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Draft Curriculum for Wales 2022: April 2019

DRAFT STATUTORY GUIDANCE
AREA OF LEARNING AND EXPERIENCE

Mathematics and Numeracy

CONTENTS

INTRODUCTION TO THE MATHEMATICS AND NUMERACY AREA OF LEARNING AND EXPERIENCE

Mathematics is an international discipline, and numeracy, the application of mathematics, plays a critical part in our private, social and civic lives, and in the economic health of the nation.

It is imperative that mathematics and numeracy experiences are as engaging, exciting and accessible as possible for learners, and that they ensure learners develop mathematical resilience (the ability to embrace challenge as a positive aspect of learning). Developing mathematical resilience contributes to the development of ambitious and capable learners.

In the early years, play forms an important part in the development of mathematics and numeracy, enabling learners to solve problems, explore ideas, establish connections and collaborate with others. In later years, learners need to have opportunities to work both independently and collaboratively to build on the foundations established in the early years.

For learners of all ages, real-life examples drawn from the local, national and international environment help them make connections between the concrete and the abstract. Real-life contexts can be used to introduce and explore mathematical concepts, as well as to consolidate them. Indeed, teaching that introduces a reasoning and problem-solving approach to all mathematics and numeracy experiences supports the development both of positive dispositions and of the four purposes of the curriculum, as well as the development of the mathematical proficiencies.

A transformational curriculum

The White Paper *Our National Mission: A Transformational Curriculum*[1] set out the detailed legislative proposals for Curriculum for Wales 2022.

The proposal is that funded non-maintained settings and schools will be required to provide a broad and balanced curriculum that meets the four purposes of the curriculum, and comprises the six areas of learning and experience. There will be statutory duties to teach Welsh, English, religious education, relationships and sexuality education, and the three cross-curricular responsibilities of literacy, numeracy and digital competence. Further information on how the Health and Well-being Area of Learning and Experience can support this is provided in the 'Developing a broad and balanced curriculum' section of this document.

Funded non-maintained settings and schools will have discretion as to how they design their school-level curriculum to meet their curriculum duties. However, in considering the exercise of that discretion, they must have regard to statutory guidance issued by Welsh Ministers. In practice, that means they should follow the statutory guidance unless they have good reason not to.

This statutory guidance for the Health and Well-being Area of Learning and Experience, which forms part of the wider Curriculum for Wales 2022 statutory guidance, is intended to provide a national framework that funded non-maintained settings and schools can build on to develop their own curriculum. It is not intended to be a comprehensive or exhaustive syllabus, nor a guide for organising timetables. It sets out:

- what funded non-maintained settings and schools should take into account in designing their curriculum and how it could be structured
- the broad expectations for learners for the Health and Well-being Area of Learning and Experience at each progression step.

[1] beta.gov.wales/our-national-mission-transformational-curriculum

How the area of learning and experience supports the four purposes of the curriculum

The development of mathematics has always gone hand in hand with the development of civilisation itself. Mathematics surrounds us and underpins so many aspects of our daily lives, such as architecture, art, music, money and engineering. And while it is creative and beautiful, both in its own right and in its applications, it is also essential for progress in other areas of learning and experience, not least in Science and Technology, which would be virtually impossible without it. What is more, numeracy – the use of mathematics to solve problems in real-world contexts – is required in almost all areas of life.

Formal mathematics is founded on basic truths and develops through rigorous logical reasoning. It involves inventing or discovering abstract objects and establishing the relationships between them. It also teaches us the difference between conjecture, likelihood and proof.

Mathematical thinking involves applying similarly logical reasoning, this time to the investigation of relations within and between concepts, along with justifying and proving findings. Indeed, understanding mathematical concepts and being able to apply and reason with the abstract representations of concepts is central to learning mathematics. And essential to this is comprehension of, and proficiency with, the symbols and symbol systems used in mathematics.

Applying mathematics requires strategic competence in the use of abstraction and modelling, and learners also develop resilience, as well as a sense of achievement and enjoyment, as they overcome the challenges involved. Subsequently, mathematical activities teach learners not to be afraid of unfamiliar or complex problems, as they can be reduced to a succession of simpler problems and, eventually, to basic computations. And as they reflect on the approaches used, and on their own mathematics and numeracy learning, learners develop metacognitive skills which help them know which steps to take to improve performance. Thus they become **ambitious, capable learners, ready to learn throughout their lives**.

Mathematics also contributes to developing **enterprising, creative contributors, ready to play a full part in life and work**. It encourages learners to be creative because it requires them to play, experiment, take risks and be flexible in tackling mathematical problems. Because mathematics is essentially abstract, it teaches learners to operate with objects that do not physically exist, using and developing their creativity to imagine and discover new realities. It also supports numerical modelling and forecasting to encourage entrepreneurial thinking.

Mathematics promotes **ethical, informed citizens of Wales and the world** by providing learners with tools to analyse data critically, enabling them to develop informed views on social, political, economic and environmental issues. It encourages clarity of thinking, allowing learners to understand and make reasoned decisions.

In mathematics and numeracy, learners encounter contexts involving health and personal finance, and develop the skills needed to manage their own finances, make informed decisions and become critical consumers. They learn to interpret information and data to assess risk, and they use their numeracy skills across the curriculum to make effective choices, becoming **healthy, confident individuals, ready to lead fulfilling lives as valued members of society**.

WHAT MATTERS STATEMENTS FOR MATHEMATICS AND NUMERACY

The number system is used to represent and compare relationships between numbers and quantities.

Numbers are the symbol system for describing and comparing quantities. This is the first abstract concept that learners meet in mathematics, and it helps to establish the principles of logical reasoning. In mathematics the number system provides a basis for algebraic, statistical, probabilistic and geometrical reasoning, as well as for financial calculation and decision-making.

Knowledge of, and competence in, number and quantities are fundamental to confident participation in the world, and provide a foundation for further study and for employment. Computational fluency is essential for problem-solving and progressing in all areas of learning and experience. Fluency is developed through using the four basic arithmetic operations and acquiring an understanding of the relationship between them. This leads to preparing the way for using algebraic symbolisation successfully.

Algebra uses symbol systems to express the structures of relationships between numbers, quantities and relations.

Algebra is the study of structures abstracted from computations and relations, and provides a way to make generalisations. Algebraic thinking moves away from context to structure and relationships. This powerful approach provides the means to abstract important features and to detect and express mathematical structures of situations in order to solve problems. Algebra is a unifying thread running through the fabric of mathematics.

Algebraic thinking is essential for reasoning, modelling and solving problems in mathematics and in a wide range of real-world contexts, including technology and finance. Making connections between arithmetic and algebra develops skills for abstract reasoning from an early age.

Geometry focuses on relationships involving properties of shape, space and position, and measurement focuses on quantifying phenomena in the physical world.

Geometry involves playing with, manipulating, comparing, naming and classifying shapes and structures. The study of geometry encourages the development and use of conjecture, deductive reasoning and proof. Measurement allows the magnitude of spatial and abstract features to be quantified, using a variety of standard and non-standard units, and can support the development of numerical reasoning.

Reasoning about the sizes and properties of shapes and their surrounding spaces helps us to make sense of the physical world and the world of mathematical shapes. Geometry and measurement have applications in many fields, including art, construction, science and technology, engineering, and astronomy.

Statistics represent data, probability models chance, and both support informed inferences and decisions.

Statistics is the practice of collecting, manipulating and analysing data, allowing representation and generalisation of information. Probability is the mathematical study of chance, enabling predictions of the likelihood of events occurring. Statistics and probability rely on the application and manipulation of number and algebra.

Managing data and representing information effectively provides the means to test hypotheses, draw conclusions and make predictions. Reasoning with statistics and probability, and evaluating their reliability, develops critical thinking and analytical skills that are fundamental to making ethical and informed decisions.

Relationships between what matters statements in Mathematics and Numeracy

The different areas of mathematics are highly interconnected and dependent on one another, and, as any teacher of mathematics knows, concepts are built up over time, drawing on prior knowledge and learning, often from more than one area of mathematics. What is important when planning to teach any specific topic is to work out the prior knowledge the learners need in order to be able to access and understand the new topic.

Algebra, geometry and statistics cannot be understood without a prior understanding of number and consistent reference to numbers, calculations and the number system. As learners progress they learn to see numerical expressions as relational rather than computational, e.g. a computation such as 2 + 8 = 10, and that this is the basis for deriving other facts, e.g. 8 + 2 = 10, 8 = 10 – 2, and so on. This lays the foundations for using algebraic symbolisation successfully. Making connections between arithmetic and algebra helps to develop tools and skills for abstract reasoning from an early age.

Measure is an aspect of geometrical thinking which is closely connected to number, and much of the development of understanding of number can emerge through increasingly sophisticated measuring. Geometric thinking involves reasoning with proportion, which connects with development in number work; it also involves transforming shapes, which relates to the use of functions and mapping in algebra.

Probability is expressed through number in various ways, using percentages, fractions and decimals, and an understanding of the different representations, and the connections between them, is necessary for effective expression of probability. Statistics involves manipulation, representation and interpretation of data, which in turn require numerical and geometric thinking.

Progression within this area of learning and experience

Progression in Mathematics and Numeracy involves the development of the following interconnected and interdependent proficiencies[2].

- Conceptual understanding; having the knowledge to understand and explain a mathematical concept.
- Communication with symbols.
- Strategic competence (i.e. formulating problems mathematically in order to solve them).
- Logical reasoning.
- Fluency.

A key point to note in any planning and assessment of Mathematics and Numeracy is that the proficiencies are interconnected and interdependent; they cannot be seen as hierarchical (e.g. strategic competence does not come after learning facts and techniques) and they can be developed alongside each other. However, it may be helpful to target certain proficiencies at certain points, if this is appropriate to ensure progression.

