

INTRODUCTION

Springboard 6 is a series of lessons designed to provide additional support in booster classes for children in Year 6 who, with intensive targeted support, can achieve Level 4 in the Key Stage 2 national tests.

Experience has shown that many Year 6 children working just below the Level 4 threshold can make the progress required to catch up by the time of the tests. Booster classes can help you to provide the help and support these children need. It is crucial for these children to reach the expected standard before entering Key Stage 3 if they are to fulfil their potential in mathematics and go on to achieve a good grade at GCSE.

SPRINGBOARD 6 LESSONS

The 22 half-hour lessons are designed for use during the Spring Term of Year 6. They can be used alongside, and in addition to the work being planned in the daily mathematics lessons for that term. The lessons follow closely the sequence of topics identified by the units in the Year 6 yearly teaching programme set out in the National Numeracy Strategy's *Framework for teaching mathematics from Reception to Year 6*. They address a number of aspects of mathematics that the target group of children finds difficult. The lessons follow the three-part model developed by the National Numeracy Strategy. The lesson plans identify objectives, vocabulary, resources, teaching points and key questions. There is a substantial proportion of direct teaching in which children are expected to participate. Each lesson sets out what children should be able to do by the end of the lesson. In the plenary the lesson ends with key statements for children to remember.

All the general principles of effective mathematics teaching apply to these lessons. Some elements are especially important for booster classes:

- a step-by-step approach;
- built-in consolidation and summaries;
- the use of direct questions and discussion about ideas and methods;
- the expression of the same mathematical ideas in a variety of ways;
- the use of demonstration by the teacher to model ideas and methods and to help children to visualise the processes involved;
- the reinforcement of key mathematical vocabulary;
- the encouragement of children to articulate their mathematical thinking.

You will need to adjust the lessons to take account of your children's current attainment levels, their progress and the responses they make.

CONTENT

The table on page 7 lists the first 11 half-hour lessons designed for use over about half a term. The next 11 lessons will be sent to you early in 2002. The table also shows how the lessons link to the units teachers are likely to be using during the Spring Term, though not necessarily in the order they are presented here. The objectives of each lesson draw on those in the *Framework*.

Each lesson has a sharp focus and is independent of the other lessons. However, there is scope to combine two half-hour lessons into a one-hour lesson as the topics are often related. This may require some minor changes to the structure and timing of the lessons.

The content of the lessons covers aspects of mathematics that the target group of children appears to find difficult. To help teachers with their lesson preparation and teaching, copies of related past Key Stage 2 national test questions have been included. With each question there is a brief analysis of children's answers, and an outline of the key features teachers might address in their planning and teaching to help children cope more successfully with the mathematics.

HELPING CHILDREN TO DO THEIR BEST IN THE NATIONAL TESTS

Schools have implemented various strategies to help children to do their best in the tests and to demonstrate their mathematical capabilities. Seven effective strategies are described in some detail, together with some test questions to highlight particular points. The strategies described include ways of building revision into ongoing teaching, helping children to help themselves and to demonstrate successfully the mathematics they know, understand and can do.

HELPING CHILDREN TO REVISE

Seven effective strategies to help children revise are described below.

1 USING PREVIOUS TEST QUESTIONS WHEN TEACHING

An effective revision strategy is to incorporate relevant test questions into the teaching of **each** unit of work. A number of schools use the QCA Testbase CD-ROM. Teachers select test items and the corresponding mark schemes from the mental and written (both calculator and non-calculator) tests to use in their lessons. The advantages for the children include:

- the opportunity to discuss and compare different approaches to the questions in order to **consolidate a strategy** that they feel confident with and can use successfully;

- a familiarisation with the different **question types**, including an understanding of what is meant by ‘show your working’ and ‘show your method’;
- an understanding of how the mark scheme works for different types of questions, including those with a ‘show your working’ or ‘show your method’ box and those worth more than one mark. Children learn how they can get ‘partial credit’ on these questions;
- a more systematic development of their confidence in their ability to answer test questions.

2 REFINING AND HONING THE SKILLS FOR EACH OF THE FOUR RULES OF NUMBER

Analysis of recent test papers suggests that a significant proportion of children, when answering test questions involving the four rules of number, use calculation strategies that they are not comfortable with and do not understand fully. Consequently they are unsuccessful. There is evidence that many of these children **could** have been successful if they had chosen a different method. An effective revision strategy is to review each of the four operations in turn, using a set of different question types for each operation, drawn from the QCA Testbase CD-ROM or elsewhere. For example, the attached revision worksheet concentrates on a variety of subtraction questions.

SUBTRACTION REVISION

Use an appropriate method to answer each of the following question. You can use different methods but you need to be above to explain your method and your reasoning for each question. Remember to check your answers to be confident that they are correct.

1. 74–48
2. 175–81
3. 3000–1997
4. 1025–336
5. 28.34–17.29
6. $150 - ? = 27$
7. Jo bought a box of cards for £6.48 and paid with a £10 note. How much change should she get?
8. Sam has a 3.5 m length of string. He cuts off 1.75 m. How much is left?

When sharing and discussing children's responses to the questions, the aim would be for the children to develop the skills and confidence to:

- use mental methods whenever appropriate, including questions on the written papers;
- examine questions and decide on the most appropriate strategy for each question, recognising that the numbers in the question often help to determine the method they feel most confident in using;
- make estimates and check their answers for reasonableness.

The role of the teacher during such revision sessions is to:

- encourage children to use mental methods as a first resort;
- show children how to record their mental and calculator methods to help with questions that require some explanation or description of the method;
- ensure that children have a secure understanding of place value, which helps them apply their methods successfully;
- enable the children to adopt the strategies they are most secure with by the time of the test;
- monitor children's methods and answers, and help children with very inefficient strategies to refine their methods as far as possible, ensuring that they understand why these are more efficient methods.

3 CHECKING ANSWERS, INCLUDING APPROPRIATE USE OF THE CALCULATOR

Children need to check their methods. It is a useful strategy to appoint a 'checker' when children are working in groups. Checking calculations also needs to be embedded in the teaching so that it becomes 'second nature' for the children. An OHP calculator can be used by the teacher to demonstrate ways of checking calculations undertaken using a calculator. In particular, the teacher can model checking strategies using inverse operations. Discussing different strategies that can be used to solve a problem should provide children with alternatives that they can use to check their own solution. This applies to both mental and written methods. For example, the answer to 'find three-quarters of 360' can be checked by finding one quarter of 360 and subtracting it from 360.

4 RELATING KEY VOCABULARY AND CONTEXT TO MATHEMATICAL OPERATIONS

In order for children to be successful in test questions such as Question 10, Test B, 2001, it is important that children can identify the **key** vocabulary and match it to the correct mathematical operation. It is important to note that the key words alone do not lead to the operation; it is also the context of the question that determines the required operation. A useful strategy is to create a display of key vocabulary and the associated operations around the classroom and then refer to this vocabulary and the context as they appear in test questions. 'How many' is often associated with addition and multiplication. In this question, because of the context, it should be linked to division.

10 

This is the cost to visit the waxworks.

Adults	£8.50
Children	£4.50

On Friday morning 12 adults and 20 children visit the waxworks.

How much do they pay altogether?

Show your method. You may get a mark.

£

Guide books cost £1.50 each.

The waxworks sells £24 worth of guide books.

How many guide books is this?

cm

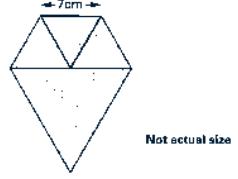
5 INTERPRETING INFORMATION AND USING ANNOTATIONS

It can be very effective to ask children to read sections of a problem and to use focused questions to establish the information that can be obtained from each section. The children should be encouraged to record this information in a way that is meaningful to them, using shorthand notes, a diagram or a flow chart. Evidence from tests suggests that children are reluctant to draw or write on published material such as test questions, apart from in the designated boxes. It is therefore important to encourage children to annotate and draw on test diagrams, graphs and tables, if it helps them to understand and answer the questions. For example, in attempting the question opposite, converting the diagram into seven small triangles would help many children to reach a solution.

20 Lauren has three small equilateral triangles and one large equilateral triangle.

The small triangles have sides of 7 centimetres.

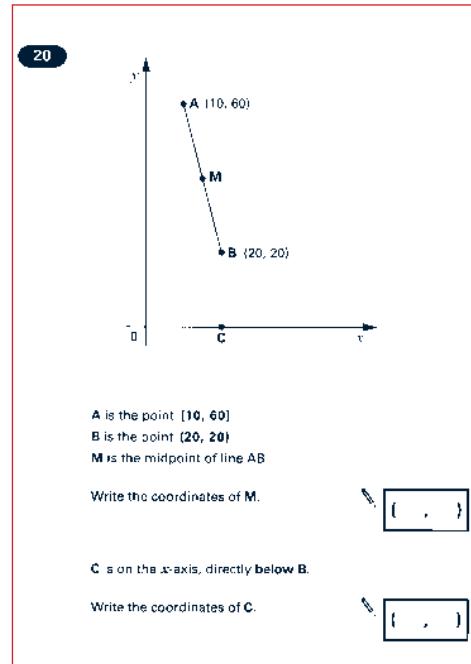
Lauren makes this shape.



Calculate the perimeter of the shape.
Do not use a ruler.

cm

Encourage children to add information to diagrams or tables from the written information provided in the question. For example, questions such as Question 20, Test A, 2001, sometimes include graphs with no scale marked on the axes. Adding information from the question to clarify the scale or to give an indication of the magnitude of the scale often helps children to see how to answer the question.



6 ORDERING NUMBERS

There is evidence that many children need a more secure understanding of the relative size of numbers. Children often have difficulty answering questions that involve the ordering of negative numbers, fractions and decimals, and questions that involve the forming of equivalent fractions. A useful revision strategy is to use visual images, such as a number line, to enable children to see where negative numbers, fractions and decimals fit, and to understand why two representations, fraction or decimal, are equivalent. Remind children that they can draw their own pictures to help them see their way through to a solution.

7 PREPARING FOR THE UNFAMILIAR

Too many children stop working before they reach the end of the test, even though there is still time available for them to attempt the remaining questions. Children need to be encouraged to try questions with which they may not be familiar. If they need to draw any diagrams to help them, they should do so. Emphasise to children that it is better to do that and tackle a question than to sit and think without recording anything.

A useful revision strategy is to change the context of particular problems with the children, establish whether this has any effect on the calculation, and if so, why. Getting children to devise problems for others to answer also helps them to interpret the unfamiliar contexts.

