extended word problems?
Do we explicitly connect skills used to teach reading across other aspects of learning such as the development and application of numeracy skills?

Further reading
Link to: PDF file: Children's Difficulties with Mathematical Word Problems
Link to: PDF file: Excellence in mathematics Report from the maths excellence group

2.3.2 Real-life problems

Real-life problems need to be carefully constructed in order that the contexts reflect the numeracy processes learners are expected to use as part of their solution. Real-life problems can be generated through focused tasks within other curriculum areas. Stronger links between numeracy and other curriculum areas can be forged through the application of numeracy skills within wider contexts.

For example, applying their numeracy skills within the context of food and health or the environment. See Appendix 5 and Appendix 6 for exemplars.

Link to: PDF file: Using Realistic Mathematics Education in UK classrooms

2.3.3 Problem solving

The mathematics 5-14 curriculum national guidelines set out a range of desirable process strategies to detail the problem-solving and enquiry skills to be developed from P1 to S2. Within this context, problem-solving often became a 'bolt-on' activity where pupils learned different strategies through a range of contexts unconnected to the development of deepening learning and understanding of key concepts and ideas. As a result, learners were often not able to think flexibly or critically to solve problems.

Within Curriculum for Excellence, a key purpose of the curriculum 3-18 is to enable all young people to solve problems. Problem-solving is a key skill to be developed across all aspects of learning. As a result, there is a change of emphasis in the development of problem solving skills within mathematics and numeracy.

> "To emphasise that problem solving is fundamental to good learning and teaching in all aspects of mathematics and its applications, problem solving will be addressed within all lines of development rather than appearing as a separate element."
>
> Building the Curriculum 1

It is evident from children and young people's responses, that there is a need to strengthen their capacity to solve problems.

Link to: ‘Skills in practice’ online resource on Education Scotland website.

Reflective questions

Within your establishment, is there agreement on how to develop children and young people’s problem solving skills?

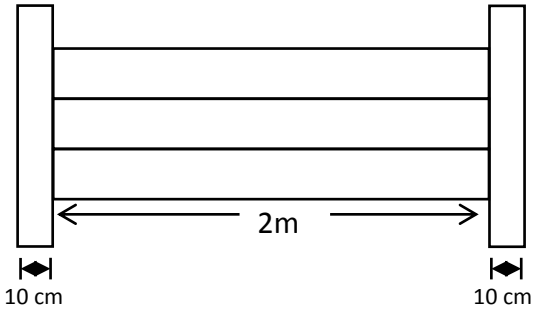
Within your establishment, how are staff planning the development of children and young people’s problem solving skills? How do you know?

How are problem solving strategies being developed progressively? For example, working backwards or reasoning logically.

Within SSLN only a few pupils were able to answer correctly the following questions.

How are you developing children and young people’s resilience and capacity to stick to a task?

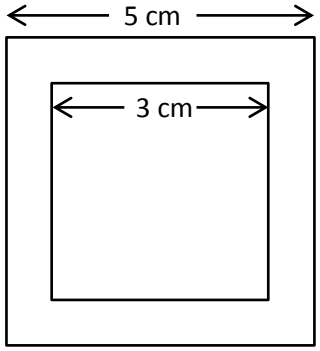
A garden fence is 8 $\frac{1}{2}$ m long. The posts are 2 m apart and 10 cm wide.



How many posts are there in a fence 8 $\frac{1}{2}$ m long?

Answer:: _____

3 cm square is cut from a piece of card which is 5 cm square.



What is the area of the card remaining after the square is cut?

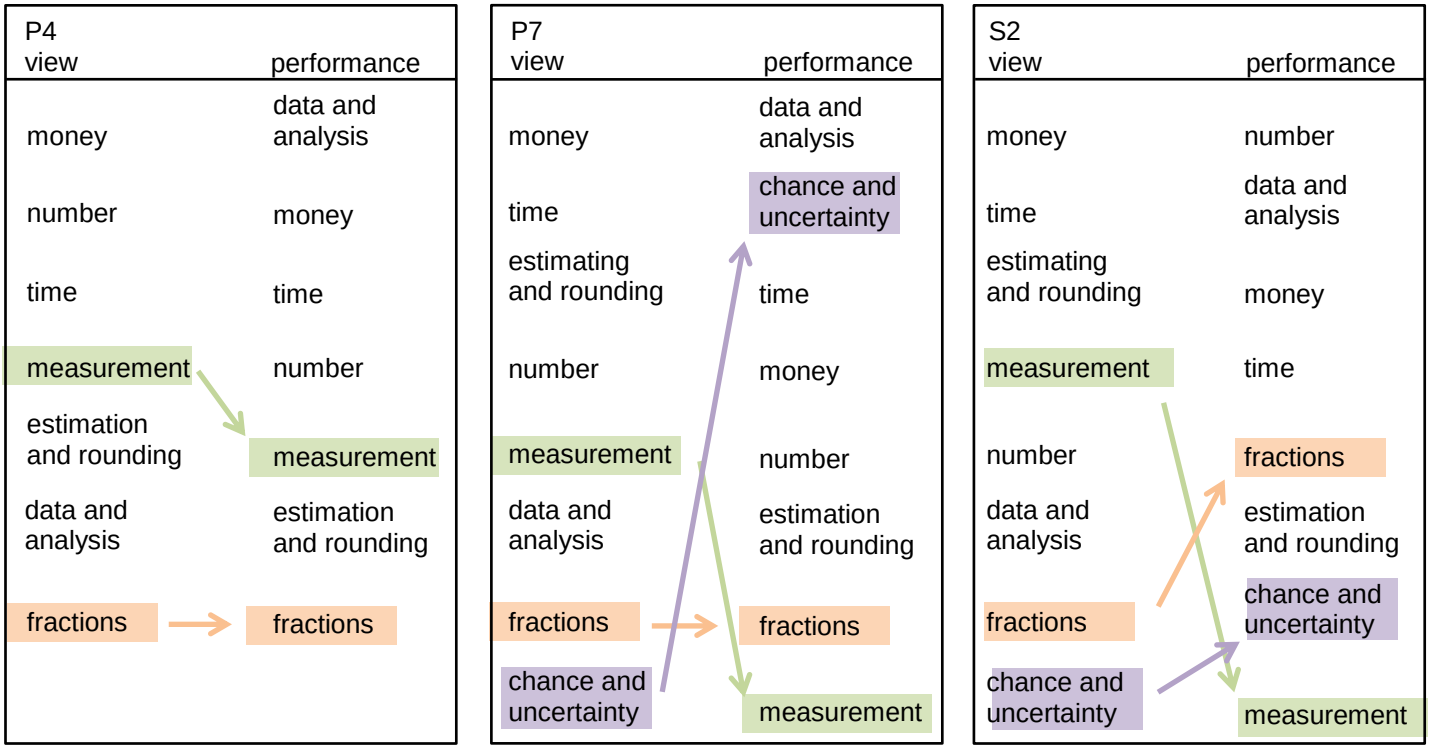
Answer: _____ cm^2

Section 2.4 Links between attitudes and attainment

Read

The SSLN results showed evidence of a strong linkage between attitudes to learning and performance. The results showed some evidence of a difference between pupils' views of their performance in the different areas of numeracy and their actual performance. The following chart from the SSLN Numeracy survey 2011 show the difference, in order of decreasing size, between the proportion of correct answers in each area and the proportion of pupils saying they thought they were 'good' or 'very good' in that area.

Difference in order of scale between percentage of correct answers and percentage of pupils saying they thought they were 'good' or 'very good', by topic:



Despite pupils having a positive view of performance in 'estimation and rounding', actual performance does not confirm this to be the case. **Estimation and rounding** should be a regular feature of numeracy practice across learning and beyond that of learning a set of procedural rules. Pupils' performance in measurement and fractions, decimal fractions and percentages was affected by pupils' inability to estimate and check the reasonableness of their answers.

Pupils' appreciation of working with numbers and their enthusiasm for learning needs to be harnessed if they are to remain motivated as they progress through school. This can be achieved through greater discussion with learners about their strengths and areas for improvement.

> "High quality interactions between learners and staff lie at the heart of assessment as part of learning and teaching. These interactions should promote thinking and demonstrate learning and development."
>
> *Building the Curriculum 5*
> *A Framework for Assessment*

Link to: Profiling: Information on purpose of profiling in the Assessment section of the Education Scotland website.

Reflective questions

Take a few minutes to think about how you can support learners to have a greater understanding of their strengths and areas for development in numeracy.

The following questions can be used to prompt your thinking**:**

What approaches do you use to make clear to learners what they are learning, what success looks like and what is expected of them in relation to numeracy?

How do you provide high quality feedback to learners about how well and how much they have learned in numeracy?

How do you demonstrate and model the range of numeracy skills which children and young people need to develop to thrive in a modern society and economy?

How can you find time for children and young people to talk about and evaluate their learning with you?

Appendix 7 provides a stimulus for discussion.

Section 3 Children and young people's performance

SSLN task items are tagged according to a main numeracy organiser with secondary organisers noted also. This provides an opportunity to explore, for example, breadth of learning across the numeracy experiences and outcomes in line with the principles of Curriculum for Excellence. In order to progress within a level, learners should be able to demonstrate confidence, proficiency and security across the three aspects of assessment (breadth, challenge, application). The numeracy organisers and experiences and outcomes specify the knowledge, understanding, capabilities and skills to be developed within each level. It is the linking of key concepts, ideas and techniques which determines whether the learner can apply their knowledge and understanding effectively within familiar and unfamiliar situations. Children's confidence in numeracy needs to be reinforced and revisited to ensure skills are steadily deepened as they progress through the levels. Developing skills in mastering integrated techniques from across the numeracy experiences and outcomes will cultivate children and young people's ability to tackle more complex problems and apply these across other curriculum areas. To achieve this goal, teachers need to know and understand the fundamental points of learning which hold the key to subsequent progress.

> "They need to agree the standards that they would expect to see from learners at each level in terms of literacy, numeracy and other essential skills for learning."
>
> *Building the Curriculum 3*

Progression is now defined in terms of 'how well' and 'how much' as well as learners' rate of progress. In terms of defining 'how well', teachers need to be clear about the fundamental points of learning and agree the pathways of progression which provide a strong foundation for further study and lifelong learning. Familiar techniques and ideas need to be constantly reinforced and revisited in order for learners to make good progress. Within this section, teachers will be able to explore children and young people's performance within measurement, fractions, decimal fractions and percentages and ideas of chance and uncertainty. Whilst there are many strengths in children and young people's performance within these organisers, there are aspects of learning which need to be strengthened at each stage in order that there is a platform on which learners are well-placed to build. In particular, where fundamental points of learning are not well-known, this impacts negatively on learner performance as they move from stage to stage.

The SSLN survey findings highlighted that learners at all stages demonstrated a lack of care and accuracy with written processes and calculations. A stronger emphasis on the importance of accuracy and checking the reasonableness of an answer is required across all areas of learning. Primary teachers, secondary school mathematics teachers and non-mathematics teachers need to agree and model the standards that they would expect to see from learners.

If learners are to improve their accuracy in calculations, teachers should also take cognisance of factors, such as working with a wider range of numbers or dealing with more than one concept, which increases the likelihood of arriving at a wrong answer. Other factors, such as learners' lack of knowledge of appropriate vocabulary or poorer reading skills, also impact on their ability to apply their numeracy skills appropriately.

In developing this resource with practitioners, they provided the following comments on what motivates children and young people to measure accurately:

> "pupils see a purpose for using measurement when doing practical tasks"
>
> *Primary teacher*

> "there appears to be a desire for pupils to get measurements correct. Incorrect measurements have a consequence, for example, the final product may not look or taste right"
>
> *Secondary teacher: technologies*

> "pupils are motivated to measure accurately within their experiments in order to develop their understanding of chemical reactions"
>
> *Secondary teacher, Science*

For example, at P4, children's lack of knowledge and understanding within particular aspects of measure track through to performance at P7 and S2.

First level	Counting squares to measure area of irregular shapes
⇩	⇩
Second level	Understanding area and perimeter
⇩	⇩
Third level	Relationship between length, area and volume as well as how to use one value to calculate the other

Reflective question
Within CPD activities, how can staff work collaboratively to share the fundamental points of learning along the pathways of progression and how these can be reinforced and strengthened across pupils' learning?

Within the SSLN survey, teachers were asked how often they had taken part in various forms of continuing professional development (CPD) in the last twelve months and, if they had, how useful they had found it.

The activities with the highest rating for level of impact were reading and discussing the numeracy experiences and outcomes with colleagues, peer observations and membership of working groups. Increasingly, teachers are working together to share standards and expectations as part of quality assurance and moderation activities.

Read

Activity
Read and discuss how a learning community are working together to share standards and expectations.

See 'Planning for transition – a cluster based approach' on Education Scotland website.

> "Assessment involves gathering, reflecting on and evaluating evidence of learning to enable staff to check on learners' progress and support further learning."
>
> *Building the Curriculum 5*

As learners move through the curriculum, they will experience a range of approaches to assessment. Some learners will proceed more slowly, some will need to revisit key concepts and techniques. In assessing learners' progress teachers need to construct tasks with different degrees of challenge and across the range of concepts within numeracy. Within the SSLN numeracy survey, tasks were designed to reflect the CfE requirements that pupils have achieved breadth, challenge and application of learning. In considering 'how well' learners are making progress it will be important for teachers to design tasks and activities which not only assess knowledge and understanding but attributes such as:

- being independent mathematical thinkers who can discuss and explain,
- being able to work as self-reliant individuals,
- being able to think flexibly and problem solve.

