USING AND APPLYING MATHEMATICS TO SOLVE PROBLEMS

Pupils should be taught to:

Solve word problems and investigate in a range of contexts

As outcomes, Year 7 pupils should, for example:

Solve word problems and investigate in contexts of number, algebra, shape, space and measures, and handling data; compare and evaluate solutions.

Problems involving money

For example:

- A drink and a box of popcorn together cost 90p.
  Two drinks and a box of popcorn together cost £1.45.
  What does a box of popcorn cost?

- Six friends went to a Chinese restaurant.
  The total cost for the set menu was £75.
  How much would the set menu cost for eight people?

- This is what a stationery shop charges for printing a book.

  Print charges
  3p per page
  75p for the cover

  Jon paid £4.35 for his book, including its cover.
  How many pages were there in his book?

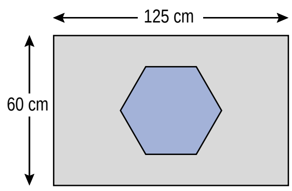
See Y456 examples (pages 84–5).

Problems involving percentages

For example:

- The value of a £40 000 flat increased by 12% in June.
  Its new value increased by a further 10% in October.
  What was its value in November?

- 20% of this flag is blue.
  What area of the flag is blue?



- Which of these statements is true?
  A. 75% of £6 > 60% of £7.50
  B. 75% of £6 = 60% of £7.50
  C. 75% of £6 < 60% of £7.50

- The price of a pair of jeans was decreased by 10% in a sale.
  Two weeks later the price was increased by 10%.
  The final price was not the same as the original price.
  Explain why.

See Y456 examples (pages 32–3).

As outcomes, Year 8 pupils should, for example:

Solve more demanding problems and investigate in a range of contexts; compare and evaluate solutions.

Problems involving money

For example:

- At Alan's sports shop, GoFast trainers usually cost £40.95, but there is ⅓ off in the sale.
  At Irene's sports shop, GoFast trainers normally cost £40, but there is a discount of 30% in the sale.
  Which shop sells the trainers for less in the sales?

- A supermarket sells biscuits in these packets:

  15 biscuits for 56p
  24 biscuits for 88p
  36 biscuits for £1.33

  Which packet is the best value for money?
  Why do you think this is the case?

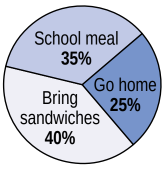
- Tina and Jill each gave some money to a charity. Jill gave twice as much as Tina, and added £4 more. Between them, the two girls gave £25 to the charity. How much did Tina give?

Problems involving percentages

For example:

- Chloe and Denise each bought identical T-shirts from the same shop.
  Chloe bought hers on Monday when there was 15% off the original price.
  Denise bought hers on Friday when there was 20% off the original price.
  Chloe paid 35p more than Denise.
  What was the original price of the T-shirt?

- The pie chart shows the lunch choices of class 8NP.



  7 pupils in class 8NP have a school meal.
  How many go home for lunch?

- The floods in Mozambique in February 2000 affected 600 000 people, one twenty-eighth of the population of Mozambique.
  350 000 people had to leave their homes.
  What percentage of the population had to leave their homes during the floods?

As outcomes, Year 9 pupils should, for example:

Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; *generate fuller solutions.*

Problems involving money

For example:

- Two families went to the cinema.
  The Smith family bought tickets for one adult and four children and paid £19.
  The Jones family bought tickets for two adults and two children and paid £17.
  What was the cost of one child's ticket?

- Thirty years ago the money used in Great Britain was pounds, shillings and pence.
  There were 20 shillings in £1.

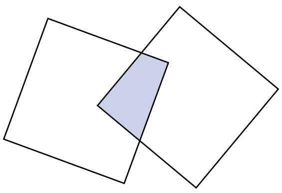
  A gallon of petrol cost 7 shillings thirty years ago.
  Today it costs about £3.80.
  How much has the cost of petrol risen in the last thirty years?

Problems involving percentages

For example:

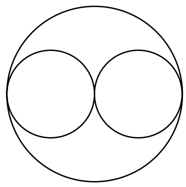
- After an advertising campaign costing £950, a firm found that its profits rose by 15% to £7200.
  From a financial point of view, was the advertising worthwhile? Justify your answer.

- The diagram shows two overlapping identical squares.



  20% of each square is shaded.
  What fraction of the whole diagram is shaded?

- *The diagram shows two identical small circles in a big circle.*



  What percentage of the big circle is filled by the two small circles?

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Solve word problems and investigate in a range of contexts (continued)

Problems involving ratio and proportion

For example:

- In a country dance there are 7 boys and 6 girls in every line.
  42 boys take part in the dance.
  How many girls take part?
- Some children voted between a safari park and a zoo for a school visit. The result was 10 : 3 in favour of the safari park.
  130 children voted in favour of the safari park.
  How many children voted in favour of the zoo?
- The risk of dying in any one year from smoking is about 1 in 200 of the population.
  The population of the UK is about 58 million.
  How many people are likely to die this year from smoking?
- A boy had £35 in birthday money.
  He spent some of the money.
  He saved four times as much as he spent.
  How much did he save?
- There are 3 chocolate biscuits in every 5 biscuits in a box.
  There are 30 biscuits in the box.
  How many of them are chocolate biscuits?
- A recipe for mushroom soup uses 7 mushrooms for every ½ litre of soup.
  How many mushrooms do you need to make 2 litres of soup?
  How much soup can you make from 21 mushrooms?
- 10 bags of crisps cost £3.50.
  What is the cost of 6 bags of crisps?
- £1 = 12.40 Danish kroner.
  How much is £1.50 in Danish kroner?

See Y456 examples (pages 26–7).

As outcomes, Year 8 pupils should, for example:

Problems involving ratio and proportion

For example:

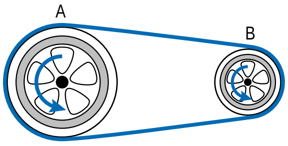
- To make orange paint, you mix 13 litres of yellow paint to 6 litres of red paint to 1 litre of white paint.
  How many litres of each colour do you need to make 10 litres of orange paint?

- Flaky pastry is made by using flour, margarine and lard in the ratio 8 : 3 : 2 by weight.
  How many grams of margarine and lard are needed to mix with 200 grams of flour?

- A square and a rectangle have the same area.
  The sides of the rectangle are in the ratio 9 : 1.
  Its perimeter is 200 cm.
  What is the length of a side of the square?

- The chocolate bars in a full box weighed 2 kg in total. Each bar was the same size.
  Eight of the bars were eaten.
  The bars left in the box weighed 1.5 kg altogether.
  How many chocolate bars were in the original box?

- A and B are two chain wheels.



  For every 2 complete turns that wheel A makes, wheel B makes 5 complete turns.
  Wheel A makes 150 turns.
  How many turns does wheel B make?

- A 365 g packet of coffee costs £2.19.
  How much per 100 g of coffee is this?

- £1 is worth 3 Australian dollars (A$).
  A girl changed £50 into Australian dollars, went on holiday to Australia and spent A$ 96.
  At the end of the holiday she changed the Australian dollars back into £.
  How much did she get?

- The cost of a take-away pizza is £6.
  A pie chart is drawn to show how the total cost is made up. The cost of labour is represented by a sector of 252° on the pie chart.
  What is the cost of labour?

- Cinema tickets for one adult and one child cost a total of £11.55.
  The adult's ticket is one-and-a-half times the price of the child's ticket.
  How much does each ticket cost?

As outcomes, Year 9 pupils should, for example:

Problems involving ratio and proportion

For example:

- 2 parts of red paint mixed with 3 parts of blue paint make purple paint.
  What is the maximum amount of purple paint that can be made from 50 ml of red paint and 100 ml of blue?

- A recipe for jam uses 55 g of fruit for every 100 g of jam.

  I want to make ten 454 g jars of jam.
  How many grams of fruit do I need?

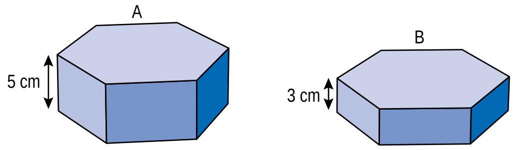
- A boy guessed the lengths of some objects.

	Guess	Actual length
A.	5 cm	7 cm
B.	40 cm	53 cm
C.	3 m	2 m
D.	10 m	7.8 m
E.	20 m	25 m

  He divided each guessed length by the measured length to produce an accuracy ratio.

  The accuracy ratio for object A is 0.71 : 1 (to 2 d.p.).
  Find the accuracy ratio for each other guess.
  Which guess was the most accurate?
  Which guess was the least accurate?

- Prisms A and B have the same cross-sectional area.



  Prism A is 5 cm high and has a volume of 200 cm^3.
  What is the volume of prism B?

- Take a two-digit number. Reverse the digits.
  Is it possible to make a number that is one-and-a-half times as big as the original number?
  Justify your answer.
  Find a two-digit number that is one-and-three-quarters times as big when your reverse its digits.

- *The Queen Mary used to sail across the Atlantic.*
  Its usual speed was 33 miles per hour.
  On average, it used fuel at the rate of 1 gallon for every 13 feet sailed.
  Calculate, to the nearest gallon, how many gallons of fuel the ship used in one hour of travelling at its usual speed.
  (There are 5280 feet in one mile.)

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving number and algebra

For example:

- I think of a number, add 3.7, then multiply by 5.
  The answer is 22.5. What is the number?

- Use each of the digits 1, 2, 3, 4, 5 and 8 once to make this sum correct:
  □□ + □□ = □□

- Use only the digits 2, 3, 7 and 8, but as often as you like.
  Make each sum correct.

□□ + □□ = 54	□□ + □□ = 155
□□ + □□ = 69	□□ + □□ = 105
□□ + □□ = 99	□□ + □□ = 110

- Here is a subtraction using the digits 3, 4, 5, 6, 7.

  365 – 47

  Which subtraction using all the digits 3, 4, 5, 6, 7 has the smallest positive answer?

- Two prime numbers are added. The answer is 45.
  What are the numbers?

- What is the largest multiple of 6 you can make from the digits 6, 7 and 8?
  What is the largest multiple of 7 you can make from the digits 5, 6 and 7?

- $\frac{4}{15}$ and $\frac{24}{9}$ are examples of three-digit fractions.
  There is only one three-digit fraction which equals $1\frac{1}{2}$.
  What is it?
  Find all the three-digit fractions that equal $2\frac{1}{2}$.
  Explain how you know when you have found them all.

- What operation is represented by each ∗ ?

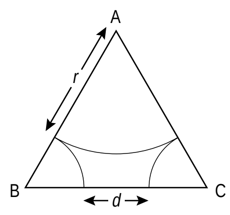
a. 468 ∗ 75 = 543	c. 468 ∗ 75 = 393
b. 468 ∗ 75 = 6.24	d. 468 ∗ 75 = 35 100

- Find two consecutive numbers with a product of 702.