SPRINGBOARD 6

FIRST 11 LESSONS

Lesson Topic	Objectives	What children should be able to do by the end of the lesson
1 Place value	Order a set of decimal numbers and identify the most significant digit when sorting numbers	Order a set of decimals
2 Multiplication and division 1	Identify and use the inverse relationship between multiplication and division	Write down the family of facts when given one fact Solve $17.4 \times \square = 40.02$ by calculating $40.02 \div 17.4$
3 Multiplication and division 2	Express a quotient as a fraction or as a decimal when dividing a whole number by 2, 4, 5 or 10 Represent halves, tenths, and fifths as fractions and decimals	Represent the remainder as a fraction, using the divisor as the denominator
4 Multiplication and division 3	Use informal written methods to support, record or explain multiplications	Use the grid method for multiplying three-digit numbers by two-digit numbers or by numbers to one decimal place
5 Problem solving 1	Identify and use appropriate operations (including combinations of operations) to solve word problems	Identify the key words and select the appropriate calculation
6 Problem solving 2	Develop calculator skills and use a calculator effectively	Solve problems in context using a calculator Interpret a calculator display
7 Fractions, decimals and percentages 1	Order fractions by converting to a common denominator	State the decimal equivalents for tenths and fifths Convert a set of fractions into a set of equivalent fractions with a common denominator
8 Fractions, decimals and percentages 2	Express percentages as simple fractions and simple fractions as percentages	Represent 43% as 0.43 and $\frac{43}{100}$ Convert $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{5}, \frac{1}{10}$ into percentages
9 Fractions, decimals and percentages 3	Calculate simple percentages of whole number quantities	Find a percentage of a quantity by halving and quartering and finding multiples of 10% Find a percentage of a quantity by first finding 1%, then multiplying
10 Fractions, decimals and percentages 4	Use a calculator to convert a fraction to its decimal equivalent and to find a fraction of a quantity	Convert fractions to decimals Calculate a fraction of a number or quantity
11 Transforming shapes	Recognise where a shape will be after a reflection in a mirror line touching the shape at a point	Reflect a shape in a mirror line that is not parallel to a side of the shape

SPRINGBOARD 6 LESSON 1

PLACE VALUE

TOTAL TIME

30
MINUTES

Objective:

- Order a set of decimal numbers and identify the most significant digit when sorting numbers

Vocabulary:

- significant digit

By the end of the lesson children should be able to:

- order a set of decimal numbers.

Resources:

- a class set of place value mats and digit cards
- a set of mixed decimal number cards, up to three decimal places
- Resource Sheet 1.1
- whiteboards and pens

ORAL AND MENTAL STARTER

10
MINUTES

Write on the board 34 052. Get children to say the number in words and discuss the value of different digits.

Q: What is the value of the digit 5?

Get the children to use the place value mat and digit cards to make 34.05 and ask the children to say the number in words.

Q: Why is there a zero in the tenths column?

Repeat this process with 280.67 and 3.004, discussing the meaning of the zeros in each case.

Reinforce this by writing on the board 105, 10.5, 1.05 and 0.105 and discuss the significance of the zeros.

Q: What is 34.05 multiplied by 10?

Demonstrate how to find the answer by moving all the cards on the mat one place to the left.

Ask the children to read the answer and give the answers to 34.05×100 and 34.05×1000 .

Q: What is 34.05 divided by 10?

Demonstrate how to find the answer by moving all the cards on the mat one place to the right.

Ask the children to read the answer, and give the answers to $34.05 \div 100$ and $34.05 \div 1000$.

Q: How would you explain to someone how to multiply a decimal number by 10?**Q: How would you explain to someone how to divide a decimal number by 10?**

Establish the idea that when multiplying a number by 10, the digits shift one place to the left and when dividing a number by 10, they shift to the right.

MAIN TEACHING ACTIVITY

Compare the two numbers 2 and 0.528 by making these numbers on the place value mats.

Q: Which is the bigger number?

Use the place value mat to demonstrate and establish that 2 is bigger because it has 2 units compared with 0.528 that has no units even though it has more digits.

Q: Which is smaller, 2.05 or 2.50?

Demonstrate and emphasise the need to look at the digits after the decimal point.

Q: Does the number of digits affect the size of the number?

Compare 1234 and 999. Ensure that children understand that when comparing whole numbers, the number of digits can be used to compare the relative size of the numbers.

Compare 0.123 and 0.3 and establish that when comparing decimal numbers less than one, the number of digits is not always significant. Emphasise that children need to compare the number of tenths and if necessary the hundredths and thousandths. Compare 21.45 and 21.65, and 7.21 and 7.211.

Q: Can you give me an example where a three-digit decimal number is smaller than a one-digit decimal number?

Refer back to 2 and 0.528.

Ask the children to make the following numbers using the mats:

8.19, 8.09, 8.91 and 8.9.

Q: Which is the most important (significant) digit (units, tenths or hundredths) to look at when ordering these numbers?

Establish that to order the numbers we first compare the units, then the tenths, then the hundredths.

Repeat for 34.7, 34.74, 37.74 and 37.47.

Take the set of decimal cards, shuffle the set, turn over four cards and stick them to the board. The children then order the set of four cards from smallest to largest and record their answer on a whiteboard. Shuffle the cards again and repeat.

PLENARY



Present the children with the following information:

Some children of Spring Hill Primary School competed in a long jump competition. Here are the results:

Sarah	1.4 m
Matthew	1.45 m
Saki	1.38 m
Bob	1.39 m

Q: Who won the competition?

Establish that the digit we need to focus on to start with is the tenths digit as the units digit is the same in each case, then the hundredths digit.

Remember:

- When ordering numbers, identify and compare the most significant digits.
- Don't be fooled into thinking that the more digits there are after the decimal point, the bigger the number.

**LESSON 1 RELATED TEST QUESTION
2000 TEST A (NON-CALCULATOR PAPER)**
11Circle **two** numbers which **add** to make **0.12**

0.1 0.5 0.05 0.7 0.07 0.2

1 mark

MARK SCHEME0.1 0.5 **0.05** 0.7 **0.07** 0.2

Accept alternative indications, e.g. the numbers crossed or underlined.

ANALYSIS OF CHILDREN'S ANSWERS

- A common error, when asked to combine two numbers to make a decimal number to two places, was to combine two numbers with one decimal place, for example, 0.5 and 0.7 to make 0.12.
- Many children did not answer this question.

IMPLICATIONS FOR PLANNING

- Decimal place value should be planned for in its own right, not just in the context of money and measures.
- Equivalent vulgar fractions can aid understanding of addition/subtraction of decimals. For example, viewing 0.12 as $\frac{12}{100}$ would help children to see that 0.05 ($\frac{5}{100}$) and 0.07 ($\frac{7}{100}$) total 0.12.
- A calculator provides a useful resource for children to see the effects of repeatedly adding 0.01 to 0.07.

SPRINGBOARD 6 LESSON 2

MULTIPLICATION AND DIVISION 1

TOTAL TIME**Objective:**

- Identify and use the inverse relationship between multiplication and division

Vocabulary:

- inverse
- factor
- multiple

By the end of the lesson children should be able to:

- write down the family of facts when given one fact;
- solve $17.4 \times \square = 40.02$ by calculating $40.02 \div 17.4$.

Resources:

- whiteboards and pens
- calculators
- OHP calculator
- OHTs 2.1 and 2.2

ORAL AND MENTAL STARTER

Write $8 \times 5 = \square$ on the board.

Establish that the answer is 40 and place it in the box.

Explain that this number sentence is part of a family of four.

Q: What are the other associated number sentences?

Collect the other three number sentences: $5 \times 8 = 40$

$$40 \div 8 = 5$$

$$40 \div 5 = 8.$$

Reinforce that for each number sentence there are usually three others. Work through $5 \times 5 = 25$ to show that in this case there is only one associated number sentence.

Remind the children that 40 is a multiple of 8 and a multiple of 5, and that 8 and 5 are factors of 40.

Write $9 \times 4 = \square$ on the board. Children use whiteboards to show an associated number sentence. Check for all possibilities from the responses.

Write $42 \div 6 = 7$ on the board and obtain the other three number sentences from the children.

Q: What is the product of 6 and 7?

Draw out the meaning of the word 'product'.

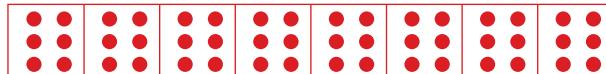
Q: How can we describe the connection between 6, 7 and 42 using the words multiple and factors?

Highlight that 6 is a factor of 42 and 42 is a multiple of 7. When discussing these, refer to the appropriate number sentence and emphasise the vocabulary being used.

MAIN TEACHING ACTIVITY

15
MINUTES

Show OHT 2.1.



Q: How many counters are there in each cell?

How many cells are there?

Q: How many counters are there altogether?

How can we represent this as a number statement?

Collect responses and write $6 \times 8 = 48$ on the board. Remind the children that each number sentence is usually a member of a family of 4.

Write $\square \div 8 = 6$ on the board, and ask the children for the other three number sentences.

$$6 \times 8 = \square \quad 8 \times 6 = \square \quad \square \div 6 = 8$$

Q: Which number sentences can you complete?

Reinforce that knowing 6 multiplied by 8 is 48, means we know that 48 divided by 8 must be 6. Ensure the children can complete all the sentences.

Show OHT 2.2 and ask the 4 questions listed above.

Write $\square \div 4 = 8$ on the board.

Q: What number goes in the empty box?

Using OHT 2.2, emphasise that there are 8 fours, and $8 \times 4 = 32$. So 32 must go in the box. Quickly ask the children to record the other three number sentences on their whiteboards using the empty box as before. Write on the board:

$$4 \times 8 = \square \quad 8 \times 4 = \square \quad \square \div 8 = 4$$

Q: Which number sentences can we complete?

Establish that knowing the multiplication fact $8 \times 4 = 32$ or $4 \times 8 = 32$ is enough. We can use this fact to fill in all the boxes. Say multiplication and division are inverse operations. The inverse operation for division is multiplication, and the inverse operation for multiplication is division.

Write a $\square \div 6.3 = 4$ on the board.

Q: What number goes in the empty box?

Emphasise that there are four 6.3s, so 6.3×4 must go in the box. Work this out using $6 \times 4 = 24$ and $0.3 \times 4 = 1.2$ so $6.3 \times 4 = 25.2$.

PLENARY

Write Question 1, $2.8 \times \square = 10.36$ on the board. Ask the children to write down the other three number sentences using the empty box notation.

Q: Which number sentence can we complete to help us find the number in the box?

Children use a calculator to work out $10.36 \div 2.8$.

Ask children to work out $17.4 \times \square = 40.02$. Discuss their strategies.

Write Question 2, $\square \div 2.5 = 5.4$ on the board. Establish that this can be worked out by calculating 5.4×2.5 . Relate this to earlier examples.

Ask children to work out $\square \div 3.8 = 7.6$. Discuss their strategies.

Q: How can we check our answer?

Establish that we can check the answer to Question 1 by putting the answer into the original number sentence. The calculation would be 2.8×3.7 . For Question 2 the check calculation is $13.5 \div 2.5$. For the other calculations the check calculations are 17.4×2.3 and $28.88 \div 3.8$.

Remember:

- When finding a missing number, it is helpful to write down the other three number sentences and then decide which one to use to find the missing number.
- Always put your answer back in the question and check that it works.

**LESSON 2 RELATED TEST QUESTION
1999 TEST B (CALCULATOR PAPER)**
17

Write in the missing number.



$$950.4 \div \boxed{} = 49.5$$

1 mark

MARK SCHEME

19.2

ANALYSIS OF CHILDREN'S ANSWERS

- On other questions children responded well to subtraction as an inverse of addition. They found the inverses of multiplication and division more difficult at all levels. More children had difficulty with recognising that the inverse of division is multiplication.
- For children who understand inverses, this was a simple question to answer using a calculator. Children who answered the question using trial and improvement methods made more errors than when a single calculation had been carried out.

IMPLICATIONS FOR PLANNING

- To help children understand inverses, it is useful to introduce simpler cases such as $12 \div \square = 3$ and get children to describe this statement in their own words and identify the family of equivalents such as: $12 = 3 \times \square$.
- During a topic on multiplication and division, inverses need to be an integral part of the teaching programme.
- Children need to be taught that when using a calculator it is useful to check answers by using the inverse of the operation in the question.

SPRINGBOARD 6 LESSON 3

MULTIPLICATION AND DIVISION 2

TOTAL TIME**Objectives:**

- Express a quotient as a fraction or as a decimal when dividing a whole number by 2, 4, 5 or 10
- Represent halves, tenths and fifths as fractions and decimals

Vocabulary:

- remainder
- fractional equivalent

By the end of the lesson children should be able to:

- represent the remainder as a fraction, using the divisor as the denominator.

