Subject benchmark statement

Mathematics, statistics and operational research

Draft for consultation June 2007

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Preface

Subject benchmark statements provide a means for the academic community to describe the nature and characteristics of programmes in a specific subject or subject area. They also represent general expectations about standards for the award of qualifications at a given level in terms of the attributes and capabilities that those possessing qualifications should have demonstrated.

This subject benchmark statement, together with others published concurrently, refers to the **bachelor's degree with honours**[1]. In addition, some subject benchmark statements provide guidance on integrated master's awards.

Subject benchmark statements are used for a variety of purposes. Primarily, they are an important external source of reference for higher education institutions (HEIs) when new programmes are being designed and developed in a subject area. They provide general guidance for articulating the learning outcomes associated with the programme but are not a specification of a detailed curriculum in the subject.

Subject benchmark statements also provide support to HEIs in pursuit of internal quality assurance. They enable the learning outcomes specified for a particular programme to be reviewed and evaluated against agreed general expectations about standards. Subject benchmark statements allow for flexibility and innovation in programme design and can stimulate academic discussion and debate upon the content of new and existing programmes within an agreed overall framework. Their use in supporting programme design, delivery and review within HEIs is supportive of moves towards an emphasis on institutional responsibility for standards and quality.

Subject benchmark statements may also be of interest to prospective students and employers, seeking information about the nature and standards of awards in a given subject or subject area.

The relationship between the standards set out in this document and those produced by professional, statutory or regulatory bodies for individual disciplines will be a matter for individual HEIs to consider in detail.

This subject benchmark statement represents a revised version of the original published in 2000. The review process was facilitated by the Quality Assurance Agency for Higher Education (QAA) as part of a periodic review of all subject benchmark statements published in this year. The review and subsequent revision of the subject benchmark statement was undertaken by a group of subject specialists drawn from and acting on behalf of the subject community. The revised subject benchmark statement went through a full consultation with the wider academic community and stakeholder groups.

QAA publishes and distributes this subject benchmark statement and other subject benchmark statements developed by similar subject-specific groups.

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[1] This is equivalent to the honours degree in the Scottish Credit and Qualifications Framework (level 10) and in the Credit and Qualifications Framework for Wales (level 6).

The Disability Equality Duty (DED) came into force on 4 December 2006[2]. The DED requires public authorities, including HEIs, to act proactively on disability equality issues. The Duty complements the individual rights focus of the *Disability Discrimination Act* (DDA) and is aimed at improving public services and outcomes for disabled people as a whole. Responsibility for making sure that such duty is met lies with HEIs.

The Disability Rights Commission (DRC) has published guidance[3] to help HEIs prepare for the implementation of the Duty and provided illustrative examples on how to take the duty forward. HEIs are encouraged to read this guidance when considering their approach to engaging with components of the Academic Infrastructure[4], of which subject benchmark statements are a part.

Additional information that may assist HEIs when engaging with subject benchmark statements can be found in the DRC revised *Code of Practice: Post-16 Education*[5], and also through the Equality Challenge Unit[6] which is established to promote equality and diversity in higher education.

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[2] In England, Scotland and Wales

[3] Copies of the guidance *Further and higher education institutions and the Disability Equality Duty*, guidance for principals, vice-chancellors, governing boards and senior managers working in further education colleges and HEIs in England, Scotland and Wales, may be obtained from the DRC at www.drc-gb.org/employers_and_service_provider/disability_equality_duty/sectoral_guidance/further_and_higher_education.aspx

[4] An explanation of the Academic Infrastructure, and the roles of subject benchmark statements within it, is available at www.qaa.ac.uk/academicinfrastructure

[5] Copies of the DRC revised *Code of Practice: Post-16 Education* may be obtained from the DRC at www.drc-gb.org/employers_and_service_provider/education/higher_education.aspx

[6] Equality Challenge Unit, www.ecu.ac.uk

Foreword

The subject benchmark statement for undergraduate programmes in mathematics, statistics and operational research (MSOR) was published by QAA in 2002. In 2006, QAA invited the Higher Education Academy Maths, Stats and OR [operational research] Network to convene a group of subject specialists to undertake a revision.

The original benchmark statement was widely welcomed by the MSOR community. It was considered to provide an excellent description of MSOR programmes at honours level in an inclusive and enabling manner. The statement was found helpful both by the MSOR community and by other users of subject benchmark statements. In the preliminary soundings taken by QAA prior to the review, opinion was unanimous that no more than minimal revision was needed.

The review group shared this opinion. The revised benchmark statement is very strongly based on the original 2002 version; indeed, most of it is unchanged. The changes that have been made are intended to increase clarity still further and, in some cases, to ensure that the statement remains up to date.

The 2002 version deliberately did not extend to programmes of MMath type. The review group recognised that there would now be value in extending the statement to cover these programmes, but also recognised that it would be a substantial task to draft and consult on such an extension. The review group therefore proceeded with the review of the benchmark statement in its existing form so as not to delay publication, and decided to follow it with an Annex to cover programmes of MMath type. The review group expects this work to be completed in 2008.

The review group acknowledges the excellent work done by the original benchmarking group. It also acknowledges the work of QAA in facilitating this revision.

May 2007

1 Preamble

1.1 The subject area MSOR is a very broad grouping of subjects, so broad that it is possible to find within it programmes that have almost nothing in common. Certainly many of the programmes included within the grouping share more with programmes outside it than with others in it. There may well be more mathematics in a programme in physics or engineering than in a programme focusing on the interpretational aspects of statistics, and more operational research in a business studies programme than in one in mathematics.

1.2 Nevertheless, statistics and OR are traditionally considered as mathematical sciences and there is a considerable degree of overlap among the subjects, even if not between every pair of programmes. The three are also linked historically and are frequently taught within the same department.

1.3 It is possible to deal with the entire subject area within a single subject benchmark statement, but only if users of this document always bear firmly in mind that there is very wide variability among the programmes that come within its scope. Further, as well as the large number of programmes that are clearly within the scope of this statement, there are many that are likely to be regarded as partially within it. This statement offers indications in this respect where appropriate, but it is important to realise that it is always for each institution to determine which standards might apply to any particular programme.

1.4 As well as a classification of programmes according to the part of the subject area on which they are focused, there is also an important classification in respect of what might be thought of as the 'style' of the programmes. Some programmes are concerned more with the underlying theory of the subject and the way in which this establishes general propositions leading to methods and techniques, which are often then applied in one or more parts of the overall subject area. In this document, these are referred to for convenience as 'theory-based programmes'. Other programmes are concerned more with accepting mathematical results, certainly with an understanding of them and the resulting methods and techniques and their strengths and limitations, and concentrating on their application to several parts of the overall subject area. These are referred to for convenience as 'practice-based programmes'.