- A function machine changes the number n to the number $3n + 1$.
  What does it do to these numbers?
  2 5 9 21 0
  What numbers must be input to get these numbers?
  10 37 100

- Triangle ABC is an equilateral triangle.
  The length of a side is a cm.
  Arcs with centres A, B and C are drawn as shown.

  Express d in terms of a and r.



See Y456 examples (pages 78–9, 82–3).

As outcomes, Year 8 pupils should, for example:

Problems involving number and algebra

For example:

- What fraction is half way between $\frac{3}{5}$ and $\frac{5}{7}$?

- Find the missing digits.
  The * need not be the same digit in each case.

  a. $\frac{*}{**} + \frac{*}{8} = \frac{3}{**}$

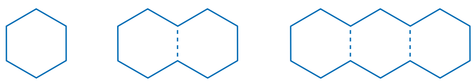
  b. $(**)^3 = ***7$

- Find four consecutive odd numbers with a total of 80.
  Find three consecutive numbers with a product of 21 924.
  Find two prime numbers with a product of 6499.

- The product of two numbers is 999.
  Their difference is 10.
  What are the two numbers?

- Calculate these, without a calculator:
  4^2 4^3 4^4 4^5 4^6
  What digit does 4^{20} end in?
  What digit does 4^{21} end in?

- This is a pattern of hexagons.
  Each side is 1 cm in length.



Find the three values marked by '**?**' in this table.

No. of hexagons	1	2	3	4	...	9	...	**?**	...	25
Perimeter	6	10	14	18	...	**?**	...	66	...	**?**

Explain why the perimeter of a pattern of hexagons could not be 101 cm.

- Each inside edge of two cube-shaped tins is 2*s* cm and *s* cm respectively.
  The larger tin is full of water; the smaller tin is empty.
  How much water is left in the larger tin when the smaller tin has been filled from it?

- A rectangle is 7*k* cm long and 3*k* cm wide.
  Find the area of a square whose perimeter is the same as that of the rectangle.
  Which has the larger area, the square or the rectangle? By how much?

- A boy gets 2 marks for each sum that he gets right and ⁻1 mark for each that he gets wrong.
  He did 18 sums and got 15 marks.
  How many sums did he get right?

As outcomes, Year 9 pupils should, for example:

Problems involving number and algebra

For example:

- 7^3 is 343. Without using a calculator, work out the units digit of 7^{12}.

- A number is a multiple of 21 and 35, and has four digits. What is the smallest number it could be?

- Here is a multiplication using the digits 3, 4, 5, 6, 7.

  $34 \times 56 \times 7$

  Which multiplication using all of the digits 3, 4, 5, 6, 7 has the smallest answer?
  Justify your answer.

- Put the digits 1, 2, 3, 4, 5 and 6 in place of the stars. Use each digit once.

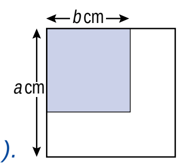
  $* \times ** = ***$

- Find the smallest number greater than 50 that has the same number of factors as 50.
  Justify your answer.

- Start with a two-digit number (TU). Find 3U + T.
  Repeat this with the new number to form a sequence.
  Try other two-digit numbers. What happens?
  What number gives the shortest sequence?
  What number gives the longest sequence?

- This fraction sum is made from four different digits.
  $\frac{3}{6} + \frac{1}{2}$
  The fraction sum is 1.
  Find as many as possible fraction sums made from four different digits and with a sum of 1.

- *The diagram shows a square of side a cm, from which a square of side b cm has been removed. Use the diagram to show that $a^2 - b^2 = (a - b)(a + b)$.*



- *I think of a number. All except two of 1 to 10 are factors of this number. The two numbers that are not factors are consecutive. What is the smallest number I could be thinking of?*

- *Two satellites circle round the Earth.*
  Their distance from the centre of the Earth is:
  Satellite A 1.53×10^7 miles
  Satellite B 9.48×10^6 miles

  What is:
  a. the minimum distance apart,
  b. the maximum distance apart,
  the satellites could be?

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

More problems involving number and algebra

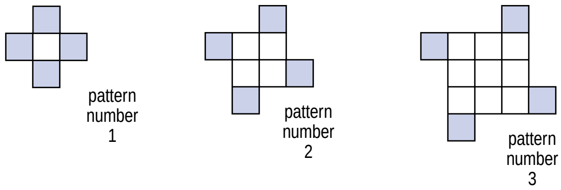
For example:

- Here is a sequence of five numbers.

  2 ☐ ☐ ☐ 18

  The rule is to start with 2 then add the same amount each time. Write in the missing numbers.

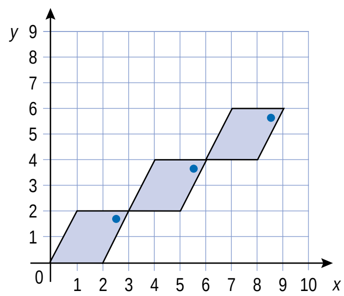
- This is a series of patterns with white and blue tiles.



  How many white tiles and blue tiles will there be in pattern number 8? Explain how you worked this out.
  Will one of the patterns have 25 white tiles?
  Give a mathematical reason for your answer.

- Does the line $y = x - 5$ pass through the point (10, 10)? Explain how you know.

- Lucy has some blue tiles, each with a marked corner. She sets them out as shown.



  Lucy carries on laying tiles. She says: '(21, 17) will be the coordinates of one of the marked corners.'

  Do you think Lucy is right? Explain your answer.

- This formula tells you how tall a boy is likely to grow.

  > Add the mother's and father's heights.
  > Divide by 2.
  > Add 7 cm to the results.
  > A boy is likely to be this height, plus or minus 10 cm.

  Marc's mother is 168 cm tall and his father is 194 cm tall. What is the greatest height that Marc is likely to grow?

- A girl at a fair tries the hoopla.
  She pays a £2 coin for 4 goes and is given change.
  The cost of each go is c pence.
  Which of these expressions gives her change in pence?
  $4c - 2$ $4c - 200$ $2 - 4c$ $200 - 4c$

See Y456 examples (pages 78–9, 82–3).

As outcomes, Year 8 pupils should, for example:

More problems involving number and algebra

For example:

- The next number in the sequence is the sum of the two previous numbers.
  Fill in the missing numbers.

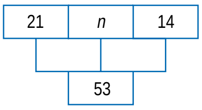
  □ □ □ 1 0 1 1 2 3 5 8

- Here is a sequence:

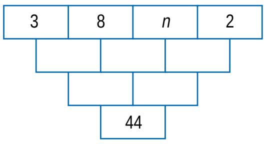
  6 19 32 45 58 71 …

  The sequence continues in the same way.
  a. Write the next three terms of the sequence.
  b. Explain the rule for finding the next term.
  c. What is the 30th term of the sequence?
  d. What is the nth term of the sequence?

- In this ‘pyramid’ each number is equal to the sum of the two numbers immediately above it.
  Find the missing number n which makes the bottom number correct.

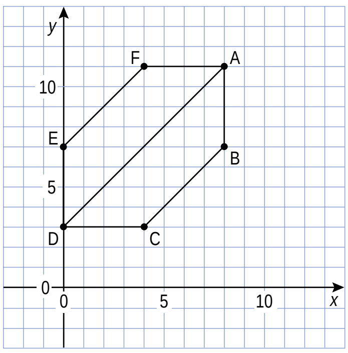


  What about this one?



- Elin played a number game. She said:
  ‘Multiplying my number by 4 then subtracting 5 gives the same answer as multiplying my number by 2 then adding 1.’
  Work out the value of Elin’s number.

- Look at this diagram.



  The line through points A and F has the equation $y = 11$.
  What is the equation of:
  a. the line through points A and B?
  b. the line through points F and E?
  c. the line through points B and C?

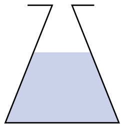
- A line with equation $y = mx + 9$ passes through the point (10, 10). What is the value of m?

As outcomes, Year 9 pupils should, for example:

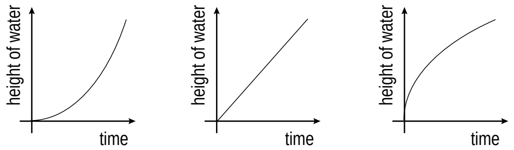
More problems involving number and algebra

For example:

- This bottle is being steadily filled with water.



  Which graph shows the relationship between time and the height of water in the bottle?

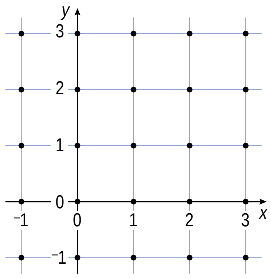


- Show that if $a(b - 4) = 0$, then either $a = 0$ or $b = 4$.

- The length of an edge of a solid cube is a cm.
  Its surface area is S cm², and its volume is V cm³.
  Prove that $S^3 = 216V^2$.

- Find a pair of numbers satisfying $7x - 2y = 38$, such that one is three times the other.
  Is there more than one answer?

- *The point (x, y) on the dotted grid is satisfied by the inequalities:*
  $x > 0$ *and* $y > 0$
  $x + y < 4$
  $x > y$
  What are the coordinates of the point?

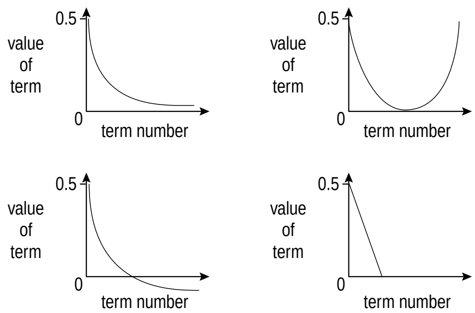


- *Look at these expressions:*

  $n - 2$ $2n$ n^2 $n/2$ $2/n$

  Which expression gives the greatest value:
  a. *when n lies between 1 and 2?*
  b. *when n lies between 0 and 1?*
  c. *when n is negative?*

- *The nth term of an infinite sequence is $n/(n^2+1)$.*
  The first term of the sequence is ½.
  Which of these four graphs has the same shape as the line joining the terms of the sequence?



Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

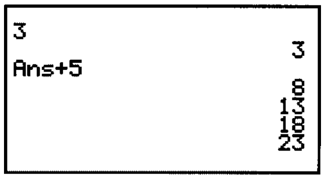
Problems to solve with a graphical calculator

For example:

- Enter these instructions (or their equivalent) on your graphical calculator.

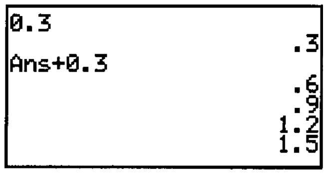
  3 EXE
  ANS + 5 EXE
  EXE EXE EXE EXE

  Describe the sequence you see. What is the rule? How many additions are needed to exceed 60?



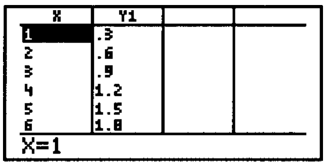
  Generate these sequences on your calculator.
  64 32 16 8 …
  1 3 9 27 …
  1 ⁻1 1 ⁻1 1 ⁻1 …
  Write down the rules that you used.

