A guide to level 3 value added in 2015 school and college performance tables

January 2015
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Summary – interpreting level 3 value added

Example of how a school or college’s value added might be displayed in a table

<table>
<thead>
<tr>
<th>Institution Name</th>
<th>VA measure based on progress between Key Stage 4 and Level 3 Qualifications (measure centred around zero and expressed in grades)</th>
<th>Limit of Level 3VA Confidence Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>0.66</td>
<td>0.97  0.35</td>
</tr>
<tr>
<td>College B</td>
<td>-0.04</td>
<td>0.27  -0.35</td>
</tr>
<tr>
<td>School C</td>
<td>-0.47</td>
<td>-0.16  -0.78</td>
</tr>
</tbody>
</table>

The higher a school or college’s VA score in a qualification type, the more progress the students in the school or college are making, with zero representing the national average.

Confidence intervals then allow us to assess whether the school or college’s VA score is significantly above or below the national average.

Displaying a school or college’s VA visually on a chart

How to interpret the information and chart

School A’s VA score is above the national average and this is statistically significant.

This is because the whole range of the confidence interval is above zero

This tells us that students in this school make more progress than average

College B’s VA score is not significantly different from the national average.

This is because the range of the confidence interval straddles the national average of zero

This tells us that students in this college make progress comparable with the average

School C’s VA score is below the national average and this is statistically significant.

This is because the whole range of the confidence interval is below zero

This tells us that students in this school make less progress than average

KEY:
- Upper Confidence Limit
- School VA Score
- Lower Confidence Limit
What is level 3 value added?

The purpose of level 3 value added (L3VA) is to measure the progress students make from the beginning to the end of their level 3 qualifications (i.e. between the end of key stage 4 (KS4) and the completion of their level 3 qualification).

When measuring how effective a school or college is, it is important to look at how well its students perform in their tests and examinations. However, when evaluating examination performance, it is also important to take into consideration that when students first join a school sixth form or college, they have varying levels of ability, i.e. students have many different starting points.

Analysis shows that there is a very strong relationship between examination performance of students at the end of KS4 and current level 3 qualifications. A value added measure uses this relationship to “estimate” how well all students, nationally, perform in the current level 3 qualifications they have opted to study.

For L3VA, an individual student’s “estimated outcome” in a level 3 qualification is calculated by looking at the actual performance of all students nationally that studied the same level 3 qualification and that demonstrated similar ability in their exams at the end of KS4.

More specifically, we estimate a student’s level 3 qualification outcome as the average points achieved in this qualification by students nationally of similar ability at KS4. This estimated outcome in the level 3 qualification can then be compared against what the student actually achieved in their level 3 qualification, to see whether or not they exceeded their estimate. The difference between a student’s actual performance and their estimated performance gives the student an L3VA score in that level 3 qualification.

The L3VA score of all students studying a particular level 3 qualification in a school or college can then be averaged to find the school or college’s L3VA score in the level 3 qualification. This score is used to identify the schools and colleges that are helping their students make more or less progress than average. The summary diagram on page 2 shows how to interpret these scores. L3VA is expressed as a proportion of a grade: as a proportion of one A level grade for an academic qualification, and as a proportion of a BTEC Level 3 Subsidiary Diploma grade for a vocational qualification.
Which students are included and how is their attainment calculated?

Level 3 value added (L3VA) is a measure that looks at the progression students make between key stage 4 (KS4) and the end of their level 3 qualification. In common with other 16-18 performance tables measures, the L3VA measure covers all students that:

- were aged 16, 17 or 18 at the beginning of the academic year (31st August 2014)
- have completed a level 3 qualification approved for 16-18 performance tables
- have reached the end of their advanced study in the 2014/15 academic year
- are in a school or college with a valid institution identifier

Additionally, to be included in the L3VA measures students must:

- have results at the end of KS4
- have completed a qualification for which a L3VA model has been calculated – see page 11 for more details

How to calculate each student’s KS4 starting point

The starting point for L3VA is the average attainment of the student at the start of their level 3 qualification, also known as prior attainment.