1.5 There are perhaps few programmes that are entirely theory-based or entirely practice-based. Most programmes have elements of both approaches, and there is a complete spectrum of programmes spanning the distance between both extremes. Further, the nature of a programme is not necessarily reflected in its title; it is the aims and objectives of the programme which make clear its position within the spectrum. The important point is that all types of programmes exist currently and are, in their different ways, valuable. This is just one aspect of the diversity of provision in this subject area.

1.6 The term 'MSOR' is used for the subject area in this document. This is not a standard acronym and is used precisely for that reason – it means the MSOR subject area covered by this subject benchmark statement and has no other connotations. This contrasts with more attractive terms, such as 'mathematical sciences'; these occur as titles of programmes of varied types and their use would also raise other issues, such as whether OR is more a mathematical science than is theoretical physics.

1.7 MSOR is a subject area whose nature is special in many ways. In terms of learning and teaching, this is largely represented in the cumulative nature of the subject and the on-going development of greater maturity in its understanding and use. This is of course true of many other subject areas, but the repeated revisiting of topics at progressively higher levels is an especially prominent feature of MSOR programmes. This implies that learning outcomes need to be set at a strategic level rather than being focused entirely on individual modules, and the success of learners should be judged primarily against outcomes at the programme level rather than the module level. There is also of course the well-known feature that assessment marks in MSOR typically span the entire mark range, and this also must be taken into account in arriving at overall judgements of learners' success. These matters are returned to in paragraphs 2.1 to 2.4 and sections 4 and 5.

Explanation of terms

1.8 This statement uses the term 'programmes' to mean entire schemes of studies followed by learners leading to an award. The term 'modules' is used to mean the individual units of study that make up programmes. Some programmes are highly structured, with a closely laid down progression of modules to be taken in each year of which many, except perhaps in the final year, are compulsory. Other programmes allow learners to assemble modules contingent only on constraints of prerequisites and timetabling (though with due guidance from staff), eventually leading to an award for the entire collection. There is a full spectrum of approaches in programme design, with most programmes containing elements of both of these extremes.

1.9 This statement sometimes refers to the duration of programmes – three years in most cases but with some important four-year exceptions. These refer, of course, to the normal lengths of time taken by full-time learners. Many MSOR programmes are available in part-time mode, including by distance learning. The time taken by learners to complete these programmes will naturally be longer, sometimes much longer (though there will often be an upper limit).

1.10 This statement applies to the whole of the United Kingdom (UK) and it is important to remember that academic traditions in Scotland are somewhat different from those in the rest of the UK. In particular, four-year honours programmes are the norm for full-time learners in Scotland, but are broadly equivalent to three-year honours programmes elsewhere. The first year of a Scottish programme is typically either broader or at a lower level than the first year elsewhere.

1.11 This statement uses the term 'department(s)' to mean the sub-unit(s) of an institution mainly responsible for the design, delivery and management of a programme. Several titles are in common use as well as 'department'; examples are 'school' and 'division'.

1.12 MSOR, then, is a very wide subject area consisting of a grouping of subjects which interact with each other and with many other subject areas. In order to put this benchmark statement into context, it is necessary to discuss these interrelationships. This is done by first considering the essential nature of the three constituents of MSOR itself.

Mathematics

1.13 Mathematics is a major intellectual discipline in its own right, with a pedigree that extends back through various cultures including the Ancient Greeks to even earlier civilisations. It has its roots in the systematic development of methods to solve practical problems in areas such as surveying, mechanical construction and commerce. The subject evolved with the realisation that such methods, when stripped of the details of the particular situations, had a wide range of application and highlighted the essential common characteristics of many different problems. Thus, generalisation and abstraction became important features of the subject. This led to logical verification of propositions concerning abstract entities. Thus, mathematics as a subject developed with watertight arguments and it became a science involving strict logical deduction with conclusions that follow with certainty and confidence from clear starting points. Consequently, mathematics has made a pre-eminent contribution to the development of human thought. While the mathematics of earlier times still remains relevant, it is now only a minute part of a rapidly developing subject. Happily, highlights such as Andrew Wiles' proof of Fermat's Last Theorem have alerted the public to the existence of continuing developments within mathematics.

1.14 The abstract study of mathematics is in itself an intellectual pursuit of value, opening up a world that contains both excitement and beauty. But the subject never loses its contact with the real world. It is its very nature as an abstract and general subject that makes it applicable to almost any discipline, since it identifies patterns that are common to many areas. Indeed, mathematics is sometimes defined as the study of patterns. It is no coincidence that those who are attracted by mathematics are often interested in music as well, for music too is about patterns. The focus of study in mathematics is not on patterns that are arbitrary, but on those that mirror correct relationships between abstract objects. To try to understand the world about us, we generally look for patterns. When we find them, or think we have found them, we can turn to mathematics to tell us more about the patterns we have identified, to tell us more about the world than we could have inferred from mere observation or unaided reasoning. Consider, for example, geometry; the word geometry originally meant measuring the earth, and this acknowledges how the subject grew out of the desire better to understand the patterns that people were noticing two and a half millennia ago.

1.15 The breadth of the applicability of mathematics is immense. Mathematics is fundamental not only to much of science and technology but also to almost all situations that require an analytical model-building approach, whatever the discipline. In recent decades there has been an explosive growth in the use of mathematics in areas outside the traditional base of science, technology and engineering. This is perhaps less well recognised as yet by the general public, but it is mathematics which, for example, lies behind computer technology, underpins such medical technology as body scanners, makes possible the cryptographic techniques needed for the security of electronic financial transactions, and enables the control of space probes.

Statistics

1.16 Statistics is a younger subject than mathematics. Two strands can be identified in tracing its origins. First, discussions about the theory of gambling in the middle of the seventeenth century led to the first attempts to found a theory of probability. Second, the gradual increase in the collection of what would nowadays be called official statistics throughout the nineteenth century led to new developments in the display, classification and interpretation of data. Many signal advances in public policy were made through the application of what might now be seen as very elementary techniques of descriptive statistics but which were at the time truly visionary. These included, famously, the identification of a single pump mainly responsible for a cholera outbreak in London, and the work of Florence Nightingale in establishing the antecedents of today's extensive medical statistics.

1.17 Probability theory has now developed into a major area rich in research and in important applications. The subject of statistics uses probability theory as part of the process of making inferences from limited data to underlying structures – looking for the patterns, as already discussed in the context of mathematics. It encompasses the whole science of collecting, analysing and interpreting data, and has become much concerned with the design processes for observational and experimental studies. It is very much a distinctive area of theory and application that uses mathematical techniques and ideas to solve problems involving randomness, chance, variability, risk and so on. Since much of the world is heavily dependent on these notions, statistics plays a major and increasing role in personal and public life, particularly in medicine, quality control and management, all areas of physical and social sciences, business and economics. It can be applied in any context where informed decisions are needed from limited data, and its extensive use has benefited from the substantial advances in computing power in recent years.