- Generate a sequence starting with 0.3 and adding 0.3.



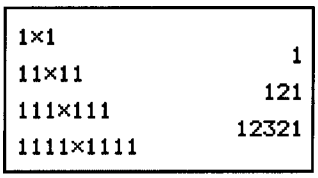
  Predict what will come next.
  Will 4.2 be in the sequence? How do you know?
  What sequence would have terms where each term is 10 times bigger than those on the screen? 100 times bigger?

  Use the table facility with 0.3*x* for y_1.



  Give the next two terms.
  Is 3 the same as 3.0?
  What is 10 × 0.3? What are 2.4 ÷ 0.3 and 2.4 ÷ 8?
  Write down other relationships you can deduce from the sequence.
  What number is half way between 0.3 and 0.6?
  Between 0.6 and 0.9? Between 0.9 and 1.2?
  Generate the sequence of ‘halfway values’.

- Enter these calculations on your calculator.



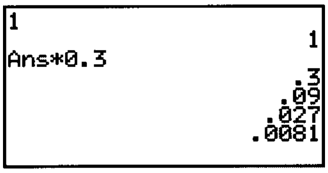
  Predict the next term.
  What would be the result of multiplying 111 111 by 111 111?
  What if you replace every digit 1 by a digit 3?

As outcomes, Year 8 pupils should, for example:

Problems to solve with a graphical calculator

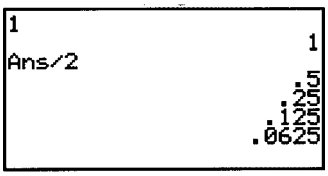
For example:

- Multiply repeatedly by 0.3.

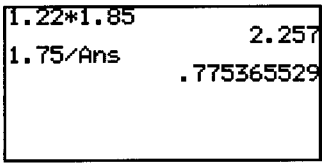


  What comes next?
  Are the numbers getting bigger or smaller?
  What fractions are equivalent to these decimals?
  Express each term using powers.

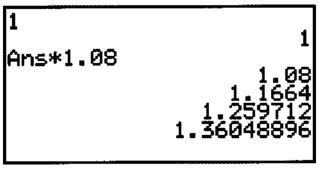
  Now try dividing repeatedly by 2.
  Will terms ever become zero?
  Or negative?



- Find the increase in depth of oil in a rectangular tank 1.22 m wide and 1.85 m long when 1750 litres of oil are added.

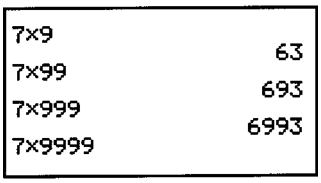


- Explore an annual growth rate of 8% linked to compound interest or population change.



  How many years will it take for the initial amount to double?
  What happens if the initial input is varied?
  How does the time to double vary with the percentage growth rate?

- Use your calculator to explore these calculations:
  7×9, 7×99, 7×999, …
  Predict the answer to $7 \times 99\,999$.
  What about $7 \times 999.........999$? Generalise.



  Explore 9×9, 9×99, 9×999, …
  Find a general rule.

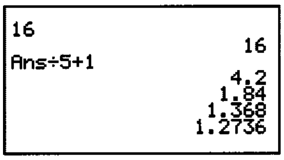
As outcomes, Year 9 pupils should, for example:

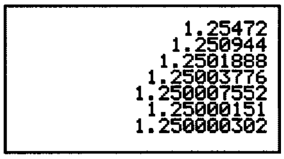
Problems to solve with a graphical calculator

For example:

- Choose a starting number.
  Divide the number by 5.
  Add 1 to the answer.
  This is now your new starting number.
  Keep repeating the process.

  Enter these instructions on your graphical calculator.
  1 6 EXE
  ANS ÷ 5 + 1 EXE
  EXE EXE EXE EXE

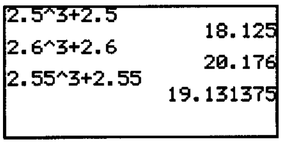


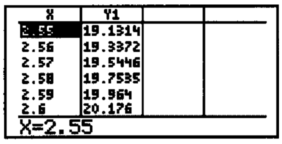


  What happens when you keep repeating the process? What value do the terms approach?
  What if you start with 6, or 300, or 3275?
  What if you divide by 4 instead of 5?
  What if you add 2 instead of 1?

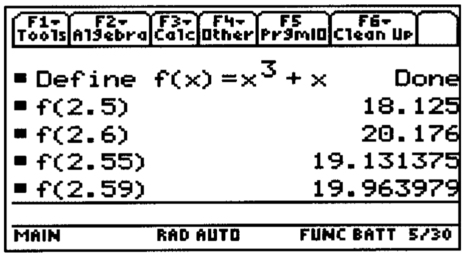
  What happens if you divide the starting number by p, then add q?
  What value do the terms approach?

- *Find an approximate solution to $x^3 + x = 20$.*





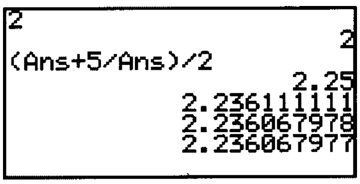
  or:



- *Find an approximate value of $\sqrt{5}$.*

  Let the first estimate be 2.
  Consider a rectangle with sides 2 and $\frac{5}{2}$.

  Explain mathematically why the mean of these two values gives a better approximation to $\sqrt{5}$, i.e. $\frac{1}{2}\left[2 + \frac{5}{2}\right]$



Pupils should be taught to:

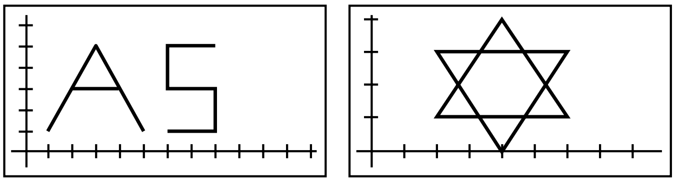
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

More problems to solve with a graphical calculator

For example:

- Use ‘plot’ and ‘line’ on a graphical calculator to draw these shapes.

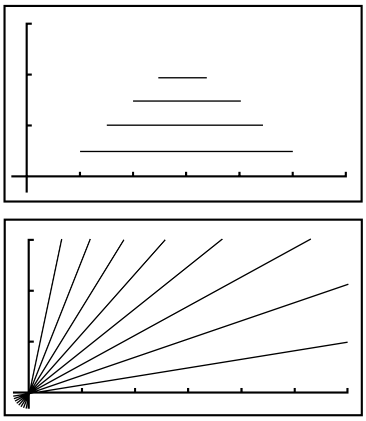


- Plot the points (3, 1) and (6, 1).
  These two points are the ends of the base of a square.
  Plot the other two points of the square.

  Clear the screen.
  Plot the points (3, 1) and (6, 1).
  These two points are now the ends of the base of a trapezium. Plot the other two points of the trapezium.
  Now plot two different trapezia with the points (3, 1) and (6, 1) as ends of their base.
  What if the base of the trapezium is (3, 1) and (5, 2)?

  Draw the line joining (3, 1) to (6, 1).
  Draw a parallelogram standing on this line.
  Now draw a hexagon standing on the line.
  Draw another shape standing on the line.
  What if the starting points are (3, 1) and (4, 5)?

- Use your graphical calculator to create displays like these by joining points.



- Use your graphical calculator to draw the outline of a skeleton cube.

- Use your graphical calculator to draw the line $y = x$.
  Now draw the lines $y = x + 1$, $y = x + 2$, and so on.
  Describe what happens.

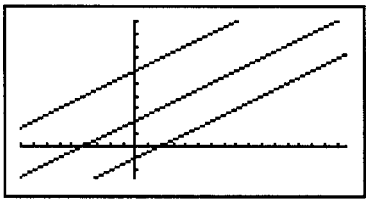
  Now draw the lines $y = x$, $y = 2x$, $y = 3x$, and so on.
  Describe what happens.

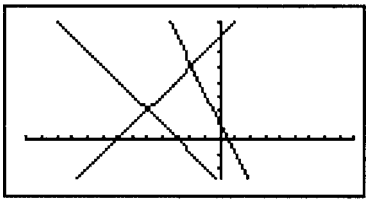
As outcomes, Year 8 pupils should, for example:

More problems to solve with a graphical calculator

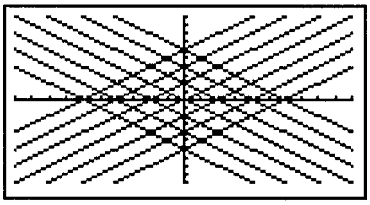
For example:

- Suggest possible equations for these straight-line graphs.

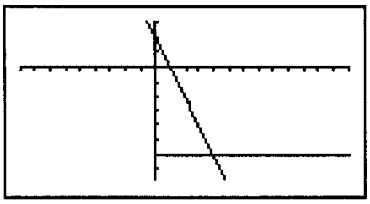




- Create a display like this with your graphical calculator.



- Draw some more straight-line graphs that pass through the point (4, $^{-}6$).

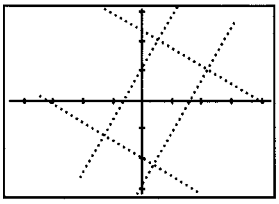
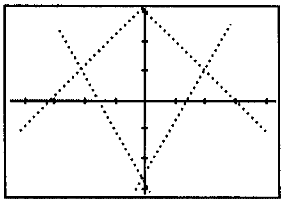


As outcomes, Year 9 pupils should, for example:

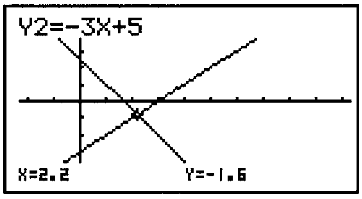
More problems to solve with a graphical calculator

For example:

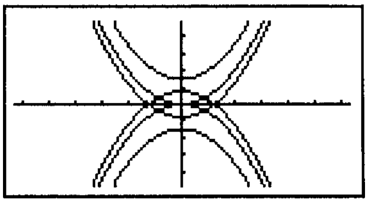
- Use a graphical calculator to draw these quadrilaterals.

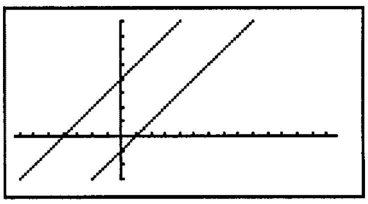
- *Solve graphically the simultaneous equations:*
  $y = 2x - 6$
  $y = {}^{-}3x + 5$



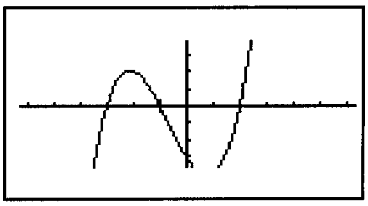
- Create a display like this with your graphical calculator.