For students aged 17 and 18, this is calculated as the average of their attainment at level 2 and below (or KS4) in exams taken up to two years before their outcome in the level 3 qualification. For students aged 16, this is calculated as the average of their attainment at level 2 and below (or KS4) in exams taken up to one year before their outcome in the level 3 qualification. This is summarised in the table below:

<table>
<thead>
<tr>
<th>Example</th>
<th>Age of student*</th>
<th>Eligible qualifications for a student’s prior attainment at key stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18</td>
<td>Level 2 qualifications and below up to age 16 (that is two years before)</td>
</tr>
<tr>
<td>B</td>
<td>17</td>
<td>Level 2 qualifications and below up to age 15 (that is two years before)</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>Level 2 qualifications and below up to age 15 (that is one year before)</td>
</tr>
</tbody>
</table>

*Age at the beginning of academic year in which the level 3 qualification was completed
Each student’s KS4 outcome is calculated by adding together the total points for their applicable KS4 qualifications and dividing the total through by the combined size of the qualifications. For example, GCSEs are equivalent to a size of 1 in performance tables measures\(^1\). All other qualifications are sized relative to one GCSE. Examples of how to calculate KS4 average point scores are shown below:

**Example A:**

Student A takes ten GCSEs at age 16 and progresses to complete a level 3 qualification at age 18. The student’s prior attainment is calculated as the total point score in the ten GCSEs divided by a volume of 10 (each GCSE is equivalent to a size of 1).

**Example B:**

Student B takes four GCSEs and an OCR Level 2 National Certificate at age 15; the student then goes on to take a level 3 qualification at age 17. The student’s prior attainment is the total point score in the four GCSEs and the OCR Level 2 National Certificate divided by a volume of 8 (that is, four GCSEs with a size of 1 and one OCR Level 2 National Certificate with a size of 4).

**Example C:**

Student C attains two Bs, three Cs and one U in GCSEs at age 15 and a merit in an OCR Level 2 National Certificate at age 15; student C then goes on to take a level 3 qualification at age 16. The student’s prior attainment is the total point score in the six GCSEs and the OCR Level 2 National Certificate (2 x 46) + (3 x 40) + (1 x 0) + (1 x 196) = 408 divided by a volume of 10 (six GCSEs with size 1 and one OCR Level 2 National Certificate with size 4). This gives an average point score of (408 / 10) = 40.8.

**How to measure achievement in level 3 qualifications for L3VA**

As with the calculation of prior attainment, a point score system is used to assign points to grades for level 3 qualifications.

\(^1\) Note that although the size of level 2 and below qualifications in KS4 performance tables measures has changed from 2014 after the vocational qualification reforms, the majority of students included in the L3VA measure in 2015 finished KS4 before this point so the points and size system applying before 2014 is used to calculate L3VA for 2015.
However, the inclusion of fails in the L3VA measure means that there are unequal gaps between the lowest grades possible in level 3 qualifications and a fail. For example, in A Levels there is a 30 point gap between each grade from A* to E but there is a 150 point gap between an E and a fail. This creates issues with the statistical modelling.

For this reason, L3VA re-bases the point scores to create even sized gaps between grades and also between the lowest grade and a fail for all qualification types. This allows fails to be included in L3VA but also retains the scale for the other grades. The diagram below demonstrates how re-basing works.

The table below shows how point scores are re-based for a selection of level 3 Value Added qualification types.

<table>
<thead>
<tr>
<th>A Level / Applied GCE Single Award</th>
<th>AS Level / Applied GCE AS Single Award</th>
<th>BTEC Level 3 Subsidiary Diploma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>Point Score</td>
<td>Rebased Point Score</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>A*</td>
<td>300</td>
<td>180</td>
</tr>
<tr>
<td>A</td>
<td>270</td>
<td>150</td>
</tr>
<tr>
<td>B</td>
<td>240</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>210</td>
<td>90</td>
</tr>
<tr>
<td>D</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
<td>E</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>FAIL</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Calculating student scores for individual qualifications

Before examining how to calculate student level 3 value added (L3VA) scores, it is important to first understand how the measure is structured.

The nature of provision in sixth forms and colleges means that there is a very wide range of different subjects offered to students as well as a wide range of different types of qualifications that students can study. To reflect this diverse range of qualifications, L3VA is structured so that there is a separate measure for every qualification (subject) offered within each qualification type. The area shaded in the diagram below shows the qualification level of the hierarchy.

The result of using the approach above is that each student will have a separate L3VA score for every level 3 qualification that they study.

In order to calculate a student’s L3VA score in a particular qualification, the first step is to use a statistical model to calculate an “estimated outcome” for the student in the qualification. This estimate is calculated based on the actual outcomes of all students nationally that have taken the same level 3 qualification and with the same level of prior achievement at the end of key stage 4 (KS4).