Operational research

1.18 OR is a more recent subject, beginning during the twentieth century. Many of its origins are to be found in the organisation of activities during the second World War. It has several foci, ranging from complex optimisation procedures with significant mathematical underpinnings to non-mathematical but academically rigorous problem-structuring methods. It finds important applications throughout industry, business and commerce, in government, the health and social services, and the armed forces. It has become one of the key quantitative management tools of modern times.

1.19 OR is perhaps less well focused as a subject than either mathematics or statistics. Model-building is crucial to OR, and this aspect is firmly rooted in both mathematics and statistics. But the problem-solving and decision-making aspects of the subject have a broader base and may, additionally or alternatively, draw on a range of non-mathematical disciplines. For example, industrial psychology and sociology might be particularly important. Because of this heritage, a relatively high proportion of programmes in the OR area are practice-based.

1.20 It should be noted that, although the name 'operational research' is generally well understood, a number of institutions use other titles for programmes in this area. One such title is 'management science'. Titles of this sort often indicate very practice-based programmes, perhaps with little mathematical content. Such programmes, by virtue of their design, might not fall entirely within the subject benchmark statement for MSOR.

Relationships within MSOR

1.21 The three subjects within MSOR have complex relationships with each other. Mathematics is the basic area. The other two areas, and indeed mathematics itself, have aspects that are focused on mathematics-based, problem-solving and model-building processes. Further, the problems that generate these processes commonly feed back into MSOR and generate new processes and even new areas of study.

1.22 Both statistics and OR may be studied with a very mathematical focus, but both may be approached from a methods-based, application-oriented focus. This forcefully brings out the distinction between what this statement refers to as theory-based and practice-based programmes.

Relationships with other disciplines

1.23 The very applicability of MSOR links it, to varying levels, with a huge swathe of subjects, including physics, chemistry, astronomy, engineering, computer science, economics, accountancy, actuarial science, finance and many others. These connections can frequently be as strong as, or even stronger than, relationships within MSOR. One aspect of the depth of these connections is that experts in MSOR not infrequently become recognised experts in these other subjects. In terms of programme design, some programmes in these areas will be heavily enough dependent on MSOR to mean that the MSOR benchmark will be of relevance, though almost certainly they will also be at least in part covered by other benchmark statements.

Career opportunities for MSOR graduates

1.24 Learners who graduate from programmes in MSOR have an extremely wide choice of career available to them. Employers greatly value the intellectual ability and rigour and the skills in reasoning that these learners will have acquired, their familiarity with numerical and symbolic thinking, and the analytic approach to problem-solving that is their hallmark.

1.25 Careers are readily available in areas where explicit MSOR skills are to the fore. For example, mathematicians work at the heart of major engineering industries and many professional statisticians work in the pharmaceutical industry. The traditional sectors of teaching and academic research remain vitally important too, of course. But other opportunities are available throughout industry, business and commerce, the public and private sectors, with large employers and in small organisations. Employers value MSOR graduates for their unique knowledge and particular skills (see paragraphs 3.21 to 3.24 and 3.25 to 3.27 of this benchmark statement).

1.26 The learned societies and professional bodies in MSOR have collaborated to provide careers websites at www.mathscareers.org.uk, at www.rss.org.uk/careers and at www.orsoc.org.uk/careers

1.27 The value of the existing diversity of MSOR provision is amply demonstrated by the ready employability of MSOR graduates from programmes of all kinds. Programmes have also been sufficiently flexible to adapt quickly to innovations; a good example is the burgeoning interest in the mathematics of finance, which is an especially important employment area. Many institutions have provided new modules, and sometimes even whole new programmes, in response to such developments. It is not the intention of the MSOR benchmark statement to constrain such flexibility and innovation in any way.

2 Nature and extent of MSOR

The cumulative nature of MSOR

2.1 The subjects included in MSOR are largely cumulative: what can be taught and learned depends very heavily and in considerable detail on previously learned material. This applies to MSOR very much more than to many other disciplines. An MSOR programme must be designed to follow a logical progression, with prerequisite knowledge always taken into account. Advanced areas of pure mathematics cannot be treated until corresponding elementary and intermediate areas have been covered. Development of application areas can often be done in parallel with other work, but it is always necessary to ensure that the required methods and techniques have been dealt with. This last may be more of a constraint in a theory-based programme than in a practice-based one, but even in the latter it will be necessary for methods to be understood before they are used.

2.2 The cumulative nature of MSOR also strongly influences the starting point of any programme - and, as an immediate consequence, the level a graduate from the programme can be expected to attain. Different programmes are designed with different entry standards. It is right that this should be so, as the population of learners who might wish to follow MSOR programmes is itself very diverse. Different levels of knowledge, skill and ability must be catered for across the subject area as a whole.

2.3 Another aspect of catering for previous knowledge arises when considering cases such as mature learners or those transferring from other institutions (possibly in other countries). It is often necessary for admissions tutors to make judgements as to what extent a learner has the appropriate prerequisite knowledge, despite a possible lack of formal qualification. Thus, some flexibility in regulations is desirable plus, of course, care when exercising it.

2.4 The cumulative nature of MSOR has a further consequence of great importance. This is that ideas can take considerable time to be assimilated: learners often do not fully understand something until some time after they have learnt it. If, however, summative assessment comes very soon after a topic has been studied, and therefore before a process of assimilation can be completed, it can be difficult to verify that a particular learning or assessment aim or objective has been achieved. This has already been discussed in paragraph 1.7, where the importance of setting learning outcomes at a strategic level is discussed, and should be recognised in assessment procedures and by those inspecting programmes. By the same token, programmes in MSOR should not restrict themselves to aims and objectives that are easily assessed.

The nature of MSOR programmes

2.5 It is appropriate to stress yet again the special nature of the MSOR area, its breadth and the diversity of programme provision within it.

2.6 MSOR programmes attract learners with a wide variety of different interests and a wide variety of academic backgrounds. As has already been pointed out, career opportunities are also very varied. The diversity of provision is invaluable and cannot be catered for within a strait-jacket. The MSOR benchmark statement is therefore enabling rather than prescriptive and allies itself strongly with QAA's view that subject benchmark statements are in no sense intended to impose a national curriculum.

2.7 Diversity of provision arises from the variation in breadth and depth between programmes and in respect of their position on the spectrum of style between being theory-based and practice-based, and also in respect of the choices available within programmes. In some programmes, certain branches of the subject are developed in considerable depth; in others, the work advances on a broad front covering a quite wide area. Graduates from strongly theory-based programmes will acquire different subject-specific knowledge and understanding from those who study the same topics in practice-based programmes. In programmes which feature some part of MSOR allied with another discipline, emphasis will often be given to subject knowledge which is particularly apposite to the other discipline, taking any overlap of coverage into account.