- *Suggest possible equations for these two lines. Find the shortest distance between them.*



- *Find a possible equation for this curve.*



- *Use your calculator to draw the curves $y = x^n$ for n = 1, 2, 3, 4, and so on. Describe what happens.*

Pupils should be taught to:

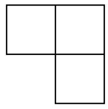
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving shape and space

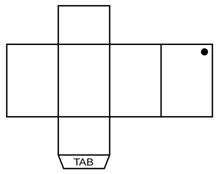
For example:

- This shape is called an L-triomino.
  Draw shapes made from two L-triominoes touching edge to edge.

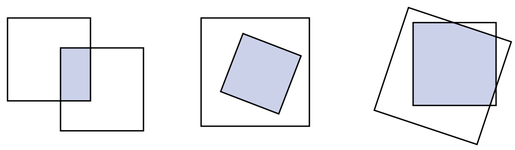
  a. Draw two different shapes, each with only one line of symmetry.
  b. Draw two different shapes, each with rotation symmetry of order 2.
  c. Draw a shape with two lines of symmetry *and* rotation symmetry of order 2.

- Here is a net with a tab to make a cuboid.
  There is a dot (●) in a corner of one of the faces.

  Imagine folding the net up.
  Write T on the edge that the tab will be stuck to.
  Draw a dot on each of the two corners that will meet the corner with the ●.

- Two squares can be overlapped to make different shapes.
  The squares can be the same size or different sizes.
  For example, you could make these shapes.

  Which of these shapes can be made by overlapping two squares: rhombus, isosceles triangle, pentagon, hexagon, octagon, decagon, kite, trapezium?
  If you think a shape cannot be made, explain why.

- Here are some instructions to draw a regular pentagon on a computer:

  REPEAT 5 [FORWARD 10, LEFT TURN 72°]

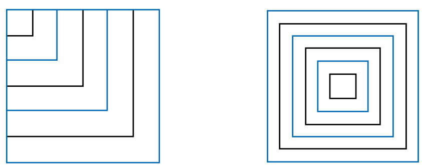
  Complete the instructions to draw a regular hexagon:

  REPEAT 6 [FORWARD 10, LEFT TURN …]

  How would you draw a regular octagon, decagon, …?

- Which letters of the alphabet have parallel lines in them?
  Use **Logo** to draw:
  a. some of the letters of the alphabet;
  b. a parallelogram.

- Use **Logo** to draw squares inside each other so that there are equal distances between the squares.

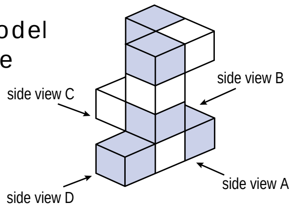
  

As outcomes, Year 8 pupils should, for example:

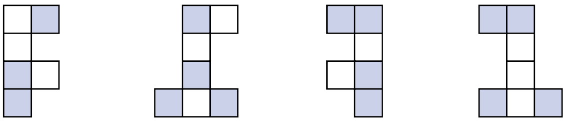
Problems involving shape and space

For example:

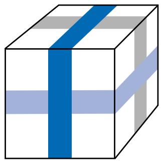
- The diagram shows a model with nine cubes, five blue and four white.



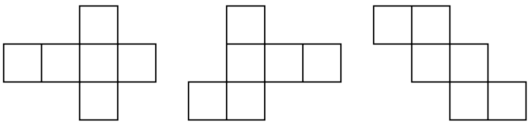
The drawings below show the four side views of the model. Which view does each drawing show?



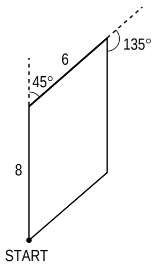
- This parcel is tied with three bands of ribbon.



Draw the bands on these three nets so that they will make the same parcel when folded up.

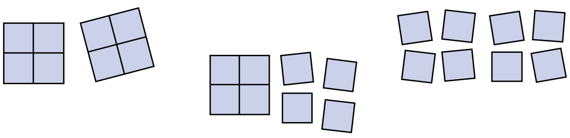


- The instructions to draw this shape are:
  REPEAT 2
  [FORWARD 8
  TURN RIGHT 45°
  FORWARD 6
  TURN RIGHT 135°]



Write the instructions to draw a kite with one right angle and two angles of 110°.

- A 4 by 2 rectangle can be cut into squares along its grid lines in three different ways.



In how many different ways can a 6 by 3 rectangle be cut into squares along its grid lines?

Can a 5 by 3 rectangle be cut into 7 squares? 8 squares? 9 squares?
Give mathematical reasons for your answers.

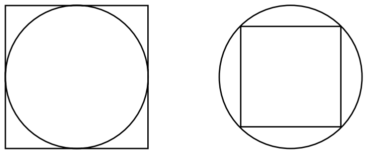
As outcomes, Year 9 pupils should, for example:

Problems involving shape and space

For example:

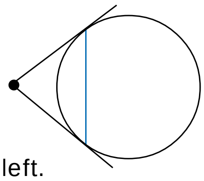
- Triangle T has vertices at (1, 2), (2, 4) and (3, 4).
  a. Draw T on squared paper.
  b. Triangle R is obtained by reflecting T in the *x*-axis. Draw R.
     What are the coordinates of its vertices?
  c. Triangle S is obtained by reflecting R in the *y*-axis. Draw S. What are its coordinates?
  d. There is a transformation that takes triangle T directly to triangle S. Describe this transformation as precisely as you can.

- Use **Logo** or **dynamic geometry software**.
  Draw a circle inside a square.
  Draw a square inside a circle.



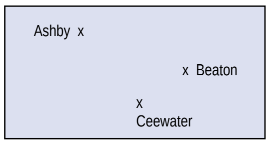
If the circles have equal diameters, what is the ratio of the areas of the squares?

- In this diagram, tangents to the circle are drawn at each end of the chord. The tangents intersect at the black dot.
  Imagine the chord moves to the left.
  What happens to the black dot?
  What if the chord moves to the right?



- The plan below shows the positions of three towns on an 8 cm by 10 cm grid.

Scale: 1 cm to 10 km.



The towns need a new radio mast. It must be:
– nearer to Ashby than Ceewater, and
– less than 45 km from Beaton.
Construct on the plan the region where the new mast can be placed.

- A blue counter and a grey counter are placed some distance apart.

Where could yellow counters be placed so that:
a. the centre of each yellow counter is equidistant from the centres of the blue and the grey counters?
b. *the centre of each yellow counter is twice as far from the centre of the blue counter as from the centre of the grey counter?*

Pupils should be taught to:

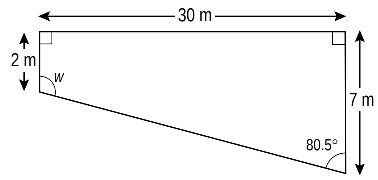
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

More problems involving shape and space

For example:

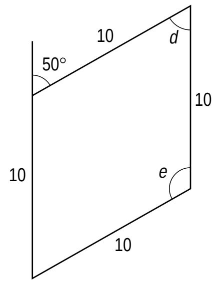
- Use this diagram to calculate angle *w*.



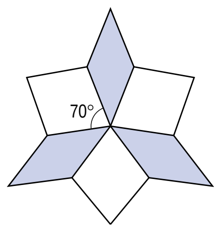
What facts about angles did you use?

- The diagram shows a rhombus.

Calculate the sizes of angles *d* and *e*.



- The shape shown below has three identical white tiles and three identical blue tiles.
The sides of each tile are all the same length.
Opposite sides of each tile are parallel.



One of the angles in the white tile is 70°.
Find the sizes of the other angles in the white tile.
Find the sizes of all the angles in a blue tile.

- Draw and label axes on squared paper, choosing a suitable scale. Plot these points.

A ($^-1$, 1)	B (2, $^-1$)	C ($^-3$, $^-1$)	D (5, $^-1$)
E (2, 2)	F (1, $^-2$)	G (5, 2)	H (2, 1)

a. Name the four points that are the vertices of:
   i. a square;
   ii. a non-square parallelogram;
   iii. a non-square trapezium.

b. Name three points that are the vertices of:
   i. a right-angled triangle;
   ii. a non-right-angled isosceles triangle.

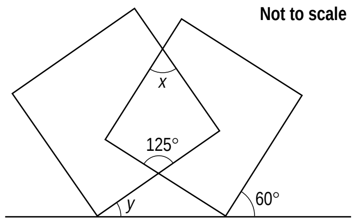
See Y456 examples (pages 110–11).

As outcomes, Year 8 pupils should, for example:

More problems involving shape and space

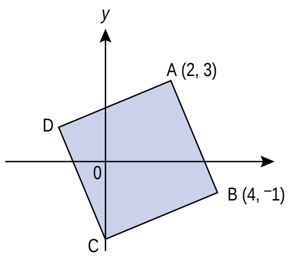
For example:

- The diagram shows two overlapping squares and a straight line.



Calculate the values of angles *x* and *y*.

- The shaded shape ABCD is a square.

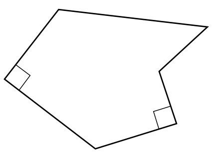


What are the coordinates of D?

- Use ruler and compasses to construct a rhombus and a square which have the same area.

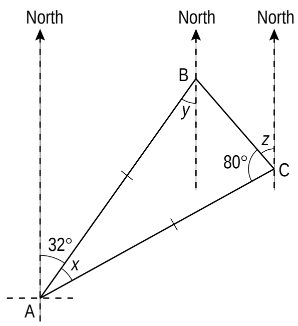
- Use ruler and compasses to construct a regular hexagon.
Draw two diagonals of the hexagon to form a right-angled triangle.
Explain why it is a right-angled triangle.

- What is the maximum number of right angles you can have in a hexagon?



What about other polygons?

- The diagram shows the positions of three points, A, B and C. The distances AB and AC are equal.



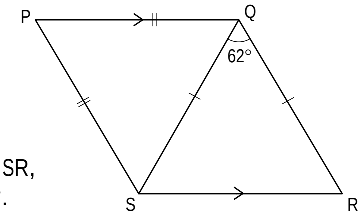
Calculate the sizes of the angles marked *x*, *y* and *z*.

As outcomes, Year 9 pupils should, for example:

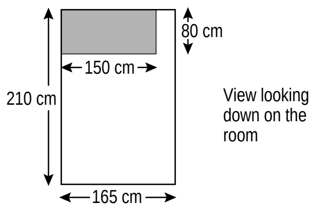
More problems involving shape and space

For example:

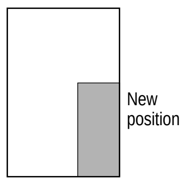
- In the diagram:
  PQ = PS,
  QR = QS,
  PQ is parallel to SR,
  angle SQR is 62°.
  Calculate the sizes of the other angles.



- *In a small room, a cupboard is in the position shown.*

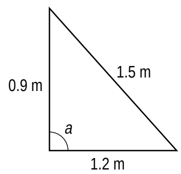


Is the room wide enough to move the cupboard to the new position shown?