The assessments used in the SSLN survey were designed to assess the wide range of knowledge, capabilities and attitudes across learning identified in the CfE experiences and outcomes.

Read

Curriculum for Excellence Briefing 2 provides guidance and advice on assessing progress and achievement in the 3-15 broad general education.

Further reading

Link to: 'Development of children's understanding of length, area and volume measurement principles'- Curry, Margaret; Mitchelmore, Michael and Outhred, Lynne 2006 - Macquarie University, Sydney, Australia

Link to: PDF file: Excellence in mathematics Report from the maths excellence group.

The following section provides further detail on children and young people's performance across the three aspects within the following organisers:

- **measurement,**
- **fractions, decimal fractions and percentages,**
- **ideas of chance and uncertainty.**

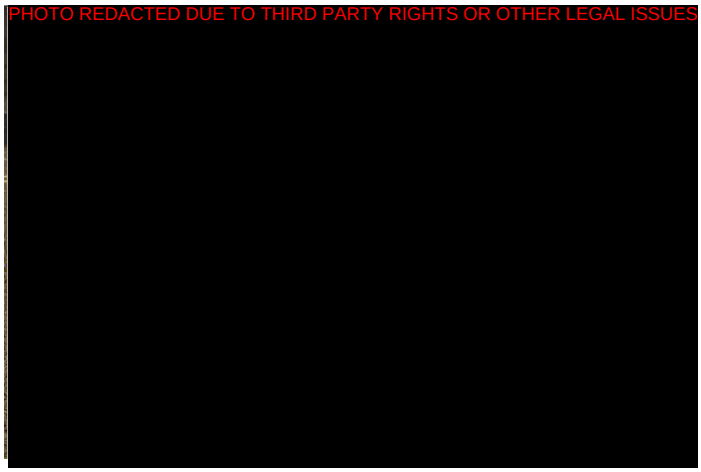
See Appendix 1 and Appendix 2 for an overview of key strengths and areas for improvement.

Learning and teaching resources to support the delivery of these numeracy organisers can be found Appendix 9, Appendix 10 and Appendix 11 as well as the links below:

- Thinking Through Mathematics on the NCETM website (National Centre for Excellence in the Teaching of Mathematics),
- Maths4Life Thinking Through Mathematics on the NRDC website (National Research and Development Centre for adult literacy and numeracy),
- Counting on a Greener Scotland - Resource pack on Adult Literacies Online.

Section 3.1 Pupil performance in measurement

P4 – First Level



Early level	First level
I have experimented with everyday items as units of measure to investigate and compare sizes and amounts in my environment, sharing my findings with others. ***MNU 0-11a***	*I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units.* ***MNU 1-11a*** *I can estimate the area of a shape by counting squares or other methods.* ***MNU 1-11b***

Secondary organisers
I have investigated how whole numbers are constructed, can understand the importance of zero within the system and can use my knowledge to explain the link between a digit, its place and its value. ***MNU 1-02a***
I can share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, then check my solution by comparing it with the estimate. ***MNU 1-01a***
Having explored fractions by taking part in practical activities, I can show my understanding of: • how a single item can be shared equally • the notation and vocabulary associated with fractions • where simple fractions lie on the number line. ***MNU 1-07a***
I have explored simple 3D objects and 2D shapes and can identify, name and describe their features using appropriate vocabulary. ***MTH 1-16a***

- **Ordering real life objects according to size and weight**

MNU 1-11a

Strengths

Overall children have a firm grasp of the learning within the early level.
For example, they can successfully order real-life objects according to size and weight. Children are able to estimate the length of objects which they can visualise within their immediate environment, e.g. a hand, glove or trainers. The following questions are provided for illustration:

Which of these is likely to weigh more than 1 kilogram?

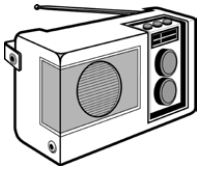



A: 2 apples B: 2 grapes C: 2 melons D: 2 plums

Answer: _____

Which of these is likely to weigh more than 5 kilograms?






A B C D

Answer: _____

Areas for improvement

Children find it more difficult to estimate the length or weight of objects when they do not have a clear understanding of how to choose an appropriate unit by which to measure. This is particularly the case where the objects concerned are not within their immediate environment, e.g. the length of a bike, or the height of a bus.

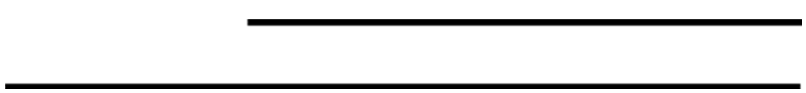
- **Practical tasks in measuring and using appropriate units**

MNU 1-11a

Strengths

Overall, children can complete simple measuring tasks, for example, finding the length of a given line and comparing lines of different lengths.
They can answer questions which involve reading a measurement from a scale, and then estimating their answer to the nearest whole unit.
This is particularly the case when the graduation on the scale is unitary.
The following questions are provided for illustrative purposes:

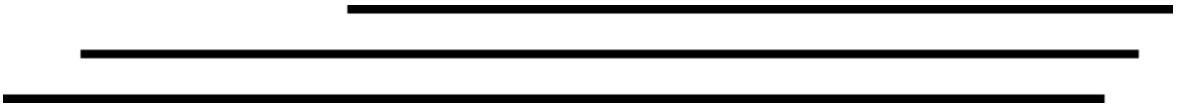
How much longer is line B than line A?



Answer: _____ cm

Use a ruler to measure these lines.

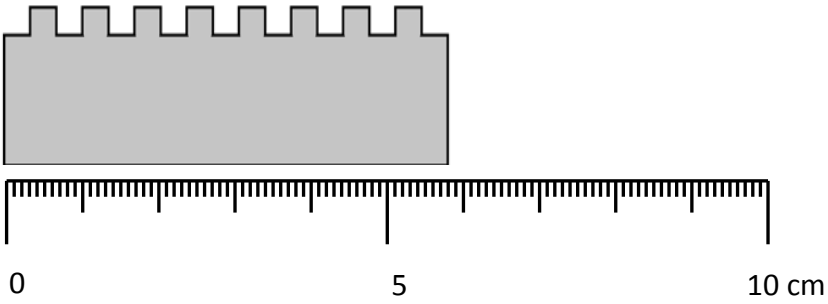
What is the length of the longest line?



Answer: _____ cm

Some of the numbers are missing from this ruler.

What is the length of this toy brick to the nearest centimetre?

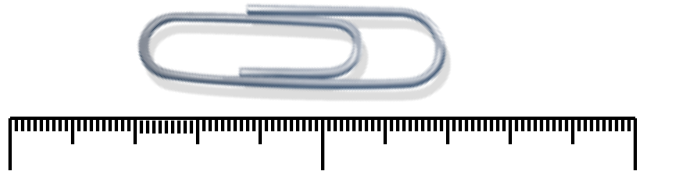


Areas for improvement

Children are not as successful in tasks which involve the identification of a strategy, prior to application of their knowledge and understanding of measurement within unfamiliar contexts and situations.

For example in the question below **only half of the pupils** recognised that a different strategy would have to be employed to achieve the correct answer.

What is the length of this paper clip to the nearest centimetre?



Answer: _____ cm

Consider:

- Is their response influenced by the number of graduations on the measuring instrument?
- Is it their skills in problem-solving which need to be developed further?

The following links provide examples of resources to support this aspect of learning:
Comparing weights unfamiliar contexts
Link to: http://www.bbc.co.uk/learningzone/clips/measurement-of-weight/846.html
Link to: http://www.bbc.co.uk/learningzone/clips/measurement-of-weight-signed/1584.html

Using scales and reading them correctly
Link to: http://www.bbc.co.uk/learningzone/clips/accurate-measurement-of-weight/253.html

- **Choosing appropriate units of measure/ using scales**

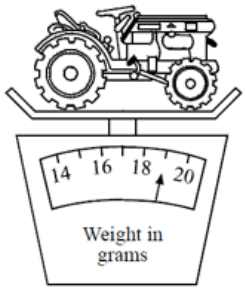
MNU 1-11a

Strengths
Children's responses show that they can choose an appropriate instrument of measure, for example, a trundle wheel, metre stick or ruler, which relate to a real-life activity.

For example, when asked to identify an appropriate instrument of measurement to find the length of a playground or the appropriate unit of measure to find the height of a bus.
It is evident that children are familiar with using different instruments to measure length, weight and height. They demonstrate a firm grasp of working with simple scales that have easily recognisable graduations.

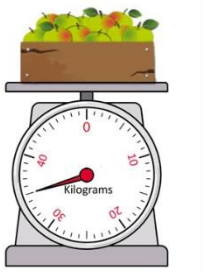
The following questions illustrate these points.

What does this toy tractor weigh?



Answer: _____ grams

A box of apples is put on a scale.



What did the box of apples weigh?

Answer: _____ kg

Areas for improvement

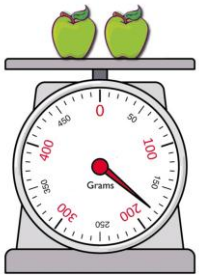
Children are not able to read scales where the value of an intermediate graduation needs to be deduced. Their knowledge and understanding of 'number and number processes' play an important role in their ability to apply their learning within this context. If the question or task involves a secondary organiser, e.g. fractions, the degree of difficulty for children increases. 'The Sciences 3-18' curriculum report' highlights the need for many children to develop further their knowledge of the units of measurement.

The following links provide exemplars of learning and teaching resources:
Link to: http://www.bbc.co.uk/learningzone/clips/measurement-reading-scales/3145.html.
Link to: http://www.bbc.co.uk/learningzone/clips/accurate-measurement-of-weight/253.html
Link to: http://www.iboard.co.uk/iwb/At-the-Vets-471

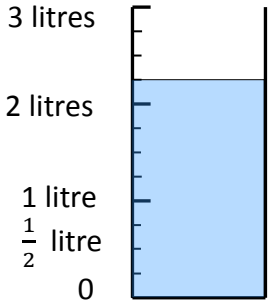
The following questions illustrate this point particularly well.

How much do the apples weigh?

A	153 g	☐
B	150 g	☐
C	180 g	☐
D	190 g	☐



Jill pours milk into a measuring jug



How much milk is in the jug?

Answer: _____ litres

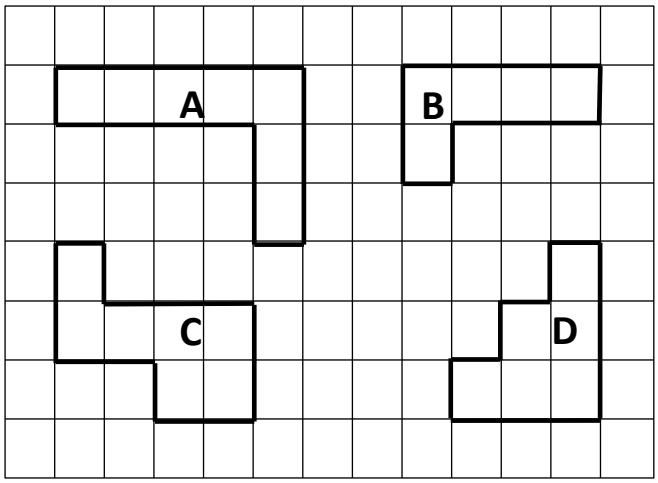
- **Estimating the area of a shape by counting squares or other methods**

MNU 1-11b

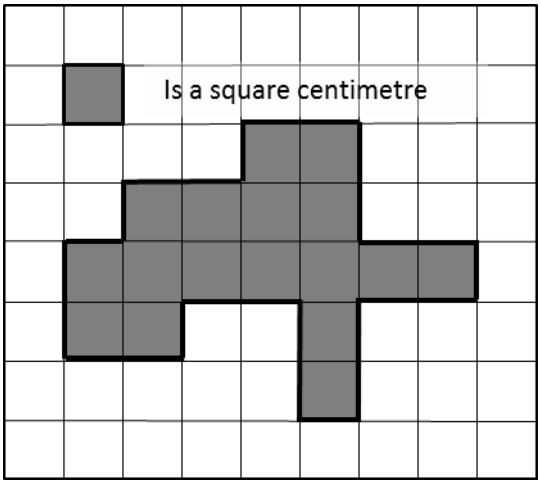
Strengths

Children's responses indicate that they have a good understanding of how to find an area of a shape by counting squares or other methods.
The more detailed results of this type of question suggest that children can count square centimetres to find an area. However, additional steps, e.g. finding two shapes of the same area, is not completed as well.
This fits in with the findings that success is reduced if more than one step is involved in the task.

Circle the letters in the two shapes that have the same area



What is the area of the shaded shape?



Answer: _____ square centimetres

Areas for improvement

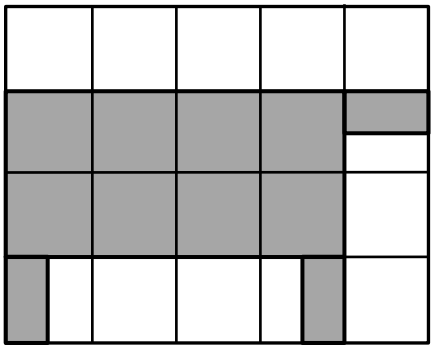
Based on children's responses, they are not as proficient in finding the area of a shape which involves counting whole and half square centimetres. This was compounded further when the shape was a simple 2D shape, for example, a triangle.