2.8 An important further source of diversity is, in many cases, the influence of the research and professional interests of the academic staff. While undergraduate programmes in MSOR are not expected to reach the frontiers of knowledge, it is a stimulating experience for a learner to be taught a subject by someone who is an active researcher or professional in the field. The choice of material presented in MSOR programmes, whilst mainly determined by its educational value, will nevertheless often be influenced in detail by the research and professional interests of the academic staff. Naturally these are not the major driving factors behind programme design, but they are of importance in providing a learning experience that is a vibrant and stimulating intellectual adventure.

2.9 Despite the diversity, general characteristics of all MSOR programmes can be identified. All programmes will aim to provide learners with a solid understanding of basic knowledge and ideas and a mastery of some areas. Whether the programme is theory-based or practice-based, there will be emphasis on the inculcation of precise understanding and the use of rigorous methods. This will come through solving challenging problems, either within MSOR itself or in other application areas. There will also be emphasis on the development of graduates capable of using their powers of analysis.

2.10 Many MSOR programmes specify a core of essential study together with a large number of options, particularly in the final year. This is elaborated on in section 3. Even in programmes that are very modular in nature, there will usually be a fundamental set of compulsory modules.

2.11 MSOR programmes include single honours programmes in each of the three separate subjects. Of these, the mathematics programmes are by far the most numerous and have by far the greatest numbers of learners. There are comparatively few programmes or learners in statistics alone and even fewer in OR alone. Exact numbers of learners in the various areas are not easy to determine from the published Universities and Colleges Admissions Service and Higher Education Statistics Agency statistics, because of the multiplicity of programme titles and definitions.

2.12 Single honours programmes in mathematics nearly always include some modules in statistics, though these might not be compulsory. There are often a few optional modules in OR as well. Single honours programmes in the other two subjects almost always include some mathematics. This may form a relatively large part of the programme; for example, this would typically be so for a theory-based statistics programme. On the other hand, there are also programmes where mathematics is only a small proportion; this might typically occur for a practice-based OR programme.

2.13 There are also many MSOR programmes that combine the study of two or all three of the separate subjects within MSOR. As an example, there are wide-ranging programmes in pure mathematics, applied mathematics and statistics. It might not be evident from the title of a programme that it is of this type. For instance, a programme might be entitled simply 'mathematics' as an indication of a broad approach, whereas this title at another institution might indicate a programme very firmly based in mathematics itself. However, the nature of the programme will be evident from its aims and objectives.

2.14 Whether such programmes are regarded as single honours or joint-honours might be largely a matter of accident in terms of the structure of the institution and whether all the teaching is done within the same department or at least within the same cost centre. For benchmarking purposes, it is clear that the MSOR benchmark statement applies to them whether they are formally single or joint.

2.15 MSOR programmes frequently offer the opportunity to take a few modules in topics from other subject areas, quite separately from application areas that are explored as part of the programme itself. Common examples are modules in accountancy or economics or modules in a foreign language. Such modules can be very important not only for vocational reasons but also because problems suitable for mathematical analysis may arise in them even though this may not have been the main reason for making them available. These modules are likely to be taught by the subject departments, in which case they may or may not be the same modules that are taught to specialists in those areas. In general terms, the MSOR benchmark statement cannot extend to 'outside' modules of this kind except, importantly, to the extent to which they affect the overall nature of a programme.

Four-year programmes

2.16 Four-year programmes of MMath type are a development resulting from deliberations of a committee of mathematicians in 1992 and subsequent representations to government. It was argued that changes in mathematics in schools made it difficult to design programmes that were an adequate preparation for postgraduate studies or for employment requiring a similar high level of mathematical knowledge and skills without, at the same time, being too demanding for the majority of learners. Consequently, the establishment of programmes of this type (usually now referred to as integrated masters programmes) was agreed to and funded in the same way as other undergraduate programmes. Similar programmes exist in physics, chemistry and engineering. The MSOR programmes commonly have the title of MMath or MSci, but some other titles also exist (for example, MMathStat). Such programmes are now offered quite widely. They are designed primarily to increase the depth more than the breadth of study. While this MSOR benchmark statement is clearly of relevance to them, it is not at present designed to include them. An annex to the benchmark statement will be developed which will include them.

2.17 A totally different type of four-year programme is the sandwich programme where learners spend a year (sometimes as two separate periods) in a supervised professional placement in industry, business or commerce. Several universities offer such placements if learners wish to take them and some programmes require learners to take them. An appropriate infrastructure of placement and tutorial support will be involved. The academic components of these programmes are usually completely unaffected by whether or not a learner takes a placement, and so the MSOR benchmark statement simply continues to apply.

2.18 Somewhat similar is the situation where learners may spend a year abroad (commonly in Europe), studying at an overseas institution in the language of that country. Such programmes can be joint ones in an MSOR subject and the foreign language, in which case they fall under the benchmarking arrangements for joint programmes discussed in paragraphs 2.29 to 2.32. They might alternatively be programmes in an MSOR subject with only a comparatively small component of language training. This situation is similar to that discussed in paragraph 2.15. Institutions must of course take special care to ensure good articulation between any MSOR studies undertaken abroad and the rest of the MSOR programme in the UK, while recognising that national differences usually preclude complete integration.

Programmes focusing on mathematics

2.19 Programmes in mathematics typically involve continuous mathematics, discrete mathematics, logical argument, problem solving and mathematical modelling. Rigorous mathematical proof will also be very important in theory-based programmes. Many programmes also contain probability theory and the fundamentals of statistics. Typical programme elements that are likely to be found are discussed in paragraphs 3.8 to 3.20. These are more likely to consist of sets of modules than single isolated modules.

2.20 Applications of mathematical theories, methods and techniques will be explored, either in other areas of mathematics or in distinct application areas or, in many cases, in both. Such areas include continuum mechanics, statistics, operational research, physics, astronomy, chemistry, biology, engineering, finance, economics, actuarial science and many others. Programmes will focus on particular areas according to their aims and objectives.

2.21 Modules in application areas might be taught by the subject departments, in which case they may or may not be the same modules that are taught to specialists in those areas. In many cases, however, the teaching is done within the home mathematics department. A common example here is mathematical physics, a subject in which a significant proportion of the researchers in the UK are located in mathematics departments. Modules taught by mathematics departments tend to be different in nature and style from those on the same topics taught by the other departments, though they need not be. The important example of mathematical physics illustrates the fact that in many ways mathematics is at least as close to physics, which has a separate subject benchmark statement, as it is to OR, which is included in the MSOR one.

Programmes focusing on statistics

2.22 Programmes in statistics typically involve many of the areas referred to in paragraph 2.19. They also typically involve understanding and managing variability through the science of data investigation, formulating probability-based models in order to make inferences from samples, statistical theory, applications of statistics in other subject areas, and communicating the results of statistical investigations.

2.23 Programme elements that are likely to be found include many of those likely to occur in a mathematics programme and in addition data visualisation, inference, likelihood, linear and non-linear modelling, experimental design, stochastic processes and time series, Bayesian methods, computational statistics, and the use of specialist statistical computing packages.