Resources:

- counting stick
- OHT 3.1

ORAL AND MENTAL STARTER

Show the children a counting stick marked 0 to 1.

Q: What steps might we count up in from 0 to 1?

Count up and back in halves, tenths and in steps that the children suggest. Using OHT 3.1, display the fraction number line 0 to 1 marked in tenths. Explain that each of the fractions has an equivalent decimal number.

Q: What are these decimal numbers?

Display the decimal number line 0 to 1 in 0.1s directly underneath the number line marked in tenths. Count up and back in steps of zero point one. Point to 0.3 and ask for the fractional equivalent. Repeat for other numbers, switching between the two number lines. Display the empty number line 0 to 1 directly below the other two number lines.

Q: How can we use either of the top two number lines to mark one half on the blank number line?

Mark $\frac{1}{2}$ on the blank number line, establishing that $\frac{1}{2} = \frac{5}{10} = 0.5$.

Q: How can we use either of the top two number lines to mark one-fifth on the empty number line?

Establish that $\frac{1}{5} = \frac{2}{10} = 0.2$.

Q: What other equivalents for fifths can you tell me?

Call out one of the numbers from any of the three number lines, e.g. $\frac{6}{10}$, and ask for the equivalents 0.6 and $\frac{3}{5}$.

MAIN TEACHING ACTIVITY



Write on the board $21 \div 5$. With the class, establish that this is 4 r 1, and write $21 \div 5 = 4 \text{ r } 1$.

Draw an empty number line on the board. Using the empty number line, jump in steps of 5 to 20. Say that there are 4 fives in 20. Jump one more step of 5 to 25 and say there are more than four fives in 21 but less than five fives. There are four and a bit fives in 21.

Write 21 as $20 + 1$. Point to the 20 and say $20 \div 5 = 4$. Point to the one and ask:

Q: How can we represent 1 divided by 5?

Establish that $1 \div 5$ is $\frac{1}{5}$ and say that $21 \div 5 = 4\frac{1}{5}$.

Record $21 \div 5$ is 4 r 1 or $4\frac{1}{5}$.

Repeat the above process for $23 \div 5$ and $24 \div 5$ and record the answers in the same way to produce:

$23 \div 5$ is 4 r 3 or $4\frac{3}{5}$.

$24 \div 5$ is 4 r 4 or $4\frac{4}{5}$.

Q: What do you think the answer to $22 \div 5$ will be?

Establish that the answer to $22 \div 5$ is 4 r 2 or $4\frac{2}{5}$.

Q: How can we write $\frac{1}{5}$ as a decimal?

Refer to the earlier work and establish that $4\frac{1}{5}$ can be written as $4 + 0.2 = 4.2$.

Repeat for $4\frac{3}{5}$ and $4\frac{4}{5}$.

Write on the board $83 \div 10$. With the class record that this is 8 r 3, and use an empty number line to demonstrate that the answer will be greater than 8 but less than 9.

Write $83 \div 10 = (80 \div 10) + (3 \div 10)$.

Point to $(80 \div 10)$. Ask for the answer. Point to $(3 \div 10)$ and ask for the answer.

Q: How could we rewrite the question $74 \div 10$?

Establish that $74 \div 10$ can be written as $(70 \div 10) + (4 \div 10) = 7 + \frac{4}{10}$.

Again emphasise that the remainder of four is divided by 10 to give $\frac{4}{10}$.

Q: How can we write $7\frac{4}{10}$ as a decimal?

Establish that $7\frac{4}{10} = 7 \text{ r } 4 = 7.4$.

In pairs, children choose numbers to divide by 10 and represent their answers in fraction and decimal form. Collect their responses and discuss their methods.

PLENARY

Remind the children that when dividing by 10 the remainder is divided by 10 and can be written as a fraction in tenths; when dividing by 5 it can be written in fifths. For each case there are decimal equivalents.

Q: How can we write $26 \div 4$, $33 \div 4$, $43 \div 4$?

Establish that the remainders will be divided by 4 and can be written as quarters. Remind them of the decimal equivalents using the number lines.

Ask the children to divide 29 by 2, 4, 5 and 10. Note their answers on the board, recording the remainders and the fractional and decimal equivalents.

Remember:

- When dividing by 10, the remainders will be tenths, a remainder of 3 will be represented as $\frac{3}{10}$ or 0.3.
- When dividing by 2 the only remainder can be 1, represented as $\frac{1}{2}$ or 0.5.
- Dividing by 4 gives quarters, and by 5 gives fifths.

**LESSON 3 RELATED TEST QUESTION
2001 TEST A (NON-CALCULATOR PAPER)**
11**Calculate $847 \div 7$** 

1 mark

MARK SCHEME

121

1 mark

ANALYSIS OF CHILDREN'S ANSWERS

- Almost half of the children who achieved Level 3 found this question difficult, a significant number not giving an answer at all.
- Most children used the short division format but many of those at Levels 3 and 4 were unable to recognise when they had produced an incorrect answer. Many incorrect answers arose because children did not use their understanding of place value during the calculation when carrying remainders.

IMPLICATIONS FOR PLANNING

- Children should be encouraged to use a simple 'chunking' method:

$$\begin{array}{r}
 7 \overline{)847} \\
 7 \\
 \hline
 14 \\
 14 \\
 \hline
 7 \\
 7 \\
 \hline
 847 = (121 \times 7)
 \end{array}$$

This will help them to identify remainders and to set them out as hundreds, tens, or units.

- Children need to have some understanding of the scale of the answer in order to check their solution, and be taught to carry out an estimate such as $700 \div 7$ or $800 \div 10$.

- When planning division questions, always include some where there are remainders to ensure children remember how to cope with them.

SPRINGBOARD 6 LESSON 4

MULTIPLICATION AND DIVISION 3

TOTAL TIME**Objective:**

- Use informal written methods to support, record or explain multiplications

Vocabulary:

- factorise

By the end of the lesson children should be able to:

- use the grid method for multiplying three-digit numbers by two-digit numbers or by numbers to one decimal place.

Resources:

- whiteboards
- Resource Sheet 4.1

ORAL AND MENTAL STARTER

Write $22 \times 10 = 220$ on the board.

Emphasise that when you multiply a number by 10, the digits move one place to the left and the 0 is put in the units place as a place holder. Explain that 'add a 0' is not acceptable as this does not work when multiplying decimals. Work through $2.2 \times 10 = 22$ to confirm this.

Write on the board $22 \times 20 = \square$.

Q: Can you work this out and explain how you did it?

Demonstrate that $22 \times 20 = 22 \times 2 \times 10$.

Point out that the 20 has been factorised as $20 = 2 \times 10$.

Write on the board $22 \times 60 = \square$.

Q: What can we multiply 22 by this time?

Write $22 \times 60 = 22 \times 6 \times 10$.

Again highlight that the 60 has been factorised as $60 = 6 \times 10$.

Ask children to work out the answers to 22×50 , 22×30 and 22×70 , presenting each of their answers in turn on their whiteboards.

Write 6×0.7 and 6×7 below one another on the board.

Q: Which one can you answer?

Establish that 7 is ten times bigger than 0.7 and that 0.7 is 10 times smaller than 7.

Q: How can we use the answer to 6×7 to work out 6×0.7 ?

Establish that we need to divide the answer to 6×7 by 10.

Write $6 \times 7 = 42$, $42 \div 10 = 4.2$

so $6 \times 0.7 = 4.2$

Q: Can you work out 6×0.07 ?

Establish that this time 42 has to be divided by 100, so

$6 \times 0.07 = 42 \div 100 = 0.42$.

Write on the board 8×3 , 8×0.3 and 8×0.03 . Ask the children to show the answers on their whiteboards.

Repeat for other examples, emphasising how one calculation can be used to obtain the other answers.

MAIN TEACHING ACTIVITY

Write 37×234 on the board.

Q: How can we estimate the answer?

Take children's estimates and explanations. Include the estimates 40×200 and 40×250 and work out the answers mentally.

Q: Which do you think is the better estimate?**Q: Why?**

Emphasise that for $40 \times 250 = 10000$, each number has been made bigger, so the answer to 37×234 will be much smaller than 10000.

Work through the left-hand grid on the board and demonstrate, using partitioning, that it represents the calculation 37×234 . Complete the grid with the children to get the right-hand grid.

×	200	30	4
30			
7			

×	200	30	4
30	6000	900	120
7	1400	210	28
	7400	1110	148

Ask questions such as:

Q: Which boxes did we fill in quickly?

Q: How can we get the answer to 37×234 from the grid?

Establish that we need to add the numbers in the columns to get the bottom row. Use column addition to add 7400, 1110 and 148. Record the answer $37 \times 234 = 8658$ and check against the estimates.

Say that you want to use the grid method to calculate 37×23.4 . Discuss the partitioning as you set out the grid on the board. Estimate the answer.

×	20	3	0.4
30			
7			

Q: What is the answer to 30×0.4 ?

Draw on the lesson's starter. Remind children that:

$$30 \times 0.4 = 3 \times 10 \times 0.4 = 3 \times 4$$

so the answer is 12.

Q: What is the answer to 7×0.4 ?

Remind them that $7 \times 4 = 28$ and as 0.4 is 10 times smaller than 4, so $7 \times 0.4 = 28 \div 10 = 2.8$.

Complete the grid and record the answer $37 \times 23.4 = 865.8$. Ask children if they can see any connection with the answer to 37×234 .

Work through another example of the type 236×4.6 . Highlight the key calculations 200×0.6 and 30×0.6 .

PLENARY

Set the children 24×51.7 . Make sure children can explain the key steps in the calculations.



Remember:

- When multiplying by a decimal such as 0.4, first multiply by 4 and then divide by 10.
- When multiplying by a decimal such as 0.04, first multiply by 4 and then divide by 100.

**LESSON 4 RELATED TEST QUESTION
1999 TEST A (NON-CALCULATOR PAPER)**
12Calculate **549 × 6**

1 mark

MARK SCHEME

3294

ANALYSIS OF CHILDREN'S ANSWERS

- Children working at Levels 3 and 4 had difficulties multiplying a three-digit number by a one-digit number. Most children tackled the calculation using the conventional vertical method for multiplication.
- Children had difficulty keeping track of the numbers they multiplied when using a compact form of multiplication.

IMPLICATIONS FOR PLANNING

- Children should always make an estimate to check whether their answer appears sensible.
- Time should be allocated to ensuring that all pupils have one written method for undertaking multiplication calculations that they can use with confidence.
- Children need to keep revisiting multiplication to rehearse and hone their mental and written methods.

SPRINGBOARD 6 LESSON 5

PROBLEM SOLVING 1

TOTAL TIME**Objective:**

- Identify and use appropriate operations (including combinations of operations) to solve word problems

Vocabulary:

- operation
- multi step

By the end of the lesson children should be able to:

- identify the key words and select the appropriate calculation to solve problems such as: 'There is space in the multi-storey car park for 17 rows of 30 cars on each of 4 floors. How many cars can park?'

Resources:

- Resource Sheet 5.1

ORAL AND MENTAL STARTER

From Resource Sheet 5.1 read Question 1 aloud and ask the children to say what they think the question is asking. Draw the following table on the board and complete the table with the children.

Question number	Information from question	Calculation
1	£2.50 for 1 hour. Cost for 3 hours?	$3 \times £2.50 = £7.50$

Use the recorded information and the children's prompts to find the solution. Get children to put the answer in a sentence such as 'The boat costs £7.50.'

