Department for Education

School efficiency metric

A technical note on the definition and calculation of school efficiency

January 2016

Contents

Summary

About this document

This document provides technical detail on how school efficiency has been defined and calculated in the School Efficiency Metric. It has been produced so that users of the tool can understand in more detail:

- How school efficiency has been defined;
- What data has been used to calculate school efficiency;
- How the School Efficiency Metric has been calculated;
- How school efficiency is presented in the Metric spreadsheet publication and how it can be interpreted.

If you are looking for **more information on why the department has published the School Efficiency Metric** and how you can use it, we recommend reading our guide.

Review date

This technical note will be amended as and when changes are made to the methodology of the School Efficiency Metric. The earliest potential review is ahead of the next publication of the School Efficiency Metric in Spring 2016.

Who is this technical note for?

This is a technical note that describes the methodology of the Metric in full detail. It is intended for people with a good general level of statistical knowledge. Our guide provides an overview of the calculation of the Metric in less technical terms.

The technical note will be of interest to anyone who wishes to understand in detail how school efficiency is measured by the department.

This technical note will also be useful for all users of the School Efficiency Metric who wish to explore the background to the Metric in more technical detail, having read the School Efficiency Metric guide and the introduction to school efficiency contained within the Metric publication.

Defining and measuring school efficiency

Definition of efficiency

Efficiency is generally defined as getting the maximum possible output from the inputs used, or producing a given level of output with the minimum of inputs. The most efficient decision-making unit (DMU) can be defined in two ways: theoretically (showing that, in principle, better results are not obtainable); and empirically (showing that no other organisation performs better). The School Efficiency Metric that the department has developed is a relative empirical measure that emphasises how each school's efficiency compares to similar schools.

There are two major components of efficiency:

- The **output** we are trying to maximise or the objective we are trying to achieve. In a car factory, the output might be the number of cars produced each month.
- The **inputs** that are required or used in the production of the output. In the car factory, this could be the cost of parts and equipment, the running costs of the factory and the number of workers or cost of labour.

Measuring school efficiency in England

Establishing a practical definition of school efficiency has been a key element of the department's work to develop a School Efficiency Metric. This section considers the two major components of efficiency in the context of schools in England, and explains the department's choices when defining and measuring school efficiency.

Output

It is sensible to choose attainment as the output when measuring school efficiency given that the objective of schools is to raise attainment to the highest possible level for all of its pupils. The challenge, therefore, is to choose the most appropriate measure of attainment. At key stage 4, for example, there are three options:

1. Percentage of pupils achieving 5A*-C at GCSEs: This measure does not take into account prior attainment, though modelling techniques could control for this. The major objection to using this measure is that it only recognises achievement over a particular threshold. It does not recognise all levels of achievement and progress: only attainment at a C grade or above is counted, and all grades from A* to C grades are considered equal.

2. Key stage 4 Average Points Scores (APS): This measure assigns points to each grade students achieve in every subject they take. The measure takes the average of the best 8 grades. It therefore overcomes, to some extent, the main objection to the 5A*-C measure. APS, however, also suffers from an absence of information on prior attainment, so any modelling would have to control for that separately.

3. Best 8 Value Added (VA) scores: This measure of attainment calculates how much progress a pupil has made compared to their expected progress, which is estimated based on their prior attainment and the relative progress of their cohort. A pupil's VA is, roughly, the difference between expected and achieved attainment. A school's VA is the average of their pupils' VA figures. The best 8 GCSE results are counted when calculating progress from the key stage 2 baseline.

Value added is currently the most appropriate measure of education output for use in assessing school efficiency. Since it accounts for prior attainment and takes a pupil's performance across a significant number of subjects (and all grades within those subjects) into account, it overcomes the major limitations of the other available measures.

For schools with a primary phase Metric, the output is Key Stage 1 to Key Stage 2 value added. For schools with a primary phase Metric, the output is Key Stage 2 to Key Stage 4 'Best 8 value added'. Once introduced, Progress 8 measures will be used to calculate efficiency scores for secondary schools. There will be no major impact on the efficiency metric from moving from VA to Progress 8.

