

SPRINGBOARD 6

SECOND 11 LESSONS

Lesson Topic	Objectives	What children should be able to do by the end of the lesson
12 Addition and subtraction 1	Choose and use the appropriate operations of addition and subtraction to solve problems, explain methods and show working	Associate key words such as 'more than', 'altogether', and 'how many', with the correct operation needed to solve real life problems
13 Addition and subtraction 2	Choose and use the appropriate operations of addition and subtraction to solve word problems involving time	Find intervals for 24-hour times and use a timetable to work out the length of a journey and departure and arrival times
14 Shape and space	Calculate angles in a triangle and around a point	Find the missing angles in a triangle and around a point
15 Measures 1	Know approximate equivalents of imperial and metric units for lb and kg, miles and km, litres and pints	Identify imperial and metric unit Order a set of imperial and metric units Know that 1 kg is just over 2 lb, that 1 litre is less than 2 pints and that 8 km is about 5 miles
16 Measures 2	Record estimates and readings from scales used to measure length, mass and capacity	Read scales and identify on the scale the interval that contains a given value
17 Measures 3	Find perimeters of simple shapes and their areas by counting squares, and begin to use the formula in words for the area of a rectangle	Find the perimeter and area of a rectangle Find the area of other shapes and compare the areas of these shapes
18 Problem solving 3	Read scales to a suitable degree of accuracy and extract and interpret data from tables and charts to solve problems	Read scales from graphs and understand how data is represented on bar charts and pie charts
19 Problem solving 4	Solve problems involving ratio and proportion	Solve simple problems involving ratio and proportion by scaling up and down, and work out the fractional part from a given ratio
20 Properties of and reasoning about numbers 1	Use tests of divisibility and factors of numbers to inform and check division calculations	Decide whether a number is divisible by 2, 3, 4 or 5 and by multiples of 2, 3, 4 and 5
21 Properties of and reasoning about numbers 2	Use a given relationship expressed in words to develop a sequence and describe in words the rule for a given sequence	Use a given rule to generate a sequence Describe a sequence in words and decide if a given number is in the sequence
22 Properties of and reasoning about numbers 3	Use knowledge of number facts and properties of numbers to solve number problems	Use a calculator and known number facts to solve simple number problems

SPRINGBOARD 6 LESSON 12 ADDITION AND SUBTRACTION 1

TOTAL TIME

 A circular icon with the number '30' in the center and the word 'MINUTES' written around the bottom edge.
Objective:

- Choose and use the appropriate operations of addition and subtraction to solve problems, explain methods and show working

Vocabulary:

- more than
- altogether
- how many

By the end of the lesson children should be able to:

- associate key words such as 'more than', 'altogether' and 'how many', with the correct operation needed to solve real life problems.

Resources:

- OHT 12.1
- Resource Sheet 12.1
- Resource Sheet 12.2
- whiteboards and pens

ORAL AND MENTAL STARTER

 A circular icon with the number '10' in the center and the word 'MINUTES' written around the bottom edge.

Give out Resource Sheet 12.1.

Q: Altogether how many Year 6 children attend Number Line Lane Primary School and Roundup Road Middle School?

Take responses and identify the key word in the question. Establish that the calculation is an addition.

Q: What other questions could we ask that will involve addition?

Q: What questions can you ask that involve subtraction?

Take responses and list the different key words that the children have used in their question.

Show the children OHT 12.1 and give each pair of children two words from the list of words on OHT 12.1.

Ask them to make up two questions using the words and the information on Resource Sheet 12.1. Children write their questions on whiteboards.

Children swap whiteboards and answer each other's questions.
Discuss the questions and identify the key words and the calculation used for each question. Check answers and correct any errors.

MAIN TEACHING ACTIVITY



Ask the children to read Question 1 on Resource Sheet 12.2. Ask the children what information they are given and what they are asked to find out.

Use the following questions to develop children's understanding of the question, and discuss their responses and reasons.

Q: Will the answer be more than or less than 1470?

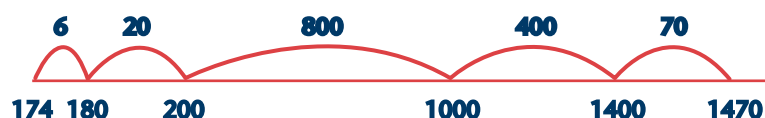
Q: Which operation do we need to carry out?

Q: How did you decide this?

Q: Can we ask the question a different way?

Agree on the correct operation and ask children to carry out the calculation $1470 - 174$ on their whiteboards. Compare the different strategies used and discuss the efficiency of each method.

Remind children who are not confident with a compact written method that the empty number line can be used in the written test. Quickly show the empty number line and work through the method of counting up from 174.



Establish that the answer is 1296.

Discuss how to reduce the number of steps needed to increase efficiency.

Remind the children that addition is the inverse of subtraction, and can be used to check the answer by calculating $1296 + 174$.

Ask the children to work through Questions 2 and 3. Discuss each problem and identify the operation required. Explore children's method and get them to check their answers using the inverse operation.

PLENARY

Discuss Question 4 on Resource Sheet 12.2.

Ask different children to read out one piece of information.

Q: What is the question asking us to calculate?

Establish that for this problem we need to work out the answer in stages.

Ask the following questions to identify the stages.

Q: Who goes to the cinema?

Q: How much does it cost the 2 adults to go?

Q: How much does it cost the 3 children to go?

Q: How much does it cost for 2 adults and 3 children to go to the cinema?

Give out calculators and ask the children to record their calculation on their whiteboards.

Collect answers and discuss methods and recordings. Establish that the cost is £10.90.