2.24 Programmes in statistics will feature problems taken from many application areas. These might include biology, chemistry, medicine, pharmaceuticals, engineering, geography, archaeology, environmental science, actuarial science, economics, management, law and many others. Separate modules in these areas might also be available; such modules would often be taught by the respective subject departments.

Programmes focusing on operational research

2.25 Programmes in OR appear under a number of different titles. Only in some cases does the phrase OR appear in the title. Other titles include business decision methods, business systems modelling and management science. Particularly in respect of the last of these, it may not be clear from the title alone whether or not it is an MSOR programme, but its aims and objectives should clarify this.

2.26 Programmes in OR are inherently concerned with the processes of modelling complex and often ill-determined situations in forms suitable for analysis. Interpretation of the results in the context of the original problems is especially important. Mathematical and non-mathematical aspects of the modelling may both be important. In respect of the mathematical aspects, the techniques of mathematical programming are likely to be central, including linear, non-linear, integer and dynamic programming. Also likely to be involved are some of the areas of mathematics (particularly discrete mathematics) referred to in paragraph 2.19 and some of the areas of statistics referred to in paragraphs 2.22 and 2.23.

2.27 Programme elements that might be found depend on the focus of the programme within the OR area. They are likely to include a selection of some from programming methods, combinatorial analysis, graphs and networks, scheduling and sequencing models, game theory, decision analysis, decision support systems, inventory control models, queueing models, simulation modelling, heuristic methods, soft systems methods, and many others.

2.28 Programmes in OR, like those in statistics, will feature problems taken from many application areas. These might include manufacturing and production, public and health services, performance measurement and management, corporate and strategic planning, transportation, distribution, logistics and location, and many others. Separate modules in these areas might also occasionally be available, usually taught by the respective subject departments.

Joint programmes

2.29 The subjects within MSOR are among the most common constituents of joint programmes. Indeed, as discussed in paragraph 2.13, there are many programmes that lie entirely within MSOR, combining elements of the different MSOR subjects, that exhibit all the characteristics of joint programmes.

2.30 In respect of joint programmes with other subjects, MSOR subjects are to be found combined in major–minor mode and/or in 50:50 mode (and sometimes even in minor–major mode) with just about every other discipline. The benchmarking of such programmes presents special problems which have been discussed elsewhere. Suffice to say that the MSOR aspects will normally fall under the MSOR benchmark statement in so far as it can be applied to programmes in which the MSOR content is less (sometimes much less) than the total.

2.31 A further complication arises with programmes that are very modular in nature, where learners assemble many modules chosen from a possibly very wide selection. Where such programmes contain MSOR elements, it should again be the case that the MSOR benchmark statement applies in so far as it can to a programme where MSOR forms much less than the total content.

2.32 A word of caution needs to be added about the nomenclature of joint programmes. In some cases, the two (or more) subject areas are developed in a planned way such that they support each other. An MSOR example might be a programme in statistics and management. In other cases, the subjects are developed in parallel and independently of each other. A programme in mathematics and a language might be of this type. Some institutions might reserve the name joint programme for the latter of these types, referring to the former type as a single honours programme in the two subjects. In any case, the two types of programme might well have identical or very similar titles. As in many other aspects, the nature of a programme will be revealed by its aims and objectives.

MSOR subjects in other programmes

2.33 MSOR subjects are unique among all disciplines in the extent to which they necessarily occur in programmes in other areas. For example, mathematics itself has a key role throughout engineering and the physical sciences, statistics is widely used in the experimental and social sciences, and both statistics and OR are important in management. The style and presentation of MSOR subjects in these other programmes is normally different from that in an MSOR programme. In many cases the MSOR subject is taught by an MSOR department, but in others it is taught by the department in which the programme is based. In general the MSOR benchmark statement is unlikely to apply to MSOR teaching of this kind, though it is invariably important that the other programmes pay due attention to the place of MSOR within them and, where appropriate, elements of MSOR should be incorporated in other benchmark statements.

2.34 MSOR subjects are also commonly included in foundation programmes that prepare learners who do not have the normal entry qualifications for study on higher education programmes. Again, the MSOR benchmark statement cannot be directly applied, but it may be useful as a reference point when designing such programmes.

Professional body accreditation of MSOR programmes

2.35 Professional body accreditation is awarded by the Royal Statistical Society to programmes in statistics that meet its criteria. Graduates from such programmes are automatically eligible, on application, for the professional award of Graduate Statistician. Graduates from non-accredited programmes may also be awarded Graduate Statistician status on application, on an individual basis, if there is a sufficient proportion of statistics in their programmes. Graduate Statisticians with appropriate professional experience may be eligible for the professional award of Chartered Statistician.

2.36 The Institute of Mathematics and its Applications approves programmes in MSOR that meet the educational requirements for Chartered Mathematician status. Graduates from such programmes are eligible for Graduate Membership on application and may apply for Corporate Membership after obtaining appropriate professional experience.

2.37 Graduates from MSOR programmes also commonly enjoy partial exemption from the examinations of many other professional bodies, such as those for accountancy and for the actuarial profession, if they have taken the appropriate modules and passed them at a sufficiently high standard.

3 Knowledge, understanding and skills

Introduction

3.1 As has been discussed in section 2, the general subject area of MSOR covered by this benchmark statement is very wide. The knowledge and skills that may be expected of graduates in the area cover a correspondingly wide range.

3.2 Such graduates will have knowledge and skills that are specific to areas within MSOR. In this statement, such knowledge and skills are referred to as subject-specific. At the higher levels of study, this knowledge and these skills will naturally vary between graduates because of the different areas of the subject(s) that different graduates pursue to the higher levels. This diversity is a natural feature of the MSOR subject area, is to be welcomed and must not be restricted in any way. Furthermore, it is dynamic and evolving, as programmes develop to encompass new areas of study. It is however possible to discern subject-specific knowledge and skills that are enjoyed by all such graduates.

3.3 Diversity requires a foundation on which to build securely. Sequential subjects such as those covered by this benchmark statement require sequential layers of foundations, each building on previous ones. This process may well continue right through a programme. While most of the foundations are normally laid in the earlier years of programmes, it must be recognised that this is not exclusively the case. Equally, the earlier years are not necessarily exclusively concerned with laying foundations; in many programmes it may be entirely proper for more advanced work or for work on applications to start at an early stage, provided always that any necessary prerequisite knowledge is in place.

3.4 For the above reasons, it makes no sense to attempt to construct a comprehensive listing of subject-specific knowledge for all graduates from programmes covered by the MSOR benchmark statement. Such a listing would be far too prescriptive, might well force unnecessary and undesirable modifications in some existing programmes and would confer no positive benefits.

3.5 However, some themes of knowledge can be identified; both horizontal themes that spread across much of the work of a year and vertical themes that run through several years. This type of consideration might help to put in perspective the discussion in paragraphs 3.8 to 3.20 of the subject-specific knowledge and understanding that graduates from the MSOR area should be expected to possess.