Contextual example 1: Learning about fractions – the case of ½

Conceptual understanding

Understanding that a half is a result of dividing something into two equal parts. This could be through connecting their concrete and/or real-life experiences of partitioning objects and numbers into equal parts with images (e.g. pictures and images on the number line) and the abstract representation of a ½ using the symbolic notation. A learner who understands what a half is might be able to give real-life or visual examples, and would also be able to explain why something might not be a half (e.g. a pizza cut into two parts which are not equal).

Communicating with symbols

Understanding the convention of how a half is written and what the symbols mean. This could also involve linking division and fractions (i.e. ½ = 1 ÷ 2) and using terms such as numerator and denominator.

Strategic competence

Being able to recognise real-life situations which involve a half; being able to represent these mathematically; being able to model situations involving halving mathematically; using pictures/images and language and symbols.

Fluency

Being able to count in steps of a half and being able to begin to recall halves of numbers.

[2] Based on *Adding It Up: Helping Children Learn Mathematics* by Kilpatrick, Swafford and Findel (National Research Council, 2001) and adapted to the Welsh context by Anne Watson, Emeritus Professor of Mathematics Education, University of Oxford, 2018.

Logical reasoning

Being able to understand the relationship between a half and a whole; being able to justify why ¾ is also a half. Being able to reason that ½ + ½ = 1, ½ x 2 = 1 and 1 ÷ 2 = ½. Being able to justify why there may be many ways of splitting a shape in half.

Through connecting these proficiencies within a learner's experience of ½, learners should develop a deep understanding of a half as an example of a fraction. The way in which these foci are introduced could vary, but ultimately, having opportunities to explore and connect these proficiencies should ensure learner progression within this idea.

Developing a broad and balanced curriculum

Literacy, numeracy and digital competence

The cross-curricular responsibilities of literacy, numeracy and digital competence support almost all learning and are essential for learners to be able to participate successfully and confidently in the modern world.

Literacy

Literacy can be developed through engaging and accessible experiences where learners are given regular opportunities to describe, explain and justify their understanding of various mathematical concepts, using appropriate mathematical vocabulary. The development of these skills can be seen in rhymes and songs through to discussions around abstract concepts. Learners will also use and develop their literacy skills in a written form in order to describe mathematical processes, such as reasoning.

Learners will increasingly use literacy skills in order to understand a range of calculation strategies, describing visualisation of shapes, studying and interpreting information in statistics and comparing alternative methods before arriving at a solution to a mathematical problem. They should use these literacy skills as they encounter practical, real-world problems.

Numeracy

Mathematics and Numeracy, by definition, has numeracy at its heart.

Numeracy involves applying and connecting the five mathematical proficiencies[3] in a range of real-life contexts, within the Mathematics and Numeracy Area of Learning and Experience, and the wider curriculum.

Real-life contexts can be used to introduce and explore mathematical concepts, as well as to consolidate them. For example, the use of percentages can be applied to annual percentage rates (APRs) to demonstrate their application to financial literacy.

Digital competence

Digital approaches will enhance learners' mathematical and numeracy skills across a range of situations that will naturally occur within the area of learning and experience. Digital competence is more than the interaction with technology. For example, collaborating to solve a problem and the development of algorithms to support the understanding of patterns will provide the opportunity to support the development of learner's digital skills. Creating a graph by using a spreadsheet, for example, will enhance digital understanding and also strengthen learners' mathematical and numerical skills.

Naturally, as learners develop and progress they will increasingly use more complex digital skills and processes, techniques and systems to create solutions to address specific problems, opportunities or needs. Aspects of collection, representation and analysis, for example, will become more sophisticated as learners progress.

[3] See 'Progression within this area of learning and experience' section.

Welsh language

The number system in Welsh (Cymraeg) is an area that highlights the distinctive nature of mathematics in a Welsh context. In the early years, in particular, it would be helpful if teachers could consider teaching learners the traditional (vigesimal) way of counting in Welsh, as well as the more modern way. The decision as to when to introduce it, however, should be left to individual schools, to reflect the context of the school. Children from homes/communities where Welsh is widely spoken, for example, may already know these from an early age, whereas for English-medium schools and some Welsh-medium schools it could be taught when learning to tell the time or date.

The Welsh number system, as with other Celtic and some European languages, traditionally used the vigesimal system, using a base of 20 instead of the modern (and common) base of 10. In comparatively recent times a decimal system, in common with other Indo-European languages, was set up to simplify the teaching of number. (For example, in the old system 11 was ‘un ar ddeg’ [1+10] and 14 was pedwar ar ddeg [4+10]. In the ‘new’ system, 11 is ‘un deg un’ [10+1] and 14 is ‘un deg pedwar’ [10+4]).

Welsh dimension and international perspective

The Welsh dimension and international perspective elements will enable learners to understand the connection between mathematics (and numeracy) and authentic real-world contexts that span across both Wales and the world. Mathematics is a universal language, but in order for learners to make sense of this language, and to understand mathematical concepts, it helps to provide examples rooted in everyday life. Learners, especially young learners, are often unable to think abstractly and find it easier to learn mathematics in more concrete terms.

Each school in Wales will have a unique environment in which they work, and schools should explore local sources and resources that might have a mathematics application. The Learned Society of Wales refers to Wales providing ‘practical contexts through which procedure, theories, and/or principles are given concrete form through examples, case study, and illustrations of real-life applications’ (The Learned Society of Wales, 2018). Emeritus Professor Gareth Ffowc Roberts, in his submission to the Learned Society paper, stated that ‘while there may be no such thing as ‘Welsh mathematics/numeracy’ there are very particular Welsh ways of experiencing mathematics/numeracy’. The challenge is to teach and learn mathematics and numeracy in Wales so that children gain ownership of them, and perceive them as being a natural part of their culture rather than believing that they are copying another community’s culture. Various local and national organisations (e.g. the National Museum of Wales and CADW) have already developed mathematics teaching resources. Since April 2019 the Welsh Government has had responsibility for deciding the rates of Income Tax paid by Welsh taxpayers. Schools are encouraged to use Welsh examples when teaching financial matters, and specifically about forms of taxation, highlighting the link between Mathematics and Numeracy, the real world and the responsibility Welsh Government has for determining Income Tax levels in Wales.

There are particular contributions to the field of mathematics from Wales and internationally that could be used to support understanding of mathematical concepts and conventions, and also to support understanding of the development of mathematics as a body of knowledge. Wales has a proud history of producing outstanding mathematicians such as Robert Recorde and William Jones. Schools should consider every opportunity to highlight their achievements and, hopefully,

inspire learners to become mathematicians themselves. Using international examples from a range of cultures could enable learners to understand the history of mathematics and its development into an international and universally applicable language. This could also promote and support cross-curricular learning.

The Mathematics and Numeracy Area of Learning and Experience has developed a set of mathematical proficiencies which permeate every aspect of what matters statements and have shaped progression in the area of learning and experience. These proficiencies build on the work of Kilpatrick et al.[4] and elsewhere (such as the Australian curriculum), but have been adapted to the Welsh context by the pioneer group following considerable work with experts. These proficiencies are fundamental to learning within the discipline and will be part of a recognisably Welsh identity of the emerging Mathematics and Numeracy curriculum.

Wider skills

The Mathematics and Numeracy Area of Learning and Experience provides opportunities for learners to develop all four of the following wider skills.

Critical thinking and problem-solving

The development of logical and critical thinking underpins learning in mathematics. Mathematics teaches us problem-solving skills, which transfer to all areas of the curriculum, to life in general and to the world of work. The work of mathematics involves solving problems, beginning by analysing requirements, asking questions and evaluating information. In the development of solutions, learners identify potential approaches and develop arguments, justifying their decisions.

Planning and organisation

Mathematical thinking requires learners to be organised and, as they progress through school, their organisational skills will develop, particularly as they plan and implement the data-handling cycle. In their mathematical problem-solving, learners are encouraged to predict and estimate solutions and then to check their answers, reflect on their results and evaluate their approaches. Increasing confidence in decision-making for mathematical problem-solving supports learners to be more aspirational in setting goals and challenges for themselves.

Creativity and innovation

Mathematical working requires and develops creativity and curiosity which also transfer to other aspects of life. Frequently in mathematical problem-solving the learner does not immediately know how to approach the problem; it takes creativity and courage to explore different approaches before deciding how to proceed. Planning and modelling tasks within mathematics develops learners' ability to turn ideas into action.

Personal effectiveness

Studying mathematics develops personal effectiveness. Everyone encounters challenges in studying mathematics at some point, and overcoming the challenges requires and develops resourcefulness and resilience.

[4] *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001)

Communicating about mathematical thinking and solving problems is a core aspect of mathematics. Mathematical communication is precise and logical and transfers to other areas of life.

Careers and work-related experiences

Learning from careers and labour market information

Mathematics and numeracy are an essential part of every aspect of our lives, whether at work or undertaking practical everyday activities. We use mathematics when we go shopping, plan holidays, decide on mortgages or evaluate and plan career pathways. Decisions in life are so often based on numerical information that, to make the best choices, we need to be numerate.

Mathematics helps to develop skills such as problem-solving, interpreting data and information, attention to detail, and accurate measuring and reasoning – all of which are highly desirable to employers. The ability and knowledge developed through mathematics and numeracy are a fundamental part of most professions. As competency increases, a wider range of opportunities can be accessed. There is substantial evidence that a lack of numeracy skills leads to poorer outcomes. Organisation for Economic Co-operation and Development (OECD) data indicates that those with poor numeracy skills are more than twice as likely to be unemployed. It also indicates a direct relationship between wage distribution and numeracy skills[5].