Inputs

Education is a complex process and many factors, or 'inputs', contribute to a pupil's achievement in school: family and social factors; early educational experiences; aptitude or ability; and every educator or educational establishment a pupil interacts with before the point at which they take a particular examination or test.

In a school efficiency context, we seek only to measure the inputs controlled by the school, and the extent to which the school has contributed to the level of pupil achievement. Ideally, we would strip out that portion of education output that is influenced by inputs outside of a school's control. We do this by focusing on progress made while a pupil is in the school.

At the highest level, the inputs within a school's control are all accounted for by total or per-pupil funding. Therefore, the School Efficiency Metric uses per-pupil funding as the measure of school inputs. The Metric shows how a school performs relative to others, given a certain level of funding.

We include only the funding that schools receive from central government and/or their local authority.[1] We do not include any funding that schools receive from other sources, such as donations, or any self-generated income. Some schools earn extra revenue by, for example, leasing their sports facilities. We do not include this stream of income as it could be seen to discourage income generation. In this document, when we refer to a school's income we refer only to the central government and local authority funding that we include in the measure of inputs.

The School Efficiency Metric methodology uses only one year of per-pupil funding. If we use, say, five years' worth of funding for secondary schools, it would take five years for any significant change in VA or funding to fully work its way through. In the year following a significant change, only around one-fifth of the change would be recorded in the Metric. In addition, counting multiple years of funding would add complexity to the calculation of the Metric.

[1] The funding items that we include are detailed in the "Summary of data used in the School Efficiency Metric" section.

Calculating the School Efficiency Metric

There are three steps to calculating the Efficiency Metric.

In the first step we **calculate an 'efficiency score' for each school** by dividing its value added (VA) by its income per pupil. As value added and income per pupil are measured on different scales, they are not directly comparable. To allow as fair a comparison between the measures as possible, we 'standardise' both before doing the calculation.

In the second step, **we identify a school's unique group of 'efficiency neighbours'**. These are its 50 statistically most similar schools in terms of the proportion of pupils with a statement of SEN or School Action Plus (% SEN) and the proportion of pupils who have been eligible for Free School Meals at any point in the last 6 years (% Ever6 FSM). In addition, a school's 'efficiency neighbours' are of the same phase and organisational type. If a school has a sixth form, its 'efficiency neighbours' are all schools with a sixth form; conversely the 'efficiency neighbours' of schools without a sixth form are all schools without a sixth form. The statistical similarity of the 'matched' schools declines as we add more criteria.

In the third step **we calculate a school's 'efficiency decile'**. We do this by comparing its 'efficiency score' to those of its 'efficiency neighbours'. In each 'efficiency neighbours' group, we band schools into deciles (10 groups of five) based on their 'efficiency score'. A school's relative efficiency is reported in terms of which decile they are located in their unique 'efficiency neighbours' group. A school in decile 1 has one of the largest five 'efficiency scores' (and is therefore one of the five most efficient schools) in its 'efficiency neighbours' group; a school in decile 10 has one of the smallest five 'efficiency scores' (and is therefore one of the five least efficient schools) in its 'efficiency neighbours' group.

In the School Efficiency Metric publication we provide schools with additional information, alongside their 'efficiency decile', to help interpret their relative efficiency. This includes an estimate of the improvement in value added required to reach higher levels of efficiency, given the school's current level of income. More details of this are available in the annex to this note.

Schools that have both a primary and secondary phase receive both a primary and secondary Efficiency Metric. As school income is allocated on a school level, we have had to make assumptions about how this income is divided between phases. Our method for assigning income to each phase of these schools is described in the annex to this note.

The three steps are now discussed in more detail.

First step: calculating an 'efficiency score'

Firstly, we 'standardise' a school's value added and income per pupil.[2] VA and income per pupil are measured on different scales and, consequently, are not directly comparable. Without adjusting either VA or income per pupil, an 'efficiency score' that divides VA by income per pupil places a greater weight on income per pupil than VA: this is due to the higher average of per-pupil income and the wider standard deviation. This would mean a one-unit change in per-pupil funding would lead to a greater change in the 'efficiency score' than a one-unit change in VA.