Link to: http://www.bbc.co.uk/learningzone/clips/calculating-the-area-of-a-shape-in-square-turtles/826.html
Link to: http://www.bbc.co.uk/learningzone/clips/calculating-the-area-of-a-shape-in-square-turtles-signed/1735.html

Reflective questions
Consider the questions illustrated below.
Now think about, either individually or with colleagues, the following:

- How often do we integrate learning of the properties and language of 2D shapes with finding the area of a 2D shape?
- How do we support learners to use their knowledge of fractions to find the area of a shape?

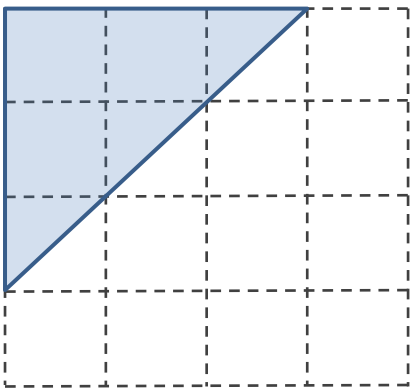
Each square has an area of 1 square centimetre



What is the area of the shaded shape?

Answer: _____ square centimetres

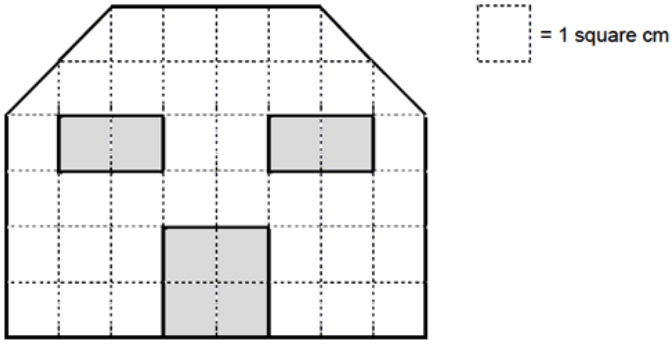
Donald draws a triangle on a grid.



What is the area of the triangle? Each square = 1 square centimetre.

Answer: _____ cm^2

Mal makes this shape in card. The house is white and the doors and windows are red.



What area is white?

Answer: _____ square centimetres

- **Estimating the amount an object holds**

MNU 1-11a

Strengths

Children can apply the strategy of counting square centimetres to find the area of a 2D shape to counting centimetre cubes to find a volume.

Areas for improvement

Within first level, children are developing their understanding of the concept of capacity and the associated language.

Children's responses indicate that they need to refine their understanding that capacity is:

- to receive or contain, e.g. this jug has a larger capacity than the tumbler
- the maximum amount or number that can be received or contained e.g. this jug has the capacity to hold 2 litres of juice.

From undertaking a range of practical activities, children should be able to estimate and compare the capacity of different containers.

Reflective questions

How do we develop pupils' spatial awareness in order that they can estimate the capacity of objects?

How do we support pupils to recognise an appropriate strategy to deal with this type of task?

The following links provide some exemplars of learning and teaching resources:
Learning resources: First level Measurement – Area on Education Scotland website.
Link to: http://www.tes.co.uk/teaching-resource/Teachers-TV-Primary-Maths-Measures-6044811/

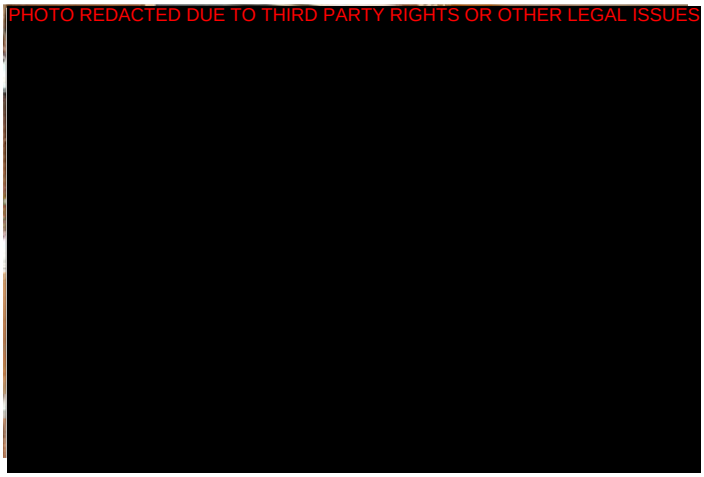
The jug contains orange juice

Estimate how many mugs can be filled with juice from the jug.

A	2	☐
B	4	☐
C	8	☐
D	12	☐

Measurement

P7 – Second Level



First level	Second level	Third level
I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units. **MNU 1-11a** *I can estimate the area of a shape by counting squares or other methods.* **MNU 1-11b**	*I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure.* **MNU 2-11a** *I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems.* **MNU 2-11b** *I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object.* **MNU 2-11c**	*I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task and using a formula to calculate area or volume when required.* **MNU 3-11a** *Having investigated different routes to a solution, I can find the area of compound 2D shapes and the volume of compound 3D objects, applying my knowledge to solve practical problems.* **MTH 3-11b**

Secondary organisers
I can use my knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if my answer is reasonable, sharing my solution with others. **MNU 2-01a** I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. **MNU 2-02a** Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others. **MNU 2-03a** I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. **MNU 2-03b**

Having explored the need for rules for the order of operations in number calculations, I can apply them correctly when solving simple problems.

MTH 2-03c

I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems.

MNU 2-07a

I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method.

MNU 2-07b

Having explored a range of 3D objects and 2D shapes, I can use mathematical language to describe their properties, and through investigation can discuss where and why particular shapes are used in the environment.

MTH 2-16a

Through practical activities, I can show my understanding of the relationship between 3D objects and their nets.

MTH 2-16b

- **Different methods to find the volume of a simple 3D object**

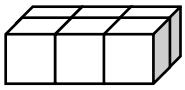
MNU 2-11c

Strengths

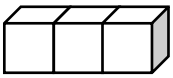
At P7, children build well on their prior learning at first level in finding the volume of a simple 3D object constructed from centimetre cubes, as illustrated below.

Which two blocks together have the same volume as D?

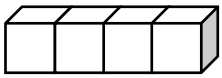
A



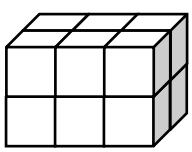
B



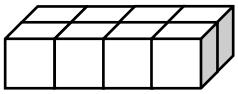
C



D



E



Answer: _____ and _____

Areas for improvement

Based on responses at P7, children's understanding of the concept of spatial awareness is not developing well from learning at first level.

Areas for improvement identified at P4 are not understood well enough or sufficiently reinforced as pupils progress through second level, for example, when children are required to realise that the 3D object is constructed in such a way that they need visualise the 3D representation.

Further support for learning and teaching of this aspect can be found at:

Link to: http://illuminations.nctm.org/ActivityDetail.aspx?ID=125

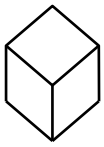
Link to: http://www.topicbox.net/mathematics/3d_shape/

Link to: http://www.bbc.co.uk/learningzone/clips/finding-the-volume-of-a-solid-cuboid/849.html

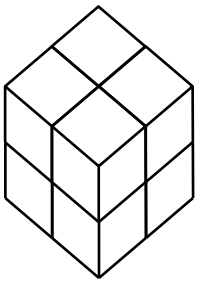
Link to: http://www.bbc.co.uk/learningzone/clips/finding-the-volume-of-a-solid-cuboid-signed/1648.html

Box B is twice as long, twice as wide and twice as high as Box A.

Box A



Box B



Box A holds 1 kilogram of tea.
How much will Box B hold?

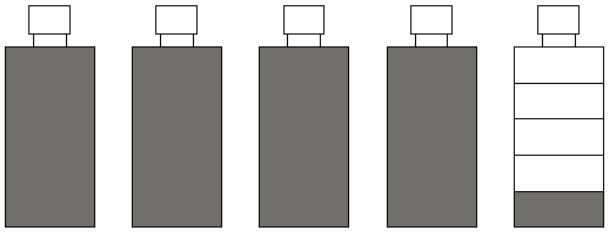
Answer: _____ kilograms

At first level, children learn to estimate the amount an object holds. At second level, this concept is developed further and involves common units of measure, e.g. millilitres and litres.

The concept of volume, capacity and container filling continues to provide challenges for pupils in this type of question at second level as well as at first level.

Link to: http://www.bbc.co.uk/learningzone/clips/capacity-and-measure/13567.html

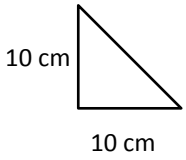
A full bottle holds 1 litre of water

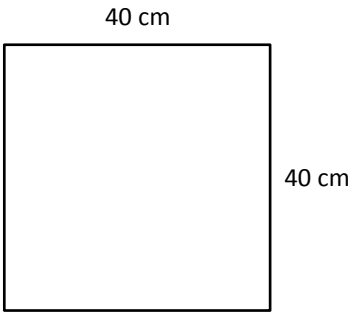


How many 200 ml containers can be filled from these bottles?

Answer: _____

How many of these triangular tiles would you need to tile an area 40 cm long and 40 cm wide?





A	4
B	8
C	16
D	32

Children's skills in recognising and identifying a strategy for finding the volume of a 3D object needs to be developed further.

They need to recognise that volume is a property with three dimensions and can be quantified by finding the total number of same sized units of volume that need to fill the space without gaps or overlap.

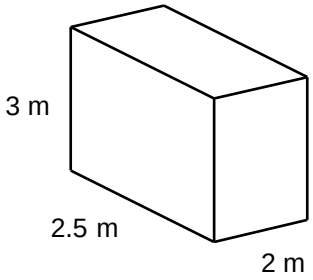
As children begin to use a formulaic approach, such as length x breadth x height, they require competence in understanding and using associative properties, for example:

30 x 20 x 10 = 3 x 2 x 10 x 10 x 10

Children do not have a strong grasp or good understanding of an effective strategy to find the volume of a cube or a cuboid. At P7 only a few children were able to answer correctly a question involving calculating the volume of a cube.

The following questions illustrate these points.

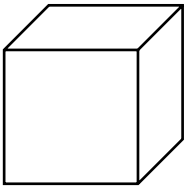
A new television arrives in a box which is 2 m wide, 2.5 m deep and 3 m tall.
What is the volume of the box



Answer: _____ m^3

Each side of a cube has length 4 cm.

Calculate the volume of the cube



Answer: _____ cm^3

- **Using common units of measure and converting between related units of the metric system**

MNU 2-11b

Strengths

At P7, children are able to order lengths and weights, when the question involves basic well known relationships between standard units, for example, ordering 3kg, 30000 g, 30000mg.

Children demonstrate a firm grasp of the learning within first level, for example, they can read scales on measuring instruments which involves finding the value of an easily deduced intermediate graduation.

There is evidence that they can convert between related units of the metric system, when the task involves using well known relationships between standard units, for example convert 3kg into grams.

Areas for improvement

It is evident from the number of children's responses that success within this aspect of measurement is influenced by their knowledge and understanding of key aspects of learning within other numeracy organisers. These are highlighted in the table below.

For example, children do not appear to be able to choose a preferred form in which to carry out a conversion or calculation within the application of measurement.

This is particularly the case when the quantities involve working with numbers in decimal fractions with up to 3 decimal places and mixed numbers. In addition, their knowledge and understanding of the link between a digit, its place and its value is not applied well within measurement.

As previously mentioned with measurement at P4, children are still not competent in reading scales on measuring instruments which involve identifying the graduations.

I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain **the link between a digit, its place and its value.**

MNU 2-02a

I have explored the contexts in which problems involving decimal fractions occur and **can solve related problems using a variety of methods.**

MNU 2-03b

I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can **carry out the necessary calculations to solve related problems**.

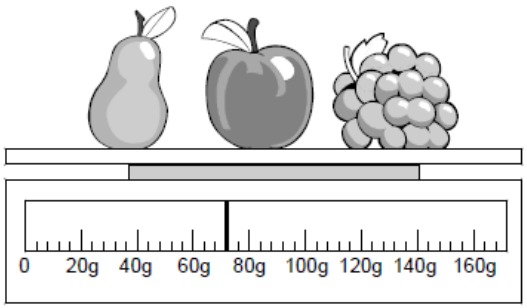
MNU 2-07a

I can show the equivalent forms of simple fractions, decimal fractions and percentages and **can choose my preferred form when solving a problem**, explaining my choice of method.

MNU 2-07b

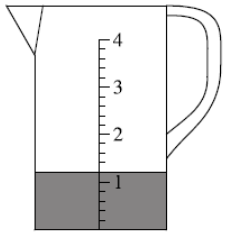
The following questions illustrate the points above:

What is the total weight of the fruit?



Answer: _____ g

This jug has some diluting juice in it
Liam adds water to make 2 litres of juice
How much water did Liam add?



Answer: _____ litre _____ ml

A jug holds $1\frac{3}{4}$ litres of water.

How many millilitres is this?

Answer: _____ ml

A piece of rope measures 3·745 metres
How many centimetres is this?