Linking the area of learning and experience to careers and work-related experiences

A number of careers specifically require the use of mathematics, including air traffic controller, surveyor, accountant, psychologist, teacher and carpenter or joiner. It is essential for learners to be aware of the wide range of careers available to them, and to be shown how the experiences, knowledge and skills made available in mathematics and numeracy can be applied in the world of work, whether in employment or as an entrepreneur, where these skills are increasingly crucial.

Incorporating careers and work-related experiences into mathematics can help contextualise learning and increase engagement. For instance, statistical analysis and probability can be used in the evaluation of labour market intelligence and information. Learners will therefore be able to effectively identify where jobs are, the qualifications and skills required, and whether this is an area of job increase or decline, thereby applying mathematics in a practical, constructive manner.

Careers and work-related experiences can help to raise aspirations and support financial planning by adding realism and context. Learners can be encouraged to plan for the future and to consider the implications of finance on career pathway decisions, for example, in considering the pay scale of various jobs. This engages and motivates learners and can be further enhanced by the incorporation of valuable lifestyle budgeting exercises and money management for independent living.

Learner progression relating to careers and work-related experiences is part of a continuum of learning for learners aged 3 to 16. Success for a young primary school learner could include:

- acting a variety of different jobs through role play
- belief that they can do any job – tackling gender stereotyping
- communicating with people in their community about the different jobs they do and the rewards that a job can bring.

[5] www.nationalnumeracy.org.uk/why-numeracy-important

By progressing learning, success for 16-year-old learners could include:

- demonstrating and applying the skills learned in relation to the world of work
- identifying interests, strengths and skills to make informed post-16 choices
- understanding and demonstrating the behaviours an employer looks for in a good employee
- evaluating risks when developing a business idea and exploring different methods of setting up and sustaining an enterprise.

Work-related experiences

Learners develop interests, strengths, skills and aspirations through experiences as part of the curriculum and in life beyond school. Employer engagement, whether by employer talks, visits to workplaces, events or practical activities are essential in adding realism to the world of work and underlining the importance of mathematics and numeracy. It can further support learner progression to enable an understanding of the labour market and trends locally, pan Wales, nationally and globally. Employers raise awareness of roles within the workplace to challenge stereotypes and preconceptions and to raise awareness of all learning and training routes available.

Incorporating careers and work-related experiences within Mathematics and Numeracy allows learners to understand, evaluate and engage with the world of work and enables greater engagement, raised aspirations, and informed, effective decision-making. In addition, it creates an environment where stereotypes can be challenged and the merits of diversity expressed.

Effective careers guidance is essential in securing the most appropriate route for learners' aspirations, informing them of the diversity of entry points and pathways into the world of work. Schools should offer opportunities to foster entrepreneurial skills and learners should be aware of the benefits of setting up enterprises.

Understanding post-16 and higher-education opportunities

It is essential for learners to be aware of all opportunities available to them post-16. Therefore, as well as understanding about employment, training and apprenticeships, learners should be provided with information and the opportunity to engage with a range of learning providers. Opportunities for engagement should include attending careers and skills fairs, talks from and visits to further and higher education providers, as well as presentations from students in further or higher education. Learners should be directed to online research tools that provide course and progression information to support their understanding of the range of learning opportunities available, to help raise their aspirations and form a basis on which informed decisions can be made.

Relationships and sexuality education

When preparing mathematics sessions we must be aware that reinforcing any stereotype around gender is potentially restricting.

Effective mathematics instruction must be conducted in an environment that encourages equal achievement in mathematics for boys and girls. Teachers must ensure that there are equal opportunities for participation and that the classroom setting is a positive factor in achievement.

When introducing famous mathematicians, care should be taken to ensure that both female and male mathematicians are presented to learners, in order to promote equality and gender positivity.

Teachers should choose activities connecting mathematics activities to careers in ways that do not reinforce existing gender stereotypes and select activities that spark initial curiosity about mathematics.

Teachers of all age-groups should be encouraging gender equality within mathematics and promoting career choices which show no bias.

Enrichment and experiences

Throughout the Mathematics and Numeracy Area of Learning and Experience there is an emphasis on actively engaging learners in their mathematical learning by reference to, for example, exploring, investigating, playing and deducing. The approach suggested through the experiences outlined in the descriptions of learning is to engage learners in meaningful problem-solving in all areas of mathematics, set in the contexts of real-world local and global situations. This will promote the learners' own desire for learning, by making its value clearer and more relevant.

The achievement outcomes refer in multiple places to the resources that could and should be used to enhance understanding and enrich the mathematical experiences of learners. The appropriate use of a range of digital technologies, manipulables, everyday objects, and concrete and abstract representations of mathematical objects helps the learners to engage with mathematical concepts from more than one perspective.

In mathematics, learners sometimes collaborate to solve problems together in groups. For example, some mathematics depends on generating several sources of data, forms of representation, and separate steps of multi-step tasks which can be shared within a group. All can, therefore, contribute to completion of something beyond that which any individual might have done alone. As they work together, they engage actively in mathematical thinking and working, communicating about mathematics. This can aid the development of their own understanding, and enhance their own experience of mathematics. However, it is also important in mathematics to encourage independent and individual work, with learners taking responsibility for their own learning. They need time to think without being swamped by others' talk; time to internalise methods, facts, etc. A balanced diet of group and individual working will enrich the learners' experience.

The learning of mathematics will be further enriched by the use of meaningful and substantial tasks, from modelling real-life numerical problems, perhaps involving financial calculations, to relating patterns in nature to mathematical sequences, to using the data-handling cycle to investigate their own research questions.

Putting the area of learning and experience into practice

The Mathematics and Numeracy curriculum will need to be used by schools to design school-level provision, with the four purposes of the curriculum central to the planning of experiences that learners will encounter. Important decisions will need to be taken about the structure and sequence of mathematics and numeracy topics, and these decisions should be informed by the hierarchic and connected nature of mathematics concepts, in order to ensure foundations are built upon and experiences are connected.

Deep understanding in mathematics and numeracy should develop through planning for all five of the mathematical proficiencies, and the connection and application of these in a range of contexts. Schools will have the autonomy to determine the length of time spent on particular aspects of mathematics and numeracy, and formative assessment should drive decisions about planning in the short term. In planning over longer periods of time, schools will need to ensure that systems of mapping and tracking provision across year groups are robust.

The Mathematics and Numeracy curriculum does not prescribe one particular pedagogy, rather it requires practitioners to collaborate and make informed decisions to ensure that progress is achieved.

WHAT MATTERS IN MATHEMATICS AND NUMERACY

The number system is used to represent and compare relationships between numbers and quantities.

Numbers are the symbol system for describing and comparing quantities. This is the first abstract concept that learners meet in mathematics, and it helps to establish the principles of logical reasoning. In mathematics the number system provides a basis for algebraic, statistical, probabilistic and geometrical reasoning, as well as for financial calculation and decision-making.

Knowledge of, and competence in, number and quantities are fundamental to confident participation in the world, and provide a foundation for further study and for employment. Computational fluency is essential for problem-solving and progressing in all areas of learning and experience. Fluency is developed through using the four basic arithmetic operations and acquiring an understanding of the relationship between them. This leads to preparing the way for using algebraic symbolisation successfully.

Learning

Principles of progression

Principles of progression are the basis on which the achievement outcomes have been developed and should guide the progression of learning within the area of learning and experience.

The following interdependent proficiencies[6] have been used to formulate the achievement outcomes and are central to progression at each stage of mathematics learning.

Numeracy involves applying and connecting these proficiencies in a range of real-life contexts, across and beyond the curriculum.

Conceptual understanding: Mathematical concepts and ideas should be dwelt on, built on, and connected together as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding by explaining and expressing concepts, finding examples (or non-examples) and by representing a concept in a variety of ways, including verbal, concrete, visual, digital and abstract representations.

Communication with symbols: Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility in their application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence: (i.e. formulating problems mathematically in order to solve them) Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to be able to solve them.

Logical reasoning: As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and proof should become increasingly abstract, moving from verbal, visual or concrete explanations to representations involving symbols and conventions.

Fluency: As learners experience, understand and apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow. As a result, facts, relationships and techniques learned previously should become firmly established, memorable and usable.

[6] Based on *Adding It Up: Helping Children Learn Mathematics* by Kilpatrick, Swafford and Findel (National Research Council, 2001) and adapted to the Welsh context by Anne Watson, Emeritus Professor of Mathematics Education, University of Oxford, 2018.

Progression step 1

Achievement outcomes

I have experienced and explored numbers in a variety of contexts and can express whole numbers in words, signs and symbols to convey size and order.

I can notice and read numbers in number-rich indoor and outdoor environments, experiencing cardinal, ordinal and nominal numbers.

I can use my experience of the counting sequence of numbers and of one-to-one correspondence to count sets reliably. I can count objects that I can touch, and ones that I cannot.

I can notice, read and write numbers in a range of media, through a multisensory approach, from zero to at least 20, ensuring they are correctly formed.

I have experienced the counting sequence of numbers in different ways, counting forwards and backwards, and starting at different points.

I have explored composing a quantity in different ways, using combinations of objects or quantities and using the corresponding mathematical language such as words like ‘more’, ‘less’, ‘most’ or ‘least’.

I can understand that a number can be partitioned in different ways.

I have experienced grouping and sharing with objects/quantities, and I can group or share small quantities into equal-sized groups.

I can use my visual sense of number to make estimates and comparisons. I can check estimates using counting or measuring.

I have used money in play and real-life situations and understand that I need to exchange money for items. I can use the language of money.

Progression step 2

Achievement outcomes

I can read, write and interpret numbers using figures and words up to at least 1,000.

I can order and sequence numbers, including odd and even numbers, in number-rich indoor and outdoor environments, and can count on and back in uniform steps of any size.