For example, a student that scored an average of 52 points at KS4 would have their estimated outcome in A level geography calculated by the statistical model based on the actual outcomes of all students nationally taking A level geography that also scored an average of 52 points at the end of KS4.

A student’s L3VA score is then calculated by subtracting their estimated outcome in the qualification from their actual outcome in the qualification. Using the same example, if a student achieves an ‘A’ in their A level...
geography but they were estimated to achieve a ‘B’ by the statistical model, then the student has an L3VA score of +1 grade.

The positive score tells us that this student has exceeded their estimated A level geography outcome. If the L3VA score was negative, then this would tell us that the student scored less than their estimated A level geography outcome. The tables below summarise the calculation described above.

**Example 1**

<table>
<thead>
<tr>
<th>Student’s actual KS4 average point score</th>
<th>Performance of all students taking A level geography with an average score of 52 at KS4 used to estimate student’s performance in A level geography (using a statistical model)</th>
<th>Student’s estimated A level geography outcome</th>
<th>Student’s actual A level geography outcome</th>
<th>Difference (actual – estimate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>52 points</td>
<td></td>
<td>B</td>
<td>A</td>
<td>+1 grade</td>
</tr>
</tbody>
</table>

**Example 2**

<table>
<thead>
<tr>
<th>Student’s actual KS4 average point score</th>
<th>Performance of all students taking A level French with an average score of 46 at KS4 used to estimate student’s performance in A level French (using a statistical model)</th>
<th>Student’s estimated A level French outcome</th>
<th>Student’s actual A level French outcome</th>
<th>Difference (actual – estimate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>46 points</td>
<td></td>
<td>Between a B and a C grade</td>
<td>B</td>
<td>+0.4 grades</td>
</tr>
</tbody>
</table>

The technical annex, calculation of exam level L3VA (section 1) see page 20, provides a more detailed description of how students’ estimated scores and their L3VA scores are calculated.

**Calculating school and college scores for individual qualifications**

Once student level 3 value added (L3VA) scores have been calculated for a particular qualification (e.g. OCR Level 3 Cambridge Technical Diploma in
A qualification type L3VA score (e.g. AS levels) is calculated by finding the average of all the qualification L3VA scores that belong to the qualification type. A school or college's AS level qualification type score would be found by averaging all the AS level qualification L3VA scores (e.g. AS level history, AS level economics, AS level maths) offered by the school. The calculation is also weighted by the number of students taking each qualification; this gives greater weight to qualifications being taken by more students.

The example below demonstrates how to calculate an AS level qualification type L3VA score for a school/college offering three AS levels:

\[
\text{AS Level VA Score} = \frac{(50 \times +0.25) + (20 \times -0.70) + (100 \times +0.35)}{50 + 20 + 100} = +0.20 \text{ grades}
\]

### Which qualification types are included?

In order for a qualification type (e.g. A levels) to be included within L3VA, it must first be a level 3 qualification type but it also must have a graded
outcome. This means that the qualification type needs to have four or more possible outcomes, for example, A levels have seven different outcomes (A*, A, B, C, D, E and FAIL). Additionally, there needs to be a minimum of 80 students and 5 institutions offering the qualification type nationally in order for it to be included in L3VA.

The list below shows some of the qualification types included in L3VA for the publication based on 2014/15 examination data.

<table>
<thead>
<tr>
<th>Qualification type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A levels</td>
</tr>
<tr>
<td>AS levels</td>
</tr>
<tr>
<td>Extended Project (Diploma)</td>
</tr>
<tr>
<td>Pre-U Qualifications</td>
</tr>
<tr>
<td>Applied GCE A Level Qualifications</td>
</tr>
<tr>
<td>International Baccalaureate</td>
</tr>
<tr>
<td>Free Standing Maths Qualifications</td>
</tr>
<tr>
<td>BTEC Level 3 Qualifications</td>
</tr>
</tbody>
</table>

**The aggregate A levels measure**

A level 3 value added measure for all types of A levels was included for the first time in the 2014 amended data. From 2015, it corresponds to the broader A level definition used in the departmental performance tables. In 2015, A level value added measures comprise of GCE A levels, GCE AS levels, Applied GCE A levels and Applied GCE AS levels as well as combined and double GCE A and AS awards\(^2\). Calculations for this measure are similar to those for school and college value added scores.