Answer: _____ cm

The following learning and teaching resources support children's learning in these aspects:
Link to: http://www.iboard.co.uk/iwb/Measure-Capacity-KS2-Version-475
Link to: http://www.bbc.co.uk/learningzone/clips/measurement-of-length/1810.html
Link to: http://www.bbc.co.uk/learningzone/clips/measurement-of-length-signed/1671.html
Link to: http://www.bbc.co.uk/learningzone/clips/capacity-and-measure/13567.html

- **Find the perimeter and area of a simple 2D shape**

MNU 2-11b

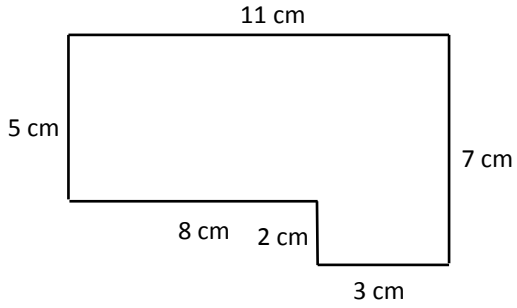
Strengths

At P7 children are able to apply their knowledge of finding the perimeter of a simple 2D shape, relatively well.

This includes simple 2D compound shapes where all the lengths are provided within a diagram.

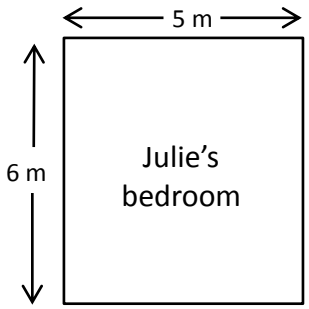
The following questions illustrate these points:

Calculate the perimeter of this shape.



Answer: _____ cm

What is the perimeter of Julie's bedroom?



Answer: _____ m

Areas for improvement

Overall, children's understanding of how to find the area of a simple 2D shape is not well developed.

Based on their responses, the following aspects are commonly misunderstood:

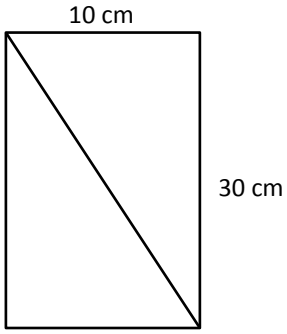
- relationship between a rectangle and a right angled triangle,
- distinction between a perimeter and an area,
- properties of a square.

Children's lack of knowledge within the above mentioned aspects impacts on their ability to use an appropriate strategy within a problem solving context. This includes word problems where children have to extract information to decide on an appropriate strategy.

The lack of understanding of the properties of a square hinders progress in finding the area of a square and surface area of a cube.
The following questions illustrate the aspects of learning in which children do not perform well:

A rectangle is cut into two triangles.

What is the area of one triangle?



Answer: _____ cm^2

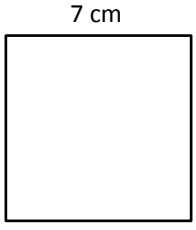
A rectangle is 30 cm long and 10 cm wide.

What is the area of the rectangle?

Answer: _____ cm^2

Why do children provide an answer of $40cm^2$ or $80cm^2$?

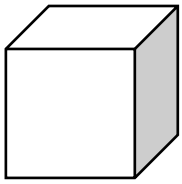
What is the area of this square?



Answer: _____ cm^2

Less than a third could answer this type of question correctly.

Each edge of a cube measures 8 cm.



Each face of the cube is to be painted white

Which calculation gives the area to be painted white in square cm?
Tick (✓) one box

A	6 x 8	☐
B	8 x 8	☐
C	6 x 8 x 8	☐
D	8 x 8 x 8	☐

Further reading
Link to: http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=12
Link to: http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=13

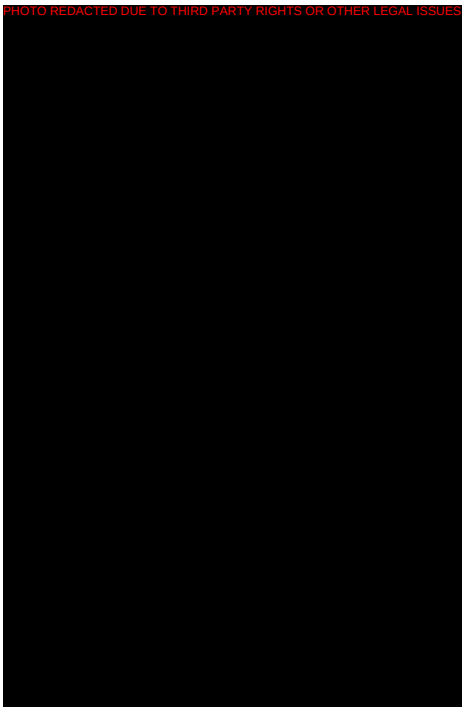
Learning resources: Second level Measurement – Area on Education Scotland website.

National Assessment Resource (NAR):

- Carleith Primary School: Primary learners practice area calculations (login required)

S2 Third Level

Measurement



Second level	Third level	Fourth level
I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure. **MNU 2-11a** *I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems.* **MNU 2-11b** *I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object.* **MNU 2-11c**	*I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task and using a formula to calculate area or volume when required.* **MNU 3-11a** *Having investigated different routes to a solution, I can find the area of compound 2D shapes and the volume of compound 3D objects, applying my knowledge to solve practical problems.* **MTH 3-11b**	*I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations.* **MNU 4-11a** *Through investigating real-life problems involving the surface area of simple 3D shapes, I can explore ways to make the most efficient use of materials and carry out the necessary calculations to solve related problems.* **MTH 4-11b** *I have explored with others the practicalities of the use of 3D objects in everyday life and can solve problems involving the volume of a prism, using a formula to make related calculations when required.* **MTH 4-11c**

Secondary organisers
I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem. ***MNU 3-01a***
I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions. ***MNU 3-03a***
I can continue to recall number facts quickly and use them accurately when making calculations. ***MNU 3-03b***
I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations. ***MNU 3-07a***
Having used practical, pictorial and written methods to develop my understanding, I can convert between whole or mixed numbers and fractions. ***MTH 3-07c***
I can create and evaluate a simple formula representing information contained in a diagram, problem or statement. ***MTH 3-15b***
Having explored the notation and vocabulary associated with whole number powers and the advantages of writing numbers in this form, I can evaluate powers of whole numbers mentally or using technology. ***MTH 3-06a***

- **Using common units of measure and converting between related units of the metric system**

MNU 2-11b

- **Choosing the appropriate units and degree of accuracy for the task**

MNU 3-11a

Strengths

Similar to the story of performance at P7, there is evidence that young people can convert between related units of the metric system when the task involves using well known relationships between standard units, for example convert 3.1kg into grams.

Areas for improvement

At S2, young people continue to experience difficulty in choosing the appropriate units to enable them to solve problems in measurement.

Often it is their lack of skill in carrying out calculations with a wider range of numbers, fractions and decimal fractions, which impacts on their ability to solve problems within familiar contexts.

Tasks which involve the integration of techniques from across the numeracy experiences and outcomes are found to be more challenging by learners than those which require the application of a direct fact.

For example, converting 2 m 30 cm to metres only requires knowledge and understanding of the relationship between metres and centimetres, whereas questions which involve multiple steps, require learners to apply skills from across their learning.
The learning within the following experiences and outcomes are integral to calculations within measurement.

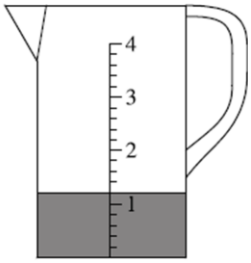
I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions.

MNU 3-03a

I can solve problems by carrying out calculations with a **wide range** of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.

MNU 3-07a

This jug can hold 4 litres of liquid.
$1\frac{3}{4}$ litres are added to the liquid already in the jug.
How much liquid is now in the jug?
Give your answer as a **decimal fraction.**



Answer: _____ litres

Subtract 93.5 grams from 1·656 kg.

Give your answer in grams.

Answer: _____ grams

What possible responses do you think young people gave for this question?

A leaking tap drips 2500 ml of water every hour.

How much water is leaked in one day? Give your answer in **litres.**

Answer: _____ litres

$7\frac{1}{4}$ m of curtain material is cut from a roll of fabric.

The roll has 10 m 4 cm on it.

After the cut has been made, what length of material will still be left on the roll?

Answer: _____ m _____ cm

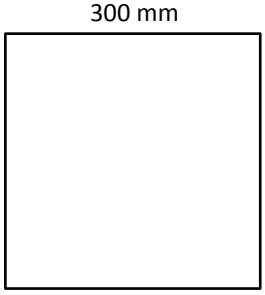
Learning and teaching resources to support young people's learning in these aspects can be found at:
Learning resources: Third level Measurement – Area and volume on Education Scotland website.
Link to: http://www.tes.co.uk/teaching-resource/Maths-Starters-Building-Yard-6049185/
Link to: http://www.backtofrontmaths.com.au/teachers/wp-content/uploads/2012/02/Y6-Unit-1-Decimals-and-Measurement.pdf

- **Find the perimeter and area of a simple 2 D shape** ***MNU 2-11c***
- **Use a formula to calculate area or volume when required** ***MNU 3-11a***
- **Find the area of compound 2D shapes and volume of 3D objects** ***MTH 3-11b***

Strengths

By S2, young people have a firmer grasp in finding the perimeter of 2D shapes when given the dimensions or set within a simple context.

Allan made a square picture frame at school.
What is the perimeter of the frame?

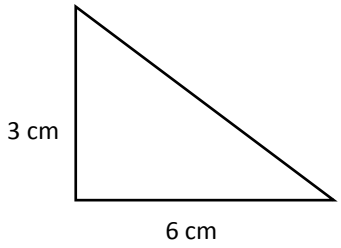


Answer: _____ mm

Areas for improvement

As mentioned within the story of measurement at P7, children's ability to find the area of a simple 2D shape or volume of a simple 3D object is not secure. This is having a detrimental impact on the learning within third level. More needs to be done to strengthen the learning within second level to bridge the learning between second and third level. In order for more children and young people to become competent in these aspects, practitioners who plan learning and teaching at second and third level need to support learners to achieve a deeper understanding of these concepts. This includes the development of young people's spatial awareness in the context of capacity, container packing and best fit algorithm. There is a stronger emphasis on this aspect of learning in the new qualifications within the senior phase.

Calculate the area of this right-angled triangle.



Answer: _____ cm^2

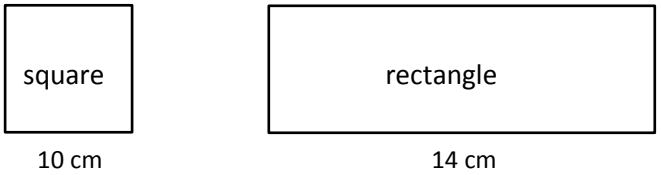
Just over a third of young people answered this type of question correctly.

In planning a range of tasks which involve area or volume, these should include appropriate exposure to the application of inverse operations.

Inverse operations serve as an excellent diagnostic of competence and confidence with direct operations. This is closely related to the development of children and young people's ability to problem solve. Only a few young people were able to answer correctly tasks which involved working backwards.

The following questions illustrate these points

The two shapes below have the same perimeter.
What is the breadth of the rectangle?

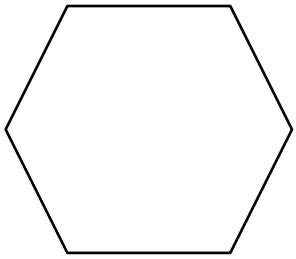


Answer: _____

Note: The diagrams are not to scale.

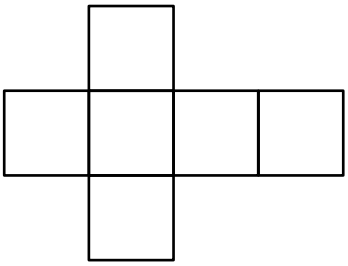
Abdul's father is building a pond in his garden.

The shape of the pond is a regular hexagon.
The perimeter of the pond is 7·5 metres.
What is the length of one side?



Answer: _____ m

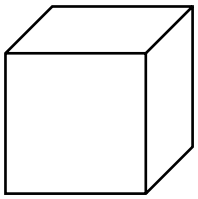
The diagram shows a piece of card. The area of the card is 96 cm^2
It can be folded to make a cube.



What would be the length of one edge of this cube?

Answer: _____ cm

A cube has a volume of 125m^3.
What is the length of one side of the cube?

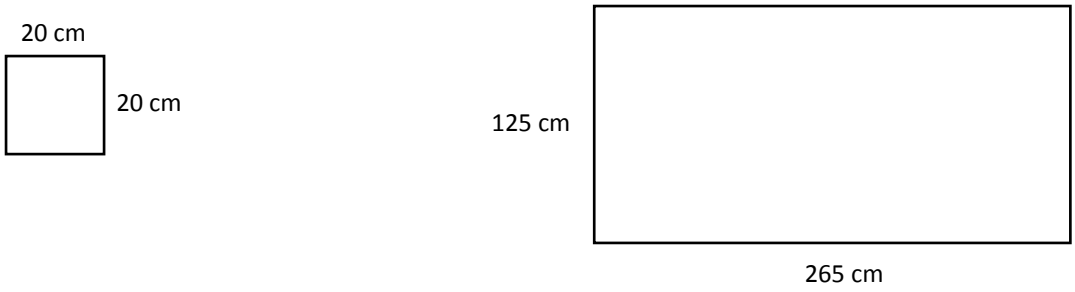


Answer: _____ metres

The length of a rectangular field is 8 metres more than **twice** its width.
If the field is 40 metres wide, what is its perimeter?