I have experienced numbers below zero in practical contexts, including temperature measurements.

I have explored composing numbers in a variety of ways and I can understand that number value can be determined by the position of the digits used.

I have explored additive relationships using a range of representations. I can use my understanding of additive relationships to add and subtract whole numbers using a variety of written and mental methods. I can estimate and check the accuracy of my answers using inverse operations when appropriate.

I have demonstrated an understanding of, and can recall, multiplication tables including 2, 3, 4, 5 and 10, and use the term 'multiples'.

I have explored multiplicative relationships, using a range of representations (including sharing/grouping and the array), and I can use my understanding of multiplicative relationships to multiply and divide whole numbers using a variety of written and mental methods.

I have engaged in practical tasks and real-life problems to estimate and round numbers to the nearest 10 and 100.

I can reflect on how reasonable answers are in the light of estimations and I have verified calculations using inverse operations.

I have demonstrated an understanding that unit fractions represent equal parts of a whole and are a way of conveying quantities and relationships.

I have experienced fractions in practical situations, using a variety of representations.

I have explored equivalent fractions and understand equivalent fraction relationships.

I have demonstrated an understanding of when to count, when to measure and when to calculate to find quantities.

I can explain that money and ways to save and pay come in different forms. I can make sensible financial transactions in familiar role-play scenarios, including making informed decisions and choices about spending and saving.

Progression step 3

Achievement outcomes

I have used a range of practical equipment to develop and secure my understanding of place value for positive and negative integers. I can read, write and interpret numbers, using figures and words up to at least one million.

I have extended my understanding of the number system, through a range of activities using non-digital and digital manipulatives, to include decimals and fractions, and I can confidently place whole numbers and fractional quantities on a number line. I have demonstrated my understanding that a fraction can be used as an operator, or to represent division. I can use place value for non-integers.

I have demonstrated my understanding that fractions (including improper fractions) as well as mixed numbers, decimals and percentages, provide different ways of representing non-integer quantities.

I have explored number patterns, connections and calculations with manipulatives and digital technology, and can demonstrate my understanding of number facts and relationships. I have used my knowledge of number facts and relationships to solve problems in mathematical and real-life contexts.

I have developed, used and discussed efficient and accurate methods when applying all four arithmetic operations to integers and decimals. I can combine these operations, in both mathematical and real-life contexts of problem-solving.

I can verify calculations and statements about number by inverse reasoning and approximation methods.

I have explored the meaning of negative numbers in meaningful and authentic contexts. I can compare the sizes of negative numbers and I can calculate the differences between any two integers. I can check my answers.

I can fluently recall times tables up to at least 10 x 10 and demonstrate my understanding by using them appropriately in applications and in mental and written arithmetic.

I have demonstrated an understanding of rounding and can solve problems requiring rounding to the nearest unit, 10, 100 and 1,000.

I have demonstrated an understanding of the equivalence of simple fractions, decimals and percentages, and I can convert between representations. I can use my knowledge of equivalence to compare the sizes of fractions. I understand the inverse relation between the denominator of a fraction and its value.

I can use my knowledge of multiplication, division, fractions and percentages to calculate proportions of a number or quantity, and to divide a number or quantity in a given ratio. I have solved problems involving ratio and proportion in real-life contexts, and I have used my knowledge of estimation and rounding to predict and check my answers.

I have demonstrated an understanding of income and expenditure, and I can calculate profit and loss. I have created and evaluated budgets for activities and events.

Progression step 4

Achievement outcomes

I can fluently and accurately apply the four arithmetic operations, in the correct order, on integers, decimals and fractions, using written, mental and digital methods. I can use my sense of number to predict and check my answers.

I have demonstrated an understanding of the equivalence of fractions, decimals and percentages and I can convert fluently between the different forms, using both written methods and a calculator. I have used my knowledge of percentages and ratio to solve problems that involve simple interest, compound interest, depreciation, and calculating bills and budgets that include basic taxation on goods and services.

I have derived and applied the rules of indices, with the exclusion of fractional indices, to calculate values and solve problems.

I have consolidated my understanding of reciprocals when dividing fractions.

I can use standard index form to represent small and large numbers and to perform calculations in appropriate real-life and mathematical contexts.

I can solve problems requiring rounding or significant figures at various stages of the calculation and give the answer, using both written and digital methods, and I can interpret the calculator outputs.

I have demonstrated an understanding of ratio and proportion and can solve numerical problems that involve direct and inverse proportion, including expressing one quantity as a proportion of another, proportional change and problems that involve foreign currencies and exchange rates.

I can use my knowledge of number to predict and check my answers.

I can justify choices based on value for money, personal well-being and global impact.

Progression step 5

Achievement outcomes

I can recognise the difference between rational and irrational numbers, and apply all four arithmetic operations to them.

I have explored the relationship between powers, roots and fractional indices, and I can derive and apply the rules to simplify and decompose surds.

I have demonstrated fluency in moving between representations of numbers, including converting a given recurring decimal to a fraction.

I have demonstrated an understanding that measurements are not always accurate and are subject to tolerance and margins of error. I can solve problems involving upper and lower bounds, and justify the outcome. I can use my knowledge of tolerance when choosing the required degree of accuracy to make real-life calculations.

I can solve problems involving repeated and inverse proportional reasoning.

I can use my knowledge of annual equivalent rate (AER) and annual percentage rate (APR) to develop models to evaluate and compare financial products.

I can calculate income tax and understand the implications of taxation.

Planning for learning

Links within this area of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across the Mathematics and Numeracy Area of Learning and Experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

Algebra uses symbol systems to express the structures of relationships between numbers, quantities and relations.

- The connections between arithmetic and algebra develop tools and skills for abstract reasoning from an early age.
- There is a strong relationship between the algorithms of arithmetic and the laws of algebra.
- The order of operations and laws of arithmetic are followed in algebra.

Geometry focuses on relationships involving properties of shape, space, and position, and measurement focuses on quantifying phenomena in the physical world.

- Number is used throughout geometry to quantify shape, size and movement.
- Measure is an aspect of geometrical thinking which is highly connected to number and much of the development of understanding of number can emerge through increasingly sophisticated measuring.
- Geometric thinking involves reasoning with proportion, which connects with development in number work.
- Geometry involves lengths, areas and volumes which are expressed as numerical quantities.
- Use of rules of number to calculate further values related to measurement and geometry.

Statistics represent data, probability models chance, and both support informed inferences and decisions.

- Probability and statistics are described and manipulated by using number; they are represented using number.
- Probability is expressed through number in various ways, e.g. through the use of percentages, fractions and decimals, and the connections between the representations is necessary for effective expression of probability.
- Statistics involves manipulation and representation of data, which involves numerical thinking.

Links with other areas of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across all the areas of learning and experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

Expressive Arts

- Scale, proportion and ratio.
- Fractions in music, rhythm.
- Songs and rhymes.

Health and Well-being

- Understanding of estimation and rounding.
- Application to making real-life decisions including financial ones.
- Fractions, percentages and proportions, e.g. balanced diet.

Humanities

- Ratio and scale.
- Finance.
- Rounding.
- Ordering.

Languages, Literacy and Communication

- Songs and rhymes.

Science and Technology

- Rounding and estimating.
- Laws of indices.
- Reading, writing and calculating in standard index form.
- Direct and inverse proportion.

Experiences, knowledge and skills

Progression step 1

Learners need to experience, to know or be able to:

- explore numbers and quantities through multisensory experiences both indoors and outdoors
- learn that the number system, and whole numbers in particular, allow us to convey size and quantity
- represent and communicate with whole numbers
- begin to understand that money is needed to pay for things
- use a variety of representations to explore number, including objects, visual representations and, where appropriate, digital representations.

Progression step 2

Learners need to experience, to know or be able to:

- extend their understanding of the number system, through experiential learning, to include large numbers, zero, negative numbers and fractions
- explore composition and equivalence of number, and learn about place value
- learn about relationships in numbers
- understand the additive relationship and be introduced to the multiplicative relationship, including using the array
- use their understanding of the equivalence and value of coins and notes to make appropriate transactions.

Progression step 3

Learners need to experience, to know or be able to:

- extend their understanding and use of the number system, through a broad range of experiences, to include negative numbers, decimals and fractions
- further develop understanding of place value
- explore the properties of numbers, including factors, multiples, primes, and the inverse relationship between squares and square roots
- become increasingly confident in using all four arithmetic operations in their calculations with whole numbers and decimals, and combine these, using distributive, associative and commutative laws where appropriate
- create and evaluate enterprise projects linked to their immediate and local environment.

Progression step 4

Learners need to experience, to know or be able to:

- become increasingly fluent in their use of number in calculations, through a broad range of experiences, in order to interpret mathematical statements and describe quantities both with and without the use of calculators
- further develop their understanding of equivalence, appreciating that any number may be represented in many ways
- use proportional reasoning to compare two quantities using multiplicative thinking, and then apply this to a new situation
- deepen their understanding of using and comparing very big and very small numbers
- select and use efficient mental, written and digital methods to perform calculations
- extend their understanding of finance to personal, local and global contexts.

Progression step 5

Learners need to experience, to know or be able to:

- become increasingly fluent in their use of number, through a broad range of experiences, in order to describe, interpret and communicate size, scale and comparisons, both within and beyond mathematics
- become increasingly fluent in their calculations, both with and without the use of calculators, and deepen their understanding of how to use rational and irrational numbers
- become critical consumers in broader financial contexts.

WHAT MATTERS IN
MATHEMATICS AND NUMERACY

Algebra uses symbol systems to express the structures of relationships between numbers, quantities and relations.