Standardising is a statistical method of putting both measures on a more equal footing. In practice, it means roughly that having better VA than 75% of schools is just as important as having lower per-pupil income than 75% of schools. We standardise VA and income per pupil so that they have a mean value of 100 and standard deviation[3] of 15.

The formulas for standardising VA and income per pupil are:

Formulas for standardised value added and standardised income per pupil

$$\text{Standardised VA} = 100 + \left[\frac{(\text{VA of school} - \mu_{\text{Value Added}}) \text{ x } 15}{\sigma_{\text{Value Added}}}\right]$$

$$\text{Standardised income per pupil} = 100 + \left[\frac{(\text{Income per pupil of school} - \mu_{\text{Income}}) \text{ x } 15}{\sigma_{\text{Income}}}\right]$$

Variable	Description
$\mu_{\text{Value Added}}$	The (weighted)[4] mean Value Added for all schools of the same phase.
$\sigma_{\text{Value Added}}$	The (weighted) standard deviation of Value Added for all schools of the same phase.
μ_{Income}	The (weighted) mean income per pupil for all schools of the same phase.
σ_{Income}	The (weighted) standard deviation of income per pupil for all schools of the same phase.

[2] The income per pupil figure used to calculate an 'efficiency score' removes any area cost uplift applied to a school's funding. However the income per pupil displayed in the Efficiency Metric spreadsheet publication includes any area cost uplift.

[3] Standard deviation is a measure that reflects how spread out a set of values is from the mean of the set. The smaller the standard deviation is, the closer the set of values is to its mean.

[4] The means and standard deviations used to standardise VA and income per pupil are weighted by school level FTE. This is explained in more detail in the annex to this note.

Secondly we calculate the school's efficiency score. This is its 'standardised' VA divided by its 'standardised' income per pupil, multiplied by 100.

Formula for the school efficiency score

$$\text{Efficiency score} = 100 \times \left[\frac{\text{Standardised VA}}{\text{Standardised income per pupil}}\right]$$

Worked example: Primary school

Consider a primary school with VA of 99.5 and income per pupil of £3000. The (weighted) mean values and standard deviations of VA and income per pupil are:

(Weighted) mean VA = 100;

(Weighted) mean income per pupil = £4000;

(Weighted) standard deviation of VA = 1.5;

(Weighted) standard deviation of income per pupil = 1000

Therefore this school's standardised VA and income per pupil are:

$$\text{Standardised VA} = 100 + \left[\frac{(99.5 - 100)\ x\ 15}{1.5}\right] = 95$$

$$\text{Standardised income per pupil} = 100 + \left[\frac{(3000 - 4000)\ x\ 15}{1000}\right] = 85$$

Its 'efficiency score' is therefore:

$$'\text{Efficiency score}' = \left(\frac{95}{85}\right)\ x\ 100 = 111.8$$

Worked example: Secondary school

Consider a secondary school with VA of 1005 and income per pupil of £6000. The (weighted) mean values and standard deviations of VA and income per pupil are:

(Weighted) mean VA = 1000;

(Weighted) mean income per pupil = £5500;

(Weighted) standard deviation of VA = 20;

(Weighted) standard deviation of income per pupil =1000

Therefore this school's standardised VA and per pupil funding are:

$$'\text{Standardised}'\ \text{VA} = 100 + \left[\frac{(1005 - 1000)\ x\ 15}{20}\right] = 103.75$$

$$'\text{Standardised}'\ \text{income per pupil} = 100 + \left[\frac{(6000 - 5500)\ x\ 15}{1000}\right] = 107.5$$

Its 'efficiency score' is therefore:

$$'\text{Efficiency score}' = \left(\frac{103.75}{107.5}\right)\ x\ 100 = 96.5$$

Second step: identifying a group of 'efficiency neighbours'

Identifying a school's 'efficiency neighbours' group requires two levels of grouping. Firstly, we group all schools with schools of the same phase and organisational type. For example, a secondary academy's efficiency neighbours are all secondary academies. Furthermore we group schools with sixth forms with other schools that have sixth forms and schools without a sixth form with other schools without a sixth form.