3.6 Graduates from the MSOR area will also have highly developed skills of a more general kind. Obvious examples are that they will be highly numerate and that graduates from many of the programmes will be thoroughly at home with applications in computing.

3.7 The sections below therefore apply to all programmes to which the MSOR benchmark statement applies. The term graduates is used as a shorthand for graduates from all such programmes, unless it is qualified by a reference to a particular subject area or style of programme within MSOR.

Subject-specific knowledge and understanding

General principles

3.8 All graduates will have knowledge and understanding of mathematical methods and techniques appropriate to their main field of study, and of some results from a range of major areas of MSOR. In addition, graduates from most programmes will have met at least one major area of application of their subject in which it is used in a serious manner and is essential for proper understanding. The nature of the application area and the manner in which it is studied might vary depending on whether the programme is theory-based or practice-based. Different skills will be developed in the graduates according to the focus of the programme.

Methods and techniques

3.9 All graduates will have knowledge and understanding of, and the ability to use, mathematical methods and techniques appropriate to their programme. Common ground for all programmes will include basic calculus and basic linear algebra. Other methods and techniques will be developed according to the requirements of the programme, which will also largely determine the levels to which the developments are taken.

3.10 As examples, graduates from programmes in OR may have considerable knowledge of constrained optimisation and its application to allocating scarce resources, or of modelling different decision-making processes; whereas graduates from programmes concentrating on applications of mathematics in physics or engineering may have correspondingly deep knowledge of methods for dealing with differential equations.

3.11 These examples have been deliberately chosen as being fairly far apart in the spectrum of MSOR programmes, but it is to be emphasised that the methods and techniques covered in them are not mutually exclusive. All programmes will cover methods and techniques that pertain to a range of areas of mathematics.
Each programme will develop in depth according to its own requirements.

Areas of mathematics

3.12 Graduates from theory-based programmes will have knowledge and understanding of results from a range of major areas of MSOR. Examples of possible areas are algebra, analysis, geometry, number theory, differential equations, continuum mechanics, mathematical physics, probability theory and statistics, but there are many others. This knowledge and understanding will support the knowledge and understanding of mathematical methods and techniques, by providing a firmly developed mathematical context.

3.13 Graduates from practice-based programmes will also have knowledge of results from a range of areas of MSOR, but the knowledge will commonly be designed to support the understanding of models and how and when they can be applied, rather than (though sometimes this will be covered as well) providing a deep understanding of the mathematical derivation of the models.

Mathematical thinking and logical processes

3.14 Graduates will have an understanding of the importance of assumptions and an awareness of where they are used and of possible consequences of their violation.

This will include an appreciation of the distinction between the roles of validity of assumptions and validity of arguments. Graduates will also appreciate the power of generalisation and abstraction in developing mathematical theories or methods to use in problem solving. Theory-based programmes may tend to emphasise the role of logical mathematical argument and deductive reasoning, often including formal processes of mathematical proof; practice-based programmes may tend to emphasise understanding and use of structured mathematical or analytical approaches to problem solving.

3.15 Knowledge and understanding under this heading will inform and underpin many other activities that may appear in various programmes, such as axiomatic approaches to advanced pure mathematics or the general role of modelling.

Numerical mathematics and mathematical computing

3.16 All graduates will have knowledge and understanding, at the level required for their programmes, of some processes and pitfalls of mathematical approximation.

3.17 All graduates of practice-based programmes and many from theory-based programmes will have some knowledge and understanding of mathematical computing, often with direct experience of one or more computer packages. They will have an awareness of the appropriateness of the package(s) to the problems being addressed and, when feasible, some awareness of the nature of the algorithms on which the package(s) are based.

Modelling

3.18 Modelling is the process of formulating problems in mathematical or statistical form using appropriate notation. All graduates will have knowledge and understanding of this process. Normally the problems will come from at least one application area, but they could also come from other areas within MSOR.

3.19 All graduates of practice-based programmes and many from theory-based programmes will have knowledge and understanding of a range of modelling techniques and their conditions and limitations, and of the need to validate and revise models. Additionally, they will know how to use models to analyse, and as far as possible solve, the underlying problem or to consider a range of scenarios resulting from modifications to it, as well as how to interpret the results of these analyses.

Depth of study

3.20 All graduates will have knowledge and understanding developed to higher levels in particular areas. The higher-level content of programmes will reflect the title of the programme. For example, graduates from programmes with titles involving statistics will have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Programmes with titles such as mathematics might range quite widely over several branches of the subject, but nevertheless graduates from such programmes will have treated some topics in depth.

Subject-specific skills

3.21 MSOR graduates will have subject-specific skills developed in the context of a very broad range of activities. These skills will have been developed to a sufficiently high level to be used after graduating, whether it be in the solution of new problems arising in professional work or in higher academic study, including multidisciplinary work involving mathematics.

3.22 A number of subject-specific skills are to be expected of all MSOR graduates. Most of these will be formally assessed at some stage during the degree programme. However, it must be recognised that some are not necessarily susceptible to explicit assessment. Some pervade all mathematical activity and will be reflected in assessments focused on many areas of subject content.

3.23 Many of the subject-specific skills to be expected of all MSOR graduates are directly related to the fundamental nature of MSOR as a problem-based subject area – whether the problems arise within MSOR itself or come from distinct application areas. Thus, graduates will have the ability to demonstrate knowledge of key mathematical concepts and topics, both explicitly and by applying them to the solution of problems. They will be able to comprehend problems, abstract the essentials of problems and formulate them mathematically and in symbolic form so as to facilitate their analysis and solution, and grasp how mathematical processes may be applied to them, including where appropriate an understanding that this might give only a partial solution. They will be able to select and apply appropriate mathematical processes. They will be able to construct and develop logical mathematical arguments with clear identification of assumptions and conclusions. Where appropriate, they will be able to use computational and more general information technology (IT) facilities as an aid to mathematical processes and for acquiring any further information that is needed and is available. They will be able to present their mathematical arguments and the conclusions from them with accuracy and clarity.

3.24 Graduates from programmes focused on particular branches of MSOR will have other subject-specific skills that are relevant to those particular branches. An exhaustive list of such skills will not be helpful, but as examples:

- graduates from programmes focusing on pure mathematics will have skills relating particularly to logical argument and solving problems in generality, and facility with abstraction, including the rigorous development of formal theories
- graduates from programmes focusing on physical applied mathematics or theoretical physics will have skills relating particularly to formulating physical theories in mathematical terms, solving the resulting equations analytically or numerically, and giving physical interpretations of the solutions
- graduates from programmes focusing on statistics will have skills relating particularly to the design and conduct of experimental and observational studies and the analysis of data resulting from them
- graduates from programmes focusing on OR will have skills relating particularly to the formulation of complex problems of optimisation and the interpretation of the solutions in the original contexts of the problems.