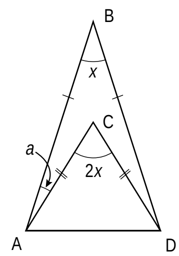
Give a mathematical justification for your answer.

- *Show that angle a is 90°.*

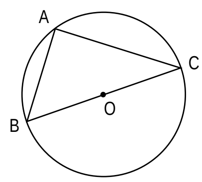


- *Two isosceles triangles have the same base AD. Angle ACD is twice the size of angle ABD.*

  Call these angles 2x and x. Prove that angle a is always half of angle x.



- *BC is a diameter of a circle centre O. A is any point on the circumference. Prove that angle BAC is a right angle.*



Hint: Join AO.

Pupils should be taught to:

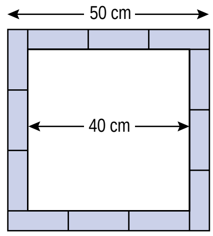
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving perimeter and area

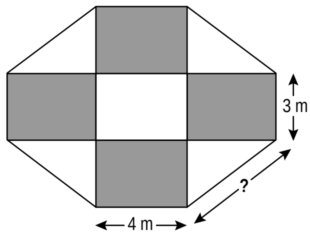
For example:

- Twelve rectangles, all the same size, are arranged to make a square outline, as shown in the diagram.



  Calculate the area of one of the rectangles.

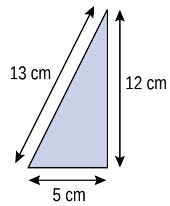
- This plan of a garden is made of rectangles and triangles.



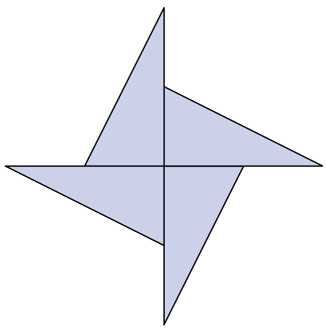
  The area of each rectangle is 12 square metres.
  What is the area of the whole garden?

  The perimeter of the garden is 34 metres.
  What is the longest side of each triangle?

- Triangles like this



  are used to make a star.



  What is the area of the star?
  What is its perimeter?

- Use **Logo** or **dynamic geometry software** to draw a square.
  Now draw a square which is one quarter of the area of the first square.
  Explain why the area of the second square is one quarter of the area of the first square.

See Y456 examples (pages 96–7).

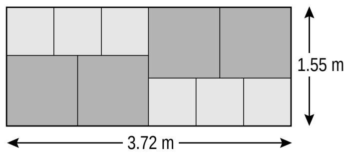
As outcomes, Year 8 pupils should, for example:

Problems involving perimeter, area and volume

For example:

- What is the smallest perimeter for a shape made of 8 regular hexagons each of side *a*?
  9 regular hexagons? 10 regular hexagons?

- Two sizes of square paving stones are used to make a path 3.72 metres long.



  Calculate the width of a small paving stone.

- The length of a rectangle is 4 cm more than its width. Its area is 96 cm^2. What is its perimeter?

- A boy has some small cubes.
  The edge of each cube is 1.5 centimetres.
  He makes a large cube out of the small cubes.
  The volume of the larger cube is 216 cm^3.
  How many small cubes does he use?

- Boxes measure 2.5 cm by 4.5 cm by 6.2 cm.
  Work out the *largest* number of boxes that can lie flat in a 9 cm by 31 cm tray.

- How many different cuboids can be made using exactly one million cubes?

- The end faces of a square prism are identical squares.
  What square prism do you end up with if you keep cutting square prisms from an 8 by 6 by 4 cuboid?

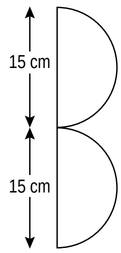
- A cuboid container holds 1 litre of water.
  Find the minimum surface area of your cuboid.
  You may find it helpful to use a **spreadsheet**.

As outcomes, Year 9 pupils should, for example:

Problems involving circumference, area and volume

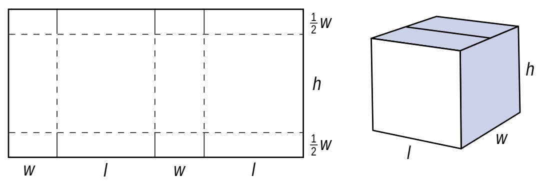
For example:

- A letter B is made out of a piece of wire.
  It has a straight side and two equal semicircles.



  Calculate the total length of the wire.

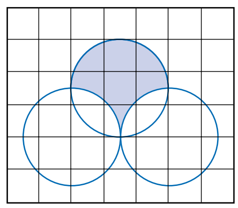
- Some boxes are made from rectangular sheets of thick card. Edges are joined with sticky tape.
  This is how to mark the cut and fold lines.
  Cuts are solid lines and folds are dotted lines.



  The net is used to make a box from a sheet of card 100 cm long and 40 cm wide. The box is 20 cm high. Calculate the volume of the box.

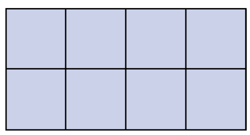
- *Cover a 5 by 5 pinboard with shapes of area two square units, with no two shapes the same.*

- *A design is made from three circles on a 1 cm grid.*



  Calculate the perimeter and area of the shaded part of the design. Give each answer correct to one decimal place.

- *What is the area of the smallest circle into which this 2 cm by 4 cm rectangle will fit?*



- *A cylinder has a diameter of 7.4 cm and a height of 10.8 cm.*
  Find the diameter and height of some other cylinders with the same volume.
  Which of these cylinders has the smallest surface area?
  *You may find it helpful to use a **spreadsheet**.*

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

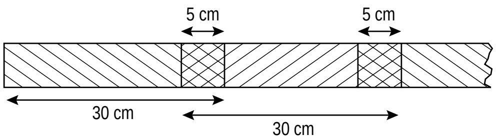
Problems involving measures

For example:

- A pupil paced the length and width of a corridor.
  She found it was 5 paces wide by 142 paces long.
  She measured one pace. It was 73 cm.
  What was the approximate length of the corridor in metres?

- A map has a scale of 1 cm to 6 km.
  A road measured on the map is 6.6 cm long.
  What is the length of the road in kilometres?

- Strips of paper are each 30 cm long.
  The strips are joined together to make a streamer.
  They overlap each other by 5 cm.

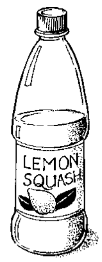


  How long is a streamer made from only two strips?

  A streamer is made that is 280 cm long.
  How many strips does it use?

- A box of figs costs £2.80 per kilogram.
  A fig from the box weighs 150 g.
  Find the cost of the fig.

- A lemon squash bottle holds 750 ml.
  The label says: *Dilute with 3 parts water.*
  Asmat likes her drink to be strong.
  She adds between 2 and 2.5 parts water to the squash to make a drink.
  How many litres of lemon drink can she make with one bottle of squash?



- A number of bars of soap are packed in a box that weighs 850 g. Each bar of soap weighs 54 g.
  When it is full, the total weight of the box and the soap is 7.6 kg. How many bars of soap are in the box?

- The maximum load in a small service lift is 50 kg.
  60 tins of food must go up in the lift. Each tin weighs 840 g.
  Is it safe to load all of these tins into the lift?

  The tins are to go in a cupboard which is 1.24 m high.
  Each tin is 15 cm high.
  How many layers of tins will fit in the cupboard?

- Every morning Ramesh catches a school bus at 8:05 a.m.
  It arrives at the school at about 8:40 a.m.
  Each Friday, the bus takes longer and it arrives at 8:55 a.m.
  How long does Ramesh spend coming to school over a school week, a term of 16 weeks, a year of 39 weeks...?

See Y456 examples (pages 86–9).

As outcomes, Year 8 pupils should, for example:

Problems involving measures

For example:

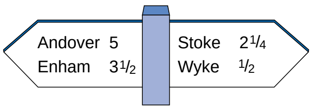
- A school expects between 240 and 280 parents for a concert. Chairs are to be put out in the hall. Each chair is 45 cm wide, and no more than 8 chairs must be put in a row together. There must be a corridor 1 m wide down the middle of the hall and 0.5 m of space between the rows for people to get to their seats. What is the minimum space needed to set out the chairs?

- The Wiro Company makes 100 000 wire coat-hangers each day. Each coat-hanger uses 87.4 cm of wire. How many km of wire are used each day? What is the greatest number of coat-hangers that can be made from 100 m of wire?

  The company decides to reduce the amount of wire used for each coat-hanger to 85 cm. How many more coat-hangers can be made from 100 m of wire?

- Mr Green sells apples at 40p per kilogram. Mrs Ball sells apples at 24p per pound. Who sells the cheaper apples? Explain how you worked it out.

- The distance to Andover is given as 5 miles.



  Do you think that this is correct to:
  P. the nearest yard,
  Q. the nearest quarter mile,
  R. the nearest half mile, or
  S. the nearest mile?

  Use your answer to state:
  a. the shortest and longest possible distances to Andover;
  b. the shortest possible distance between Stoke and Enham.

- Using a map, measure the bearings and distances 'as the crow flies' between the centres of York, Sheffield, Derby and Nottingham. Make a scale drawing of the positions of these four cities using a scale of 1 : 500 000.

As outcomes, Year 9 pupils should, for example:

Problems involving measures

For example:

- Here is an old recipe for egg custard with raisins. Change the amounts into metric measures.

  Egg custard with raisins
  $\frac{1}{4}$ pound of raisins
  1 pint of milk
  3 eggs

  Put the raisins in an 8 inch bowl.
  Mix the eggs and milk, and pour over the raisins.
  Bake in an oven at 320° Fahrenheit for about 1 hour.

- A plank of wood weighed 1.4 kg. 25 centimetres of the plank were cut off its length. The plank then weighed 0.8 kg. What was the length of the original plank?

- The diameter of a red blood cell is 0.000 714 cm and the diameter of a white blood cell is 0.001 243 cm.

  Work out the difference between the diameter of a red cell and the diameter of a white cell. Give the answer in millimetres.

  Calculate how many white cells would fit across a full stop which has a diameter of 0.65 mm.

- *A world-class sprinter can run 100 m in about 10 seconds. A bus takes 15 minutes to go 3 miles to the next town. Which average speed is faster, that of the sprinter or the bus?*

- *Alice walked 800 m to school. She timed herself with a stop watch. It took her 10 minutes 27.6 seconds. What was her average speed? How accurately do you think it is sensible to give your answer?*

- *Pluto takes 248 years to circle the Sun at a speed of 1.06×10^4 miles per hour. Assuming Pluto's orbit is a circle, how far is it from the Sun?*

- *Liquid for cleaning jewellery is stored in a cuboid container with a base of 5 cm by 4 cm. A gold brooch weighing 48.25 g is to be cleaned by immersing it in the liquid. The density of gold is 19.3 g/cm³. By how much will the liquid in the container rise?*

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving probability

For example:

- Samir spins a fair coin and records the results.
  In the first four spins, 'heads' comes up each time.