Algebra is the study of structures abstracted from computations and relations, and provides a way to make generalisations. Algebraic thinking moves away from context to structure and relationships. This powerful approach provides the means to abstract important features and to detect and express mathematical structures of situations in order to solve problems. Algebra is a unifying thread running through the fabric of mathematics.

Algebraic thinking is essential for reasoning, modelling and solving problems in mathematics and in a wide range of real-world contexts, including technology and finance. Making connections between arithmetic and algebra develops skills for abstract reasoning from an early age.

Learning

Principles of progression

Principles of progression are the basis on which the achievement outcomes have been developed and should guide the progression of learning within the area of learning and experience.

The following interdependent proficiencies[7] have been used to formulate the achievement outcomes and are central to progression at each stage of mathematics learning.

Numeracy involves applying and connecting these proficiencies in a range of real-life contexts, across and beyond the curriculum.

Conceptual understanding: Mathematical concepts and ideas should be dwelt on, built on, and connected together as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding by explaining and expressing concepts, finding examples (or non-examples) and by representing a concept in a variety of ways, including verbal, concrete, visual, digital and abstract representations.

Communication with symbols: Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility in their application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence: (i.e. formulating problems mathematically in order to solve them) Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to be able to solve them.

Logical reasoning: As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and proof should become increasingly abstract, moving from verbal, visual or concrete explanations to representations involving symbols and conventions.

Fluency: As learners experience, understand and apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow. As a result, facts, relationships and techniques learned previously should become firmly established, memorable and usable.

[7] Based on *Adding It Up: Helping Children Learn Mathematics* by Kilpatrick, Swafford and Findel (National Research Council, 2001) and adapted to the Welsh context by Anne Watson, Emeritus Professor of Mathematics Education, University of Oxford, 2018.

Progression step 1

Achievement outcomes

I have explored patterns through a range of hands-on activities and by using a variety of concrete, visual and digital resources. I can recognise, copy and generalise patterns and sequences around me.

I have demonstrated an understanding of the concepts of equality and inequality, using objects.

I can talk about how sets of objects change when objects are added to and taken away from them.

Progression step 2

Achievement outcomes

I have explored patterns of numbers and shape using concrete, paper-based and digital materials. I can recognise, copy and generate sequences of numbers and visual patterns.

I have demonstrated an understanding of the concepts of equality and inequality within a number equation. I can use the equals sign to indicate that both sides of a number sentence have the same value and I can use inequality signs when comparing quantities.

I have explored commutativity with addition and multiplication and I can recognise when two different numerical expressions describe the same situation but are written in different ways.

Progression step 3

Achievement outcomes

I can recognise equalities, inequalities and equivalence of expressions, and also when commutativity, distributivity and associativity can be used to state a simple expression in a different way.

I have explored patterns of numbers and shapes, using digital and non-digital methods. I can interpret, explain in words and generalise numerical sequences and spatial patterns.

I can create equations to model problems, using symbols or words to represent unknown values. I can use inverse operations to find unknown values in simple equations using mental, written and digital methods, and manipulatives. I can check my answers.

I have explored the notion of function, including the use of digital function machines.

I have demonstrated an understanding of the idea of input, application of a rule (including inverse operations) and output, using a function machine or other appropriate methods.

Progression step 4

Achievement outcomes

I have demonstrated an understanding of the concept of a variable and I have used letters to represent variables in forming linear algebraic expressions.

I can manipulate algebraic expressions fluently by simplifying, expanding and factorising by extracting a common factor. I can also substitute values and change the subject of a formula when the subject appears in one term.

I can distinguish between algebraic expressions, equations and inequalities. I have used a variety of methods, including trial and improvement where appropriate, to solve equations and inequalities in the first degree, which may include brackets and unknowns on one or both sides. I can check my answers using substitution.

I have used equations and inequalities in the first degree to represent and model real-life situations and solve problems. I can interpret my answers and check that they make sense in context.

I can recognise linear sequences and can generalise them using algebra. I can find, describe and use the *n*th term. I can describe and use the term-to-term rule for simple iterative sequences. I can apply my knowledge of sequences to solve real-life and mathematical problems.

I can recognise, draw, sketch and interpret linear graphs, and investigate graphs, using written and digital methods. I can demonstrate an understanding of each of the terms in the equation of a straight line. I can explore the effect on the line when the constant or coefficient is changed.

Progression step 5

Achievement outcomes

I can fluently manipulate algebraic expressions by expanding double brackets, factorising quadratic expressions and changing the subject of a formula where the subject appears in more than one term. I can simplify and manipulate algebraic fractions.

I can solve a range of linear and higher-order equations, and inequalities, including simultaneous, quadratic and trigonometric equations, using numerical, graphical and algebraic methods where appropriate. I can then interpret the meaning of the answer, or answers, checking for reasonableness.

I have used equations and inequalities, and relevant graphs, to model and solve problems in real-life and mathematical contexts, including those which describe proportion and exponentiation, and I can use my knowledge of the real world and number sense to predict and check my interpretations of these.

I can recognise and generalise simple non-linear sequences using algebra.

I can understand the concept of an identity and can translate statements describing mathematical relationships into algebraic models, using expressions and equations.

I have investigated a variety of non-linear graphs (including quadratic, cubic and reciprocals), using written and digital methods. I can demonstrate an understanding of the effect of the coefficients, indices and constants on the shape of the graph. I can determine the gradient at a point and the area under a graph, and understand what these represent. I can use graphs to solve problems.

Planning for learning

Links within this area of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across the Mathematics and Numeracy Area of Learning and Experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

The number system is used to represent and compare relationships between numbers and quantities.

- The connections between arithmetic and algebra develop tools and skills for abstract reasoning from an early age.
- There is a strong relationship between the algorithms of arithmetic and the laws of algebra.
- The order of operations and laws of arithmetic are followed in algebra.

Geometry focuses on relationships involving properties of shape, space, and position, and measurement focuses on quantifying phenomena in the physical world.

- Algebra and geometry are connected principally through the expression of shape, measure and movement through algebraic expressions, equations and formulae.
- An equation is an algebraic concept, which, when graphed, becomes a geometric concept. The variables within the equation refer to geometric concepts.
- Co-ordinates, geometrically represented on the Cartesian plane, are defined by algebraic functions.
- Functions and mappings in algebra can be used to describe transformations.
- Algebraic formulae and equations are used to connect the geometric concepts of triangles with measures of angles and sides.

Statistics represent data, probability models chance, and both support informed inferences and decisions.

- Algebra is used within probability and statistics to express generalities and develop formulae.
- In probability, algebra allows us to use the concept of a variable and we can apply this in probability by using a random variable; this is a parameter or event with a random outcome.
- In statistics, general formulae are written using algebra.
- Algebra and statistical analysis are interlinked; use of algorithms and formulae to calculate further statistical measures to analyse data.

Links with other areas of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across all the areas of learning and experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

Expressive Arts

- Sequences and patterns.

Science and Technology

- Concept of the variable.
- Equations and formulae.
- Direct and inverse proportion.
- Patterns and graphs.

Experiences, knowledge and skills

Progression step 1

Learners need to experience, to know or be able to:

- explore, copy, repeat, continue and generate a range of simple patterns and sequences using a multisensory approach
- start to predict what will come next, through shared stories, pictures and rhymes that have a pattern element
- through active experience, demonstrate an understanding of equality and inequality, and how these can be preserved when numbers are changed, using the language of comparison and equality.

Progression step 2

Learners need to experience, to know or be able to:

- express and use, through experiential learning, general laws and rules of arithmetic
- explore and describe sequences of whole numbers with a common difference between terms
- find missing terms and continue sequences
- understand that Robert Recorde's equals sign is placed between two or more expressions that have the same value
- recognise commutativity of addition and multiplication, using objects, diagrams and numbers
- reason logically, using equality and the laws of arithmetic.

Progression step 3

Learners need to experience, to know or be able to:

- through a broad range of experiences, describe relations which depend on equality, equivalence and the commutative, associative and distributive laws
- describe and generate, in words, visual patterns and numerical sequences
- model simple problem situations, using words and symbols to create equations from which they can find an unknown value
- understand that a function can transform a set of numbers to a new set of numbers, according to a rule
- move between concrete, visual and abstract representations throughout their mathematical work.

Progression step 4

Learners need to experience, to know or be able to:

- through a broad range of experiences, appreciate the power of mathematics to express relationships concisely and in forms that are universally understood
- express the laws of arithmetic using algebraic notation, developing their conceptual understanding of a variable
- explore, generate, identify and represent linear patterns in a variety of contexts
- model real-life situations, using equations and inequalities to solve problems, checking the reasonableness of their solutions
- explore equations graphically, using digital technologies
- explore numerical and physical sequences, using written and digital methods
- investigate linear graphs in realistic situations.

Progression step 5

Learners need to experience, to know or be able to:

- through a broad range of experiences, create multiple representations of a mathematical concept
- explore connections between equations and expressions with geometric, numerical and graphical representations, and understand their equivalence
- explore, generate and identify non-linear patterns, and express them algebraically and graphically
- develop and apply their knowledge of a range of methods such as factorisation, simplification and inverse functions to change the subject of formulae involving two or more variables
- model real-life situations, identifying variables and constructing polynomials.

WHAT MATTERS IN
MATHEMATICS AND NUMERACY

Geometry focuses on relationships involving properties of shape, space and position, and measurement focuses on quantifying phenomena in the physical world.

Geometry involves playing with, manipulating, comparing, naming and classifying shapes and structures. The study of geometry encourages the development and use of conjecture, deductive reasoning and proof. Measurement allows the magnitude of spatial and abstract features to be quantified, using a variety of standard and non-standard units, and can support the development of numerical reasoning.