Repeat with other problems. Ensure that the children are able to identify key 'bits' of information in the question.

Questions 5 and 6 include irrelevant information to ensure that the children listen carefully.

MAIN TEACHING ACTIVITY

Show the following statement:
'Guidebooks cost £1.50 each.'

Q: What questions can we ask using the above information?

Explore a range of questions and incorporate key words and phrases such as:

'How many ...';
'Increase in price ...';
'How much change from ...'.

Get the children to identify the calculation they would need to make to answer these questions.

Now include the statement:
'A shop sells £24 worth of guidebooks.'

Q: What questions could we ask now?

Encourage questions such as: 'How many guidebooks were sold?'
'How many more guidebooks would need to be sold for total sales of £33?'
'If the shop sold another 15 guidebooks, how much money would the shop take?'

Get the children to identify the calculation they would need to make to answer these questions.

Children work in pairs to develop a question based on the above information. They then work out the solution and exchange their question with another pair, who answer the question and check it against the solution.

Collect and discuss children's questions and answers.

PLENARY

Select another example from Resource Sheet 5.1. Work through it to exemplify the strategy set out below.

1. Read the problem carefully (twice or more).
2. Identify key words to help you think about what the problem is asking.
3. Put the problem into your own words or use pictures to help you understand the question.
4. Decide what information you need and what operations you will use.
5. Record your calculation and your solution.

6. Check you are using the correct units for measures and money.
7. Read the question again and check the reasonableness of your answer, making an estimate to help you.

Q: What are some of the important words to look for when reading through a problem?

Collect and note these down on the board. For each, give an example using small numbers, and ask the children for the calculation they need to carry out.

Remember:

Look for the key words in the questions to help you to decide what operations you have to use and the calculations you need to carry out.

LESSON 5 RELATED TEST QUESTION 2000 TEST A (NON-CALCULATOR PAPER)

7



Some children go camping.
It costs £2.20 for each child to camp each night.
They go for 6 nights.

How much will each child have to pay for the 6 nights?

Show your working. You may get a mark.

£

2 marks

There are 70 children.
Each tent takes up to 6 children.

What is the least number of tents they will need?

Show your working. You may get a mark.

tents

2 marks

ANALYSIS OF CHILDREN'S ANSWERS

- Children working at Levels 3 and 4 found it difficult to know what calculation was needed.
- Many children failed to interpret questions that required them to put the answer into a context.
- They did not recognise that the remainder in a context needed to be included in the calculation, and that this might require some rounding.
- Division questions in context are some of the most difficult for children to complete. One in six children could not identify the correct operation to use.

MARK SCHEME

- (a) Award **TWO** marks for the correct answer of £13.20.

Accept £13.20p **OR** £13.20 **OR** £13.20p **OR** £13.20p. **Do not** accept for **TWO** marks incorrect representations of money values, e.g. £1320 **OR** £13.2 **OR** £1320p.

If the answer is incorrect, award **ONE** mark for evidence of appropriate working, e.g. $\frac{2.20}{6} \times 6 =$ wrong answer.

*Calculation must be performed for the award of the mark. Accept £13.2 **OR** £1320p **OR** £1320 – for **ONE** mark.*

Up to 2 marks

- (b) Award **TWO** marks for the correct answer of 12.

If the answer is incorrect, award **ONE** mark for evidence of appropriate working, e.g. $70 \div 6 =$ answer other than 12.

Accept as appropriate working $11\frac{2}{3}$ **OR** unrounded or inappropriately rounded calculations of $70 \div 6$, e.g. 11 **OR** 11 remainder 4 **OR** 11.6 even if no method is shown **OR** the two consecutive multiples of 6 which straddle 70, i.e. 66 and 72.

Up to 2 marks

IMPLICATIONS FOR PLANNING

- The vocabulary used in a problem-solving lesson needs to be identified and discussed during the plenary.
- Children need to be presented with different contexts which require them to round an answer with a remainder either up or down.
- Problems involving division need to be given greater attention. Children should be given the opportunity to discuss strategies for solving problems requiring the operation of division.
- Children need to be given strategies to solve two-stage problems leading to multi-stage calculations.

SPRINGBOARD 6 LESSON 6

PROBLEM SOLVING 2

TOTAL TIME**Objective:**

- Develop calculator skills and use a calculator effectively

Vocabulary:

- round up
- round down

By the end of the lesson children should be able to:

- solve problems in context using a calculator;
- interpret a calculator display when solving problems such as – '196 children and 15 adults went on a school trip. Buses seat 57 people. How many buses are needed?'

Resources:

- Resource Sheets 6.1 and 6.2

ORAL AND MENTAL STARTER

Give the children Resource Sheet 6.1.

In pairs, ask them to sort the questions into three sets.

Q: Which questions can you do in your head?

Q: Which questions definitely require a calculator?

Q: Which questions are you not sure you can do mentally?

Emphasise that 'mentally' can include jottings for working out.

Q: Which questions would we definitely answer mentally?

Discuss the questions the children suggest and ask for their mental methods. Ask the children if they needed to make any jottings and to explain their methods.

Q: Which questions would we definitely answer using a calculator?

Discuss the questions and ask the children why they need a calculator.

Discuss any remaining questions in turn to establish whether any of the children can offer a mental method. Identify the questions that you would want the children to answer using mental strategies, e.g. 15% of £4.80. Emphasise those mental strategies that are appropriate for such questions and the role of jottings.

MAIN TEACHING ACTIVITY

Refer to Question 4 on Resource Sheet 6.1. Read it through with the children.

Q: What information is in the question?

Get the children to identify the cost of the tickets and discuss the separate cost for the 7 adults and the cost for the 17 children.

Q: What calculation do we need, to find the total cost for the 7 adults?

Discuss the operation required and record: $\text{£}3.60 \times 7$. Establish that this is less than £28 ($\text{£}4 \times 7$).

Q: What calculation do we need, to find the total cost for the 17 children?

Discuss the operation required and record: $85\text{p} \times 17$ children. Establish that this is less than £17 ($\text{£}1 \times 17$).

Q: What do we need to do now to answer the question?

Get the children to describe the steps, by adding the cost for the adults to the cost for the children and that the total will be less than £45 ($\text{£}28 + \text{£}17$).

Now ask the children to use their calculators to calculate the exact cost for 7 adults.

Q: What is the cost for 7 adults?

Ensure children can interpret 25.2 on the calculator as £25.20. Record the answer.

Q: What would 25.7 and 25.07 on the display mean if the answers were in pounds?

Check that children can interpret these correctly.

Q: What is the cost for 17 children?

If the children produce a display of 1445, establish that this is a number of pence and that to get the answer in pounds they needed to enter 85p as 0.85.

Alternatively, divide 1445 by 100 to change the pence into pounds. Record the answer.

Q: What is the total cost for the tickets?

Record £25.20 + £14.45 and work through the addition with the children.

Compare the answer £39.65 to the estimate of £45.

Give the children Resource Sheet 6.2. In pairs, get the children to answer each question with a sentence and show their working to each part.

Work through the question on Resource Sheet 6.2, with the children using the prompts on the sheet to help them. Discuss what they record and what methods they use. Establish that two steps are required to solve it.

PLENARY



Present the following two questions on the board.

'300 children are going on a trip by coach. Each coach can hold 45 children. How many coaches are needed?'

'Sarah has saved £48. How many posters can she buy for £3.75 each?'

Ask the children how they would solve these problems, and record their suggested calculations on the board.

Q: What estimates could we make for each question?

Discuss possible estimates such as $300 \div 50 = 6$ coaches and $48 \div 4 = 12$ posters. In each case discuss whether the children think the estimate is too small or too big.

Get the children to carry out the calculations using a calculator:

$300 \div 45 = 6.6666666$ coaches; $48 \div 3.75 = 12.8$ posters. Discuss each in turn.

Q: What does 6.6666666 coaches mean?

Establish that the answer means 6 coaches are not enough by calculating 6×45 . Explain that we need to round up.

Q: Does 12.8 posters mean that Sarah can buy 13 posters?

Establish that Sarah does not have enough money to buy 13 posters by calculating $13 \times £3.75$. Explain that we need to round down.

Remember:

Always re-read the question to decide if you round the answer on the display up or down. Always check you are right by doing another calculation.

LESSON 6 RELATED TEST QUESTION 2000 TEST B (CALCULATOR PAPER)

9



Chris saves 50p coins.

He has saved 45 of them.

How much money has Chris saved?

1 mark

Michelle has saved £8.40 in 20p coins.

How many 20p coins does Michelle have?

Show your method. You may get a mark.

2 marks

MARK SCHEME(a) £22.50 **OR** 2250pAccept £22.50p **OR** 22.50 **OR** 2250 **OR**
22.50. **Do not** accept £2250 **OR** £22.5.

1 mark

(b) Award **TWO** marks for the correct answer of 42.If the answer is incorrect, award **ONE** mark for evidence of appropriate method, e.g.

$$840 \div 20 \text{ } \mathbf{OR} \text{ } 8.4 \div 0.2.$$

Accept for **ONE** mark, £42 **OR** 42p as evidence of an appropriate method.

Answer need not be obtained for the award of the mark.

No method mark is awarded for $8.40 \div 20$ alone.

Up to 2 marks

ANALYSIS OF CHILDREN'S ANSWERS

- Children think that 'show your method' means that they must not use the calculator.
- Children demonstrated that they could use the calculator for calculating amounts but then misinterpreted the answer displayed on the calculator.
- There were children who used efficient written methods rather than an appropriate calculator method.

IMPLICATIONS FOR PLANNING

- In lessons where calculators are available, children should be encouraged to record those calculations they carried out.
- Oral and mental starters should include calculator-based activities where children discuss their methods and solutions.
- Calculator lessons should include time for children to interpret calculator displays, for example, for unit conversions, time and money calculations.
- Lessons should be planned to compare mental, written or calculator methods, and include teaching on the effective and efficient use of a calculator, in particular emphasising how and when it should be used.

SPRINGBOARD 6 LESSON 7

FRACTIONS, DECIMALS AND PERCENTAGES 1

TOTAL TIME

30
MINUTES

Objective:

- Order fractions by converting to a common denominator

Vocabulary:

- equivalent
- numerator
- common denominator

By the end of the lesson children should be able to:

- state the decimal equivalents for tenths and fifths;
- convert a set of fractions into a set of equivalent fractions with a common denominator.

Resources:

- counting stick marked 0 to 1
- OHT 3.1
- Resource Sheet 7.1
- OHT 7.2

5
MINUTES

ORAL AND MENTAL STARTER

Using a counting stick marked 0 to 1, get the children to count up in tenths and fifths. Stop at four fifths and ask for the equivalent fraction in tenths.

Q: Which is bigger, $\frac{3}{5}$ or $\frac{7}{10}$?

Using OHT 3.1, display the first number line 0 to 1 marked in tenths. Point to one-tenth.

Q: How would you write one-tenth as a decimal?

Remind children that one tenth can be represented as 0.1. Using OHT 3.1 now show the second number line marked in steps of 0.1, and count in steps of 0.1. Establish that $\frac{1}{5} = \frac{2}{10} = 0.2$. Count forwards and backwards in steps of 0.2, stopping at various points and asking for the fractional equivalent in tenths and fifths where appropriate.

MAIN TEACHING ACTIVITY**Q: Which fractions do you know are equivalent?**

Give out Resource Sheet 7.1. Ask children to look at the patterns in the rows of numbers.

Write the following fractions on the board:

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{6}{10} \quad \underline{\hspace{1cm}}$$

Ask the children for the missing numbers.