Answer: _____ metres

Paul is tiling part of a kitchen wall. The area to be tiled is 265 cm long and 125 cm high.



How many 20 cm square tiles are needed to tile this area?

Answer: _____ tiles

How many centimetre cubes will fit into a cube of side 10cm?

Answer: _____

Learning and teaching resources to support young people's learning can be found at the following links:

National Assessment Resource (NAR):

- Westhill Academy: Pupils consider the cost of flooring the rooms of a house (login required)

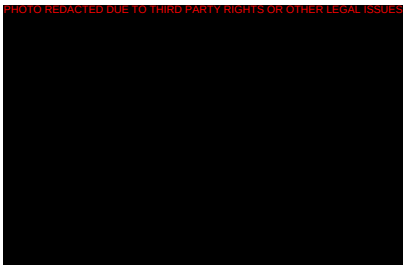
Learning resources: Third level Measurement – Area and volume on Education Scotland website.
Link to: http://www.bbc.co.uk/learningzone/clips/understanding-area-and-perimeter/13556.html

Further reading

Link to: Development of Children's Understanding of Length, Area and Volume Measurement Principles

Section 3.2 Pupil performance in fractions, decimal fractions and percentages, including ratio and proportion

P4 First Level



Early level	First level
I can share out a group of items by making smaller groups and can split a whole object into smaller parts. **MNU 0-07a**	*Having explored fractions by taking part in practical activities, I can show my understanding of:* • *how a single item can be shared equally* • *the notation and vocabulary associated with fractions* • *where simple fractions lie on the number line.* **MNU 1-07a** *Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division.* **MNU 1-07b** *Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent.* **MTH 1-07c**

Secondary organisers
I have investigated how whole numbers are constructed, can understand the importance of zero within the system and can use my knowledge to explain the link between a digit, its place and its value. **MNU 1-02a**
I can use addition, subtraction, multiplication and division when solving problems, making best use of the mental strategies and written skills I have developed. **MNU 1-03a**
I have explored a variety of ways in which data is presented and can ask and answer questions about the information it contains. **MNU 1-20a**
Using technology and other methods, I can display data simply, clearly and accurately by creating tables, charts and diagrams, using simple labelling and scale. **MTH 1-21a**
I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units. **MNU 1-11a**
I can estimate the area of a shape by counting squares or other methods. **MNU 1-11b**

Strengths

Overall, children have an appropriate knowledge and understanding of the core concepts within the early level, for example, sharing out a group of items by making smaller groups and splitting a whole object into smaller parts. This understanding extends into the aspects of learning highlighted in blue in the above table.
However, children's knowledge in other aspects of learning within these experiences and outcomes is not as strong. These are highlighted in red in the above table.

- **Understanding the concept of a fraction and using common fractions to represent parts of a whole or a set**

  MNU 1-07a

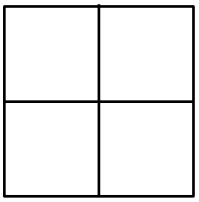
- **Understanding how groups of items can be shared equally**

  MNU 1-07b

Children demonstrate their understanding of how a single item can be shared equally when aided by a pictorial representation. Children can also recognise how to represent a fraction of a whole within a pictorial representation. They cope better with shading sections within a 2D shape diagram with equal sections than they are at selecting equal fractions from a range of pictorial representations. This indicates that children have learnt the basic concept of identifying a fraction of a whole but have not understood fully that equivalent fractions can be represented in different ways.

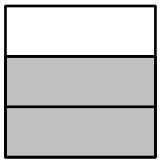
Children are familiar with shading parts of a simple diagram to represent a fraction of a whole or a set. These diagrams tend to be simple 2D shapes such as squares, rectangles or a circle. Other 2D shapes such as a kite or rhombus involve children considering the properties of the shape in order to reason out the parts to shade. Children do less well when more than one step is required.

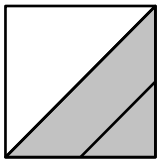
Shade three quarters of the shape.

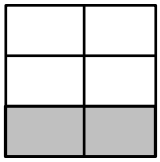


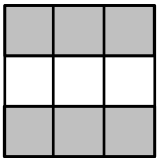
Cross out (×) half of these shapes.

Two of the diagrams show $\frac{2}{3}$ shaded. Tick both.

A  ☐

B  ☐

C  ☐

D  ☐

Typically, over a third of children considered B as an option. Why might this be the case?

Children can share out a group of items equally given a pictorial representation. Pictorial representations enable learners to visualise splitting groups of items into equal amounts.

The following question illustrates the above point:

Kelly cuts all these oranges into quarters.

How many quarters does she have altogether?

A	4	☐
B	8	☐
C	12	☐
D	16	☐

- **Finding a fraction of an amount**

MNU 1-07b

Children can apply their knowledge of division to find a fraction of an amount given simple fractions in word form such as a third, or a quarter within their knowledge of multiplication tables. Their understanding of fractions written in word or mathematical form needs to be strengthened along with knowledge of division in order to achieve greater success in this aspect. They can carry out finding a fraction of an amount given a pictorial representation and within a structured question.

For example:

Find a $\frac{1}{3}$ of 24 stars given pictorial representations of 24 stars.

However, tasks involving additional steps, for example, extracting from the information in the question prior to carrying out the calculation resulted in a lower response rate.

For example:

Find an $\frac{1}{8}$ of the following:

Areas for improvement

- I can find a fraction of an amount by applying my knowledge of division ***MNU 1-07b***
- Understanding the concept and notation of fractions ***MNU 1-07a***

If children do not have a good grasp of multiplication tables, this impacts on their ability to find a fraction of an amount. Their responses indicate that they do not recognise how to find a fraction of an amount when the fraction is expressed in mathematical notation within a simple word problem.

For example, at P4, **less than half of pupils** answered the following questions correctly:

A baker drops a box of 15 eggs.

$\frac{1}{3}$ of the eggs break.

How many of the eggs break?

Answer: _____ eggs

Jack buys 55 plants for his garden.

$\frac{1}{5}$ of them are violets.

How many violets does Jack buy?

Children need to be more fluent in direct operations before being successful at corresponding inverse operations. Performance decreased further, with around a third or fewer answering correctly, with **around a quarter not attempting,** when tasked to carrying out a division calculation involving an amount out-with the multiplication tables.
For example:

There are 51 pupils in Primary 7 at Beach Primary School.

$\frac{1}{3}$ of them can swim.

How many of the Primary 7 pupils can swim?

Answer: _____ pupils

84 pupils are taking part in the school's sports day.
$\frac{1}{6}$ are competing in the long jump.

How many pupils are taking part in the long jump?

Answer: _____ pupils

The car park has 90 spaces.
$\frac{1}{10}$ of the spaces are for disabled drivers.

How many spaces are for disabled drivers?

Answer: _____ spaces

Children's knowledge and understanding of core concepts within other numeracy organisers impacts on their ability to complete tasks relating to fraction work.

Common misunderstandings and misconceptions include:

- Understanding numerator and denominator
- Knowledge of multiplication tables and related division facts particularly the 8, 9 and 10 times tables

In particular, the MNU 1-03a experience and outcome and related learning impacts on their ability to work with fractions.

Their fluency and confidence in the following aspects needs to be strengthened:

I can use addition, subtraction, multiplication and division when solving problems, making best use of the mental strategies and written skills I have developed.

MNU 1-03a

In particular:

- Developing their understanding of the meaning of mathematical notation
- Calculating and solving …..multiplication and divisions problems, with an emphasis on using a range of 'mental' strategies
- Understanding that calculations can be set out differently
- Solving word problems involving the four number operations
- Developing their knowledge of multiplication (not number stations) and related division facts and applying these

So 8 x 4 = 32; 4 x 8 = 32; 32 divided by 4 = 8; 32 divided by 8 = 4; $\frac{1}{8}$ of 32 = 4; $\frac{1}{4}$ of 32 = 8; 32 ÷ ? = 4; 32 ÷ ? = 8.

- Understanding multiplication and division as inverse processes and using the bonds which they know to derive families of eight facts

Learning and teaching resources to support children's learning in this area can be found at

- Thinking Through Mathematics on the NCETM website (National Centre for Excellence in the Teaching of Mathematics)
- Maths4Life Thinking Through Mathematics on the NRDC website (National Research and Development Centre for adult literacy and numeracy)

- **Understanding the concept and notation of where fractions lie on a number line**

MNU 1-07a

- **Using common fractions to represent parts of a whole**

MNU 1-07a

To understand these concepts, learners require a good understanding of place value and how to use this to compare numbers.

Learners will also require support to be able to estimate where a number lies on the number line, to enable their understanding of place value.

Less than half managed to answer correctly questions relating to placing a fraction on a number line. In particular, children's ability to work with mixed numbers on a number line impacted on their success in reading measurements or weights depicted on a range of measuring instruments (***MNU 1-11a***).

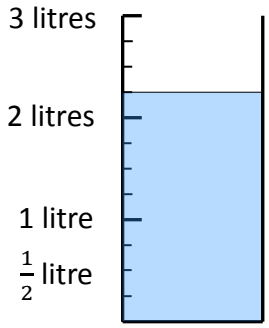
Children's understanding of where fractions lie on a number line is related to their understanding of fractions as part of a whole.

Reflective question

How often do we discuss with learners fractions larger than a quarter and smaller than a half to support understanding of size?

In our interactions with children, do we mainly describe fractions as 'parts of a whole' relating to slices of cake or pizza more often than emphasising other representations and aspects of fraction work?

Jill pours milk into a measuring jug



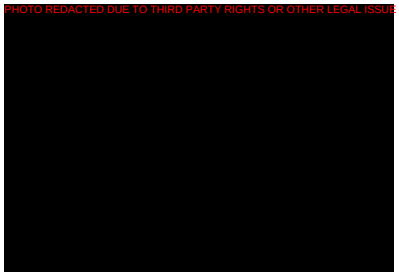
How much milk is in the jug?

Answer: _____ litres

Learning and teaching resources to support the delivery of these organisers can be found at:
Link to: http://www.mathsisfun.com/numbers/fractions-match-frac-line.html
Link to: http://www.bbc.co.uk/learningzone/clips/ordering-fractions-on-a-number-line-signed/1662.html

Fractions, decimal fractions and percentages including ratio and proportion

P7 Second Level



First level	Second level	Third level
Having explored fractions by taking part in practical activities, I can show my understanding of: • *how a single item can be shared equally* • *the notation and vocabulary associated with fractions* • *where simple fractions lie on the number line.* ***MNU 1-07a*** *Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division.* ***MNU 1-07b*** *Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent.* ***MTH 1-07c***	*I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems.* ***MNU 2-07a*** *I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method.* ***MNU 2-07b*** *I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions.* ***MTH 2-07c***	*I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.* ***MNU 3-07a*** *By applying my knowledge of equivalent fractions and common multiples, I can add and subtract commonly used fractions.* ***MTH 3-07b*** *Having used practical, pictorial and written methods to develop my understanding, I can convert between whole or mixed numbers and fractions.* ***MTH 3-07c*** *I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts.* ***MNU 3-08a***

Secondary organisers
I can use my knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if my answer is reasonable, sharing my solution with others. ***MNU 2-01a*** I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. ***MNU 2-02a*** I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. ***MNU 2-03b*** Having discussed the variety of ways and range of media used to present data, I can interpret and draw conclusions from the information displayed, recognising that the presentation may be misleading. ***MNU 2-20a***

Strengths

By P7, children have a firmer grasp of first level concepts. This includes using common fractions to represent parts of a whole e.g. if ¾ of the coffee granules have been used what fraction remains?

• **Unitary fraction of an amount within a simple word problem**	By P7, they are more able to find a fraction of an amount, including working out with the multiplication tables by using associated facts e.g. 1/7 of 63g = 9g so 1/7 of 630g = 90g
• **Finding equivalent fraction, decimal fractions and percentages and using the preferred form in solving problems**	Common equivalences such as 1/10 = 0.1 = 10% and ½ = 50% = 0.5 are relatively well known. Items involving simple percentages and fractions are completed reasonably well. For example, Find 10% of 350g.
• **I can show the equivalent forms of simple fractions, decimal fractions and percentages**	Children are able to show the equivalent forms of common fractions such as 1/10, 2/10 and a ½. The following examples illustrate working with equivalent forms. • Which of the following numbers are less than 2/10? – 0.18 0.208 0.2 0.02 • Write 10/75 as a percentage. • If one in five people are left-handed what percentage is this? • Find 25% of 2kg.

However, children are not particularly familiar with how to apply their knowledge of equivalences to other less common values.
For example, if pupils know the following facts well:

½ = 0.5 = 50%

And that ¼ = 0.25 = 25%

Then is it reasonable that they could work out that ⅛ = ? = ? using useful tools and resources such as a number line.

Only 5% of children could reason from known facts that 3/8 was 0.375 as a decimal fraction.

Reflective question

How do we encourage our learners to use known facts to reason and work out the unfamiliar?

- Develop efficient estimation strategies
- Developing efficient mental and written strategies for addition/subtraction/multiplication and division calculations
- Understanding and using decimal notation and place value in decimal fractions
- Problems involving decimal fractions can solve related problems using a variety of methods**.**

Children are more able to carry out addition and subtraction calculations involving decimal fractions when the decimal fractions are balanced and involve 1/10 or 1/100.
For example:

Calculate

0·33 + 3·30 + 30·33 + 3·03

Answer: _____

Multiplication and division calculations which involve decimal fractions are performed successfully when children have a good grasp of multiplication tables. Calculations involving multiplication are more accurately carried out than those involving division.