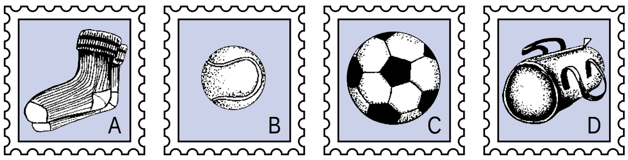
1st spin	2nd spin	3rd spin	4th spin
head	head	head	head

  Samir says:
  'A head is more likely than a tail.'
  Is he correct? Give a reason for your answer.

- There are six balls in a bag.
  The probability of taking a red ball out of the bag is 0.5.
  A red ball is taken out of the bag and put to one side.
  What is the probability of taking another red ball out of the bag?

- In each box of cereal there is one free card.
  You cannot tell which card will be in a box.
  Each card is equally likely.

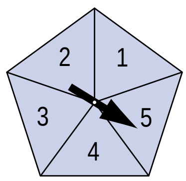
  Altogether there are four different cards.
  When you have them all, you can send for free sports socks.



  Zoe needs card A. Paul needs cards C and D.
  They buy one box of cereal.
  What is the probability that:
  a. the card is one that Zoe needs?
  b. the card is one that Paul needs?

  Their mother opens the box.
  She tells them the card is not card A.
  What is the probability now that:
  c. the card is one that Zoe needs?
  d. the card is one that Paul needs?

- A fair spinner has five sections numbered 1, 2, 3, 4, 5.
  What is the probability of getting a prime number from one spin?



  What about a fair spinner with 6 sides? 7 sides?

  Draw a bar-line graph to show the probability of getting a prime number from one spin of a spinner with 4 to 15 sides.

- Some children choose six tickets numbered from 1 to 200.
  Kay chooses numbers 1, 2, 3, 4, 5 and 6.
  Zak chooses numbers 14, 45, 76, 120, 137 and 182.
  Mary then picks a number at random from 1 to 200.
  Is Kay or Zak more likely to have Mary's number? Explain why.

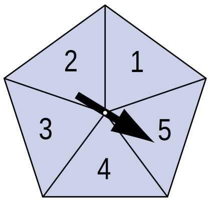
See Y456 examples (pages 112–13).

As outcomes, Year 8 pupils should, for example:

Problems involving probability

For example:

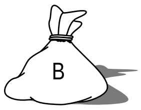
- Here is a spinner with five equal sections.



  Jane and Sam play a game.
  They spin the pointer many times.
  If it stops on an odd number, Jane gets 2 points.
  If it stops on an even number, Sam gets 3 points.
  Is this a fair game?
  Explain your answer.

- Bag A contains 12 red counters and 18 yellow counters.

  Bag B contains 10 red counters and 16 yellow counters.



  I am going to take one counter at random from either bag A or bag B.
  I hope to get a red counter.
  Which bag should I choose? Justify your choice.

- All the cubes in a bag are either red or black.
  The probability of taking out a red cube at random is $\frac{1}{5}$.
  One cube is taken at random from the bag and placed on the table. The cube is red.
  What is the smallest number of black cubes there could be in the bag?

  Another cube is taken from the bag and placed beside the first cube. The second cube is also red.
  From this new information, what is the smallest number of black cubes there could be in the bag?

- The names of all the pupils, all the teachers and all the canteen staff of a school are put in a box.
  One name is taken out at random.
  A pupil says:
  'There are only three choices.
  It could be a pupil, a teacher or one of the canteen staff.
  The probability of it being a pupil is $\frac{1}{3}$.'

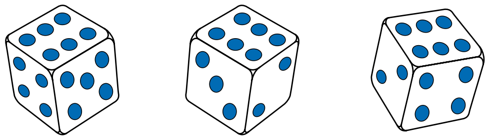
  The pupil is wrong. Explain why.

As outcomes, Year 9 pupils should, for example:

Problems involving probability

For example:

- Some pupils threw three fair dice.

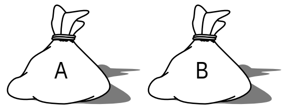


  They recorded how many times the numbers on the dice were the same.

Name	No. of throws	Results		
		all different	two the same	all the same
Morgan	40	20	12	2
Sue	140	81	56	3
Zenta	20	10	10	4
Ali	100	54	42	0

  Write the name of the pupil whose data are most likely to give the best estimate of the probability of getting each result. Explain your answer.

- Two bags, A and B, contain coloured cubes.



  Each bag has the same number of cubes in it.
  The probability of taking a red cube at random out of bag A is 0.5.
  The probability of taking a red cube at random out of bag B is 0.2.

  All the cubes are put in an empty new bag.
  What is the probability of taking a red cube out of the new bag?

  What if bag A has twice the number of cubes that are in bag B?

- Karen and Huw each have three cards, numbered 2, 3 and 4. They each take one of their own cards. They then add together the numbers on the four remaining cards.
  What is the probability that their answer is an even number?

- *John makes clay pots.*
  Each pot is fired independently. The probability that a pot cracks while being fired is 0.03.

  a. *John fires two pots.*
     Calculate the probability that:
     i. *both pots crack;*
     ii. *only one of them cracks.*

  b. *John has enough clay for 80 pots.*
     He gets an order for 75 pots.
     Does he have enough clay to make 75 pots without cracks? Explain your answer.

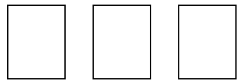
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Solve word problems and investigate in a range of contexts (continued)

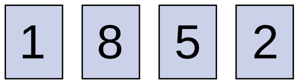
Problems involving handling data

For example:

- Write a different number in each of these boxes so that the mean of the three numbers is 9.



- James has four number cards. Their mean is 4.
  James takes another card.
  The mean of his five cards is now 5.
  What number is on the new card?

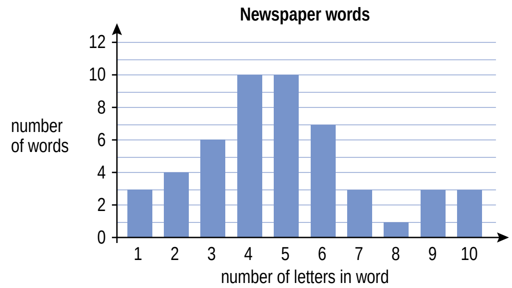


- Jeff has played two of the three games in a competition.

	Game A	Game B	Game C
Score	62	53	

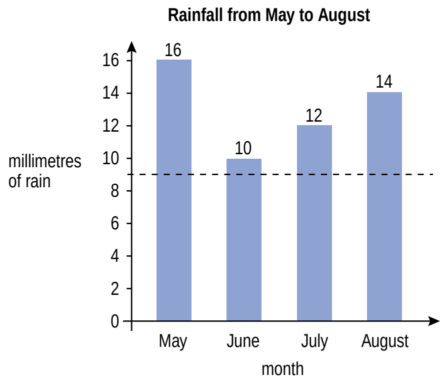
  To win, Jeff needs a mean score of 60.
  How many points does he need to score in game C?

- Kelly chooses a section of a newspaper. It has 50 words in it. She draws a bar chart of the number of letters in each word.



  Kelly says:
  '23 of the 50 words have fewer than 5 letters. This shows that nearly half of all the words used in the newspaper have fewer than 5 letters in them.'
  Explain why she could be wrong.

- Here is a bar chart showing rainfall.



  Kim draws a dotted line on the bar chart. She says:
  'The dotted line on the chart shows the mean rainfall for the four months.'
  Use the chart to explain why Kim *cannot* be correct.

See Y456 examples (pages 114–17).

As outcomes, Year 8 pupils should, for example:

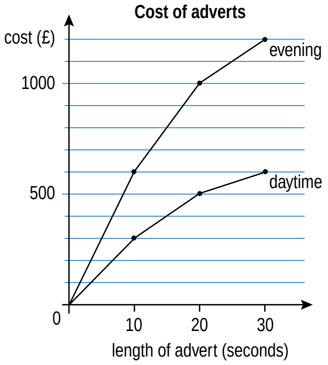
Problems involving handling data

For example:

- Imran and Nia play three different games in a competition. Their scores have the same mean.
  The range of Imran's scores is twice the range of Nia's scores.
  Fill in the missing scores in the table below.

Imran's scores		40	
Nia's scores	35	40	45

- This chart gives the cost of showing advertisements on TV at different times.



  An advertisement lasts 25 seconds. Use the graph to estimate how much cheaper it is to show it in the daytime compared with the evening.

  An advertisement was shown in the daytime and again in the evening. The total cost was £1200.
  How long was the advertisement in seconds?

- The cost of exporting 350 wide-screen TV sets was:

Item	Cost
freight charge	£371
insurance	£49
port charges	£140
packing	£280

  a. Find the mean cost of exporting a wide-screen TV.
  b. Construct a pie chart showing how the export costs in the table make up the total cost.

- Fifteen pupils measured an angle.
  Here are their results.

Angle measured as	Number of pupils
45°	5
134°	3
135°	4
136°	3

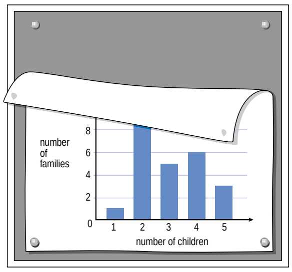
  Use the results to decide what the angle is most likely to measure.
  Give your reasons.

As outcomes, Year 9 pupils should, for example:

Problems involving handling data

For example:

- A class collected information about the number of children in each of their families.
  The information was displayed in a frequency chart, but you cannot see it all.



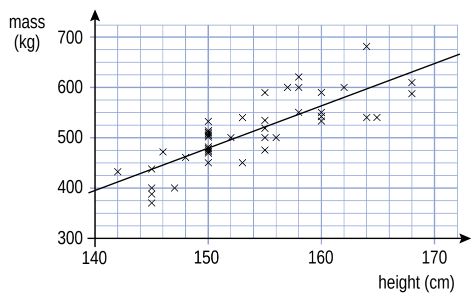
  Let n be the number of families with 2 children.
  a. Show that the total number of children in all families is $55 + 2n$.
  b. Write an expression for the total number of families.
  c. The mean number of children is 3.
  What is the value of n?

- Here are some data about the population of the regions of the world in 1950 and 1990.

Regions of the world	Population in millions in 1950	Population in millions in 1990
Africa	222	642
Asia	1558	3402
Europe	393	498
Latin America	166	448
North America	166	276
Oceania	13	26
World	**2518**	**5292**

  a. In 1990, for every person who lived in North America, how many people lived in Asia?
  b. A pupil thinks that from 1950 to 1990 the population of Oceania went up by 100%.
  Is the pupil right? Explain your answer.

- This scatter graph shows heights and masses of some horses. It also shows a line of best fit.



  a. What does the scatter graph show about the relationship between the height and mass of horses?
  b. The mass of a horse is 625 kg. Estimate its height.
  c. Laura thinks that the length of the back leg of a horse is always less than the length of its front leg. If this were true, what would the scatter graph look like? Draw a sketch.