Repeat with the following fractions and discuss how the children use Resource Sheet 7.1 to form the missing fractions.

$$\frac{1}{5} \quad \frac{2}{10} \quad \frac{3}{15} \quad \underline{\hspace{1cm}} \quad \frac{5}{\underline{\hspace{1cm}}} \quad \frac{6}{\underline{\hspace{1cm}}}$$

Explain that the rows of numbers are multiples. Use Resource Sheet 7.1 to construct other lists of equivalent fractions.

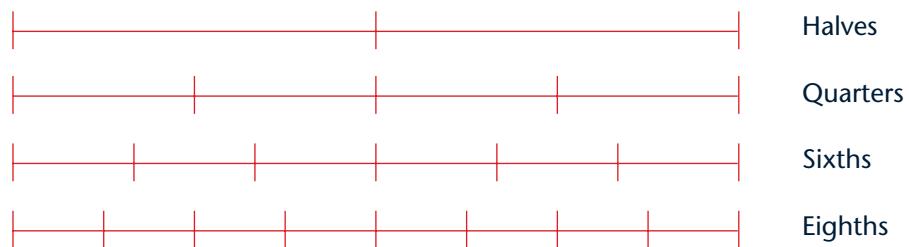
Emphasise that the fractions are produced by multiplying the numerator and denominator of the first fraction by 2, then by 3, then 4, and so on.

Using a 0 to 1 number line, explain that another way of thinking about this is that multiplying the denominator increases the number of intervals or steps. Multiplying by 2 changes fifths to tenths, so $\frac{1}{5}$ is equivalent to 2 tenths.

Refer to the first list of fractions and ask:

Q: Which of these fractions are equivalent?

Show OHT 7.2. On the first number line, divide into 2 and identify $\frac{1}{2}$. On the second divide into 4 and identify $\frac{2}{4}$, on the third divide into 6 and identify $\frac{3}{6}$, and on the fourth divide into 8 and identify $\frac{4}{8}$. Discuss the resulting picture and the equivalents.



With the children, record this information in the following table:

Fractions	Equivalents	Shared denominator
$\frac{1}{2}$	$\frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14}$	$\frac{1}{2} = \frac{5}{10}$
$\frac{1}{5}$	$\frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{5}{25}$	$\frac{1}{5} = \frac{2}{10}$

Circle the fractions in each row which share the same denominator and fill in the end column. Ask children to continue the equivalents for $\frac{1}{2}$ up to $\frac{10}{20}$ and add $\frac{10}{20}$ to the third column. Similarly, for $\frac{1}{5}$, add $\frac{4}{20}$ to the third column.

Q: Which is the bigger fraction, $\frac{1}{2}$ or $\frac{1}{5}$?

Get the children to compare the equivalents. Say that we have answered the question by being able to change $\frac{1}{2}$ and $\frac{1}{5}$ into fractions whose denominators are the same. They share a common denominator.

Write $\frac{3}{4}$ and $\frac{5}{6}$ on the board and get the children to work out sets of equivalent fractions for each fraction to establish which is the smaller fraction.

PLENARY



Write $\frac{1}{3}, \frac{1}{6}, \frac{1}{4}$ on the board.

Q: How can we put these fractions in order?

Establish that we need to write each fraction with the same (common) denominator.

Write on the board $\frac{1}{3} \quad \frac{2}{6} \quad \frac{3}{9} \dots \dots$

Invite the children to continue the list using Resource Sheet 7.1.

Repeat for $\frac{1}{6}$ and $\frac{1}{4}$.

Ask a child to circle a fraction from each list of fractions that have the same or common denominator.

Q: What is the connection between the original denominators and the common denominator?

Emphasise the need to multiply the numerators and denominators. Get the children to use their knowledge of multiplication tables to identify the common denominator.

Remember:

You can convert any fraction into another equivalent fraction by multiplying the numerator and denominator by the same number.

LESSON 7 RELATED TEST QUESTION
2001 TEST A (NON-CALCULATOR PAPER)

19

Complete these fractions to make each equivalent to $\frac{3}{5}$



10

15

12

1 mark

MARK SCHEME

Fractions completed as shown below:

$$\begin{array}{ccc} \boxed{9} & \boxed{6} & \boxed{12} \\ \hline 15 & 10 & 20 \end{array}$$

All three fractions must be correct for the award of the mark.

ANALYSIS OF CHILDREN'S ANSWERS

- Nearly 90% of children at Level 5 completed this successfully but the majority of children at Levels 3 and 4 were not able to identify all three fractions correctly.
- A common error was for children to write 3s and 5s in the boxes to try to make fractions equivalent using the digits in the original fraction $\frac{3}{5}$.

IMPLICATIONS FOR PLANNING

- Lessons should be planned to include making and using 'fraction families' for the common fractions.
- There should be planned teaching activities using an enlarged number line to show children how to locate fractions and to understand that larger numerators and denominators do not necessarily change the value of the fraction.
- Oral and mental starters should include counting activities that involve children in identifying equivalent fractions.

SPRINGBOARD 6 LESSON 8

FRACTIONS, DECIMALS AND PERCENTAGES 2

TOTAL TIME**Objective:**

- Express percentages as simple fractions and simple fractions as percentages

Vocabulary:

- equivalent
- denominator

By the end of the lesson children should be able to:

- represent 43% as 0.43 and $\frac{43}{100}$;
- convert $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{10}$ into percentages.

Resources:

- Resource Sheet 8.1
- Resource Sheet 8.2
- OHT 3.1

ORAL AND MENTAL STARTER

Remind the children that percentage means per 100.

Establish that 100% means 100 per 100 or 1 whole.

Explain that 50% means 50 per 100 and can be written as 50/100 as in m/s (metres per second), and $\frac{50}{100}$ or $\frac{1}{2}$.

Q: What is 25% in hundredths?**Q: What is 75% in hundredths?**

Remind the children that they should know that $25\% = \frac{25}{100} = \frac{1}{4} = 0.25$, and that $75\% = \frac{75}{100} = \frac{3}{4} = 0.75$.

Q: How many hundredths is 10%?

Establish that $10\% = \frac{10}{100}$ and this can be written as 0.10 or 0.1.

Q: How many hundredths is 20%?

Establish that $20\% = \frac{20}{100}$ and can be written as 0.20 or 0.2.

Q: How many hundredths is 30%?

Establish that $30\% = \frac{30}{100}$ and can be written as 0.30 or 0.3.

MAIN TEACHING ACTIVITY

Display OHT 3.1 with the four number lines directly below each other. Explain that these represent:

- 0 to 1 fraction line
- 0 to 1 decimal line
- 0 to 1 empty line
- 0 to 100% percentage line

Remind children that 100% represents 100 per 100 or the whole, so it is equivalent to 1.

Invite the children to locate $\frac{1}{2}$, $\frac{1}{10}$, $\frac{3}{10}$ and $\frac{7}{10}$.

Draw a straight line from each fraction on the fraction line to the equivalent percentage on the percentage line. The line will pass through the decimal number line.

Using the percentage line, invite children to locate the percentage equivalents of $\frac{1}{2}$, $\frac{1}{10}$, $\frac{3}{10}$ and $\frac{7}{10}$.

Ask children to identify the midpoint of the decimal number line.

Establish that the number is 0.5 and remind them it can be written as $\frac{50}{100}$ by referring to the same position on the percentage line.

Repeat for the points on the decimal number line connecting $\frac{1}{10}$ to 10%, $\frac{3}{10}$ to 30% and $\frac{7}{10}$ to 70%.

Ask the children where to mark $\frac{1}{4}$ on the fraction line.

Establish that this mark is halfway between 0 and $\frac{1}{2}$.

Draw a straight, vertical line from $\frac{1}{4}$ to the percentage line.

Q: What is $\frac{1}{4}$ as a percentage?

Get the children to confirm that this is 25%.

Q: How can we use the same method to find $\frac{3}{5}$ as a percentage?

Ask the children to convert $\frac{3}{5}$ to tenths, and draw the vertical line to the percentage line from $\frac{6}{10}$, to establish that $\frac{3}{5} = \frac{6}{10} = 60\%$. Repeat for $\frac{2}{5}$ and $\frac{4}{5}$.

Explain that it is sometimes easier to convert fractions to percentages rather than to decimals and use what they know to work out other percentages,
 e.g. $\frac{1}{2} = 50\%$ so $\frac{1}{4} = 25\%$, $\frac{1}{8} = 12.5\%$ and $\frac{1}{16} = 6.25\%$;
 e.g. $\frac{1}{10} = 10\%$ so $\frac{3}{10} = 30\%$, $\frac{7}{10} = 70\%$.

PLENARY

Hold up a card from the shuffled set of fraction and percentage cards from Resource Sheets 8.1 and 8.2. Invite children to give the fraction or percentage equivalent. Ask the children to explain how they know the two are equivalent. Shuffle and repeat.

Q: What is 43% as a fraction?

Establish that the answer is $\frac{43}{100}$ and remind them that any percentage can be written as a fraction with a denominator of 100.

Q: What is 33% as a fraction?

Invite children to write the answer $\frac{33}{100}$ on the board. Ask them to find 3×33 . Draw out that $3 \times 33\% = 99\%$ which is nearly 100% so 33% is almost $\frac{1}{3}$.

Q: What is $\frac{2}{3}$ as a percentage?

Establish by doubling that the answer is about 66% or 67% and discuss why both answers are reasonable estimates.

Remember:

- Percent means per hundred and percentages such as 45%, 63% can be written as $\frac{45}{100}$, $\frac{63}{100}$.
- Knowing that $\frac{1}{2} = 50\%$, means that you can work out that $\frac{1}{4} = 25\%$ and $\frac{1}{8} = 12.5\%$.
- Knowing that $\frac{1}{10} = 10\%$ means that you can work out that $\frac{3}{10} = 30\%$.
- Knowing that $\frac{1}{3}$ is about 33% means that you can work out that $\frac{2}{3}$ is about 66%.

LESSON 8 RELATED TEST QUESTION
1999 MENTAL ARITHMETIC TEST

Put a ring around the fraction on your answer sheet which is equivalent to 40%.

13

- $\frac{1}{40}$ $\frac{40}{60}$ $\frac{1}{4}$ $\frac{4}{10}$ $\frac{1}{400}$

1 mark

MARK SCHEME**13**

- $\frac{1}{40}$ $\frac{40}{60}$ $\frac{1}{4}$ $\frac{4}{10}$ $\frac{1}{400}$

**ANALYSIS OF CHILDREN'S ANSWERS**

- Many children do not answer questions with decimals, fractions or percentages.
- The relationship between percentages and fractions was poorly interpreted by children at all levels. The common response to this question was $\frac{1}{40}$.

IMPLICATIONS FOR PLANNING

- Percentages should be discussed by referring to the number of parts per hundred. Children should be taught that 100% represents 100 per 100 or a whole, and equivalent fractions developed through later discussions about hundredths, tenths and special cases.
- The relationship between fractions, decimals and percentages should be a regular feature of oral and mental starters, for example, by using a counting stick to establish equivalents.

SPRINGBOARD 6 LESSON 9

FRACTIONS, DECIMALS AND PERCENTAGES 3

TOTAL TIME**Objective:**

- Calculate simple percentages of whole number quantities

Vocabulary:

- hundredths
- percentage
- equivalent

By the end of the lesson children should be able to:

- find a percentage of a quantity by halving and quartering and finding multiples of 10%;
- find a percentage of a quantity by first finding 1%, then multiplying.

Resources:

- whiteboards
- OHP calculator
- calculators

ORAL AND MENTAL STARTER

Remind children that $\frac{1}{2}$ and 50% are equivalent.