Areas for improvement

- **Understanding and using inverse relationships of adding, subtracting, multiplying and dividing when simplifying calculations and solving problems**
- **Carry out the necessary calculations to solve related problems**

MNU 2-07a

Based on their responses, children's ability to carry out calculations involving non-unitary fractions of an amount needs strengthened.

Often, children were not clear enough about the role of the numerator or denominator in carrying out the calculation. Typical errors include:

- Carrying out one operation e.g. division or multiplication
- Confusing the role of the numerator and denominator e.g. dividing by the numerator, multiplying by the denominator

Examples of questions where learners did not respond well:

The vet treated 120 animals last week.

$\frac{9}{10}$ of the animals were rabbits.

How many rabbits did the vet treat last week?

Answer: _____ rabbits

A school has a roll of 588 pupils

$\frac{3}{7}$ of them are boys.

How many boys are there?

Answer: _____ boys

- **Finding equivalent fractions**

MTH 2-07c

Comparing and ordering fractions - locating where they sit on the number line

MTH 2-07c

Further support for learning and teaching of this aspect can be found at:

Link to:
http://www.bbc.co.uk/learningzone/clips/ordering-fractions-on-a-number-line/38.html

Reflective questions

- How do we develop children's understanding of fractions as a value and help them gain a sense of size?
- How do we help children to link their learning of equivalent fractions and ordering fractions?
- How often in our interactions with learners do we use questions such as:

  How many fractions lie between a $\frac{1}{3}$ and a $\frac{1}{2}$? or

  to construct equivalent fractions e.g. $\frac{?}{8} = \frac{1}{2} = \frac{3}{?}$

  Only **around a third** of P7 pupils could demonstrate a firm grasp of equivalence.

  How can we support P7 children to have a stronger grasp of these concepts to ensure they can progress well?

Link to:
http://www.bbc.co.uk/learningzone/clips/ordering-fractions-on-a-number-line-signed/1662.html

- **Understanding the relationship between simple proportion and ratio**

MNU 2-07a

Link to:
http://www.tes.co.uk/teaching-resource/Teachers-TV-Problem-solving-and-Other-Topics-6046026/

Link to:
http://www.bbc.co.uk/skillswise/topic/ratio-and-proportion

Shahnaz had exams in Maths, English and French. Her results are shown below:

Maths: $\frac{36}{40}$ English: $\frac{16}{20}$

French: $\frac{50}{80}$

Put her results in order starting with her best result.

Answer: 1. _____ 2. _____ 3. _____

Children often learn to compose equivalent fractions such as

$\frac{1}{2} = \frac{?}{4} = \frac{?}{8}$ but do not readily identify that $\frac{12}{24}$ belongs to the same family as they have learned by heart rather than understanding the concept.

Using true/false statements can help clarify children's thinking.
For example;
Is the following statement true or false, explain your answer :

- The larger the denominator the smaller the value of the fraction
- $\frac{4}{9}$ is within the same family as $\frac{16}{18}$

Aspects of fraction work such as equivalence act as a basis for applications of multiplicative reasoning using ratio, percentages and proportionality at third level. The simplification of fractions at second level needs to prepare the progression pathway to concepts within third level such as increasing and decreasing proportionally.

- **'…decimal fractions are used and can carry out the necessary calculations to solve related problems.'**

MNU 2-07a

Secondary organisers

- ***I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value.***

MNU 2-02a

- ***I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods.***

MNU 2-03b

Children's knowledge and understanding of the following aspects from other numeracy experiences and outcomes impacts on their ability to carry out calculations involving decimal fractions.
For example:

- estimate, calculate then check an answer to decide if it is reasonable
- understand and use decimal notation and place value in decimal fraction calculations
- use efficient written strategies for addition, subtraction, multiplication and division calculations
- demonstrate fluency and confidence with multiplication tables, particularly the 7, 8, and 9 times tables
- demonstrate fluency in direct operations in order to be successful at corresponding inverse operations.

Children's responses to questions indicate an application of a 'rule' learnt at either an earlier stage or within another concept being applied to decimal fraction calculations. These include:

- Partitioning e.g. 3.7 + 4.4 = 7.11 or 50.26 x 4 = 200.104
- 'Taking the smaller number from the larger number' e.g. 12.35 – 6.897 = 6.547
- Add a zero 23.1 x 100 = 23.100

Children are not as accurate with decimal fraction calculations which involve:

- balancing the calculation to carry out the required operation
- tenths and thousandths.

For example:
If six tenths as a decimal fraction is 0.6.
How would you write as a decimal fraction:
- three hundredths
- eleven thousandths
- eleven tenths

Four tenths is the same as ------- hundredths
How many different numbers could you write down which lie between 0.41 and 0.42?

Improving on these skills within second level will provide a stronger platform on which to build and develop subsequent learning within decimal fraction work. Evidence from the analysis of children's responses suggests that their skills in carrying out decimal fraction calculations accurately, remains an area for improvement. Too many children remain unsure as to what to do with the following types of calculations:

10 – 2.086 9.07 – 6.531	(place value/balancing)
976 divided by 5	(forming a decimal fraction)
3 divided by 4	(knowing how to tackle division involving a whole number requiring forming a decimal fraction)

A significant minority of children can complete decimal fraction operations involving addition, subtraction, multiplication and division but do not place any importance on using decimal notation within their answer. Similarly, a significant minority of children are confused with the terminology of a decimal fraction. For example, write 25% as a **decimal fraction** was not well understood.

Reflective question

Across the learning community are we consistent in our:

- **expectations in the terminology decimal fractions;**
- **emphasis on rounding to the nearest tenth, hundredth or thousandth ?**

- **Having discussed the variety of ways and range of media used to present data, I can interpret and draw conclusions from the information displayed.**

MNU 2-20a

By the end of P7, children have explored a variety of ways to present, interpret and draw conclusions from data. However, within the SSLN survey, they were not as adept at interpreting and drawing conclusions from pie charts involving percentages.

As children will be exposed to this type of graph frequently across the curriculum at third level, it is important that they have a firmer grasp of this aspect.

Further resources to support learning and teaching with this aspect can be found at:
Link to: http://www.tes.co.uk/teaching-resource/Teachers-TV-Problem-solving-and-Other-Topics-6046026/
Link to: http://www.bbc.co.uk/bitesize/ks2/maths/number/fractions_to_decimals/play/
Link to: http://nzmaths.co.nz/resource/modeling-numbers-decimals
Link to: http://www2.nzmaths.co.nz/LearningObjects/numbers/numbersEndecimal.html

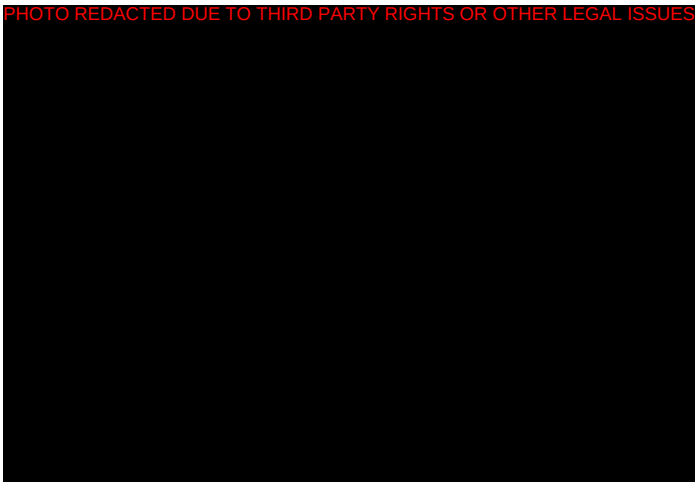
The following NAR exemplar provides an illustration of children's experiences in choosing from an extended range of tables, charts, diagrams and graphs.

National Assessment Resource (NAR):

- Wallace Hall Primary: Pupils learn about WWII Fatalities (login required)

Fractions, decimal fractions and percentages including ratio and proportion

S2 Third Level



Second level	Third level	Fourth level
I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems. **MNU 2-07a** *I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method.* **MNU 2-07b** *I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions.* **MTH 2-**	*I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.* **MNU 3-07a** *By applying my knowledge of equivalent fractions and common multiples, I can add and subtract commonly used fractions.* **MTH 3-07b** *Having used practical, pictorial and written methods to develop my understanding, I can convert between whole or mixed numbers and fractions.* **MTH 3-07c** *I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts.* **MNU 3-08a**	*I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices.* **MNU 4-07a** *I can solve problems involving fractions and mixed numbers in context, using addition, subtraction or multiplication.* **MTH 4-07b** *Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems.* **MNU 4-08a**

Secondary organisers
I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions. **MNU 3-03a** I can continue to recall number facts quickly and use them accurately when making calculations. **MNU 3-03b**

Strengths

By S2, young people have a stronger grasp of the following concepts:

- Finding a fraction of an amount using common fractions e.g. $\frac{1}{7}$of 630g
- Carrying out calculations involving common percentages e.g. 10%, 25%, 50% within simple word problems
- Using equivalent forms of simple fractions and percentages e.g. 75% = $\frac{3}{4}$
- Increasing or decreasing proportionally quantities within straightforward contexts
- Carrying out calculations with decimal fractions (just over half can do this accurately).

Areas for improvement

- **Carrying out calculations with a wide range of fractions, decimal fractions and percentages**

MNU 3-07a

Young people are not particularly adept at carrying out calculations involving a wider range of fractions. A significant majority cannot complete accurately calculations of the form:

$\frac{3}{7}$ of 80·5? = ? $\frac{7}{30}$ of 150 = ? $\frac{20}{100}$ of 4350 = ?

And related inverse problems such as $\frac{?}{5}$ of 60 = 24

Inverse operations serve as a diagnostic tool in assessing learners' confidence and fluency in direct operations. They are important for subsequent progress to other areas of learning including algebraic thinking.

At the P7 stage, it was noted that children needed to strengthen their knowledge and understanding of finding equivalent fractions, decimal fractions and percentages and using the preferred form in solving problems. Children's confidence and mastery needs to be developed to ensure that abilities and skills can be steadily deepened and understood within third level. At S2, young people are not competent in working with a wider range of fractions, decimal fractions and percentages.

For example, typically less than a fifth could convert

- an $\frac{1}{8}$ to a decimal fraction or a percentage.
- 0.01 to a fraction or
- $\frac{3}{25}$ to a percentage.

At S2, questions involving a percentage of an amount involved learners in applying their knowledge of widely used percentages to a wider range of percentages. The assumption being 'If you know this fact' then you know how to find.'

For example, if you know how to find 1% and 10% then you know how to find a multiple of 10% and break down other percentages using related facts e.g. 37.5%.
Young people's knowledge of these widely used percentages should support them in carrying out calculations involving multiples of 10% or those percentages which require to be broken down e.g. 12.5%.

Further support for learning and teaching of this aspect can be found at:
Learning resources: Third level – Fractions on Education Scotland website.
Learning resources: Third level – Fractions, decimal fractions and percentages on Education Scotland website.

Link to: http://www.tes.co.uk/teaching-resource/Percentages-Price-Discount-on-Suitcase-6039016
Link to: http://www.tes.co.uk/teaching-resource/Percentages-VAT-on-Suitcase-6085219

- **Understanding the relationship between simple proportion and ratio and using these concepts to solve problems in context**

MNU 3-08a

Young people are more able to carry out 'stepped-out' ratio questions than those requiring identification of a strategy, extraction of information, logical thinking and organisation of thought.

However, learners often misrecognise ratio problems as additive and so they will need experience in a wide range of different contexts.

For example:

30 litres of water are mixed with 5 litres of orange concentrate to make an orange drink.

Work out the ratio of water to orange concentrate.

Write the ratio in its simplest form.

Answer: _____

Reflective questions

How are we linking and integrating techniques from fraction work at the earlier level to support learners' understanding of ratio and proportion?

How are we linking the use of ratio and proportion to learning across other curriculum areas?

How do we steadily build learners understanding of the relationship between two quantities and its relevance to life and areas such as science?

How do we progress learning from tabular, unitary methods to extracting ratios and fractional multiples?

Further support for learning and teaching of this aspect can be found at:

Link to: http://www.bbc.co.uk/learningzone/clips/the-apprentice-buying-ingredients-for-sausages/11300.html

Section 3.3 Ideas of chance and uncertainty

The implementation of Curriculum for Excellence (CfE) has brought about important changes to the curriculum and learning and teaching with an increased emphasis upon numeracy, and expectations higher than previously. Practitioners are aware of the need to revisit the statements of experiences and outcomes where the focus and level of demand has been raised. Within the Information Handling 'Ideas of chance and uncertainty' numeracy organiser the statements of experiences and outcomes reflect this increased expectation with lines of development being cumulatively grown from an earlier stage than previously.

Collaboration with colleagues in relation to these increased expectations and pathways of progression will encourage a shared understanding of these revised standards as well as support the planning of relevant contexts for learning. The increased expectations in relation to the 'ideas of chance and uncertainty' organiser reflect the skills young people need to apply within their daily lives and in the world of work.