General skills

3.25 Graduates from the MSOR area will have acquired many general skills honed by their experiences of studying MSOR subjects. All these subjects are essentially

problem-solving disciplines, whether the problems arise within MSOR itself or come from areas of application. Thus the graduates' experiences will be embedded in a general ethos of numeracy and of analytical approaches to problem solving.
In addition, an important part of most MSOR programmes is to take theoretical knowledge gained in one area and apply it elsewhere. The field of application is often a significant topic of study in its own right, but the crucial aspect of the process is the cultivation of the general skill of transferring expertise from one context to another.

3.26 A number of general skills are to be expected of all MSOR graduates, though in some cases they are likely to be developed more in graduates from some programmes than others. Even more than in the case of the subject-specific skills, it must be recognised that some are not susceptible to explicit assessment and indeed some are better not assessed so as to avoid creating imbalances.

3.27 MSOR graduates will possess general study skills, particularly including the ability to learn independently using a variety of media which might include books, learned journals, the internet and so on. They will also be able to work independently with patience and persistence, pursuing the solution of a problem to its conclusion. They will have had the opportunity to develop general skills of time management and organisation. They will be adaptable, in particular displaying readiness to address new problems from new areas. They will be able to transfer knowledge from one context to another, to assess problems logically and to approach them analytically. They will have highly developed skills of numeracy, including being thoroughly comfortable with numerate concepts and arguments in all stages of work. They will typically have general IT skills, such as word processing, the ability to use the internet and the ability to obtain information. They will also have general communication skills, typically including the ability to work in teams, to contribute to discussions, to write coherently and to communicate results clearly. Where appropriate, they will have knowledge of ethical issues, including the need for sensitivity in handling data of a personal nature. All of these competencies enhance the general employability of MSOR graduates; see paragraphs 1.24 to 1.27.

4 Teaching, learning and assessment

Introduction

4.1 MSOR subjects are characterised by the need for a high degree of conceptual and abstract thinking within the learning process. While this poses considerable challenges to the learner, it also offers correspondingly great rewards. It is often intellectually very demanding to achieve understanding at the necessary degree of abstraction; yet, achieving deep understanding of complex ideas is often a more than adequate reward for the effort required. To master a new concept is a satisfying experience in itself and gives the learner who has achieved it the confidence to take on similar challenges in the future. It often has direct practical benefits as well because, once a concept is understood, its particular instances are easily learned.

4.2 MSOR graduates are rightly seen as possessing considerable skill in abstract reasoning, logical deduction and problem solving, and for this reason they find employment in a great variety of careers and professions, as discussed in paragraphs 1.24 to 1.27. Teaching and learning methods should assist the development of these skills, by encouraging not merely the capacity for abstract reasoning, but also the learners' capacities for independent and self-motivated learning, problem-solving skills, and some of the knowledge and skills which are common to employment in many fields.

4.3 As in all other disciplines, MSOR teaching has developed following changes in wider educational practice. This situation is continually evolving and departments will continue to respond to changing circumstances. Even though innovation might be risky and by no means straightforward to manage, departments should not feel discouraged from introducing developments which are recognised as having clear merits. Therefore, those judging performances of departments should not be unduly critical of detail in such instances.

4.4 Advances in e-learning have led to a variety of imaginative and useful software products for supporting learning throughout MSOR, with many software developments aimed at quite general MSOR teaching. In addition, use of computers and software to carry out technical MSOR work has been widespread for some time. Examples of this are the use of standard spreadsheet software for MSOR purposes, the use of computer algebra systems, the use of sophisticated programs for advanced numerical analysis and numerical solution of equations, the use of statistical packages for data analysis and model building, and the use of mathematical programming packages for formulating and solving OR problems.

4.5 These various changes have led to a variety of learning, teaching and assessment methods. These may differ markedly depending on the style of the programme, the subject matter, the level and background of the learners, the special interests of the department, and the resources that are available to it.

4.6 Institutions may choose to adopt a range of learning, teaching and assessment methods. By the nature of the subject area, learners benefit from seeing arguments developed by teachers in 'real time' in lectures, with appropriate emphases. It is commonly necessary to see extended arguments at length and this is likely to require considerable space for display of the teacher's material. Thus, the traditional board based lecture continues to have substantial merit. Typically such lectures may be supported by problem classes, tutorials and seminars; these may be provided by formal support centres within the institution. Further methods that institutions may use include, for example, workshops, group work and projects. Whatever methods are chosen, they should reflect the needs of learners, the aims and learning outcomes of the programme or the module and the resources available.
Learning, teaching and assessment methods should be regularly reviewed.

Teaching and learning

4.7 Institutions will always have in mind that learning opportunities and teaching methods should be designed so as to be appropriate for the aims and objectives of the programme and/or module. They should foster learners' knowledge of and enthusiasm for the subject and stimulate engagement and participation in the learning process. They should encourage learning in depth, and also encourage learners to reflect on and take responsibility for their own learning and to develop their academic self-confidence. These are of course very general criteria that should pervade all of an institution's work rather than being regarded as specifically part of the MSOR benchmark statement itself.

4.8 Learners within MSOR are likely to meet a range of learning activities including, importantly, regular opportunities to undertake appropriate exercises, as this is a key to achieving good understanding. All programmes, unless based on distance learning, will include lectures and tutorials of various kinds. The detailed nature of these, and of any other types of activities that are used, will be determined appropriately for particular subject areas within MSOR, bearing in mind also the style of the programme. Some institutions might also choose to develop some skills by extra-curricular activities, placements or ancillary modules. Whatever learning activities and teaching methods are used, it is essential that they are appropriate both for the learners and for achieving the desired learning objectives. What is taught and how well it is learnt are more important than the method of delivery.

Assessment

4.9 As with teaching and learning (see paragraph 4.7), institutions will always have very general criteria about assessment in mind, too general to be regarded as specifically part of the MSOR benchmark statement. These criteria refer to such features as validity, reliability and fairness, assessing appropriate aspects of knowledge and skill, supporting the aims of learning and teaching, providing opportunities for feedback wherever appropriate, and being open to external scrutiny (for example, by external examiners) as appropriate.

4.10 As with learning activities, the learners are likely to meet a range of methods of assessment, depending on the aims and learning outcomes of the programme or module and on the knowledge and skill being assessed. Assessment will be carried out according to context and purpose and, where different methods of assessment are in use, will recognise that learners may exhibit different aptitudes in different forms of assessment.

4.11 The assessment of MSOR subjects is not necessarily restricted to the assessment of mathematical knowledge and understanding alone. As examples, many programmes will also assess, according to their aims and objectives, the ability to use mathematical ideas in the context of an application, the ability to carry out an extended statistical investigation or the ability to communicate effectively in the context of MSOR.

5 Benchmark standards

Introduction

5.1 Because the general subject area covered by the MSOR benchmark statement is very wide, the standards that may be expected of graduates in the area can only be specified in a fairly general way.