Pupils should be taught to:

Identify the information necessary to solve a problem; represent problems mathematically in a variety of forms

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
investigate, explore, solve, explain… true, false…
problem, solution, method, answer, results, reasons, evidence…

Identify the necessary information; represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.
For example, solve:

- Birthday candles
  Mrs Sargent is 71 years old.
  Every year since she was born she has blown out the corresponding number of candles on her birthday cake.
  How many birthday candles has she blown out altogether?

  Related objectives:
  Explain and justify methods and conclusions, orally and in writing.

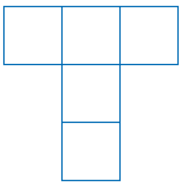
  Link to expressing functions in words then symbols (pages 160–1); simple mappings (pages 160–1).

- Challenging calculators
  Find at least two different solutions to these problems.
  a. The 7 key doesn't work.
     Make your calculator display 737.
  b. None of the keys that are odd numbers work.
     Make the calculator display 975.

  Related objectives:
  Break a complex calculation into simpler steps; choose and use appropriate and efficient operations, methods and resources, including ICT.

  Link to rapid recall of number facts, including complements to 100 and multiplication and division facts (pages 88–9); using the order of operations, and brackets (pages 86–7).

- Tiles
  A T-shape is made from 5 square tiles.
  The length of a side of a tile is w cm.
  Write an expression for the area of the T-shape.
  If the area of the T-shape is 720 cm^2, what is the length of its perimeter?



  Another T-shape is made from 5 square tiles.
  The length of a side of a tile is v cm.
  Write an expression for the perimeter of the T-shape.
  If the perimeter of the T-shape is 120 cm, what is its area?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to using the formula for the area of a rectangle; calculating the perimeter and area of compound shapes made up of rectangles (pages 232–3).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
best estimate, degree of accuracy…
justify, prove, deduce… conclude, conclusion…
counter-example, exceptional case…

Identify the necessary information; represent problems in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.
For example, solve:

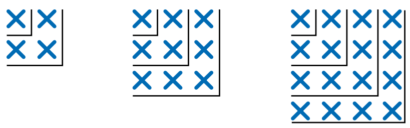
- Estimating lengths
  Who is best at estimating 10 centimetres?
  Decide how to find out. For example, each pupil could cut 20 pieces of string to an estimated 10 cm, then measure each piece to find the 'error' to the nearest half centimetre. Find the mean error for each pupil.

  Confirm by estimating 20 different lengths of ribbon held up at the front of the class, including four lengths of 10 cm.

  Related objectives:
  Give solutions to an appropriate degree of accuracy in the context of the problem.

  Link to comparing two distributions using the range and one or more of the measures of average (pages 272–3);
  interpreting tables, graphs and diagrams for both discrete and continuous data and drawing inferences; relating summarised data to the questions being explored (pages 268–71).

- Crosses
  How many crosses are there in each pattern?



  a. Find the value of:
     i. 1 + 3   ii. 1 + 3 + 5   iii. 1 + 3 + 5 + 7
  b. Draw the next pattern in the sequence.
     What is the value of 1 + 3 + 5 + 7 + 9?
  c. What is the sum of:
     i. the first 6 odd numbers?
     ii. the first 20 odd numbers?
     iii. the first *n* odd numbers?
  d. Add the crosses in each pattern along diagonals. Prove from the third pattern that
     $1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2$
     Generalise this result.

  Related objectives:
  Solve problems and explore pattern and symmetry in a range of contexts.

  Link to generating sequences from practical contexts and finding the general term in simple cases (pages 154–7).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
trial and improvement…
generalise…

Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem. For example, solve:

- Five coins
  A game involves tossing five coins for a 10p stake. If you score exactly two heads you win 20p and get your stake back; otherwise you lose.
  Give mathematical reasons to justify whether this is a sensible game to play.

  Related objectives:
  Present a proof, making use of symbols, diagrams and graphs and related explanatory text; *justify generalisations and choice of presentation, explaining selected features.*

  Link to identifying all the mutually exclusive outcomes of an experiment; knowing that the sum of probabilities of all mutually exclusive outcomes is 1 (pages 278–81);
  comparing experimental and theoretical probabilities (pages 284–5);
  appreciating the difference between mathematical explanation and experimental evidence (pages 284–5).

- Painted cubes
  A cube of side 3 cm is made up of 27 individual centimetre cubes. The cube is dipped into a pot of paint, so that all the exterior sides are covered in the paint. The cube is then broken up into the individual 27 centimetre cubes.

  How many of the cm cubes have 3 sides painted? 2 sides painted? 1 side painted? 0 sides painted?
  Explore for different sized cubes.

  Generalise. Justify your generalisation.
  Explore further.

  Related objectives:
  Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples, explaining why; *pose extra constraints and investigate whether particular cases can be generalised further.*

  Link to generating sequences using term-to-term and position-to-term definitions, on paper and using ICT (pages 148–51);
  ***finding the next term and nth term of a sequence and exploring its properties* (pages 152–3).**

Pupils should be taught to:

Break problems into smaller steps or tasks; choose and use efficient operations, methods and resources

As outcomes, Year 7 pupils should, for example:

Break a complex calculation into simpler steps, choosing and using appropriate and efficient operations, methods and resources, including ICT. For example, solve:

- Square shuffle
  This activity is based on a slider puzzle.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Seat 15 pupils in a 4 by 4 array of chairs.
The aim is to move the pupils across the chairs so that the person in the back left seat (labelled 13) is moved to the front right seat (labelled 4) in as few moves as possible.

The rules are:
- The person in front of, behind or alongside the empty chair can move into it.
- No one else can move.
- Diagonal moves are not allowed.

Explore for different sizes of squares.

Related objectives:
Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

Link to generating and describing simple integer sequences (pages 144–7); expressing simple functions in words then symbols, and representing simple mappings (pages 160–3).

- Number problems
  a. Make up word problems to reflect statements such as:
     $36 \times 18 \times 45 = 29\,160$
     $(36 + 45) \div 2 = 40.5$
     20% of 450 = 90

  b. What operation is represented by each ∗ ?
     905 ∗ 125 = 1030   905 ∗ 125 = 780
     905 ∗ 125 = 7.24   905 ∗ 125 = 113 125

Related objectives:
Represent problems mathematically, making correct use of symbols and words.

Link to written calculations (pages 104–7); understanding operations (pages 82–5).

See Y456 examples (pages 74–5).

As outcomes, Year 8 pupils should, for example:

Solve more complex problems by breaking them into smaller steps, choosing efficient techniques for calculation, algebraic manipulation and graphical representation, and resources, including ICT.
For example, solve:

- Missing digits
  Find the missing digits represented by the $\ast$ in examples such as:
  $(\ast 5)^2 = \ast\ast\ast$ $\quad (\ast\ast)^2 = \ast\ast 25$
  $(\ast\ast)^3 = \ast\ast\ast 7$ $\quad (\ast\ast\ast)^2 = \ast 44 \ast 44$
  $(3 \times \ast\ast)^2 = 54 \ast 56$

  Make up some examples of your own.
  Make sure that someone else can solve them!

  Related objectives:
  Solve problems in a range of contexts (number).

  Link to factors, powers and roots (pages 52–5).

- Hand luggage
  An airline specifies that hand baggage must meet this requirement:

length + width + depth must be less than 1 m

  What dimensions for the hand baggage would give the most space for the contents?

  Related objectives:
  Solve problems in a range of contexts (measures).

  Link to knowing the formula for the volume of a cuboid; calculating volumes and surface areas of cuboids and shapes made of cuboids (pages 238–41).

- Everything 15% off!
  In a gift shop sale everything is reduced by 15%.
  A quick way of calculating the sale price is to multiply the original price by a number.
  What is the number?
  Give a mathematical reason to justify your answer.

  After two weeks, the sale price is reduced by a further 15%. Show that this means the original price has been reduced by 27.75%.

  Related objectives:
  Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.

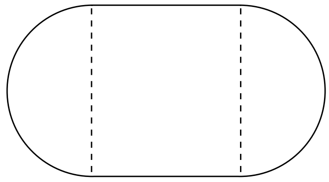
  Link to expressing a given number as a percentage of another; using the equivalence of fractions, decimals and percentages to compare proportions; calculating percentages and finding the outcome of a given percentage increase or decrease (pages 70–7).

As outcomes, Year 9 pupils should, for example:

Solve substantial problems by breaking them into simpler tasks, using a range of efficient techniques, methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious. For example, solve:

- Running track
  Design a running track to meet these constraints:

  – The inside perimeter of the track has this shape.

  – Both straights must be at least 80 metres.
  – Both ends must be identical semicircles.
  – The total inside perimeter must be 400 m.

  What is the greatest area the running track can enclose?

  What if the track has to be 8 metres wide?
  What is the smallest rectangular field needed to contain it?

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to using the formulae for the circumference and area of a circle (pages 234–7).

- Number puzzle
  A number plus its square equals 30.
  How many different solutions can you find?

  What if a number plus its cube equals 30?

  Related objectives:
  Suggest extensions to problems, conjecture and generalise, identify exceptional cases or counter-examples, explaining why;
  pose extra constraints and investigate whether particular cases can be generalised further.

  Link to simplifying algebraic expressions (pages 116–17); expanding the product of two linear expressions (pages 118–21); using systematic trial and improvement methods and ICT tools to find approximate solutions of equations (pages 132–5).

Pupils should be taught to:

Present and interpret solutions, explaining and justifying methods, inferences and reasoning

As outcomes, Year 7 pupils should, for example:

Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing. For example, solve:

- Number cell puzzles

2	5			

The empty cells are filled by adding the two preceding numbers, for example:

2	5	7	12	19

Using the same rule, find the missing numbers for these cells:

6				24

10				23

What about these?

10				24

				24

Related objectives:
Suggest extensions to problems by asking 'What if ...?'; understand the significance of a counter-example when looking for generality.

Link to adding/subtracting simple fractions (pages 66–9); calculating fractions of quantities and measurements (whole-number answers) (pages 66–9); multiplying a fraction by an integer (pages 66–9); extending mental methods of calculation to include decimals, fractions and percentages (pages 92–101).

- Square totals
Start with a 1–100 square.
Choose a 2 by 2 square, for example:

1	2
11	12

Because the first number in the square is 1, call the sum of the four numbers in the square S_1.
$S_1 = 1 + 2 + 11 + 12$, so $S_1 = 26$.
What about $S_2, S_3, S_4, \ldots$?
What do you notice?
Why does this happen?

Try a 2 by 3 rectangle, or a 3 by 3 square. Explore further.

Related objectives:
Suggest extensions to problems by asking 'What if...?'

Link to generating simple linear sequences (pages 144–7); expressing functions in words then symbols (pages 160–1); simple mappings (pages 160–1).