> "Statistics cannot be ignored in either mathematics, or in a wide range of other taught subjects where problem definition, data collection, analysis and interpretation skills are essential to strengthen subject-specific knowledge."
>
> Professor Valerie Isham
> President of The Royal Statistical Society

In the Scottish Survey of Literacy and Numeracy (SSLN) 2011, teachers reported back, through the teacher questionnaires, high levels of confidence in delivering the Curriculum for Excellence numeracy experiences and outcomes. The one exception was the area of 'ideas of chance and uncertainty', where primary school teachers and secondary non-maths teachers expressed less confidence. Secondary non-mathematics teachers, particularly within the arts and humanities curriculum areas reported the least confidence in delivering this aspect of numeracy. It is important to note that the introduction of CfE in secondary schools started with S1 in 2010/2011 and those teachers who responded to the questionnaire were therefore not yet following the CfE with their S2 classes. It should also be noted that the demands and opportunities to promote numeracy across all areas of the curriculum may not necessarily have an even or smooth profile across CfE levels or the topics and themes being taught. Numeracy is best promoted when it is not forced into every lesson but when it occurs naturally within relevant and real-life contexts. It is therefore,

appropriate that the 'ideas of chance and uncertainty' organiser has an uneven profile across curriculum areas.

This professional learning resource provides the opportunity to explore further:

- relevant contexts for learning, and
- children and young people's performance in 'ideas of chance and uncertainty'.

Activity 4

Reflective question

Individually, or with your colleagues, take a few minutes to explore and note down your understanding of the purpose and key ideas behind the learning within the numeracy organiser 'ideas of chance and uncertainty'.

Now consider the following statements:

Read

> "From working out the best deals in the supermarket to understanding trends and probabilities that affect decisions in business and politics, people's ability to interpret data and their sources has never been more important. A good grounding in the application and use of statistics is essential to everyday life ad future education…"
>
> *Jane Curtis*
> *President of the Institute and Faculty of Actuaries*

> Statistics is about using data as the evidence on which to make decisions and to solve problems. It has widespread applications in policy, society, the economy and the academic world.
>
> Digital technology is providing data on a scale that was unimaginable just a few years ago, and this trend is set to continue. Our national prosperity is closely linked to our ability to control, understand and make use of this supply of data.
>
> *The Future of Statistics in our schools and colleges,*
> *Roger Porkess, The Royal Statistical Society and Actuarial Profession*

> My learning in numeracy enables me to:
>
> - Interpret numerical information appropriately and use it to draw conclusions, assess risk, make reasoned evaluations and informed decisions.
>
> *Numeracy across learning Principles and Practice Numeracy*

Reflecting on the above consider:

- **How well does your collective view mirror that expressed within the statements above?**

In order to develop fully, children and young people's understanding of 'ideas of chance and uncertainty', it is important to acknowledge that this is so much more than learning a number of procedural rules. Developing and promoting the learning within the 'ideas of chance and uncertainty' organiser enriches learning within other curriculum areas, for

example, within social subjects, judging reliable sources of evidence against unreliable sources of evidence to discuss an event or within science carrying out simple investigations, making predictions, taking account of the importance of carrying out a fair test.

The following examples provide a few illustrations of relevant contexts to promote the development of 'idea of chance and uncertainty' across the curriculum. These can be adapted to reflect the age group of learners

Social Studies
What do people do?
What decisions do we make about how we work, form our families, pass our leisure, live our lives?
How do we shape the society we live in?
http://www.significancemagazine.org

Modern Studies
Probability of Parties winning elections
http://www.parliament.uk/education/online-resources/subject-guides/elections/

Geography
Weather – Adult Literacies Online
http://www.aloscotland.com/alo/viewresource.htm?id=225

Climate change and numeracy

- **Resource 4:** Climate change evidence
- **Resource 8:** Climate change weather

Expressive Arts
'My statistician could have painted that?'
A statistical enquiry into modern art.
http://www.significancemagazine.org

Drama
http://wickedthemusicaleducation.com.au/pdfs/WICKED_Themes_Choices.pdf
Choices and consequences of characters (e.g. bullying)
Links to Health and wellbeing.

Sciences
What makes a fair experiment?
The Higgs Boson – Certainty
http://www.significancemagazine.org

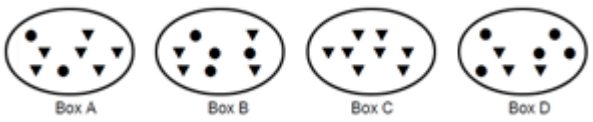
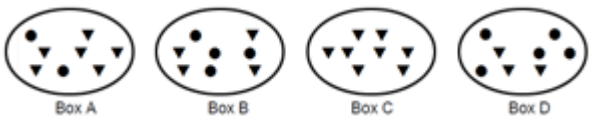
Exploring variables and informed decisions/ reasoning behind predictions (considering all factors which effect results – problem solving logical thinking)
http://www.conceptcartoons.com/

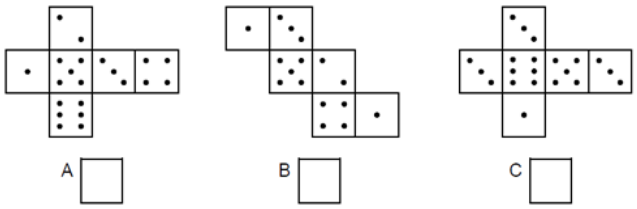
Thinking about maths - chance and uncertainty
http://www.schoolscience.co.uk/_db/_documents/cc_maths_dice.pdf

Health and wellbeing
Forecasting / Human sporting achievements /
Babies, bottles and charts
Effects of alcohol/ behaviour etc
http://www.significancemagazine.org

Health & wellbeing risk and chance
http://choicesforlifeonline.org/

Children and young people's strengths and areas for improvement within 'ideas of chance and uncertainty'

First level	Comment
I can use appropriate vocabulary to describe the likelihood of events occurring, using the knowledge and experiences of myself and others to guide me. ***MNU 1-22a***	Whilst there is no specific E & O at the early level, children can gain an understanding of 'the idea of chance and uncertainty' through the use of appropriate vocabulary e.g. fair.
Early level ***becoming aware of how the vocabulary of chance (such as likely and unlikely, never, always) is used in daily routines.***	Children have a good understanding of an introductory level of the vocabulary of chance (such as, likely and unlikely, never, always, certain, impossible).
First level ***understanding concepts such as likely, probable, unlikely; certain, never; possible and impossible and using this vocabulary in daily life***	Their understanding of concepts such as probable, possible ,equally likely and the application of more than one condition is not as well-founded. At this level, it is important to make connections to using this vocabulary in daily life.
Marie takes a shape out of a box without looking.  It is certain that the shape is a triangle. Which box is it? Answer: _____	At first level, children's familiarity with expressing 'ideas of chance and uncertainty' from written to numerical form needs to be developed further. For example, expressing one in six as 1/6
Marie takes a shape out of a box without looking.  It is very likely but not certain that the shape is a triangle. Which box is it? Answer: _____	http://nrich.maths.org/content/id/7556/preview/# A number of starter activities which develop the language and concept of chance and uncertainty. http://nrich.maths.org/7312 This article gives teachers advice on approaches to introducing chance and uncertainty in the classroom.

Second level	Comment
I can conduct simple experiments involving chance and communicate my predictions and findings using the vocabulary of probability. ***MNU 2-22a*** ***Second level*** ***assigning numerical values to the likelihood of the occurrence of simple events understanding, for example:*** • ***equal chance*** • ***fifty-fifty*** • ***one in two, two in three, etc*** • ***percentage chance*** ***becoming aware of how the implications of chance are used in daily routines, decision making and the media*** Each of these can be cut out and folded to make a dice.  Which one offers the best chance of throwing an even number? Tick (□) one box. See: Appendix 8	Children have a good grasp of concepts within the first level, as well as being able to assign numerical values to the likelihood of the occurrence of simple events. Integrating techniques and skills from other numeracy organisers, for example, working with fractional representations, presents more of a challenge. http://nrich.maths.org/6107 Activities which question/probe the nature of randomness. The Monty Hall problem http://www.bbc.co.uk/learningzone/clips/the-monty-hall-problem-probabilities-and-game-shows-explained/11261.html

Third level	Comment
I can find the probability of a simple event happening and explain why the consequences of the event, as well as its probability, should be considered when making choices. ***MNU 3-22a*** ***Third level*** ***understanding and using the probability scale 0-1 in simple experiments*** ***understanding that the probability of an event not happening is (1 minus the probability of it occurring), for example the probability of throwing a 1 on a die is 1/6, the probability of not throwing a 1 is (1 - 1/6) i.e. 5/6 identifying all possible mutually exclusive outcomes of a single event and assigning numerical values to the probability of each outcome using practical activities to develop an understanding of the link between the frequency of an event occurring and the probability of it occurring*** ***investigating real-life situations which involve making decisions based on the likelihood of events occurring*** ***beginning to understand the concept of a 'random' outcome (i.e. equal probability of an occurrence happening)*** ***discussing how methods of collecting information may affect the nature of the data collected and the conclusions drawn or predictions made, e.g. effects of sample size or 'bias', robustness of data*** See: Appendix 8	It is important to note that the introduction of CfE in secondary schools started with S1 in 2010/2011 and therefore teachers were not yet following CfE with their S2 classes. As previously mentioned, there is an increased emphasis with regard to 'ideas of chance and uncertainty' than in the 5-14 mathematics curriculum. However, the survey still acts as a benchmark of young people's views and experiences. As teachers develop and embed the key ideas and concepts in this numeracy organiser within their programmes of learning, young people will become increasingly competent and confident in their understanding of 'ideas of chance and uncertainty'. This includes their understanding of 'sampling' http:www.rgs.org (search sampling) http://nrich.maths.org/1077 When you roll two ordinary six-faced dice like these and add together the two numbers, what results could you get? Do you have more chance of getting one answer than any other? If so, what is that answer? And why? Extension activities. http://nrich.maths.org/7222 Do you feel lucky? http://www.tes.co.uk/teaching-resource/KS3-Maths-Lesson-Starters-Scrapyard-Maths-6020119/

Additional resources

The Royal Institution Christmas Lectures (2006)
The Num8er My5teries: the secret of the winning streak.
http://www.rigb.org/contentControl?action=displayContent&id=00000000386
What is the best tactic for surviving on a game show? Which numbers will win me the lottery? This video lasts 40 minutes.

All levels of support (progression & pedagogy)

- http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M12501P.htm
- http://www.mathrealm.com/Books/RL_Probability.pdf
- http://www.bbc.co.uk/skillswise/topic/probability

Education Scotland website: Ready for emergencies.

National Assessment Resource (NAR):

- Lochgilphead joint campus pupils assess marketing materials (login required)

Appendix 1

Performance overview in Measurement

From the more detailed analysis of children and young people's performance within **measurement**, the following strengths and areas for development were identified:

Key strengths Pupils demonstrate ability in:

First level	Second level	Third level
MNU 1-11a estimate and measure • ordering real life objects according to size • ordering real life objects according to weight • estimating the length of the objects they can visualise within their immediate environment • reading measurements from a unitary scale. • estimating a measurement to the nearest whole unit. • simple measure tasks – measuring length of a line or object. • choosing appropriate instrument for measuring • working with scales involving easily recognisable graduations **MNU 1-11b area of 2D shape** • counting whole square centimetres to find area of simple shapes. • counting centimetre cubes to find volume	***MNU 2-11a estimate measure*** ***MNU 2-11b conversion and calculation of units*** • Converting and ordering standard units of measure where relationship between units is familiar. • Reading scales involving easily deduced graduations. ***MNU 2-11c perimeter and area of a 2D shape, volume of a 3D object*** • finding the perimeter of a simple 2D shape. • finding the perimeter of a compound 2D shape where all lengths are provided. • finding the volume of a simple, 3D object where cubes can be seen.	***MNU 3-11a Choose appropriate unit and degree of accuracy using a formula to calculate area and volume.*** • Converting related units of measure, including notation involving decimal fractions, where well known relationships between units exist. **MTH 3-11b find the area and volume of complex 2D shapes and 3D objects** • finding the perimeter of a 2D shape.
Areas for Improvement: Pupils require support to develop:		
MNU 1-11a estimate and measure • estimating length or weight when unit of measure is unknown or unfamiliar • estimating length or weight when objects cannot be seen (not within immediate environment) • identifying a successful strategy to apply their knowledge within measure in an unfamiliar situation. • **reading scales where the value of an intermediate graduation needs to be deduced.** ***MNU 1-11b area of a 2D shape*** • finding the area of shapes with half square centimetres. • **estimation of capacity of containers.**	***MNU 2-11a estimate measure*** ***MNU 2-11b conversion and calculation of units*** • **reading scales, where the value of an intermediate graduation needs to be deduced** ***MNU 2-11c perimeter and area of a 2D shape, volume of a 3D object*** • **visualisation of unseen cubes in 3D representations when finding the volume of an object.** • understanding of associative properties of multiplication • **distinction between perimeter and area.** • **knowledge of the relationship between a rectangle and a right angled triangle.** • **knowledge of the properties of a square** • **understanding of the concept of capacity.**	***MNU 3-11a Choose appropriate unit and degree of accuracy using a formula to calculate area and volume.*** • Carrying out calculations with a wider range of numbers • Choosing appropriate units to solve problems **MTH 3-11b find the area and volume of complex 2D shapes and 3D objects** • Understanding of area of 2D shape or volume of simple 3D object. **spatial awareness of capacity, container packing and best fit** • Understanding of how to find the surface area of a cube or cuboid

Appendix 2

Performance overview in Fractions, Decimal Fractions and Percentage

From the more detailed analysis of children and young people's performance within **fractions, decimal fractions and percentages**, the following strengths and areas for development were identified.