In the second level of grouping, we identify the 49 other schools that have the most similar proportions of % Ever6 FSM and % SEN. These are a school's 'efficiency neighbours'. These schools are identified using the Euclidean Distance Matching method. These statistically similar schools, as well as the school that we are grouping, form the 'efficiency neighbours' group of 50.

Technical explanation: Euclidean Distance Matching method

This is a statistical technique used to calculate the "distance" between two data points across, theoretically, many dimensions. In creating 'efficiency neighbours' groups, we use it to calculate the 'distance' between two schools in terms of % SEN and % Ever6 FSM. The smaller this 'distance', known as the 'Euclidean distance', the more similar the schools are in these characteristics.

The calculation of the 'Euclidean distance' involves a comparison of % SEN and % Ever6 FSM. As with VA and income per pupil, these two characteristics are measured on different scales. % SEN = 20% is relatively large, whereas % Ever6 FSM = 20% is relatively small. To overcome the issues that this causes, these values are 'standardised'.

$$\text{Standardised value of \% SEN} = \frac{(\text{\% SEN of school} - \text{Mean value of \% SEN across phase})}{\text{Standard deviation of \% SEN across phase}}$$

$$\text{Standardised value of \% Ever6 FSM} = \frac{(\text{\% Ever6 FSM of school} - \text{Mean value of \% Ever6 FSM across phase})}{\text{Standard deviation of \% Ever6 FSM across phase}}$$

The Euclidean distance between two schools, school X and school Y, is calculated using the following formula.

$$\text{Euclidean Distance between schools X and Y} = \sqrt{(\text{SEN}_x - \text{SEN}_y)^2 + (\text{Ever6}_x - \text{Ever6}_y)^2}$$

Where

Variable	Description
SEN_x	Standardised value of % SEN of school X
SEN_y	Standardised value of % SEN of school Y
Ever6_x	Standardised value of % Ever6 FSM of school X
Ever6_y	Standardised value of % Ever6 FSM of school Y

We repeat these steps to find the Euclidean distance between school X and all other schools. The schools corresponding to the 49 smallest Euclidean Distances are the 49 statistically most similar to School "X" in terms of % SEN and % Ever6 FSM and become its 'efficiency neighbours'. Each school has a unique 'efficiency neighbours' group of 50 including the school itself and its 49 'efficiency neighbours'.

There are 50 schools in each 'efficiency neighbours' group. This achieves a balance between having a manageable number of comparator schools for schools to benchmark against, and ensuring that most schools have at least one 'efficiency neighbour' within 25 miles. Group sizes of 50 ensure that almost 90% of schools have an 'efficiency neighbour' within 25 miles.

Worked example of Euclidean Matching

Consider **school A**. It is a secondary academy, with a sixth form. Its % SEN = 10% and its % Ever6 FSM is 30%.

Consider four potential 'efficiency neighbours' for school A. These are:

School B: A primary academy. Its % SEN is 12% and its % Ever6 FSM is 27%.
School C: A secondary maintained school without a sixth form. Its % SEN is 8% and its % Ever6 FSM is 35%.
School D: A secondary academy with a sixth form. Its % SEN is 12% and its Ever6 FSM is 36%.
School E: A secondary academy with a sixth form. Its % SEN is 23% and its Ever6 FSM is 15%.

The first level of grouping is to identify the schools of the same phase and organisational type as school A. Also, as school A is a school with a sixth form, we also identify the schools that have sixth forms.

School B is a primary academy. Although it is of the same organisational type, it is in a different phase to school A. Therefore it can't be an 'efficiency neighbour' to school A.

Similarly school C cannot be an 'efficiency neighbour' to school A as it is of a different organisational type – it is a maintained school whereas school A is an academy – and doesn't have a sixth form, whereas school A does.

Schools D and E can be 'efficiency neighbours' to school A, as, like school A, they are secondary academies with sixth forms.