\(^2\) The full list of approved level 3 qualifications can be accessed in the 16-18 performance tables List of A level, Academic and Vocational Qualifications Document
For example:

\[
\text{Aggregate A-levels score} = \frac{(50 \times +0.25) + (40 \times +0.10)}{(50 + 40)} = +0.18 \text{ grades}
\]

\[
\begin{align*}
\text{A level History} \\
\text{VA score} &= +0.25 \\
\text{Cohort Size} &= 50
\end{align*}
\]

\[
\begin{align*}
\text{Vocational A level Business} \\
\text{VA score} &= +0.10 \\
\text{Cohort Size} &= 40
\end{align*}
\]

**Calculating school and college academic and vocational scores**

It is also possible to group together school and college level 3 value added (L3VA) scores into a score for all academic qualifications and a score for all vocational qualifications.

Aggregating to academic and vocational level involves combining L3VA scores from different qualification types, for example, if a school has an L3VA score in AS level maths and in A level biology, both L3VA scores would contribute to the school’s academic L3VA score. Similarly if a college has an L3VA score in BTEC Level 3 Subsidiary Diploma in Business and an L3VA score in Level 3 Cambridge Technical Diploma in Sport, then both of these would count towards the college’s vocational value added score. The relative size of the qualification type is also taken into account in the calculation of academic and vocational L3VA scores. Academic L3VA scores are given as a proportion of one A level grade and vocational L3VA scores are given as a proportion of one BTEC Level 3 Subsidiary Diploma grade.

The example below demonstrates how to calculate an academic L3VA score for a school/college offering one A level and one AS level:
Interpreting school and college scores

When evaluating a school or college’s performance we must be careful to note that it is based on a given set of students’ results.

A school or college could have been equally effective and yet the same set of students might have achieved slightly different results in their qualifications, and the school would almost certainly have shown different results with a different set of students. This element of uncertainty needs to be taken into account when interpreting a value added score; 95% confidence intervals are provided as a proxy for a range in which you can be 95% certain the true value added score lies.

A school or college’s confidence interval is always centred on the school or college’s level 3 value added (L3VA) score. For example, if a school or college’s L3VA score is +1 and the size of their confidence interval is 0.5 grades, then the confidence interval ranges between +0.5 and +1.5 (i.e. half a grade either side of the L3VA score).

The size of the confidence interval is largely determined by the number of students in the school or college that completed the level 3 qualification. Schools and colleges with fewer students completing the qualification have wider confidence intervals because their L3VA score is based on a smaller number of students, so there is less evidence on which to judge the school or college’s effectiveness.

To judge a school or college’s effectiveness, both the L3VA score and the associated confidence interval need to be taken into account. If the whole range of the confidence interval is above the national average of zero (i.e. the

Academic Qualifications

\[
\frac{(50 \times +0.5) + (100 \times -0.15)}{(50 \times 1.0) + (100 \times 0.5)} = +0.10 \text{ A Level Grades}
\]

A Level Chemistry
VA Score = +0.50
Cohort Size = 50
Qualification type size = 1.0

AS Level English Literature
VA Score = -0.15
Cohort Size = 100
Qualification type size = 0.5
lower bound is greater than zero), we can say the school/college score is above the national average and this is statistically significant. Therefore, we can be confident the school/college is helping its students make better than average progress in the qualification. An illustration of how to interpret school/college L3VA scores is given on page 16.

Similarly, when the entire range of the confidence interval is below zero (i.e. the upper bound is less than zero), we can say the school/college score is below the national average for the level 3 qualification and this is statistically significant.

Finally, if the confidence interval straddles the national average of zero, then we can say that the school/college is *not significantly different* from the national average for the qualification, in other words, we cannot confidently say that the school or college’s L3VA score is definitely above or definitely below the national average for the qualification.

The table and diagram below shows how a school/college’s L3VA score and confidence intervals should be interpreted to reach one of the three definitions above. School A is an example of a school that is significantly above national average, School B is not significantly different from national average, and School C is significantly below national average.

<table>
<thead>
<tr>
<th></th>
<th>School A</th>
<th>School B</th>
<th>School C</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3VA score</td>
<td>+1.0</td>
<td>+0.1</td>
<td>-1.0</td>
</tr>
<tr>
<td>Upper confidence limit</td>
<td>+1.5</td>
<td>+0.6</td>
<td>-0.5</td>
</tr>
<tr>
<td>Lower confidence limit</td>
<td>+0.5</td>
<td>-0.4</td>
<td>-1.5</td>
</tr>
</tbody>
</table>
For more information on the calculation of confidence intervals, please see the technical annex at pages 20 to 27.
When and where will level 3 value added be published?