Children use whiteboards to show the answers to the following questions:
Show me 50% of 48, 72, 140, 460, 8000 and 89.

Q: If we know 50% of something, how can we find 25%?

Emphasise the halving of 50%. Ask children to show 25% of 80, 64, 72, 140, 460 and 8000.

Ask the children to think of a multiple of 10 that is less than 1000. Say that you will find 10% of their number very quickly. Take numbers from children and give the 10%.

Q: How am I able to find the answer so quickly?

Discuss how to find 10% of any quantity including finding 10% of numbers less than 100, e.g. 35, 46.

Q: If we know 10% how can we work out 15%?

Discuss how the children can find 10% and then half of 10% to get 5%. Work through finding 15% of 300.

Using whiteboards ask children to find 15% of 400, 60, and 120.

MAIN TEACHING ACTIVITY



Q: How can we find 1% of a quantity?

Ask children what 1% means and how $\frac{1}{100}$ helps them to find 1%.

Ask children to find 1% of the following quantities:

200 km, 300 g, £680, 3500 cl.

Confirm the answers by demonstrating the calculation with an OHP calculator.

Discuss each of the answers and highlight that the answer must include the units.

Record the results on the board.

1% of 200 km = 2 km, 1% of 300 g = 3 g, 1% of £680 = £6.80,

1% of 3500 cl = 35 cl.

Using the above information ask the children to find the answers to the following:

4% of 200 km, 3% of 300 g, 2% of £680, 4% of 3500 cl.

Ask the children how they used the answers on the board to calculate these percentages. Draw out that the method is to find 1% by dividing by 100 and then multiplying by the percentage. Say that this always works. Give out calculators and work through two of the questions using this method.

Q: How can we find 27% of £387 using a calculator?

With the children, divide 387 by 100 to find 1% and then multiply by 27 to find 27%.

Record as follows:

$$1\% \text{ of } £387 = £3.87, \quad 27\% \text{ of } £387 = £3.87 \times 27 = £104.49$$

Ask the children to find 17% of £45 and 4% of £66.50.

PLENARY



Q: What strategy would you use to find 50%, 25%, 10%, 20%, 60% of a given amount?

Encourage the children to use halving, doubling and quartering, dividing by 10 to

find 10% and then multiplying by 6 to find 60%.

To find 60% compare different methods: $6 \times 10\%$ and $50\% + 10\%$.

Present the following question:

'79% of children at a school thought that the school dinners were good. If there are 340 children in the school, how many children thought that the dinners were good?'

Q: What calculation is required to answer the question?

Get the children to work out the answer. Using an OHT calculator, draw out the fact that to find 79% of 340 the calculation can be written as $(340 \div 100) \times 79$.

Q: How do we interpret the answer 268.6 children?

Discuss the fact that we can only have a whole number of children. Establish that a sensible answer would be 269, as the percentage was only approximate.

Q: Why is 269 a more accurate answer than 268?

Remind the children when they need to round up or down.

Remember:

When finding a percentage of a quantity:

- first find 1%;
- then multiply this answer by the percentage;
- decide if you need to round up or down;
- include the units.

LESSON 9 RELATED TEST QUESTION
2000 MENTAL ARITHMETIC TEST
13

What is two per cent of three hundred?

1 mark

MARK SCHEME

6

ANALYSIS OF CHILDREN'S ANSWERS

- Many children did not give an answer to questions related to decimals, fractions or percentages.
- When asked to calculate a percentage of a number above one hundred the response rate drops. It was very low for this question and the common errors included dividing 300 by 2 or, in fewer cases, subtracting 2 from 300.

IMPLICATIONS FOR PLANNING

- Plan to end plenaries with some quick-fire questions involving percentages, fractions and decimals.
- When setting work involving percentages always include numbers bigger than 100.

24

Calculate 15% of 460



MARK SCHEME

69

1 mark

ANALYSIS OF CHILDREN'S ANSWERS

- Almost half of children working at Level 3 did not give a response; they had no strategies for starting the question.
- Most of the children working at Level 5 succeeded with a more formal written method. Others used a less formal strategy, finding 10% and then adding half of it to give 15%.

IMPLICATIONS FOR PLANNING

- Children should be taught to use familiar skills such as dividing by 10, halving and doubling when tackling percentage questions.
- Children should be taught to break up a percentage into 10%, 5%, 1% and use what they know to rebuild the percentage.

SPRINGBOARD 6 LESSON 10

FRACTIONS, DECIMALS AND PERCENTAGES 4

TOTAL TIME**Objective:**

- Use a calculator to convert a fraction to its decimal equivalent and to find a fraction of a quantity

Vocabulary:

- fraction
- decimal fraction
- numerator
- denominator

By the end of the lesson children should be able to:

- convert fractions to decimals;
- calculate a fraction of a number or quantity.

Resources:

- calculators
- whiteboards

ORAL AND MENTAL STARTER

Write some fractions on the board, e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{10}$, $\frac{3}{8}$.

Q: Do you know how to write any of these fractions as decimals?

Remind the children that they should know that $\frac{1}{2} = 0.5$ and that $\frac{1}{10} = 0.1$. Explain that 0.5 and 0.1 are called decimal fractions.

Q: How can we use a calculator to show that $\frac{1}{2} = 0.5$ and that $\frac{1}{10} = 0.1$?

Give out calculators. Establish that $\frac{1}{2}$ means 1 divided by 2 and use the calculators to demonstrate this with the children.

Repeat for $\frac{1}{10}$.

Q: How do we convert $\frac{3}{8}$ to a decimal fraction?

Children work out the equivalent decimal fraction using a calculator. Collect their responses and get them to use their calculator to work out the decimal fraction equivalent of $\frac{7}{16}$. Children show their answer by holding up whiteboards.

Repeat for different fractions.

MAIN TEACHING ACTIVITY

Ask the children 'What is half of 40?' Ask what they divided by to get the answer 20. Repeat this by asking 'What is $\frac{1}{3}$ of 90?' and 'What is $\frac{1}{4}$ of 80?' up to 'What is $\frac{1}{10}$ of 700'?

Establish that when finding a unit fraction we divide by the denominator.

Q: If $\frac{1}{4}$ of 80 is 20, what is $\frac{3}{4}$ of 80?

Establish that we multiply 20 by the numerator, 3, to get 60.

Q: What is $\frac{3}{10}$ of 250?

Get the children to find one tenth ($250 \div 10 = 25$) and then three tenths ($25 \times 3 = 75$).

Record this on the board as:

(Find $\frac{1}{10}$) $250 \div 10 = 25$ (Find $\frac{3}{10}$) $25 \times 3 = 75$

Write 680 on the board. Ask the children to find $\frac{1}{10}$ mentally and write (Find $\frac{1}{10}$) $680 \div 10 = 68$, then to use their calculator to work out $\frac{4}{10}$, $\frac{9}{10}$, $\frac{3}{10}$ of 680, recording their method and answers on whiteboards.

Discuss the calculations the children did mentally and those they did using a calculator.

Q: How can we find $\frac{5}{6}$ of 300?

Establish that this can be done first by finding $\frac{1}{6}$ of 300, and then multiplying this answer by 5 to get $\frac{5}{6}$. Record as (Find $\frac{1}{6}$) $300 \div 6 = 50$ (Find $\frac{5}{6}$) $50 \times 5 = 250$.

Set other questions and get the children to use a mix of mental and calculator methods.

PLENARY

Write 490 kg on the board.

Q: How can we find $\frac{4}{7}$ of this quantity?

Take children's responses and show how the two earlier calculations can be written as one calculation: $(490 \div 7) \times 4 = 280$

Q: The answer is 280 what?

Highlight the need to include the units in the answer.

Ask the children to now work out $\frac{2}{7}$ of 490 kg.

Write down:

$$\frac{1}{7} \text{ of } 490 \text{ kg} = 70 \text{ kg}$$

$$\frac{2}{7} \text{ of } 490 \text{ kg} = 140 \text{ kg}$$

$$\frac{4}{7} \text{ of } 490 \text{ kg} = 280 \text{ kg}$$

Q: What do the answers add up to?

Q: Why is the answer 490 kg?

Remind the children that they have found $1 + 2 + 4$ sevenths altogether, this is seven sevenths, and the total is 490 kg.

Remember:

You find a fraction of a number or quantity by first dividing the quantity by the denominator and then multiplying by the numerator.

Always include units in your answer.

**LESSON 10 RELATED TEST QUESTION
1998 TEST B (CALCULATOR PAPER)**
20

Calculate 24% of 525



1 mark

MARK SCHEME

126

ANALYSIS OF CHILDREN'S ANSWERS

- The most common error was to regard the percentage as a fraction $\frac{1}{24}$. Only about half of children working at Level 5 and 13% at Level 4 responded correctly.
- Children had few strategies to solve this question using a calculator.

IMPLICATIONS FOR PLANNING

- Children need to be taught how to use the calculator to find percentages of a quantity. They need to be taught when it is appropriate to do so and when a mental method might be better, for example, when finding 10% of 500.
- Children need to be taught a checking strategy, in this case recognising that 24% is close to $\frac{1}{4}$ and $\frac{1}{4}$ of 500 is 125.
- Children who recognise that 24% can be represented as $\frac{24}{100}$ should be encouraged to find 24% by finding $\frac{24}{100}$ using $(525 \div 100) \times 24$.

SPRINGBOARD 6 LESSON 11

TRANSFORMING SHAPES

TOTAL TIME

30
MINUTES

Objective:

- Recognise where a shape will be after a reflection in a mirror line touching the shape at a point.

Vocabulary:

- reflection
- mirror line
- parallel
- perpendicular
- image

By the end of the lesson children should be able to:

- reflect a shape in a mirror line that is not parallel to a side of the shape.

Resources:

- OHP
- OHT 11.1
- Resource Sheet 11.2
- tracing paper
- mirrors

ORAL AND MENTAL STARTER

5
MINUTES

Show the children the grid on OHT 11.1. Mark on a mirror line. Explain that a mirror line is marked down the centre. Using four square tiles, arrange them to make a shape that touches the mirror line. (An alternative to tiles would be to have a pre-drawn shape on an acetate overlay.) Invite children to come to the OHP and make the image that is a reflection of the shape in the mirror line. For each example, ask:

Q: Is this the correct reflection of the shape in the mirror line?

Children respond with thumbs up or thumbs down.

Repeat, making other shapes where the shape does not touch the mirror line.

MAIN TEACHING ACTIVITY**20
MINUTES**

Give each child Resource Sheet 11.2. For Question 1, ask the children to make a reflection of the two shapes in the mirror line. Suggest they might start with the square nearest the mirror line and work outwards to the tile furthest from the mirror line.

Collect responses and quickly work through using the grid on OHT 11.1.

Q: How can you be sure that the reflection you have drawn is correct?

Discuss suggestions from the children as to how they can check their work, including the use of mirrors and tracing paper.

Children work in pairs on Question 2.

Q: When is it easy to reflect a shape?**Q: Why?**

Establish that for one reflection for Question 2 this can be built up by first drawing the part of the shape that is touching the mirror line. The second shape needs another approach.

Check each answer by using tracing paper and a mirror, and discuss their methods.

Emphasise that the children can move the sheet around so that the mirror line is best placed for them. Encourage the children to 'be the object' looking in the mirror.

Repeat for Questions 3 and 4.

PLENARY**5
MINUTES**

Begin by discussing children's work and methods from the main activity. Invite children to check one another's answers, using tracing paper and mirrors.

Now explain that the distance from each point on the image to the mirror line is exactly the same distance as the distance from the same point on the shape to the mirror line. Demonstrate that this must be the perpendicular distance by drawing a shape using the grid on OHT 11.1. Identify the right angles involved. Invite a child to draw the image. Identify a point on the shape and the equivalent point on the image. Count the number of squares from both points to the mirror line and establish that they are the same.