In the second level of grouping, we calculate the Euclidean Distances between school A and the schools that pass the first level of grouping (schools D and E), in terms of % SEN and % Ever6 FSM. Their school level characteristics in standardised form are:

School A: Standardised SEN = 0.10. Standardised Ever6 FSM = -0.20
School D: Standardised SEN = 0.11. Standardised Ever6 FSM = -0.10
School E: Standardised SEN = 0.18. Standardised Ever6 FSM = -0.40

Therefore:

$$\text{Euclidean Distance between schools A and D} = \sqrt{(0.10 - 0.11)^2 + (-0.20 - 0.10)^2} = 0.10$$

$$\text{Euclidean Distance between schools A and E} = \sqrt{(0.10 - 0.18)^2 + (-0.20 - -0.40)^2} = 0.22$$

The "Euclidean distance" between schools A and D is smaller than between schools A and E. This means that school D is statistically more similar to school A than school E is, in terms of

Worked example of Euclidean Matching

% SEN and % Ever6 FSM.

If these are amongst the 49 smallest "Euclidean distances" between school A and all potential 'efficiency neighbours', school D and school E will be amongst school A's unique 'efficiency neighbours' group.

Third step: calculating a school's 'efficiency decile'

A school's relative efficiency is reported in terms of in which 'efficiency decile' they located within their unique 'efficiency neighbours' group. Every 'efficiency neighbours' group of 50 is split into deciles (10 groups of five) that are based on schools' 'efficiency scores'. How a school's 'efficiency score' translates into its 'efficiency decile' is shown in the following table.

'Efficiency decile'	'Efficiency score' in 'efficiency neighbours' group
1	Largest to 5th largest
2	6th largest to 10th largest
3	11th largest to 15th largest
4	16th largest to 20th largest
5	21st largest to 25th largest
6	26th largest to 30th largest
7	31st largest to 35th largest
8	36th largest to 40th largest
9	41st largest to 45th largest

‘Efficiency decile’	‘Efficiency score’ in ‘efficiency neighbours’ group
10	46th largest to 50th largest

Worked example: Primary school

Consider the primary school from the earlier worked example. Its ‘efficiency score’ is 111.8. In its ‘efficiency neighbours’ group, its ‘efficiency score’ is the 8th largest. Therefore its ‘efficiency decile’ is **decile 2**.

Summary of data used in the School Efficiency Metric

This section summarises the data used in calculating the School Efficiency Metric.

Value added

For the primary phase, we use the key stage 2 overall value added measure for the 2013/14 cohort taking the tests. This measures the progress made by pupils in reading, writing and mathematics between key stage 1 and key stage 2.

For the secondary phase we use the best 8 value added measure for the 2013/14 exam taking cohort. This measures the progress made by pupils between key stage 2 and key stage 4 in the subjects that make up their best 8 GCSE (or equivalent) exam results, including English and Mathematics, at key stage 4.

Income per pupil

We include only the funding that comes from central government and/or their local authority in the measure of income per pupil. Where applicable, we include:

- Schools block funding (which schools receive from the dedicated schools grant given to local authorities);
- 16-19 funding;
- Pupil premium funding – including deprivation and service child pupil premium;
- Year 7 literacy and numeracy catch-up premium;
- PE and sports premium;
- Summer schools programme funding;
- Start-up grants for academies;
- Education services grant;
- Academy insurance.

These funding items were received by maintained schools during the financial year 2013-14 and by academies during the academic year 2013/14. In calculating the Efficiency Metric, we remove the area cost uplift (i.e. the Hybrid Area Cost Adjustment used in school funding calculations) applied to funding received during the period.[5]
This means that schools in high cost areas aren't in a low 'efficiency decile' simply because of their location.

We don't include any funding that schools receive from other organisations, such as donations. Neither do we include any self-generated income.

[5] The data used to populate graphs in the School Efficiency Metric publication is the funding actually received by the schools, including any area cost uplift.

School characteristics

Schools' names, phase and organisational type are correct as at September 2013.

We characterise a school as having a sixth form if it either gets 16-19 funding for the financial year 2013-14 (academic year 2013/14 for academies) or has full-time equivalent pupils in key stage 5. The latter is taken from the School Census of January 2014, and so is correct as of this date.