There are two level 3 value added (L3VA) publications based on 2014/15 examination data; the first available at the end of October 2015 based on unamended examination data; the second in January 2016 updated with amended examination data.

"Unamended" publication

The L3VA publication based on unamended data is made available on the Performance Tables Checking Website. An Excel L3VA report and a L3VA Summary PDF report are located on the documents page in folders called ‘Level 3 VA Report’ and ‘Level 3 VA Summary’. Schools and colleges that are part of a consortium also have anonymised Excel and PDF reports combining results from all schools and colleges in the consortium.

“Amended” publication

The L3VA publication based on amended data is published on two websites:

- The Performance Tables Checking Website
- The Performance Tables Website

As with the unamended publication, the Excel L3VA report and the L3VA summary PDF report will be available in the same locations on the Performance Tables Checking website. Schools and colleges in a consortium will again receive two further anonymised reports showing L3VA data for all schools and colleges within the consortium.

L3VA scores based on amended data are also published on the Department for Education’s Performance Tables website. The performance tables website shows academic and vocational L3VA scores, as well as qualification type L3VA scores for every school and college (e.g. an A level L3VA score or a BTEC National Diploma L3VA Score).

Other useful resources

As well as this guide, there are a number of other useful resources on level 3 value added (L3VA). The resources detailed below can all be found on the checking website until the publication of amended data, when they will also be located on the 16-18 performance tables website.
There is also a mailbox which you can contact if you have any questions on L3VA though we advise that you check the FAQs document first:

Level3VA.HELPDESK@education.gsi.gov.uk

**Frequently asked questions:**

A list of frequently asked questions can be found alongside this guide on the checking website.

**L3VA ready reckoner and guide:**

An L3VA ready reckoner is available on the checking website. Unlike the L3VA reports, the ready reckoner allows the user to input their own data to calculate L3VA scores for specific groups of students. For consistency, the output from the ready reckoner is in the same format as the L3VA reports.

A guide on how to use and interpret the L3VA reports and ready reckoner can also be found on the checking website.

**What is new in level 3 value added for 2014/15?**

1) **Aggregate A-level measure now includes AS levels and applied equivalents**

The 2015 data includes an aggregate A level measure that comprises of results for A levels, AS levels, Applied A levels, Applied AS levels and combined and double A and AS level awards. This aggregate measure was included in the 2014 amended release but AS levels were not included. As a result, the number of exam entries included in this measure will be larger.

**No further changes**

There are no further changes to 2015 level 3 value added (L3VA).

However, in 2016 there will be more significant changes to L3VA as part of government reforms to 16-18 performance tables. In 2016, the current multi-level modelling methodology for value added will be replaced with a new methodology based on percentile banding. More information is available in the 16 to 19 accountability headline measures technical guide published here:

[16-to-19-accountability-headline-measures-technical-guide](#)
Note that in July 2015 the department shared shadow 2014 L3VA scores with schools and colleges, which used the new percentile banding methodology that will apply in 2016 performance tables. The department intends to share shadow 2015 L3VA scores with schools and colleges, calculated using the new methodology, in 2016.
Technical Annex

Section 1 – calculation of exam level L3VA

1. Before L3VA scores can be calculated for schools and colleges, an initial stage is required to calculate individual exam level L3VA scores for students. These exam level L3VA scores provide the building blocks for calculating the aggregated school and college L3VA scores.

2. The first step in calculating a student’s L3VA score in an individual exam is to derive their estimated point score in the qualification. The student’s estimated attainment is calculated using the national lines produced as part of the statistical modelling. The student’s qualification L3VA score is then found by subtracting their estimated point score from their actual point score (both re-based) for that qualification.

3. The information required to perform this calculation is detailed below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma = \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$</td>
<td>Coefficients for given qualification at national level</td>
</tr>
<tr>
<td>$S$</td>
<td>National covariance matrix</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>National variance of error</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of entries within school or college within given qualification and subject</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Variance ratio for school or college for given qualification and subject</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Shrinkage factor for school or college for given qualification and subject</td>
</tr>
<tr>
<td>$x_n$</td>
<td>Prior attainment for $n_{th}$ exam record</td>
</tr>
<tr>
<td>$y_n$</td>
<td>Outcome attainment for $n_{th}$ exam record</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>Array of exam record L3VA scores</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Standard error for school or college for given qualification and subject</td>
</tr>
</tbody>
</table>
**Variable**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^2$</td>
<td>Scalar variance for school or college for given qualification and subject (intermediate value)</td>
</tr>
</tbody>
</table>