Ask children to identify some equivalent points on their shapes and images. Mark on a strip of paper the distance between two equivalent points and the position of the mirror line. Establish that the perpendicular distance of each point to the mirror line is the same.

Remind children that the shape has not changed, only its position, and that edges that are parallel to the mirror line stay parallel to it.

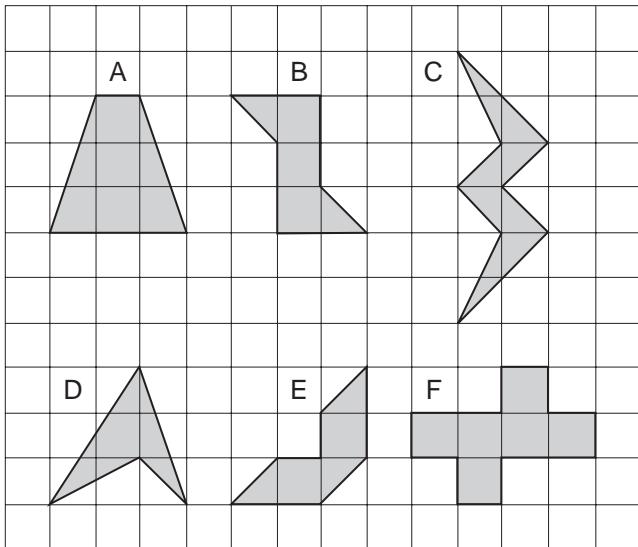
Remember:

- Each point on the original shape reflects at right angles to the mirror line.
- Each point in the image must be the same perpendicular distance away from the mirror line as it was for the original shape.
- Any point or line on the mirror line will not move when reflected.

LESSON 11 RELATED TEST QUESTION 2000 TEST A (NON-CALCULATOR PAPER)

9

Here are some shaded shapes on a grid.



MARK SCHEME

Award **TWO** marks for the correct answer of A, C, E.

Accept letters in any order.

Accept for **ONE** mark either three correct letters and one additional letter **OR** two correct letters and up to one incorrect letter.

Accept alternative, unambiguous indications, e.g. ticks or mirror lines drawn on the correct shapes.

Up to 2 marks

Which **three** shapes have **reflective symmetry**?

You may use a mirror or tracing paper.

ANALYSIS OF CHILDREN'S ANSWERS

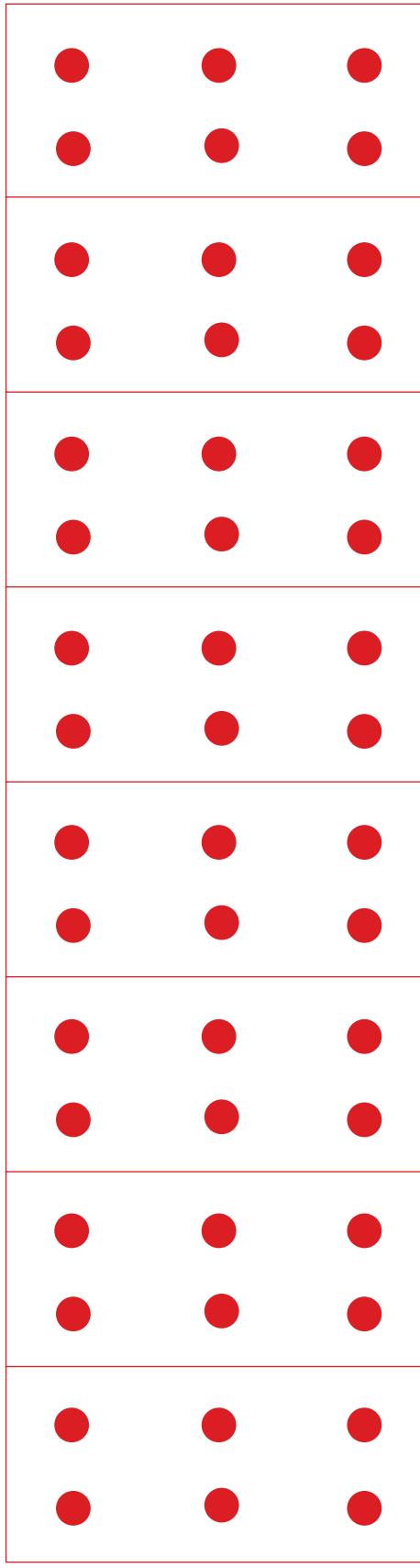
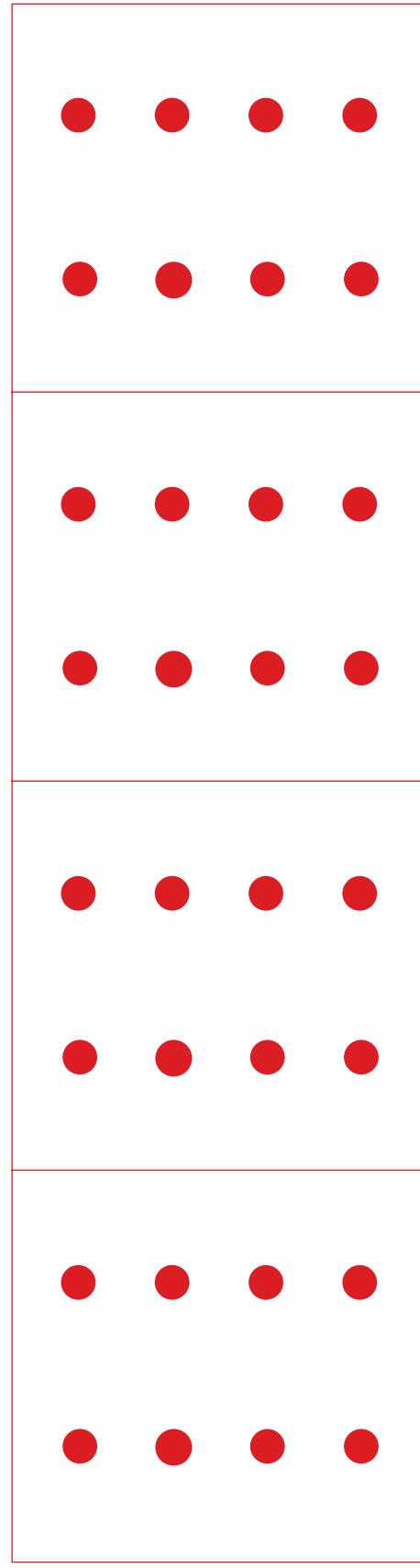
- Children found it is easier to identify shapes with vertical lines of symmetry than shapes with horizontal lines of symmetry. Shapes with diagonal lines of symmetry were the most difficult to identify.
- Children find reflection in the oblique angle more difficult than in the horizontal or vertical.

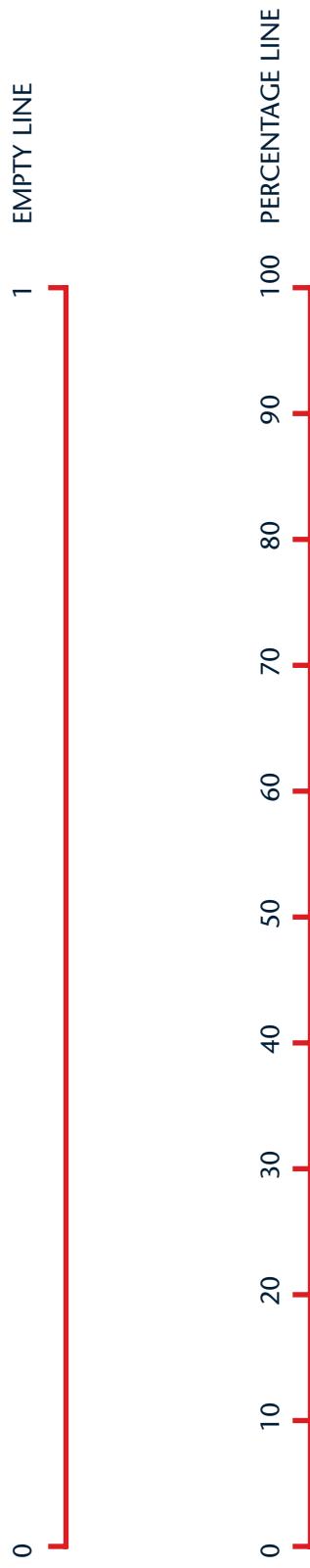
IMPLICATIONS FOR PLANNING

- Plan to include more visualisation work in oral and mental starters.
- Lessons on reflection should include opportunities for children to reflect shapes in horizontal and diagonal lines, and should give activities that help them understand the line of symmetry moves with the shapes.
- Children need opportunities to move shapes around mirror lines to decide what is correct or not, as well as drawing on grids.

LESSON 1 PLACE VALUE**RESOURCE SHEET 1.1**

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousands

LESSON 2 MULTIPLICATION AND DIVISION 1**OHT 2.1****OHT 2.2**

**LESSON 3 MULTIPLICATION AND DIVISION 2
LESSONS 7 AND 8 FRACTIONS, DECIMALS AND PERCENTAGES 1 AND 2****OHT 3.1**

LESSON 4 MULTIPLICATION AND DIVISION 3**RESOURCE SHEET 4.1**

X			

X			

X			

X			

LESSON 5 PROBLEM SOLVING 1**RESOURCE SHEET 5.1**

1. Boat hire costs £2.50 for 1 hour.
How much does it cost to hire a boat for 3 hours?

3. There are 36 children on a bus.
That is 10 more than yesterday.
How many children were on the bus yesterday?

5. Crisps cost 25p for a small packet, and 42p for a large packet. Nigel buys 3 large packets and pays with a five pound note. How much change will he get?

7. A matchbox is exactly 11.5 cm long. A table is 1.5 m long. How many matchboxes can be placed end to end along the table?

9. Number 59 buses leave the bus station at 9.10 a.m. and every 20 minutes thereafter. How many buses will leave the station between 9.20 a.m. and 11.55 a.m.?

2. Plants are sold in trays of 20. If you buy 7 trays of plants, how many plants is this?

4. Plants are sold in trays of 20. If you wanted to buy 240 plants, how many trays would you need to buy?

6. Roy's mobile phone cost 15p, 25p or 40p per minute depending on the time of day. His monthly bill comes to £146. He also pays £27.50 per month in rental charges. How much would a call of 1 hour cost at the most expensive rate?

8. Chicken must be cooked for 50 minutes for every kg. An oven needs to be on for 30 minutes to reach the correct temperature. How long must the oven be on for to cook a chicken weighing 3.5 kg?

10. A box contains 39 eggs. If the eggs are in trays of 6, how many trays are full?

LESSON 6 PROBLEM SOLVING 2**RESOURCE SHEET 6.1**

1. Hot dogs cost 75p each. The total money collected from hot dog sales is £695.25.
How many hot dogs are sold?

2. A full box of matches contains 130 matches.
How many matches would be needed to fill 165 boxes?

3. 19×3

4. Tickets for a concert cost £3.60 for adults and 85p for children.
How much does it cost for 7 adults and 17 children?

5. In three innings, a cricketer scores 70, 80 and 40 runs.
How many runs does he score in total?

6. 4.3×6.8

7. $400 \div 50$

8. There were 57 children in the playground. 29 of them went inside.
How many children stayed outside?

9. There are 36 boys and 25 girls in Year 6.
How many children altogether?

10. What is 24% of 525?

11. What is 15% of £4.80?

12. How many days are there in 34 weeks?

LESSON 6 PROBLEM SOLVING 2

RESOURCE SHEET 6.2

'On a sheet of stamps there are 18 rows. In each row there are 12 stamps. How many stamps are there on 15 sheets of stamps?'