5.2 Benchmark standards for MSOR are defined at threshold and typical levels.

5.3 In MSOR, the distinction between the two levels lies largely in the depth of the learner's understanding of concepts or techniques, the breadth of the learner's knowledge, the amount of support and guidance the learner requires to undertake an extended task, the complexity of the problems that the learner can solve or model, the learner's ability to construct and present a reasoned argument or proof and how far the learner can progress through it, and the facility with which the learner performs calculations or manipulations.

5.4 Interpretation of adjectives such as 'reasonable' in the benchmark standards is a matter of professional judgement for the internal and external examiners.

5.5 Professional judgement of examiners is of fundamental importance. It is predicated partly on collective experience, taken in conjunction with the knowledge of the ready employability of MSOR graduates.

5.6 Setting of work for assessment in MSOR subjects requires a very great deal of professional judgement. The questions to be asked in an MSOR written examination have to be thought through with very great care. In MSOR, differentiation by outcome of learners' responses will be unlikely to rescue a question that was ill-constructed in the first place. The questions must be appropriate in level and in content so that reasonably-prepared learners can be expected to solve them with the MSOR knowledge at their disposal in the time available. The questions must also be editorially sound and unambiguous (even if, in some cases, deliberately loosely-defined); this applies, of course, to all questions, but all the more so to problem-solving questions set in applied contexts.

5.7 Marking of work presented by learners in assessments of MSOR subjects is often thought to be entirely deterministic with little or no room for professional judgement. Such however is frequently not the case. For example, where a learner has to describe a model-building process, or discuss the results of an analysis of some data, professional judgement of the worth of the learner's response will be an inherent and essential part of the assessment process. Important issues of professional judgement also arise in awarding appropriate credit where learners present solutions that are flawed, but not wholly incorrect, or have clearly tried to use a correct method but have not been fully successful.

5.8 For many types of assessments in MSOR, the variation in marks achieved by learners of undoubted commitment is typically greater than in other subject areas. This is particularly so in written examinations and tests. Markers' experience shows that, on the one hand, perfect or near-perfect solutions fully deserving of very high marks arise more often than in most other subjects; while on the other hand, the problem-solving nature of MSOR can lead to weaker learners, and sometimes not-so-weak learners, having difficulty even starting some questions. It is therefore not unusual for marks in MSOR assessments to span the full mark range, and this would be a correct representation of the relative values of the learners' work. Institutions should accept that the patterns of marks achieved in MSOR assessments are likely to differ substantially from those achieved in other subjects, and therefore that global assessment regulations might be less applicable in MSOR than elsewhere.

5.9 It is also an inherent characteristic of the subject that an individual learner's performance might vary greatly over different modules. Some learners, of course, do uniformly well in all modules. But even the best learners frequently find some particular area(s) of MSOR difficult to grasp, and this may lead to the blemish of a few modules having quite low marks in a profile that is clearly of overall excellence with consistently very good performances elsewhere. In such a case, a department's internal and external examiners might well judge the learner to be of high quality overall.

5.10 In similar vein, learners of modest but nevertheless worthwhile attainment frequently present profiles where a number of modules are failed, perhaps only narrowly, whereas other modules are passed at a comfortable if moderate level. The examiners might well judge that such a learner, considered overall, has demonstrated a reasonable knowledge of the subject and that there is positive achievement, admittedly at a fairly lowly level, worthy of reward.

5.11 For these reasons, overall assessment of MSOR learners often relies on averaging or preponderance systems. In such systems, an overall view is taken of a learner's achievements. A level of success on each individual module that is commensurate with the overall performance might not be required, and it is entirely acceptable that this should be so. As in paragraph 5.8, global assessment regulations might be less applicable in MSOR than elsewhere and could seriously infringe professional judgement.

Threshold standard

5.12 The points made in paragraphs 5.1 to 5.11 must be borne in mind when interpreting the threshold benchmark standard for MSOR. It is intended that learners should meet this standard in an **overall** sense, not necessarily in respect of each and every of the statements listed.

5.13 A graduate who has reached the threshold level should be able to:

- demonstrate a reasonable understanding of the basic body of knowledge for the programme of study
- demonstrate a reasonable level of skill in calculation and manipulation within this basic body of knowledge
- apply core concepts and principles in well-defined contexts, showing judgement in the selection and application of tools and techniques
- understand logical arguments, identifying the assumptions and conclusions made
- demonstrate a reasonable level of skill in comprehending problems, formulating them mathematically and obtaining solutions by appropriate methods
- present straightforward arguments and conclusions reasonably accurately and clearly
- demonstrate appropriate general skills
- demonstrate the ability to work professionally under guidance, seeking assistance when needed.

Typical standard

5.14 The points made in paragraphs 5.1 to 5.11 must be borne in mind when interpreting the typical benchmark standard for MSOR. It is intended that learners should meet this standard in an **overall** sense, not necessarily in respect of each and every of the statements listed.

5.15 A graduate who has reached the typical level should be able to:

- demonstrate a reasonable understanding of the main body of knowledge for the programme of study
- demonstrate a good level of skill in calculation and manipulation of the material within this body of knowledge
- apply a range of concepts and principles in loosely-defined contexts, showing effective judgement in the selection and application of tools and techniques
- develop and evaluate logical arguments
- demonstrate skill in abstracting the essentials of problems, formulating them mathematically and obtaining solutions by appropriate methods
- present arguments and conclusions effectively and accurately
- demonstrate appropriate general skills
- demonstrate the ability to work professionally with a degree of independence, seeking assistance when needed.

Appendix A – Membership of the review group for the subject benchmark statement for mathematics, statistics and operational research

Professor Peter Giblin	Heads of Departments of Mathematical Sciences in the UK
Gerald Goodall	Royal Statistical Society
Michael Grove	Higher Education Academy Subject Centre for MSOR
Dr Stuart Johns	Operational Research Society
Professor Duncan Lawson (chair)	Higher Education Academy Subject Centre for MSOR
Dr Niall MacKay	London Mathematical Society
Professor Nigel Steele	Institute of Mathematics and its Applications

Appendix B – Membership of the original benchmark statement group for mathematics, statistics and operational research

Details below appear as published in the original subject benchmark statement for mathematics, statistics and operational research (2002).

Professor Rob Archbold	University of Aberdeen
Professor Russell Cheng	University of Southampton
Professor Neville Davies	The Nottingham Trent University
Dr John Erdos	King's College London
Dr Judy Goldfinch	Napier University
Mr Gerald Goodall	The Royal Statistical Society
Mr Tony Palmer	De Montfort University
Professor Chris Robson (chair)	University of Leeds
Dr Stephen Ryrie	University of the West of England
Professor Peter Saunders	King's College London
Dr Stephen Siklos	University of Cambridge
Professor Joan Walsh	University of Manchester (retired)