Reasoning about the sizes and properties of shapes and their surrounding spaces helps us to make sense of the physical world and the world of mathematical shapes. Geometry and measurement have applications in many fields, including art, construction, science and technology, engineering, and astronomy.

Learning

Principles of progression

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Communication with symbols: Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility in their application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence: (i.e. formulating problems mathematically in order to solve them) Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to be able to solve them.

Logical reasoning: As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and proof should become increasingly abstract, moving from verbal, visual or concrete explanations to representations involving symbols and conventions.

Fluency: As learners experience, understand and apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow. As a result, facts, relationships and techniques learned previously should become firmly established, memorable and usable.

[8] Based on *Adding It Up: Helping Children Learn Mathematics* by Kilpatrick, Swafford and Findel (National Research Council, 2001) and adapted to the Welsh context by Anne Watson, Emeritus Professor of Mathematics Education, University of Oxford, 2018.

Progression step 1

Achievement outcomes

I can understand and apply the language of time in relation to my daily life and in relation to events that happen around me, including naming the days of the week, the months of the year and the seasons, all in meaningful contexts.

I have used a variety of objects to measure lengths, masses and capacities, and I can understand the need to repeat the same physical unit without any gaps when measuring.

I can make estimates and comparisons with measures. I can use language and non-standard units to discuss my sense of size.

I have explored shapes through investigative play, and I can categorise and sort shapes using their properties. I can use the language of shapes to describe objects.

I have explored movements and directions, both physically and by using digital technology, and I can use mathematical language to describe position.

Progression step 2

Achievement outcomes

I can tell the time on an analogue clock (in at least 15-minute intervals) and I can connect this to time displayed digitally. I have explored and used different ways of showing the passing of time, including calendars, timelines, simple timetables and schedules.

I have estimated and measured length, volume, capacity, mass, temperature and time in practical situations, using non-standard units.

I have used a variety of measuring devices from different starting points.

I can apply standard units in practical situations to measure length, volume, capacity, mass, temperature and time accurately.

I have explored and named two-dimensional and three-dimensional shapes in a range of contexts. I can sort and categorise regular and irregular two-dimensional shapes in different ways, according to their properties. I can sketch two-dimensional shapes and make models of three-dimensional objects.

I can identify reflective symmetry in a range of contexts and can identify symmetry in two-dimensional shapes.

I have explored the concept of rotation, both physically and using digital technology, and can use simple fractions of a complete rotation to describe turns.

I have described and quantified the position of objects relative to other objects through active learning experiences, using appropriate language, including the words ‘left’ and ‘right’.

I can follow and create instructions related to movement, using a range of approaches and resources (including digital technologies) to demonstrate my understanding.

Progression step 3

Achievement outcomes

I can read analogue and digital clocks accurately and I can make calculations involving the passing of time. I have used timetables and schedules to make calculations involving time.

I have estimated and measured length, capacity, mass, temperature and time, using appropriate standard units.

I can convert between metric units in mathematical and real-life problem-solving contexts, and I can check my answers to make sure they make sense.

I have explored properties of two-dimensional shapes, using concrete, paper-based and digital resources, and I can use mathematical language to describe and compare the properties (including number of sides and symmetry) of given shapes, and to classify them accordingly. I can name different types of triangles.

I have explored vertices, edges and faces of three-dimensional shapes and I can use these characteristics to describe a three-dimensional shape. I have explored the relationship between a three-dimensional shape and two-dimensional nets, using concrete, paper-based and digital resources, and I can recognise nets of common three-dimensional shapes.

I have used a range of hands-on activities to explore the perimeter and area of shapes. I have derived the respective formulae for finding the area of a rectangle and of a right-angled triangle, and have applied them in mathematical and real-life problem-solving contexts, using estimation to predict and check my answers.

I can demonstrate my understanding of angle as a measure of rotation and I can recognise, name and describe types of angles.

I have developed an understanding of why we need co-ordinates and I can use them to locate and plot points in the first quadrant of the Cartesian plane. I can use my knowledge of co-ordinates to solve problems involving shape, length, angle and position in mathematical and real-life contexts.

Progression step 4

Achievement outcomes

I can represent and use compound measures, using standard units, and I can demonstrate an understanding of the relationship between a formula representing a measurement and the units used.

I can create and use conversion graphs to solve problems set in local and global contexts.

I have explored symmetries and other properties of regular and irregular two-dimensional and three-dimensional shapes. I can construct two-dimensional representations of three-dimensional shapes in order to investigate properties further. I can classify two-dimensional and three-dimensional shapes according to their mathematical properties.

I have explored all four transformations of two-dimensional shapes, using a variety of approaches, including digital technology. I can use my understanding to predict and describe how shapes will change under a given transformation.

I can use co-ordinates to plot points in the four quadrants and deduce the location of additional points.

I can use a protractor to measure and draw angles. I have modelled and solved problems involving bearings. I can use reasoning to calculate the size of angles in triangles and quadrilaterals. I have explored angles formed by parallel lines and by a transversal, and I can use my understanding to calculate angles in these contexts.

I have calculated the areas or surface areas of two-dimensional and three-dimensional simple and compound shapes, including circles, and have demonstrated an understanding of pi (π) as the ratio of the circumference of a circle to its diameter. I have derived the formulae for the volume of simple three-dimensional prisms and I can calculate the volumes of three-dimensional shapes to solve problems.

I have demonstrated an understanding of the relationship between right-angled triangles and squares in the context of Pythagoras' theorem, and I have used it to solve problems in mathematical and real-life contexts.

Progression step 5

Achievement outcomes

I can explain why two or more shapes are similar, congruent, or neither. I have used my knowledge of congruency and similarity to solve problems involving angles and lengths, both in mathematical and real-life contexts.

I can calculate the perimeter, area or surface area and volume of compound two-dimensional and three-dimensional shapes, and I can rearrange formulae to find missing lengths. I have demonstrated an understanding of the effect of scale when comparing measurements and shapes in all three dimensions, and I have used my knowledge of scale and ratio to calculate the lengths and areas of fractions of shapes, including arcs and segments of circles.

I have located and described the locus of points defined by a range of different criteria, using digital and non-digital technologies.

I have demonstrated an understanding of trigonometric ratios in right-angled triangles and I have used, in mathematical and real-life contexts, my knowledge of the trigonometric ratios to solve problems involving lengths, angles and area of any triangle.

I have used reasoning and logical arguments, along with my knowledge of polygons, intersecting lines, angle and the circle theorems, to solve problems, deduce and calculate angles and lengths in diagrams that involve combinations of these.

Planning for learning

Links within this area of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across the Mathematics and Numeracy Area of Learning and Experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

The number system is used to represent and compare relationships between numbers and quantities.

- Number is used throughout geometry to quantify shape, size and movement.
- Measure is an aspect of geometrical thinking which is highly connected to number and much of the development of understanding of number can emerge through increasingly sophisticated measuring.
- Geometric thinking involves reasoning with proportion, which connects with development in number work.
- Geometry involves lengths, areas and volumes which are expressed as numerical quantities.
- Use of rules of number to calculate further values related to measurement and geometry.

Algebra uses symbol systems to express the structures of relationships between numbers, quantities and relations.

- Algebra and geometry are connected principally through the expression of shape, measure and movement through algebraic expressions, equations and formulae
- An equation is an algebraic concept, which, when graphed, becomes a geometric concept. The variables within the equation refer to geometric concepts.
- Co-ordinates, geometrically represented on the Cartesian plane, are defined by algebraic functions.
- Functions and mappings in algebra can be used to describe transformations.
- Algebraic formulae and equations are used to connect the geometric concepts of triangles with measures of angles and sides.

Statistics represent data, probability models chance, and both support informed inferences and decisions.

- Geometry involves graphical thinking which is central to statistical representations and analysis.
- Graphs are used to convert from one value to another, and to represent statistical information including spread, quantity and central tendency.
- Graphical techniques are used to make connections between different sets of data.
- Data generated by measure can be analysed using statistics.

Links with other areas of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across all the areas of learning and experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

Expressive Arts

- Shape symmetry in movement and artwork.
- Scale.
- Use of appropriate equipment to measure accurately.

Health and Well-being

- Use of appropriate equipment to measure accurately.

Humanities

- Use of appropriate equipment to measure accurately.
- Scale.
- Time and chronological ordering.

Languages, Literacy and Communication

- Use of prepositions to describe the location of selves and objects.

Science and Technology

- Use of appropriate equipment to measure accurately.
- Units – use of appropriate unit, converting between units, and links between units and formulae.
- Scale.

Experiences, knowledge and skills

Progression step 1

Learners need to experience, to know or be able to:

- through exploring situations in their everyday life, develop an understanding of measurement, using non-standard units with developing accuracy
- through play and discovery, categorise and recognise shapes
- use the language of direction, location and position.

Progression step 2

Learners need to experience, to know or be able to:

- through experiential learning and solving problems in real-life and more abstract contexts, develop an understanding of standard measurement and of the properties of shapes
- select and use appropriate equipment and units to measure accurately
- describe location, movement and position, using mathematical terminology.

Progression step 3

Learners need to experience, to know or be able to:

- through a broad range of experiences and through solving problems in real-life and abstract contexts, refine their understanding of standard measurement and the relationship between units of measure
- select and use appropriate equipment and units to measure accurately
- refine their understanding of the properties of shapes
- use the mathematical concepts of angle, co-ordinates and distance to describe location, movement and position.

Progression step 4

Learners need to experience, to know or be able to:

- through a broad range of experiences and through solving problems within real-life and abstract contexts, develop their understanding of standard and compound measurement, the properties of shapes and solids, and location, movement and position
- select and use appropriate equipment and units to measure accurately
- use digital technologies to explore shape and space, developing and testing conjectures

- use angle and shape facts to deduce further features and relationships
- recognise pi (π) as the ratio of the circumference of a circle to its diameter and appreciate the significance of William Jones's contribution.