**Formula for calculation of individual exam level L3VA scores**

4. Individual exam level L3VA scores are calculated using the following equation:

$$\bar{u} = Y - (\Gamma X^T)^T$$

5. $\bar{u}$ is a $n \times 1$ matrix of the individual exam record L3VA scores and $\Gamma$ (upper case gamma) is a $1 \times 5$ matrix containing the five coefficients for this particular qualification. $Y$ is an $n \times 1$ matrix of the outcome attainment $y_n$ of each exam record for this particular qualification and $X$ is an $n \times 5$ matrix, where each row of $X$ is given

$$\left(1, x_n, x_n^2, x_n^3, x_n^4 \right)$$

6. For example, for a single case, if a learner had an average prior attainment score of $x_n = 45$ and an outcome attainment of $y = 210$ and the key coefficients were:

$$\gamma_0 = -4255$$
$$\gamma_1 = 103$$
$$\gamma_2 = -0.09$$
$$\gamma_3 = 0.00007$$
$$\gamma_4 = -0.0000001$$

Then applying the equation for a single student would give:

$$\bar{u} = Y - (\Gamma X^T)^T$$

$$\bar{u} = y_n - (\gamma_0 + x_n \gamma_1 + x_n^2 \gamma_2 + x_n^3 \gamma_3 + x_n^4 \gamma_4)$$
Would be equal to:

\[ \bar{u} = 210 - (-4255 + 45 \times 103 + 2025 \times -0.09 + 91125 \times 0.00007 + 4100625 \times -0.00000001) \]

\[ \bar{u} = 210 - 204.05 \]

\[ \bar{u} = 5.95 \]

7. Expressed as a matrix of three cases this would be equivalent to:

\[
Y = \begin{pmatrix} 210 \\ 240 \\ 270 \end{pmatrix}, \quad X = \begin{pmatrix} 1 & 45 & 2025 & 91125 & 4100625 \\ 1 & 40 & 1600 & x_n^3 & x_n^4 \\ 1 & x_n & x_n^2 & x_n^3 & x_n^4 \end{pmatrix}
\]

Therefore an example of \( \bar{u} \) would be:

\[
\bar{u} = \begin{pmatrix} 5.95 \\ -10.8 \\ 46.7 \end{pmatrix}
\]

8. If a candidate has achieved the highest possible grade for a qualification then their L3VA score should not be lower than 0. In some cases the “national line” of expected attainment fitted by the statistical model can lead to students with high prior attainment values being given an estimated attainment value above the maximum grade.

9. For example, on an AS level, a student could be given an estimated attainment value of 76 rebased performance points when the maximum attainable grade, an A grade, is only worth 75 points. In this case the student would usually be given a score of -1 however this is overwritten with a score of 0. Students whose estimates are below the maximum grade do not have their scores modified.
Section 2 – calculation of qualification L3VA

10. Now that the exam level L3VA scores have been calculated for each student, it is now possible to calculate school and college L3VA scores in each qualification that they offer. Here, the term ‘qualification’ means a subject within a qualification type, for example, A level French.

11. A qualification L3VA score for a school or college is calculated by finding the average of all the exam level L3VA scores in that qualification and in that institution. The result is multiplied by a shrinkage factor.

12. The shrinkage factor is a number between zero and one. If the number of students taking the qualification is large, the shrinkage factor is close to one, and thus has little effect on the school or college’s L3VA score. If the number of students taking the qualification is small, the shrinkage factor is close to zero, and brings the school or college’s qualification L3VA score towards the national average. By doing this, the results of qualifications with low student numbers are now more reliable and less volatile, while the L3VA scores for qualifications with large numbers of students remain virtually unchanged.

Calculation of institution L3VA score for a qualification

13. The first step is to calculate an average (mean) value added score using the exam level L3VA scores for each individual in the school or college that has taken the qualification.

\[ VA_{avg} = \frac{\sum_{1}^{n} \bar{u}_n}{n} \]

\( \bar{u}_n \) denotes the L3VA score of the \( n \)th exam record.