A school's overall FTE is taken from the School Census of January 2014, and so is correct as of this date.

A school's SEN information is taken from the School Census and the SEN2 data return of January 2014, and so is correct as of this date.

A school's Ever 6 FSM information is taken from the School Census of January 2014, and so is correct as of this date.

Feedback

The December 2015 publication of the Efficiency Metric tool (covering the 2013/14 school year) is the first time the department has provided schools with an overall indicator of relative efficiency. We plan to publish for the second time using updated performance data in Spring 2016. To help us make the tool as effective as possible, we are seeking users' feedback ahead of the second publication.

We are running an online survey on the School Efficiency Metric from 12 January 2016 until the end of February 2016. This survey will take approximately 15 minutes to complete. It will cover:

- How you located the publication and how easy it was to use
- How well you understood the Metric calculation
- How you valued the graphical information in the tool
- Your views on the comparison schools
- The next steps you took once you formed an idea of your relative efficiency

Annex

Assigning income per pupil to each phase of schools with primary and secondary phases

For schools with only one phase, calculating income per pupil is relatively simple. This becomes more complicated when considering the income per pupil of schools with both primary and secondary phases.

One possible method for calculating the income per pupil in these cases is to use the same method used to calculate it for schools with only one phase, which is to divide total income by total number of pupils, and assign the resulting figure to each phase. However, schools tend to receive higher funding per pupil for their secondary pupils than for their primary pupils. If we were to use this method, the calculated income per pupil figure is likely to be an overestimate of the figure received for the school's primary pupils, and an underestimate of the figure received for their secondary pupils. This becomes clear in the worked example below.

Therefore, we have devised another method for assigning income per pupil to each phase of such schools. The method is described in the steps below. This method assigns a more realistic income per pupil figure to each phase than the alternative method of splitting funding according to pupil numbers.

The steps used to assign phase-specific funding are:

Method for assigning income per pupil to each phase of schools with more than one phase

1) **Calculate total income of the school.** This is the sum of the individual funding items described in the "Summary of data used in the school efficiency metric" calculation.
2) **Remove the area cost uplift applied to funding.**
3) **Calculate local authority age weighted pupil units (LA AWPUs) as an implied percentage.** One of the funding factors used by local authorities (LAs) when allocating funding to schools is a "basic entitlement" factor. In allocating the funding under this factor, LAs decide on an amount per pupil to give to primary and secondary pupils. This per pupil amount is the age weighted pupil unit (AWPU). Using this, we calculate a phase-specific LA AWPU %, using the following formula.

$$\text{Primary LA AWPU \%} = \left(\frac{\text{Primary AWPU of LA}}{\text{Primary AWPU of LA} + \text{Secondary AWPU of LA}}\right)$$

$$\text{Secondary LA AWPU \%} = \left(\frac{\text{Secondary AWPU of LA}^{6}}{\text{Primary AWPU of LA} + \text{Secondary AWPU of LA}}\right)$$

Method for assigning income per pupil to each phase of schools with more than one phase

4) **Calculate phase specific "units".**

$$\text{School primary units} = \text{Primary LA AWPU \% of school's LA x School's primary FTE}$$

$$\text{School secondary units} = \text{Secondary LA AWPU \% of school's LA x School's secondary FTE}$$

5) **Calculate total funding per unit.**

$$\text{School total funding per unit} = \frac{\text{School's total funding}}{\text{School primary units} + \text{School secondary units}}$$

6) **Calculate implied total phase funding of school.**

$$\text{School implied primary funding} = \text{School total funding per unit x school primary units}$$

$$\text{School implied secondary funding} = \text{School total funding per unit x school secondary units}$$

7) **Calculate implied phase funding per pupil of school.**

$$\text{School implied primary funding per pupil} = \frac{\text{School's implied primary funding}}{\text{School's primary FTE}}$$

$$\text{School implied secondary funding per pupil} = \frac{\text{School's implied secondary funding}}{\text{School's secondary FTE}}$$

Worked example: All through school

[6] Local authorities have specific primary, key stage 3 and key stage 4 AWPUs. The secondary AWPU used here is an average of key stage 3 and 4 AWPUs, weighted by pupil numbers in the key stages across the local authority.