Progression step 5

Learners need to experience, to know or be able to:

- through a broad range of experiences, describe, represent, analyse and explain properties of shapes in two-dimensional and three-dimensional space
- use digital technologies to explore shape and space, movement and position, developing and testing conjectures
- work in real-life and mathematical contexts, using local examples where possible.

WHAT MATTERS IN MATHEMATICS AND NUMERACY

Statistics represent data, probability models chance, and both support informed inferences and decisions.

Statistics is the practice of collecting, manipulating and analysing data, allowing representation and generalisation of information. Probability is the mathematical study of chance, enabling predictions of the likelihood of events occurring. Statistics and probability rely on the application and manipulation of number and algebra.

Managing data and representing information effectively provides the means to test hypotheses, draw conclusions and make predictions. Reasoning with statistics and probability, and evaluating their reliability, develops critical thinking and analytical skills that are fundamental to making ethical and informed decisions.

Learning

Principles of progression

Principles of progression are the basis on which the achievement outcomes have been developed and should guide the progression of learning within the area of learning and experience.

The following interdependent proficiencies[9] have been used to formulate the achievement outcomes and are central to progression at each stage of mathematics learning.

Numeracy involves applying and connecting these proficiencies in a range of real-life contexts, across and beyond the curriculum.

Conceptual understanding: Mathematical concepts and ideas should be dwelt on, built on, and connected together as learners experience increasingly complex mathematical ideas. Learners demonstrate conceptual understanding by explaining and expressing concepts, finding examples (or non-examples) and by representing a concept in a variety of ways, including verbal, concrete, visual, digital and abstract representations.

Communication with symbols: Learners should understand that the symbols they are using are abstract representations and should develop greater flexibility in their application and manipulation of an increasing range of symbols, understanding the conventions of the symbols they are using.

Strategic competence: (i.e. formulating problems mathematically in order to solve them) Learners should become increasingly independent in recognising and applying the underlying mathematical structures and ideas within a problem, in order to be able to solve them.

Logical reasoning: As learners experience increasingly complex concepts, they should also develop an understanding of the relationships between and within these concepts. They should apply logical reasoning about these relationships and be able to justify and prove them. Justifications and proof should become increasingly abstract, moving from verbal, visual or concrete explanations to representations involving symbols and conventions.

Fluency: As learners experience, understand and apply increasingly complex concepts and relationships, fluency in remembering facts, relationships and techniques should grow. As a result, facts, relationships and techniques learned previously should become firmly established, memorable and usable.

[9] Based on *Adding It Up: Helping Children Learn Mathematics* by Kilpatrick, Swafford and Findel (National Research Council, 2001) and adapted to the Welsh context by Anne Watson, Emeritus Professor of Mathematics Education, University of Oxford, 2018.

Progression step 1

Achievement outcomes

I have investigated and collected data found in my environment. I can record data, giving meaning to marks which represent number and where each quantity corresponds to something in the real world, keeping simple records which include tallying and pictograms.

I have grouped sets of objects or pictures into categories, through practical experiences, and I can classify by one criterion or more. I can talk about the rule(s) I have used and can reclassify according to new criteria.

I can present data using non-digital and digital methods. I have created simple charts and graphs and I have talked about what they mean.

Progression step 2

Achievement outcomes

I can collect and organise data for posing and answering questions in relevant situations.

I can sort and classify using more than one criterion, using Venn diagrams and Carroll diagrams in practical situations.

I have used digital and non-digital methods to record and present data in a variety of ways, including the use of tally charts, frequency tables, and block graphs when appropriate axes and scales are provided.

I can interpret and analyse graphs, charts and data.

I can explain what I have found out, justify my reasoning and I can evaluate how well my method worked.

Progression step 3

Achievement outcomes

I have used a sequential data-handling cycle to ask and answer appropriate questions in meaningful situations.

I can pose and answer sensible questions and I have demonstrated an understanding of the importance of collecting relevant data that can be used to answer my questions.

I have demonstrated an understanding of the kinds of data I need, including discrete and continuous data.

I can find and use the mean of a simple set of data in meaningful contexts.

I have used appropriate methods to answer my questions by collecting, analysing and summarising my data and interpreting my results. I can evaluate my methods and suggest different or better ways to approach investigations in the future.

I have represented information by creating a variety of appropriate charts of increasing complexity, including tally charts, frequency tables, bar graphs, line graphs, with and without the use of digital technologies. I have created pie charts using digital technology.

I can use different scales on axes to extract and interpret information from a range of diagrams, tables (including databases) and graphs, including pie charts with simple fractions and proportions.

I have investigated simple statistics, presented in the media and elsewhere, to support an argument and I can explain how the statistics do or do not support the argument.

I can recognise validity and trends, and can discuss how anomalies may affect conclusions when evaluating results.

I have explored the possibility of given outcomes and have used the language of probability to describe the chance of an event occurring.

I have played games that involve flipping coins, rolling dice and using spinners in order to simulate and discuss chance.

I have hypothesised and anticipated outcomes of chance experiments in a range of contexts, recording my findings in a systematic and appropriate way.

Progression step 4

Achievement outcomes

I have used a sequential data-handling cycle to ask and answer appropriate questions.

I can choose a sensible hypothesis to investigate and I have demonstrated an understanding of the methods I can use to collect appropriate data.

I can design and have critiqued questionnaires to ensure that the data to be collected will enable the sequential data-handling cycle to be used.

I can calculate the mean, median, mode and range of a set of data and I have made comparisons between small sets of data using summary statistics.

I can select and justify an appropriate way to use my data to investigate my hypothesis. I have explored different ways to understand and summarise my data, including using averages to make comparisons between large data sets, with grouped frequency distributions for discrete and continuous data. I can use a scatter diagram to analyse two sets of variables and investigate correlation between them. I can make predictions and identify trends and anomalies in data sets.

I have presented my data in the form of appropriate graphs, charts and tables, and I have used digital technologies, taking into account the purpose of the data and nature of the audience. I have discussed the relative advantages and disadvantages of each presentation method, and I can justify my choice of method.

I have used my data to draw conclusions about my hypotheses and I have communicated my findings clearly. I can critique my own methods and findings, and consider what I may have done differently or better at each stage of the sequential data-handling cycle.

I have critically analysed statistics published in the media and elsewhere to consider what it means and how it does, or does not, support any findings reported. I can pose relevant questions to check the credibility of the findings.

I can explain randomness, and I have investigated chance by modelling and by comparing theoretical and experimental probabilities.

I have explored all the possible mutually exclusive outcomes of successive and combined events. I can work systematically making use of lists and sample space diagrams, and I have shown an understanding that the sum of probabilities of all mutually exclusive outcomes is 1. I have demonstrated an understanding of when it is appropriate to add or multiply probabilities.

I can make meaningful real-life judgements based on outcomes of experimental data and risk.

Progression step 5

Achievement outcomes

I have used a sequential data-handling cycle to ask and answer appropriate questions.

I have explored different sampling methods and I have demonstrated an understanding of the need to select a sample when collecting data. I can evaluate, choose and use different sampling techniques, including random sampling, stratified sampling and systematic sampling.

I have experimented with different approaches to presenting data, including cumulative frequency, box and whisker, and histograms, to interpret measures of central tendency and measures of spread. I can select appropriate approaches when comparing data sets, justifying and evaluating my choices.

I have critically analysed statistics in the media, considering how data is presented, its reliability, and whether and how the data has been manipulated to tell a particular story. I can make informed decisions based on statistical evidence, identifying bias and anomalies.

I can solve problems involving probabilities of mutually exclusive, independent and dependent events in real-life and mathematical contexts. I can use a variety of strategies, including using Venn and tree diagrams, to solve problems in local and wider contexts.

I have played and created games to understand the relationship between relative frequency and theoretical probabilities, making judgements on outcomes of experimental data and risk.

I can use probabilistic arguments, drawing on theory, information, research and experimentation to support my conclusion.

Planning for learning

Links within this area of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across the Mathematics and Numeracy Area of Learning and Experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

The number system is used to represent and compare relationships between numbers and quantities.

- Probability and statistics are described and manipulated by using number; they are represented using number.
- Probability is expressed through number in various ways, e.g. through the use of percentages, fractions and decimals, and the connections between the representations is necessary for effective expression of probability.
- Statistics involves manipulation and representation of data, which involve numerical thinking.

Algebra uses symbol systems to express the structures of relationships between numbers, quantities and relations.

- Algebra is used within probability and statistics to express generalities and develop formulae.
- In probability, algebra allows us to use the concept of a variable and we can apply this in probability by using a random variable; this is a parameter or event with a random outcome.
- In statistics, general formulae are written using algebra.
- Algebra and statistical analysis are interlinked; use of algorithms and formulae to calculate further statistical measures to analyse data.

Geometry focuses on relationships involving properties of shape, space, and position, and measurement focuses on quantifying phenomena in the physical world.

- Geometry involves graphical thinking which is central to statistical representations and analysis.
- Graphs are used to convert from one value to another, and to represent statistical information including spread, quantity and central tendency.
- Graphical techniques are used to make connections between different sets of data.
- Data generated by measure can be analysed using statistics.

Links with other areas of learning and experience

This section suggests where learning can be enriched through drawing links between other what matters statements across all the areas of learning and experience. It also suggests where different elements of learning could be considered together in order to support more holistic learning.

Health and Well-being

- Sorting and classifying.
- Lines of best fit.
- Analysing and inferring from data.
- Spotting trends and anomalies.
- Averages and range.
- Drawing and interpreting a range of graphs.

Humanities

- Sorting and classifying.
- Lines of best fit.
- Analysing and inferring from data.
- Spotting trends and anomalies.
- Averages and range.
- Drawing and interpreting a range of graphs.