14. Next calculate the average prior attainment \( x_{avg} \) for the same set of students using the formula below:

\[ x_{avg} = \frac{\sum_{1}^{n} x_n}{n} \]

15. The next step is to calculate the shrinkage factor denoted by \( \lambda \). First an intermediate tau squared value \( \tau^2 \) must be calculated using the formula below:
\[ \tau^2 = \bar{x} \cdot S \cdot \bar{x}^T \]

In this formula, \( \bar{x} \) is defined as \( (1, x_{avg}, x_{avg}^2) \) and \( S \) is the National Covariance matrix for the particular qualification for which the calculations are being performed.

16. Then calculate the variance ratio \( \phi \) using the error term \( \sigma^2 \) from the MLM output:

\[ \phi = \frac{\sigma^2}{\tau^2} \]

17. Finally, calculate the shrinkage factor \( \lambda \):

\[ \lambda = \frac{n}{(n + \phi)} \]

18. The overall institution L3VA score \( U \) for the given qualification is then given by:

\[ U = VA_{avg} \times \lambda \]

**Calculation of confidence intervals around a school or college’s qualification L3VA score**

19. Using the standard error, it is possible to calculate confidence intervals around a school or college’s qualification L3VA score. Confidence intervals represent the range within which we can be confident that the school or college’s true L3VA score lies. The standard error of \( U \) is given by:

\[ \psi = \sqrt{\frac{\sigma^2}{(n + \phi)}} \]

20. The 95% confidence interval around a school or college’s qualification L3VA score is then given by:

\[ U \pm 1.96\psi \]
Section 3 – calculation of qualification type L3VA

21. This section describes how to calculate qualification type L3VA scores for a school or college. An example of a qualification type would be A levels or BTEC National Diplomas.

22. A qualification type L3VA score is calculated by finding the weighted average (based on number of students) of all the school/college qualification L3VA scores that belong to the qualification type. For example, if a school offered A level geography and A level maths, then the school’s A level qualification type L3VA score would be calculated by finding the weighted average of the school’s A level geography and A level maths L3VA scores.

23. The information required to perform this calculation is detailed below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$VA_{QualSubj}$</td>
<td>School or College’s VA score for given qualification and subject (e.g. A level maths VA score)</td>
</tr>
<tr>
<td>$VA_{Qual}$</td>
<td>School or college’s VA score for given qualification type (e.g. A level VA score)</td>
</tr>
<tr>
<td>$n_{QualSubj}$</td>
<td>Number of entries within school or college within given qualification and subject</td>
</tr>
<tr>
<td>$n_{Qual}$</td>
<td>Number of entries within school or college within given qualification type</td>
</tr>
<tr>
<td>$\Psi_{QualSubj}$</td>
<td>Standard error for a given qualification and subject at school or college level</td>
</tr>
<tr>
<td>$\Psi_{VA_{Qual}}$</td>
<td>Standard error for the VA score for the given qualification type</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Weighting factor for selected subject. $\omega = 1$ for all subjects, except General Studies, where $\omega = 0.5$.</td>
</tr>
</tbody>
</table>

24. The formula below is used to calculate aggregate L3VA scores for qualification types for a school or college. The formula should be used for each qualification type that a school or college offers.

$$VA_{Qual} = \frac{\sum_{Subj}^{TotalSubjs} VA_{QualSubj} \cdot n_{QualSubj} \cdot \omega}{\sum n_{QualSubj} \cdot \omega}$$
25. It is then possible to calculate 95% confidence intervals around the school or college’s qualification type L3VA score. The first step in doing this is to calculate the standard error for the qualification type:

\[ \Psi_{VA_{Qual}} = \sqrt{\sum_{1}^{n_{QualSubj}} \left( \frac{n_{QualSubj}}{n_{Qual}} \right)^2 \cdot \Psi_{QualSubj}^2} \]

26. The final step is then to use the qualification type standard error to calculate confidence intervals around the L3VA score using the following equation:

\[ VA_{Qual} \pm 1.96 \cdot \Psi_{VA_{Qual}} \]
Section 4 – calculation of academic and vocational L3VA

27. It is also possible to calculate academic and vocational L3VA scores that go across qualification types. For example, it is possible to calculate a vocational L3VA score that aggregates together qualification value added scores from different qualification types such as BTEC Subsidiary Diploma and BTEC Extended Diploma.