Consider an all through school with the following information.

$$\text{Primary FTE} = 300$$

$$\text{Secondary FTE} = 700$$

$$\text{Total funding} = £4{,}000{,}000$$

$$\text{Local authority area cost uplift} = 1$$

$$\text{Total funding after removing the area cost uplift} = £4{,}000{,}000$$

$$\text{Primary AWPU of school's LA} = £4{,}000$$

$$\text{Secondary AWPU of school's LA} = £6{,}000$$

The LA AWPUs as an implied percentage for this school are therefore:

$$\text{Primary LA AWPU \%} = \left(\frac{4{,}000}{4{,}000 + 6{,}000}\right) = 0.4 = 40\%$$

$$\text{Secondary LA AWPU \%} = \left(\frac{6{,}000}{4{,}000 + 6{,}000}\right) = 0.6 = 60\%$$

Using these percentages, the school's phase units are:

$$\text{School primary units} = 40\% \text{ x } 300 = 120$$

$$\text{School secondary units} = 60\% \text{ x } 700 = 420$$

The school's total funding per unit is:

$$\text{School total funding per unit} = \frac{£4{,}000{,}000}{120 + 420} = £7407$$

Using this total funding per unit, the school's implied phase funding per unit is:

$$\text{School implied primary funding} = £7407 \text{ x } 120 = £888{,}889$$

$$\text{School implied secondary funding} = £7407 \text{ x } 420 = £3{,}111{,}111$$

These implied phase funding levels are then used to calculate an implied phase funding per pupil of the school.

$$\text{School implied primary funding per pupil} = \frac{£888{,}889}{300} = £2{,}963$$

$$\text{School implied secondary funding per pupil} = \frac{£3{,}111{,}111}{700} = £4{,}444$$

If we were to use the alternative method of dividing total income by total number of pupils, the funding figure assigned to each phase would be:

$$\text{School alternative primary and secondary funding per pupil} = \frac{£4{,}000{,}000}{1000} = £4{,}000$$

This alternative income per pupil figure of £4,000 is larger than the implied primary figure of £2,963, and smaller than the implied secondary figure of £4,444. As schools tend to receive more funding per pupil for their secondary pupils than their primary pupils, the implied primary and secondary income per pupil figures are likely to be a more accurate reflection of what this

Weighted mean and standard deviation formulas

In calculating a school's 'efficiency score', we 'standardise' its VA and income per pupil using their 'weighted' means and standard deviations. These are weighted by schools' FTE, as in the following formulas.

Formulas for weighted mean and standard deviations

$$\textbf{Weighted mean of VA} = \frac{\sum_{i=1}^{n}(\mathrm{FTE_i\ x\ VA_i})}{\sum_{i=1}^{n}(\mathrm{FTE_i})} = \frac{(\mathrm{FTE_1 x\ VA_1}) + (\mathrm{FTE_2 x\ VA_2}) + \cdots + (\mathrm{FTE_n x\ VA_n})}{\mathrm{FTE_1} + \mathrm{FTE_2} + \cdots + \mathrm{FTE_n}}$$

$$\textbf{Weighted mean of income per pupil} = \frac{\sum_{i=1}^{n}(\mathrm{FTE_i\ x\ Income_i})}{\sum_{i=1}^{n}(\mathrm{FTE_i})} = \frac{(\mathrm{FTE_1 x\ Income_1}) + (\mathrm{FTE_2 x\ Income_2}) + \cdots + (\mathrm{FTE_n x\ Income_n})}{\mathrm{FTE_1} + \mathrm{FTE_2} + \cdots + \mathrm{FTE_n}}$$

$$\textbf{Weighted standard deviation of VA} = \sqrt{\frac{\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{FTE_i\ x\ (VA_i} - \overline{\mathrm{VA}})^2}{\frac{\mathrm{M}-1}{\mathrm{M}}\ \mathrm{x}\ \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{FTE_i}}}$$

$$\textbf{Weighted standard deviation of income per pupil} = \sqrt{\frac{\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{FTE_i\ x\ (Income_i} - \overline{\mathrm{Income}})^2}{\frac{\mathrm{M}-1}{\mathrm{M}}\ \mathrm{x}\ \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{FTE_i}}}$$