Science and Technology

- Sorting and classifying.
- Lines of best fit.
- Analysing and inferring from data.
- Spotting trends and anomalies.
- Averages and range.
- Drawing and interpreting a range of graphs.

Experiences, knowledge and skills

Progression step 1

Learners need to experience, to know or be able to:

- through play and discovery in stimulating and relevant local contexts, collect and sort a variety of simple data and answer questions
- use appropriate digital and non-digital methods to represent and interpret data
- talk about their results.

Progression step 2

Learners need to experience, to know or be able to:

- through experiential learning in stimulating and relevant local contexts, use a sequential data-handling cycle to pose questions, then collect and sort a variety of purposeful data to answer questions
- use appropriate digital and non-digital methods to analyse, represent and interpret data
- explain their results and evaluate their methods.

Progression step 3

Learners need to experience, to know or be able to:

- through a broad range of experiences in stimulating and relevant local and global contexts, use a sequential data-handling cycle to pose questions, collect and sort a variety of purposeful data
- use appropriate digital and non-digital methods to analyse, summarise, represent and interpret data
- through using different approaches, compare, check and evaluate their results, explaining what they have found
- experiment with chance.

Progression step 4

Learners need to experience, to know or be able to:

- through a broad range of experiences in stimulating and relevant local and global contexts, use a sequential data-handling cycle to pose questions, collect and sort a variety of purposeful data
- use appropriate digital and non-digital methods to analyse, summarise, represent and interpret data to draw conclusions

- check and evaluate their results and recognise anomalies and trends in data
- extend their understanding of chance by expressing theoretical probability numerically.

Progression step 5

Learners need to experience, to know or be able to:

- through a broad range of experiences in stimulating and relevant local and global contexts, use a sequential data-handling cycle to pose questions, select a sample, and collect and sort a variety of purposeful data
- use appropriate digital and non-digital methods to analyse, summarise, represent and interpret data to draw conclusions
- check and evaluate their results
- engage critically with statistics in the media
- calculate probabilities of combined events and consider probabilities of real-life events.

GLOSSARY

Word/phrase	Definition
Abstract	Existing in thought or as an idea but not having a physical or concrete existence.
Additive relationship	Quantities that can be expressed as related to each other through addition or/and subtraction.
Algorithm	A sequence of steps required in order to perform a computation or to solve a problem.
Array multiplication	A diagram showing a multiplication sum arranged as an array. For example 5 × 4 can be thought of as 20 counters arranged into 5 rows and 4 columns.
Associative	This is a law which is used to describe mathematical operators that give the same answer when grouped in different ways. The addition and multiplication of numbers are associative, e.g. 2 x (4 x 3) = (2 x 4) x 3
Cardinal number	A number denoting quantity (one, two, three, etc.), as opposed to an ordinal number which describes the position of something in a sequence (first, second, third, etc.).
Cartesian plane	The Cartesian plane is defined by two perpendicular number lines: the x-axis, which is horizontal, and the y-axis, which is vertical. Co-ordinates such as (4, -2) can be placed onto a Cartesian plane.
Coefficient	A coefficient is a number that appears before the variable or variables in an algebraic term. In the term $5x^2$, the coefficient is 5 and the variable is x. In the term y^3, the coefficient (not written) is 1 and the variable is y.
Commutative	This is a law which is used to describe mathematical operators that give the same answer, even if the order in which we input the numbers are changed. The addition and multiplication of numbers are commutative, i.e. a + b = b + a and a × b = b × a
Compound measure	A compound measure combines two or more different units of measure to form a new unit of measure. For example, distance can be measured in miles and time in hours. We can combine these measures to form the compound measure speed which is measured here in miles per hour (mph).

Word/phrase	Definition
Concrete	Existing in a material or physical form; not abstract.
Conjecture	An opinion or conclusion formed on the basis of incomplete information.
Constant	This is the term for a value which doesn't vary, e.g. a fixed number.
Continuous data	If data is continuous, any number within a specific range can appear in that set. A person's height or weight is an example of continuous data.
Degree of accuracy	The degree of accuracy is a measure of how close and correct a stated value is to the actual, real value being described, e.g. we may be given a value that is rounded to the nearest 100, to one decimal place, or to two significant figures.
Discrete data	If data is discrete, only specific numbers can appear in that set. The number of learners in a class is an example of discrete data because it isn't possible to have half a learner.
Distributive	This is a law that states that multiplying a number by a group of numbers which have been added together, is the same as adding each multiplication separately: $a(b + c) = a \times b + a \times c$
Exponentiation	The operation of raising one quantity to the power of another.
First degree	First degree equations are equations where the highest exponent is 1. Linear equations are examples of first degree equations.
Function	A function is an algebreic relationship where each input has a single output.
Identity	An identity shows that the value of two expressions is always equal, regardless of the value of any variables used. The symbol for an identity is ≡. Note that it has three horizontal lines, not two as does an equals sign. An example of an identity is $2(x + 3) \equiv 2x + 6$
Improper fraction	This is the name given to a common fraction where the numerator is greater than the denominator, e.g. $^{8}/_{5}$ and $^{13}/_{11}$
Inequality	An inequality contains two expressions separated by one of the following symbols: <, >, ≤ or ≥. The symbol < represents 'less than'. The symbol > represents 'greater than'. The symbol ≤ represents 'less than or equal to'. The symbol ≥ represents 'greater than or equal to'.

Word/phrase	Definition
Integer	An integer is any number without a fractional or decimal part, for example 8 or –4. Integers include the whole numbers, zero, and the negative of the whole numbers.
Inverse	Mathematical inverse is the opposite. We can think about this in the context of mathematical operations. Addition is the inverse of subtraction, e.g. 5 + 6 = 11. We can reverse this by subtracting: 11 – 6 = 5. Therefore, 5 + 6 – 6 = 5
Irregular	The term irregular is used to describe a polygon in which not all the sides are equal, or not all the angles are equal. Rectangles and rhombi are examples of irregular quadrilateral polygons. All the angles of a rectangle are equal (right angles), but not all of the sides are equal. All the sides of a rhombus are equal, but not all of the angles are equal.
Iterative	Repetition of a mathematical procedure applied to the result of a previous application.
Large data set	A large data set is a data set that requires the use of technology for comprehension, typically containing 50 items or more.
Laws of arithmetic	There are three laws of arithmetic. The associative law, the commutative law, and the distributive law.
Linear	The word linear refers to an algebraic term or terms to the power 1 (the 1 isn't usually written down). Examples of linear terms are x, 4y and –9z. The opposite of 'linear' is non-linear.
Manipulatives	Manipulatives are physical tools of teaching, engaging learners visually and physically with objects such as coins, blocks, rods, puzzles, etc.
Measure of central tendency	Measures of central tendency are mean, median and mode.
Measure of spread	A measure of spread is used to describe the variability in a sample or population, e.g. the range.
Multiple representations	Representing something in different ways, e.g. representing a function symbolically and graphically.
Multiplicative relationship	Quantities that are related to each other through multiplication or/and division.

Word/phrase	Definition
Mutually exclusive	This is the term used to describe events which can't happen at the same time. For example, when we throw a dice, throwing 6 and throwing 1 are mutually exclusive.
Non-example	A non-example doesn't satisfy a rule or a definition. It provides more information about what is and what is not included in a rule or definition. Non-examples of a triangle would include a square and an open three-sided polygon.
Non-standard unit of measure	Non-standard units of measurement are units of measurement that aren't typically used, such as a pencil, an arm or a shoe.
One-to-one correspondence	Each member of one set is associated with a member of another set.
Operator	A symbol (such as +, −, ×, ÷) that shows an operation.
Ordinal number	An ordinal number is a number which describes the position of something in a sequence. For example: first (1st), second (2nd), third (3rd), fourth (4th), tenth (10th), twentieth (20th) and so on.
Partitioning a number	Splitting a number into parts.
Polynomial	An expression of two or more algebraic terms, especially the sum of several terms that contain different powers of the same variable.
Proof	A mathematical proof is an argument that deduces the statement that should be proved, from other statements that are known to be true.
Proportional change	When a quantity is changed using a fraction, percentage or decimal. For example, increasing £100 by 35%.
Proportional reasoning	Solving problems involving proportional change in which quantities have a multiplicative rather than an additive relationship.
Randomness	The property of lacking a pattern or principle of organisation; unpredictability.
Reciprocal	The reciprocal is the multiplicative inverse of any number, excluding zero. (Zero does not have a reciprocal.) For example: The reciprocal of 3 is ⅓, and the reciprocal of ⅔ is $\frac{3}{2}$
Recurring decimals	A recurring decimal is a decimal which has an infinite number of digits. An example is 0.133333333….It could be written as $0.1\dot{3}$

Word/phrase	Definition
Regular	'Regular' is used to describe a polygon with equal sides and equal angles.
Sequential data-handling cycle	The process of (1) specifying the problem and planning; (2) collecting data from a variety of sources; (3) processing and representing data; (4) interpreting and discussing data; (5) evaluating the process.
Significant figures	Each of the digits of a number that are used to express it to the required degree of accuracy, starting from the first non-zero digit.
Tolerance	An allowable amount of variation of a specified quantity.
Transversal	A transversal is a line that crosses two other lines.
Unit fractions	A fraction where the numerator is 1
Variable	A variable is a symbol or a letter which either represents an unknown value or whose value can vary.
Whole numbers	The numbers 0, 1, 2, 3, 4, and so on are whole numbers. They are similar to integers, but whole numbers do not include negative numbers. Whole numbers do not have fractional parts or decimals left over.

Mae'r ddogfen yma hefyd ar gael yn Gymraeg. This document is also available in Welsh.