28. The information required to perform this calculation is detailed below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VA_{ACVQ}</td>
<td>School or College’s overall academic / vocational value added score</td>
</tr>
<tr>
<td>N_{QualACVQ}</td>
<td>The number of academic / vocational qualifications for that school or college</td>
</tr>
<tr>
<td>VA_{Qual}</td>
<td>School or college’s VA score for a given academic or vocational qualification type (e.g. A level VA score)</td>
</tr>
<tr>
<td>\mu_{Qual}</td>
<td>National average VA score for a given academic or vocational qualification</td>
</tr>
<tr>
<td>n_{Qual}</td>
<td>Number of entries within school or college within a given academic or vocational qualification type</td>
</tr>
<tr>
<td>Vol_{Qual}</td>
<td>The volume of the qualification type for the given academic or vocational qualification, in relation to A Levels (for academic qualifications) / BTEC Level 3 Subsidiary Diplomas (for vocational qualifications)</td>
</tr>
</tbody>
</table>

29. The formula below is used to calculate aggregated value added scores for academic and vocational qualifications. As this value added score combines information from different qualification types, the \( Vol_{Qual} \) variable is included in the formula.

\[
VA_{ACVQ} = \frac{\sum_{1}^{N_{QualACVQ}}(VA_{Qual} - \mu_{Qual}) \cdot n_{Qual}}{\sum_{1}^{N_{QualACVQ}}(n_{Qual} \cdot Vol_{Qual})}
\]

This step includes a small adjustment to correct for aggregation error, which means that the student average VA score is zero rather than the institution average. This may mean there is a small inconsistency with qualification type and individual qualification scores. For example, if an institution only offered A Levels, then their A Level score could be slightly
different from their aggregate academic score, even though they are both calculated from the same results.

30. It is also possible to calculate confidence intervals around each SSA (across qualification types) value added score. To do this, the standard error must first be calculated which is given by the formula below, with additional variables required in the table after:

\[
\psi_{VA_{ACVQ}} = \sqrt{\sum_{1}^{N_{QualACVQ}} \left( \frac{n_{Qual} \cdot Vol_{Qual}}{\sum_{1}^{N_{QualACVQ}} (n_{Qual} \cdot Vol_{Qual})} \right)^2 \cdot \left( \frac{\psi_{Qual}}{Vol_{Qual}} \right)^2}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_{VA_{ACVQ}})</td>
<td>Standard error of overall academic / vocational value added score</td>
</tr>
<tr>
<td>(\psi_{Qual})</td>
<td>Standard error for the VA score for the given academic / vocational qualification type</td>
</tr>
</tbody>
</table>

31. The final step is to then use the academic / vocational standard error to calculate confidence intervals around the value added score using the following equation:

\[
VA_{ACVQ} \pm 1.96 \cdot \psi_{VA_{ACVQ}}
\]
Section 5 – calculation of statistical significance of L3VA

Statistical significance at qualification level

32. A school or college qualification value added score (denoted $U$) is defined to be below the national average and statistically significant when their value added score is below 0 and the upper end of the 95% confidence interval is below zero. This can be expressed formulaically as:

$$U < 0 \quad \& \quad (U + 1.96\psi_U) < 0$$

33. A school or college qualification value added score is defined to be above the national average and statistically significant when their value added score is above 0 and the lower end of the 95% confidence interval is above zero. This can be expressed formulaically as:

$$U > 0 \quad \& \quad (U - 1.96\psi_U) > 0$$

Statistical significance at qualification type level

34. A school or college qualification type value added score is defined to be below the national average and statistically significant when their value added score is below 0 and the upper end of the 95% confidence interval is below zero. This can be expressed formulaically as:

$$VA_{Qual} < 0 \quad \& \quad (VA_{Qual} + 1.96\psi_{VA_{Qual}}) < 0$$

35. A school or college qualification type value added score is defined to be above the national average and statistically significant when their value added score is above 0 and the lower end of the 95% confidence interval is above zero. This can be expressed formulaically as:

$$VA_{Qual} > 0 \quad \& \quad (VA_{Qual} - 1.96\psi_{VA_{Qual}}) > 0$$

Statistical significance at academic / vocational level

36. A school or college academic / vocational value added score is defined to be below the national average and statistically significant when their value added score is below 0 and the upper end of the 95% confidence interval is below zero. This can be expressed formulaically as:
\[ VA_{ACVQ} < 0 \& (VA_{ACVQ} + 1.96 \cdot \psi_{VA_{ACVQ}}) < 0 \]

37. A school or college academic / vocational value added score is defined to be above the national average and statistically significant when their value added score is above zero and the lower end of the 95% confidence interval is above zero. This can be expressed formulaically as:

\[ VA_{ACVQ} > 0 \& (VA_{ACVQ} - 1.96 \cdot \psi_{VA_{ACVQ}}) > 0 \]