Key strengths Pupils demonstrate ability in:

First level	Second level	Third level
• Understanding how a single item can be shared equally when aided by a pictorial representation • Shading parts of a simple diagram to represent a fraction of a whole or a set • Applying knowledge of division to find a fraction of an amount given simple fractions in word form, within their knowledge of the times tables • Finding a fraction of an amount given a pictorial representation and within a structured question	• Finding a fraction of an amount within a simple word problem (unitary fraction) • Finding equivalent fractions, decimal fractions and percentages using the preferred form in solving problems • show equivalent forms of simple fractions, decimal fractions and percentages • Addition and subtraction calculations involving decimal fractions when the decimal fractions are balanced • Multiplication calculations involving decimal fractions	• Finding a fraction of an amount using common fractions • Carrying out calculations involving common percentages • Using equivalent forms of simple fractions and percentages • Increasing/decreasing proportional quantities within straightforward contexts • Carrying out calculations with decimal fractions • Carrying out 'stepped-out' ratio and proportion questions
Areas for Improvement: Pupils require support to develop:		
• Finding a fraction of an amount by applying my knowledge of division -understanding numerator and denominator -knowledge of multiplication tables and related division facts of an amount out with the multiplication tables • Understanding the concept and notation of where fractions lie on a number line, including working with mixed numbers • Their knowledge of other representations and aspects of fraction work	• their knowledge of equivalences • carrying out non-unitary fraction calculations • Role of the numerator and denominator • Comparing and ordering fractions – locating where they sit on a number line • Understanding the relationship between simple proportion and ratio • Accuracy in using decimal fraction notation • Accuracy in decimal fraction calculations • their skills in interpreting and drawing conclusions from pie charts involving percentages • understand and use inverse relationships.	• Carrying out calculations with a wider range of fractions, decimal fractions and percentages • Understanding the relationship between simple proportion and ratio and using these concepts to solve problems in context

Appendix 3

Observing numeracy across learning in curriculum area

Learners know, understand and use aspects of their learning in numeracy to:

Numeracy organiser Aspects of numeracy being observed e.g. measurement, using appropriate units of measure / using scales in technology.	
Estimate, calculate, check • estimate and round appropriately • calculate accurately, where appropriate, using efficient mental and written strategies • check the reasonableness of their answer	
• work in groups or individually to solve problems • apply skills and understanding, creatively and logically to solve problems • explain their thinking and share their approaches and solutions • form and respond to questions • interpret numerical information appropriately and use it to draw conclusions, assess risk, make reasonable evaluations and informed decisions	

Appendix 4

Estimation **More or Less** **P7**

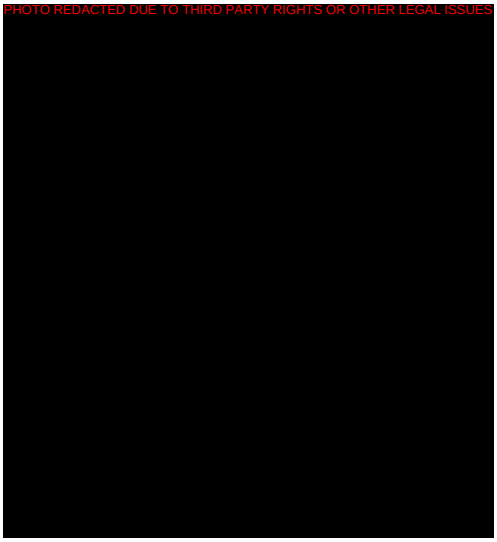
HOW FAST CAN HE RUN?

Usain Bolt is

THE FASTEST MAN IN THE WORLD

Olympic Gold Medal 2012

100 metres in 9.58 seconds



Estimate
Can he run 1000 metres in 97 seconds?

Appendix 5

CHICKEN DINNER

Contamination in Raw Chickens

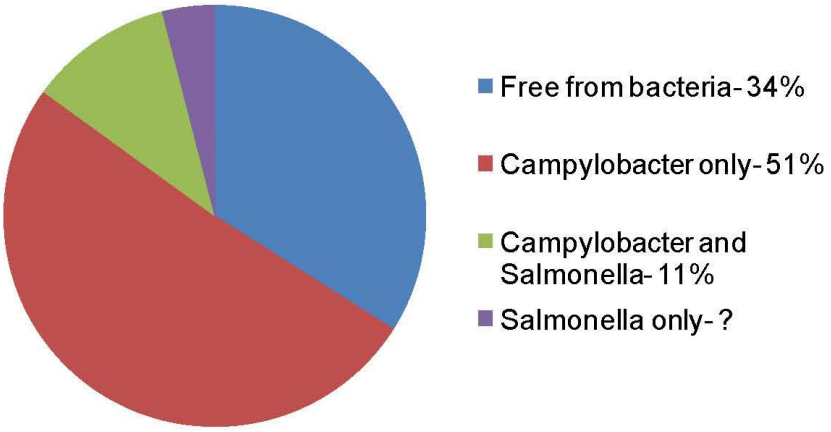


PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Cooking Chicken Safely

- Salmonella and campylobacter are destroyed by heat so it is very important to cook meats like chicken properly
- Chicken should be cooked in a preheated oven at 180-190ºC for 45 minutes per kg plus 20 minutes
- The meat should be steaming hot all the way through
- When you cut into the meat, none of it should be pink
- If juices flow out when you pierce the meat they should be clear

Salmonella and Campylobacter

- Salmonella and campylobacter are bacteria that can cause food poisoning
- If you eat food infected with salmonella the chances of being ill are 100%

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SSLN 2011 (Numeracy)

Appendix 6

Carbon dioxide (CO_2) emissions in Scotland

Carbon dioxide – CO_2 – is a greenhouse gas. It contributes to global warming.

Scotland is trying to reduce the amount of CO_2 it produces.

Table 1 shows Scotland's CO_2 emissions between 1990 and 2005.

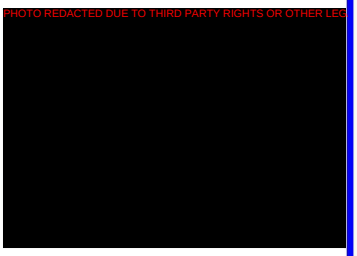

Table 1: Carbon dioxide (CO_2) emissions by sector in Scotland (million tonnes of carbon dioxide equivalent)

Sector	1990	1995	2000	2005
Energy Supply	20·57	24·27	24·02	19·50
Agriculture, business, industrial processes and waste management	11·92	8·28	8·85	8·28
Public and residential	9·05	8·91	8·91	8·65
Transport	10·99	10·88	11·20	11·89
Net land use change and forestry	-2·54	-3·72	-3·94	-4·58
Total net emissions	**50·00**	**48·62**	**49·04**	**43·75**

(Note: Trees remove CO_2 from the atmosphere)

Between 1990 and 2005, Scotland cut its CO_2 emissions by just over 12%

SCOTLAND'S TARGETS

- by the year **2020** - to reduce CO_2 emission by **40%** based on 1990 levels
- by the year **2050** - to reduce CO_2 emission by **80%** based on 1990 levels

SSLN 2011 (Numeracy)

Appendix 7
Numeracy Learning Log

Name: .....

Mental and written calculation methods		
My learning	**Examples of questions I can answer**	**My working and answers**
I can multiply and divide whole numbers and decimals by 10, 100 or 1000	I divided a number by 100. The answer was 2.08 What was my number? A pack containing 1000 sheets of paper is 9.8 cm thick. What is the approximate thickness of one sheet? Explain how you can use the fact $7 \times 9 = 63$ to find the answer to $6.3 \div 0.9$.	
I can calculate with whole numbers and decimals, using mental and written methods as appropriate	Make up an example of an addition or subtraction, involving decimals, that you would do in your head. Now make up an example you would do on paper. Explain the reasons for using these two methods. Work out: $100 - 3 \times 22.5$.	
I can find fractions and percentages of numbers and quantities	Explain how you would find 45% of £80, without using a calculator. Alan says: 'I think three-eighths of a day is 10 hours.' Is he right? Work out which is larger: $^3/_5$ of 380 kg or $^7/_8$ of 360 kg. Write in the missing numbers: 40% of 60 is ☐ 40% of ☐ is 60	
I can describe a problem and identify the mathematics I need to use to solve it		
I can explain my mathematical thinking clearly and systematically, using words, diagrams, numbers and symbols		

My understanding of fractions, ratio and proportion		
My learning	**Examples of questions I can answer**	**My working and answers**
I can solve problems using ratio and proportion and use mathematical language to describe my method		
I can solve problems involving fractions and percentages		
I can simplify fractions and ratios	Write $^{12}/_{30}$ in its simplest form. What did you do to simplify this fraction? What clues do you look for to reduce fractions to their simplest form? How do you know when you have the simplest form of a fraction? The ratio of fruit to cereal in a packet of Fruity Corn pops is 40 : 60. Write this ratio in its simplest form. The manufacturer wants to reduce the ratio of fruit to 35 : 65. Simplify this ratio.	
I can find equivalent fractions, decimals and percentages	Would you rather have $^{3}/_{4}$ or $^{5}/_{6}$ of the same bar of chocolate? Explain your choice. Which of these represent equivalent amounts? 0.4 $^{1}/_{3}$ 60% 3/4 0.2 90% 40% 0.3 3/5 0.3 0.75 0.6 0.25 0.9	

My solving of multi-step problems		
My learning	**Examples of questions I can answer**	**My working and answers**
I can solve problems involving more than one step, identifying the appropriate operation for each step		
I can check that my answer to a problem sounds sensible		

I can present my solutions to a problem clearly, both orally and in writing	The area of a rectangle is 36 cm^2. One of the sides is 3 cm long. What is the perimeter of the rectangle? If another rectangle with the same area had a side of 4 cm, would the perimeter be bigger too? Explain your thinking and record how you worked out the answer to this problem. I think of a number. I find $^1/_5$ of it then add 70. My answer is 97. What number did I think of?	

Appendix 8

Further reading
Education Scotland publication: Curriculum for Excellence Briefing 4: Interdisciplinary Learning

Excellence in Mathematics - Report from the Maths Excellence Group

Digital Education Resource Archive (DERA) publication: Learning together: Mathematics http://dera.ioe.ac.uk/995/

Journey to Excellence: Learning Together: Developing Literacy and Numeracy across learning
http://www.educationscotland.gov.uk/resources/p/professionaldevelopmentpacks/genericresource_tcm4645078.asp

Education Scotland publications: Development and progression in mathematics
Three papers covering:-

- estimation and rounding; number and number processes; and fractions, decimal fractions and percentages;
- analysis; and ideas of chance and uncertainty;
- algebraic thinking.

http://www.educationscotland.gov.uk/resources/d/genericresource_tcm4622917.asp

Education Scotland online resources:
Forest Schools Initiative – Tree mensuration (video)
Messy Outdoor Maths - measurement / weight / problem solving (video)
Improving engagement by developing literacy and numeracy in social studies
Connected magazine article: Making it all add up – making connections between numeracy and other curriculum areas.

National Assessment Resource (NAR):

- Armadale Primary School learn about measurement (login required)
- Dunbar Primary School pupils learn about units of measurement by growing plants (login required)
- Search the NAR database for other 'SSLN numeracy' exemplars.

Appendix 9

First level – P4

SQA Questions - First Level P4				
	Task Numbers			
Measurement				
Area	4024	2603		
	4489	2598		
	5196			
Fractions, Decimal Fractions + Percentages				
Fractions	4229	4284	4342	
	4230	2431	4250	
	4342	2423	2876	
	4183	2781		
	4185			
Ideas of Chance + Uncertainty				
Ideas of Chance + Uncertainty	3679	10039	10037	
	10040			

Source: National Assessment Resource

Appendix 10

Second level – P7

SQA Questions - Second Level P7					
	Task Numbers				
Measurement					
Length	4408	4395	2448		
Height	1745	4188	4733		
Length + Perimeter	1865	5190	1869		
	4657	4619	5190		
	1869	1868			
Height	4777	5425	1734		
	4858	2597	764		
Volume	1829				
Area	2443				
	5394				
Fractions, Decimal Fractions + Percentage					
Fractions	9268				
	1838				
Decimal Fractions	1934	9407	9406	5460	5428
	5406	1832	824	764	
	1734	1755	1803	5406	5284
	5255	5253	5224	5173	4246
	4733	4770	4777	4833	4856
	4859	4860	4864	5148	3345
	9213				
Percentages	5495	1913	1919	5495	
	5430	917	3361	892	
	928	3361			
Ideas of Chance + Uncertainty					
Ideas of Chance + Uncertainty Task	10017a				
	10029				

Source: National Assessment Resource

Appendix 11

Third level – S2

SQA Questions - Third Level S2				
	Task Numbers			
Organiser Measurement				
Length + Perimeter	760			
Volume	4209	4090	5147	
Organiser: Fractions, Decimal Fractions & Percentages				
Fractions:	833	4150	10005	4151
	9355	3666	4123	838
	9345	9288	4049	
Decimal Fractions:	925	929	1993	10006
	9216	3694	3695	3762
	909	925	9209	5480
	3722	3725	3670	3750
	3664			
Percentages:	729	794	795	954
	5164	3355	3357	3677
	9286			
Organiser: Ideas of Chance + Uncertainty				
Ideas of Chance + Uncertainty	3684	4763	5165	
	4793	3904	3693	
	3687	3686	3685	
	4799	10017a	10017b	

Source: National Assessment Resource

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