As outcomes, Year 8 pupils should, for example:

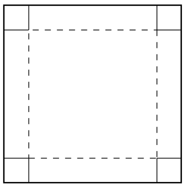
Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.
For example, solve:

- Consecutive sums
  Prove that the sum of any five consecutive numbers is always divisible by 5.

  Related objectives:
  Represent problems in algebraic, geometric or graphical form.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

- Max box
  Open-top boxes can be made from paper by cutting identical squares from each corner and folding up the sides.

  Start with a 20 cm square.
  Plan to make an open-top box with the greatest possible capacity.
  What are its dimensions?

  Explore for other sizes of squares.

  Related objectives:
  Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples.

  Link to ordering decimals (pages 40–1); constructing linear functions arising from real-life problems and plotting their graphs (pages 172–3); interpreting graphs arising from real situations (pages 174–7).

- Pizza
  These are the ingredients for a pizza for 4 people.

½ oz dried yeast	2 oz mushrooms
½ pint water	2 tomatoes
1 lb of plain flour	4 oz cheese
½ teaspoon of salt	6 black olives
8 oz ham	

  Adapt the recipe for 6 people.
  Convert the recipe to metric measurements.

  Related objectives:
  Solve problems in a range of contexts.

  Link to solving simple word problems involving ratio and direct proportion (pages 78–81); converting imperial to metric measures (pages 228–9).

As outcomes, Year 9 pupils should, for example:

Present a concise, reasoned argument, using symbols, diagrams, graphs and text; give solutions to an appropriate degree of accuracy; *recognise limitations on accuracy of data and measurements; give reasons for choice of presentation, explaining features, showing insight into the problem's structure.*
For example, solve:

- Perimeter
  The perimeter of a triangle is 48 cm. The length of the shortest side is s cm, and of another side is $2s$ cm. Prove that $12 > s > 8$.

  Related objectives:
  Represent problems in algebraic, geometric or graphical form.

  Link to solving problems using properties of triangles (pages 184–9).

- Round table
  At Winchester there is a large table known as the Round Table of King Arthur.

  The diameter of the table is 5.5 metres.
  A book claims that 50 people can sit around the table. Do you think this is possible?
  Explain and justify your answer.
  State all the assumptions that you make.

  Related objectives:
  Solve substantial problems by breaking them into simpler tasks, using efficient techniques, methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious.

  Link to using circle formulae (pages 234–7).

- Seeing the wood for the trees
  Estimate the number of trees that are needed each day to provide newspapers for the UK.

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts.

  Link to discussing how data relate to the problem, identifying possible sources; identifying possible bias and planning to minimise it (pages 250–1); communicating results using selected tables, graphs and diagrams in support, using ICT as appropriate (pages 272–5); examining results critically, recognising the limitations of any assumptions and their effect on conclusions drawn (pages 272–5).

Pupils should be taught to:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples

As outcomes, Year 7 pupils should, for example:

Suggest extensions to problems by asking 'What if...?'; begin to generalise and to understand the significance of a counter-example.

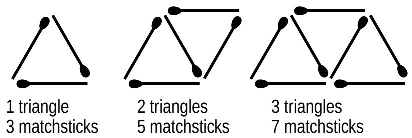
Carry out simple investigations, explain the approach and results, and generalise outcomes. For example:

- Factors
  Find the smallest number with exactly 3 factors.
  Now find the smallest number with exactly 4 factors.
  What about other numbers of factors? For example, can you find a number with exactly 13 factors?

  Related objectives:
  Choose and use efficient operations, methods and resources.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

- Matchstick shapes
  Rows of triangles can be generated using matchsticks.

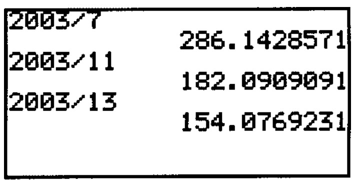


  One triangle can be made from three matchsticks, two from five matchsticks and so on. How many matchsticks are needed for 5 triangles? For 10 triangles? For 50 triangles?
  Find the rule for the number of matchsticks in any number of triangles in a row.
  Explore rows of other shapes made from matchsticks.

  Related objectives:
  Identify the necessary information; represent problems mathematically.
  Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

  Link to generating sequences (pages 144–57); expressing functions in words then symbols (pages 160–1) and representing mappings (pages 160–1).

- Prime numbers
  Is 2003 a prime number?
  How can you be sure?
  Explore other four-digit numbers.



  Related objectives:
  Explain and justify methods and conclusions, orally and in writing.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

As outcomes, Year 8 pupils should, for example:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples.

Carry out investigations, explain the approach and results, and generalise outcomes. For example:

- Multiplication squares
  Start with a multiplication square.
  Choose a 2 by 2 square of numbers, for example:

12	16
15	20

  Multiply the two pairs of numbers on opposite corners, i.e.

  $12 \times 20 = 240$
  $15 \times 16 = 240$

  Are both products always equal?
  Prove that for all such squares, the products of the numbers in opposite corners are equal.
  Try bigger squares and rectangles.
  Explore further.

  Related objectives:
  Use logical argument to establish the truth of a statement; present problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

  Link to multiples and factors (pages 52–5).

- Triangles in cubes
  Triangles are made by joining three of the vertices of a cube. How many different shapes can the triangles have? How many of the shapes are isosceles triangles? How many are equilateral triangles?

  Which of the triangles has the largest area? Justify your answer.

  Related objectives:
  Identify the necessary information to solve a problem; represent problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

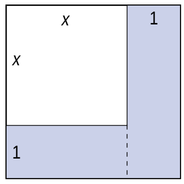
  Link to solving geometrical problems using side and angle properties of triangles, explaining reasoning with diagrams and text (pages 184–9); using the formula for the area of a triangle (pages 234–7).

As outcomes, Year 9 pupils should, for example:

Suggest extensions, conjecture and generalise; identify exceptional cases or counter-examples, with explanation; *justify generalisations, arguments or solutions; pose extra constraints and investigate whether particular cases can be generalised further.*

Carry out investigations, explain the approach and results, and generalise outcomes. For example:

- Difference of two squares
  Find two consecutive whole numbers whose squares differ by 37.
  Use the diagram to help.



  Use a diagram to simplify $(n + 1)^2 – (n – 1)^2$.
  Find two consecutive odd numbers whose squares differ by 72.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to simplifying expressions (pages 116–21); generating sequences, on paper and using ICT (pages 148–53).

- Making statements
  Adding two numbers and then squaring is the same as squaring each number and then adding.
  Is this always true, never true, or sometimes true?
  Justify your answers.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to using formulae; substituting numbers into expressions and formulae; deriving a formula and changing its subject, *extending to more complex formulae* (pages 138–43).

- Splitting 17
  If the sum of two numbers is 17, what is the greatest product they can have?
  What if there are three numbers? Explore.

  Related objectives:
  Solve substantial problems by breaking them into simpler tasks, using a range of methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious.

  Link to using index notation for integer powers and the index laws (pages 56–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples (continued)

- Perimeter and area

  The larger the perimeter of a rectangle, the larger its area.

  Is this statement true?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to using the formula for the area of a rectangle; calculating the perimeter and area of shapes made from rectangles (pages 234–7).

- Evens

  The sum of four even numbers is divisible by 4.

  When is this statement true? When is it false?

  Related objectives:
  Solve mathematical problems in a range of contexts.
  Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

  Link to multiples and factors (pages 52–5).

- Paraffin jugs

  A hardware shop has only a 5 gallon jug and a 3 gallon jug to measure out paraffin for customers.
  How can the shop assistant measure 1 gallon without wasting any paraffin?

  What if the assistant has a 7 gallon jug and a 4 gallon jug?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to rapid recall of addition and subtraction facts (pages 88–9).

- Classifying quadrilaterals

  Copy this table on to a large piece of paper.

		Number of pairs of parallel sides		
		0	1	2
Number of pairs of equal sides	0			
	1			
	2			

  Draw and name quadrilaterals in the appropriate spaces.
  Will any of the spaces remain empty? If so, explain why.

  Related objectives:
  Identify the necessary information; represent problems mathematically making correct use of symbols, words, diagrams, tables or graphs.

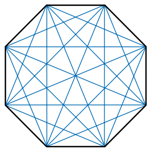
  Link to identifying and using angle, side and symmetry properties of triangles and quadrilaterals (pages 184–9).

See Y456 examples (pages 78–9).

As outcomes, Year 8 pupils should, for example:

- Diagonals
  Prove that the number of diagonals d in a regular polygon with n sides is

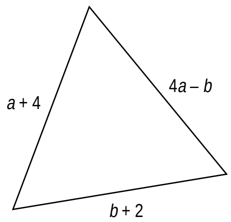
  $d = ½n(n – 3)$



  Related objectives:
  Identify the necessary information; represent problems mathematically making correct use of symbols, words, diagrams, tables or graphs.

  Link to expressing simple functions in symbols (pages 112–13); using linear expressions, justifying the form by relating to the context (pages 154–7).

- Perimeter of a triangle
  The diagram shows the lengths of the sides of a triangle in centimetres.
  The triangle is equilateral.
  Find its perimeter.



  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to finding the perimeter of a triangle (pages 234–7); solving linear equations with integer coefficients (pages 122–5); substituting integers into simple formulae (pages 138–43).

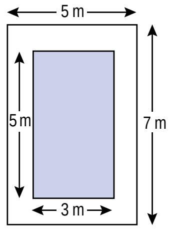
- True or false?
  Use logical argument to establish whether these statements are true or false.
  a. An isosceles triangle is made up of two identical right-angled triangles.
  b. A rhombus is a parallelogram but a parallelogram is not a rhombus.

  Related objectives:
  Present problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

  Link to explaining and justifying inferences, deductions and conclusions using mathematical reasoning (pages 184–9).

As outcomes, Year 9 pupils should, for example:

- Garden path
  A metre wide garden path surrounds a rectangular lawn.



  The area of the path can be found by subtracting the area of the lawn from the whole area, i.e. $5 \times 7 - 3 \times 5 = 20$.
  So the area of the path is 20 m².

  However, $5 + 7 + 3 + 5 = 20$.

  Prove that the sum of the inner and outer width and length will always give the numerical value of the area.

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; *generate fuller solutions.*
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to taking out single-term common factors; *squaring a linear expression, expanding the product of two linear expressions, establishing identities* (pages 116–21); deriving and using a formula and changing its subject, *extending to more complex formulae* (pages 140–3).

- Prove it
  Prove these statements:
  a. Any three-digit whole number is divisible by 9 if the sum of the digits is divisible by 9.
  b. The difference between a two-digit number and its reverse is always a multiple of 9.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form.

  Link to relevant content.

- Hexagons
  A regular hexagon has perimeter of 60 cm. Calculate its area.
  Find a method to calculate the area of any regular hexagon given its perimeter.
  What about other regular polygons?

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; generate fuller solutions.

  Link to using mathematical reasoning and applying Pythagoras' theorem (pages 184–9); using sine, cosine, tangent (pages 242–7).