$\mathrm{FTE_i}$ = Number of full time equivalent pupils in school i.
$\mathrm{VA_i}$ = Value added in school i.
$\mathrm{Income_i}$ = Income per pupil in school i.
$\overline{\mathrm{VA}}$ = weighted mean of VA
$\overline{\mathrm{Income}}$ = weighted mean of income per pupil
M = number of non zero values of $\mathrm{FTE_i}$

Estimating the improvement in VA required to reach higher levels of efficiency

The efficiency metric publication provides schools with additional information to their 'efficiency decile'. This includes estimates of the improvement in value added that they would have to achieve in order to reach the levels of efficiency of schools in higher deciles. To calculate this, we assume all schools' funding remains at the current level. For schools with 'efficiency deciles' of 1, who are amongst the most efficient five schools

in their ‘efficiency neighbours’ group, the suggested higher levels of efficiency are achieved by schools that may not be one of their ‘efficiency neighbours’ – and are thus not statistically similar. However for schools with ‘efficiency deciles’ of 2 to 10, we show estimates of improvements in VA required to reach levels of efficiency already achieved by their ‘efficiency neighbours’.

Any estimated improvements that suggest schools need to increase their VA beyond 105 (for primary schools) and 1050 (for secondary schools) are capped at these values. Roughly 95% of schools achieve value added below these capped values. Achieving value added as high as the capped values could therefore be considered ‘normal’ in a statistical sense.[7]

The following steps show how to estimate the improvement in VA required for a school to reach an ‘efficiency decile’ of 1. The formulas can be altered easily to estimate the improvement in VA required for a school to reach any ‘efficiency decile’.

The first step is to identify the level of efficiency that the school needs to improve its VA to reach. In this case this is the ‘efficiency score’ of the fifth most efficient ‘efficiency neighbour’.

The second step is to calculate the standardised VA that corresponds to the school reaching this level of efficiency. This is calculated using the following formula. Note that this is just a rearrangement of the formula to calculate a school’s ‘efficiency score’, described in the “Calculating the School Efficiency Metric” section.

Formula to work out the standardised value added needed to reach decile 1

$$\left(\frac{\text{Efficiency score required to reach decile 1}}{100}\right) \text{ x School's standardised income per pupil}$$

The final step is to use this value of standardised VA to calculate the VA required for the school to reach an ‘efficiency decile’ of 1 in its ‘efficiency neighbours’ group. This is calculated using the following formula (which is, again, a rearrangement of the formula to calculate standardised VA, described in the “Calculating the School Efficiency Metric” section).

Formula to work out the value added needed to reach efficiency decile 1

[7] The precise percentage of schools that achieve these value added scores varies each year. However 95% is a good approximation, and a sensible percentage of scores to be considered ‘normal’ in a statistical sense.

$$\left[\left(\frac{\text{School's standardised VA} - 100}{15}\right) \text{ x } \sigma_{\text{Value Added}}\right] + \mu_{\text{Value Added}}$$

Variable	Description
$\mu_{\text{Value Added}}$	The (weighted) mean VA for all schools of the same phase.
$\sigma_{\text{Value Added}}$	The (weighted) standard deviation of VA for all schools of the same phase.

Further information

Links related to this note

- Survey to provide feedback on first publication of the Efficiency Metric

Other relevant departmental guides

- Guide on value added scores for primary schools
- Guide on value added scores for secondary schools

Links to efficiency tools and resources

- School financial benchmarking (academies)
- School financial benchmarking (local authority maintained schools)
- School financial benchmarking guide (academies)
- School financial benchmarking guide (local authority maintained schools)
- Schools financial health and efficiency webpage
- Education and Endowment Fund Teaching and Learning Toolkit

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