What information are we given?

How many stamps are there on one sheet? Remember to include a calculation.

How many sheets are there?

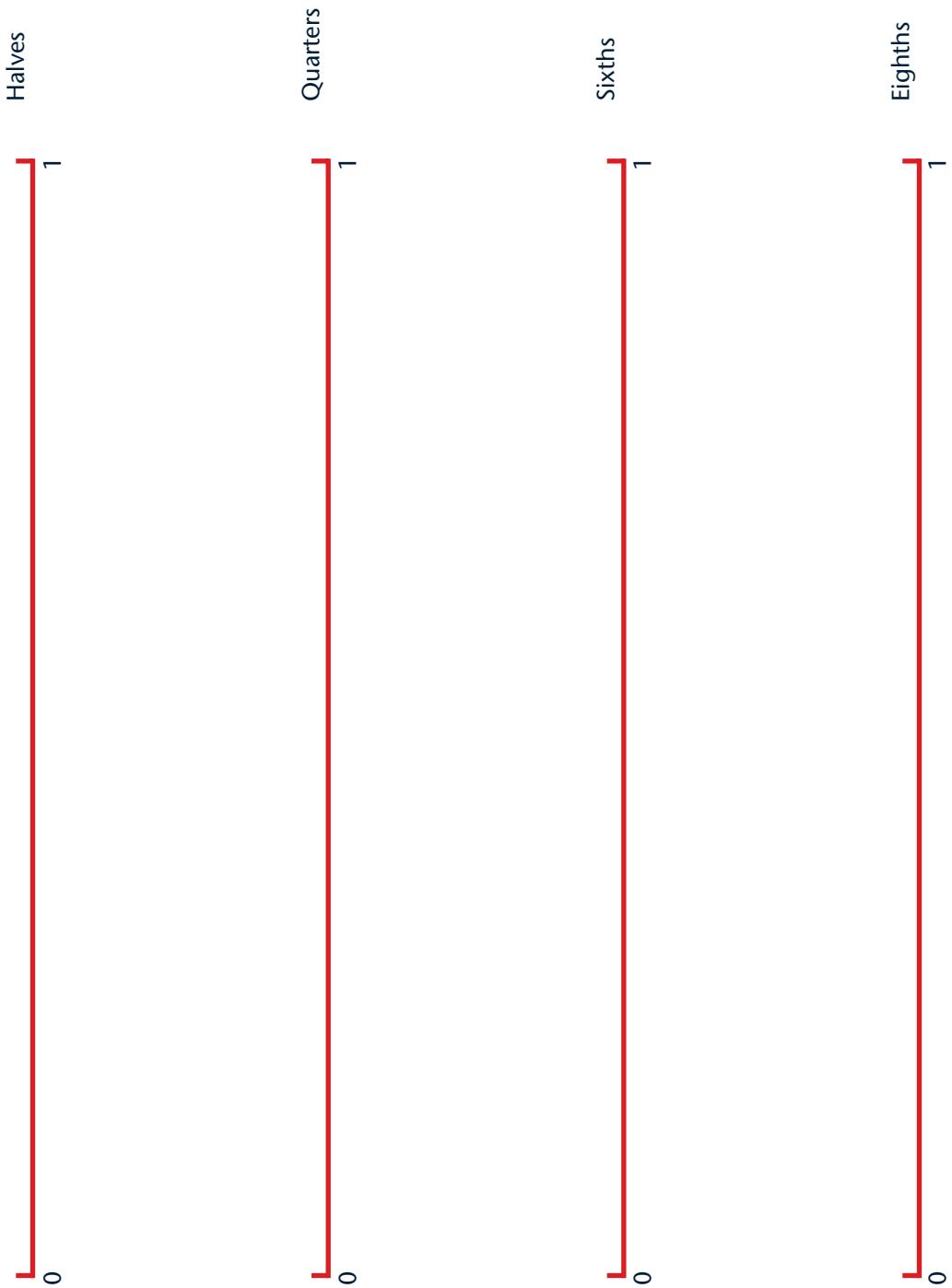
How many stamps are there altogether? Remember to include a calculation and then to write your answer in a sentence.

LESSON 7 FRACTIONS, DECIMALS AND PERCENTAGES 1**RESOURCE SHEET 7.1**

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

LESSON 7 FRACTIONS, DECIMALS AND PERCENTAGES 1

OHT 7.2

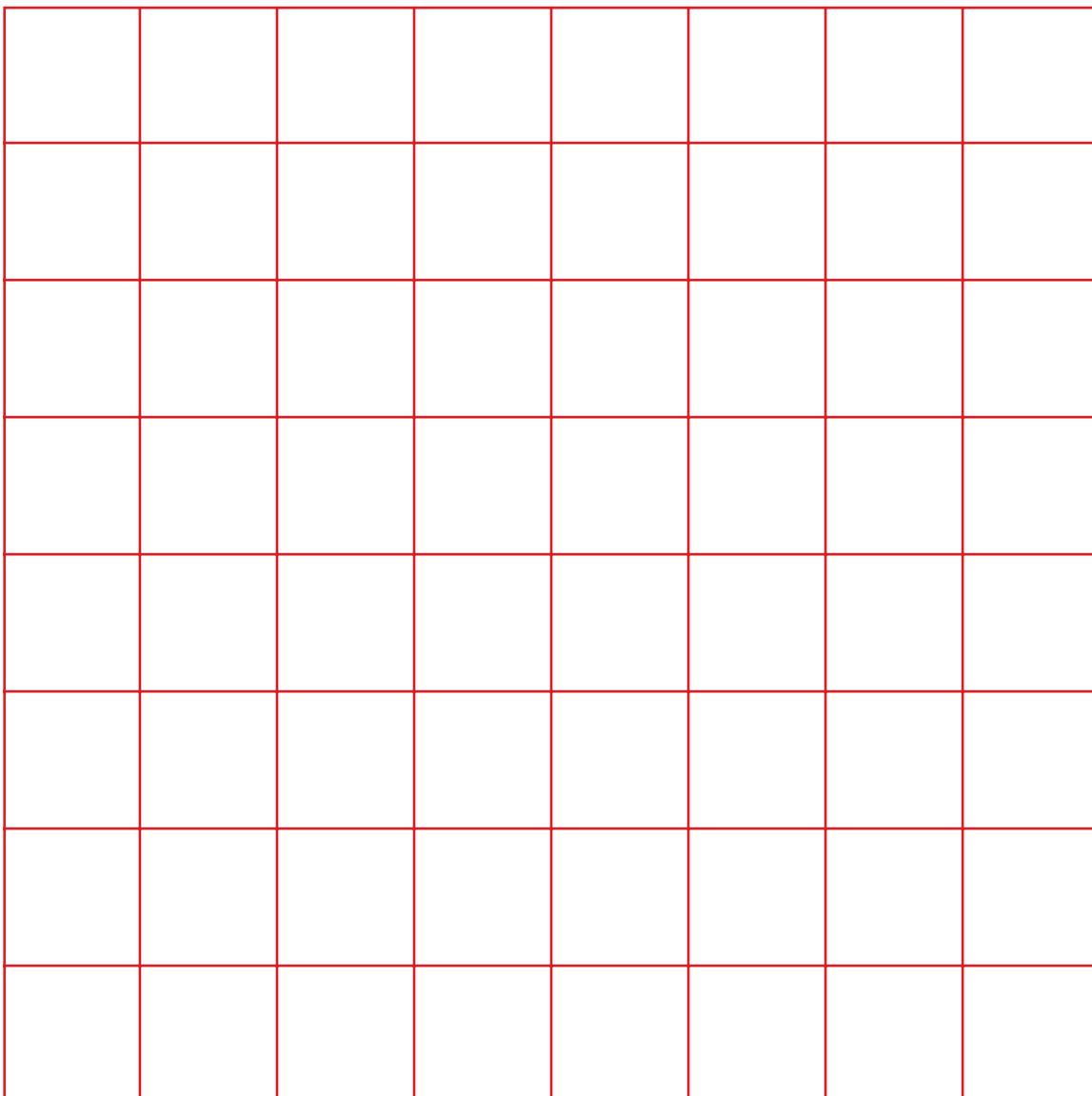


LESSON 8 FRACTIONS, DECIMALS AND PERCENTAGES 2**RESOURCE SHEET 8.1**

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	1
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{8}$
$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{1}{16}$
$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	2

LESSON 8 FRACTIONS, DECIMALS AND PERCENTAGES 2**RESOURCE SHEET 8.2**

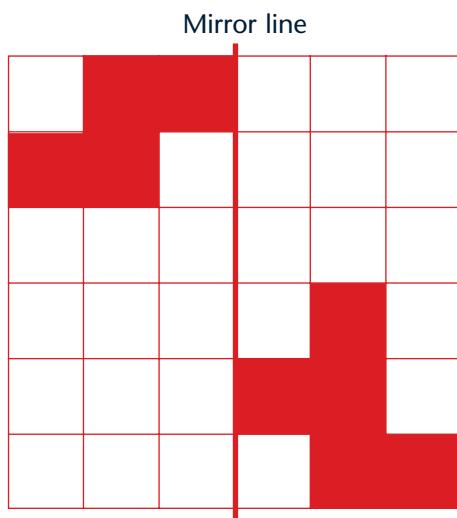
50%	25%	75%	100%
10%	20%	30%	12.5%
40%	50%	60%	6.25%
70%	80%	90%	200%

LESSON 11 TRANSFORMING SHAPES**OHT 11.1**

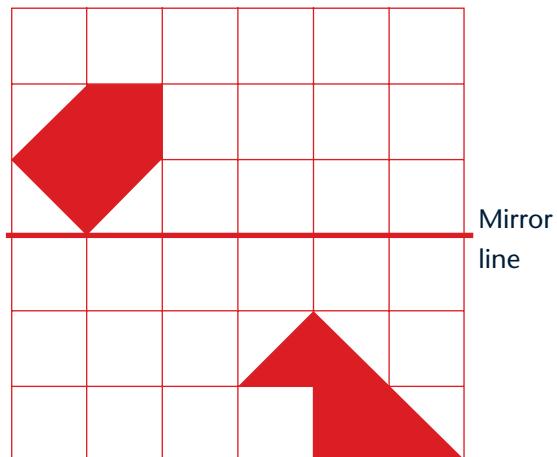
LESSON 11 TRANSFORMING SHAPES

RESOURCE SHEET 11.2

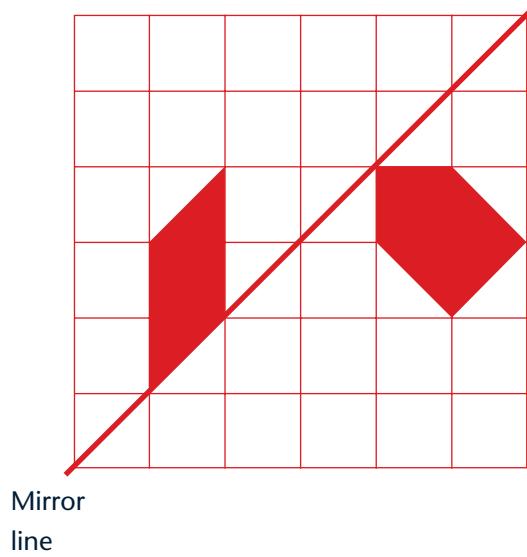
Question 1



Question 2



Question 3



Question 4