Q: How many ice creams are bought? What is the cost?

Q: What else is bought?

Establish that a bag of popcorn and 3 cans of cola are bought and that the total cost of the ice creams, drinks and popcorn is $£1.50 + £1.80 + £1.10 = £4.40$.

Collect all the costs and ask:

Q What change will there be from the £20?

Agree that the change is $£20 - (£10.90 + £4.40) = £4.70$.

Ask the children for the steps they have gone through to work out the problem.

Remind the children that some test questions have a box where they record the calculations they do, even when they have a calculator.

Remember:

- Read the question carefully and look for key words to help you understand the problem.
- Decide what information you need for your calculations.
- Use the calculation method you understand and write down the calculation.

**LESSON 12 RELATED TEST QUESTION
1998 TEST B (CALCULATOR PAPER)**

5 One length of a swimming pool is 25 metres.



(a) How many lengths are there in a 150 metre race?

1 mark

Six children swim a 50 metre race.

Lane	Name	Time in seconds
1	Bryn	92.4
2	Craig	86.3
3	Fiona	90.4
4	Harun	85.1
5	Jody	84.7
6	Dean	89.2

(b) Who finished first?

1 mark

(c) How many seconds faster was Dean than Fiona?

1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
5a	6	
5b	Jody	Accept 84.7 or Lane 5 or 5.
5c	1.2	

ANALYSIS OF CHILDREN'S ANSWERS

- The questions were accessible to the majority of children and most children working at level 4 could extract the required information from the simple tables.
- Children's interpretation and use of the information they were given was less secure. Some children responded with values that were not derived from the table; other children gave wrong answers as they made false assumptions and had not read the question correctly.

**LESSON 12 RELATED TEST QUESTION
1999 TEST A (NON-CALCULATOR PAPER)**

3 This table shows the cost of sending a letter.

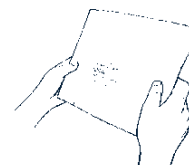
Mass	Cost in pence	
	first class	second class
up to 60g	26	20
61g to 100g	39	31
101g to 150g	49	38
151g to 200g	60	45
201g to 250g	70	55

Paul is sending a letter.
It costs 38p second class.

(a) How much would it cost him to send it first class?

1 mark

Jenny has a letter with a mass of 170g.



(b) What does it cost to send it first class?

1 mark

Question	Requirement	Additional Guidance
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3a	49	Accept £0.49p
3b	60	Accept £0.60p

For 3a and 3b, accept unambiguous indications of answers on the diagram.

If 49 and 60 are indicated on the diagram but it is not clear to which parts of the question they refer, then award the mark for 3b only.

IMPLICATIONS FOR PLANNING

- Children should be taught to read questions carefully, and be given the opportunity to discuss the information in the question to help them identify what the question is asking.
- Use past test questions to generate associated questions, for example 'A letter costs 60p to send first class – what does it weigh?' Incorporate the interrogation of secondary data in lesson plans.
- During the main part of the lesson introduce children to the language used in test questions. Encourage them to generate their own questions using this language.

SPRINGBOARD 6 LESSON 13 ADDITION AND SUBTRACTION 2

TOTAL TIME**Objective:**

- Choose and use the appropriate operations of addition and subtraction to solve word problems involving time

Vocabulary:

- analogue, digital
- colon

By the end of the lesson children should be able to:

- find intervals for 24-hour times and use a timetable to work out the length of a journey and departure and arrival times.

Resources:

- Resource Sheet 13.1
- whiteboards and pens

ORAL AND MENTAL STARTER

Count forwards and backwards in steps of 6, then in steps of 60.

Q: How many minutes are there in an hour? How many hours in a day?

Q: How many minutes are there in a day?

Write on the board 60×24 . On their whiteboards get the children to work this out using the grid method.

×	20	4	
60	1200	240	1440

Count in time intervals forwards and backwards using different steps: from 9 a.m. to 4 p.m. in half-hour steps and quarter-hour steps; from 11 a.m. to 1:30 p.m. in 15-minute, 10-minute, 5-minute steps.

Q: How many hours and minutes are there in 90 minutes?

Confirm that 90 minutes is 1 hour and 30 minutes.

Repeat this process for 75 minutes, 105 minutes, 119 minutes.

Q: How many hours and minutes are there in 150 minutes, 290 minutes, 340 minutes?

Children write their answers on whiteboards.

Collect children's responses and discuss different strategies. Refer back to counting in steps of 6 and 60.

MAIN TEACHING ACTIVITY



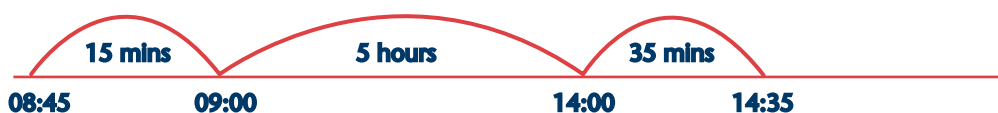
Remind children how time appears on analogue and digital clocks. Emphasise how we distinguish between morning (a.m.) and afternoon (p.m.) for the analogue clock and use 24 hours for the digital clock.

Q: What time does 14:00 represent?

Emphasise that the colon separates hours and minutes. Establish that 14:00 is 2:00 p.m. and quickly repeat with 17:30, 09:10, to practise the conversion of 24-hour time to 12-hour time.

Draw an empty time line on the board and say that you are going to use it to solve the following question: 'The car race began at 08:45 and finished at 14:35. How long did the race last?'

Demonstrate the method using a time line by bridging through the hours.



Establish that the race lasted 5 hours and 50 minutes.

Write on the board:

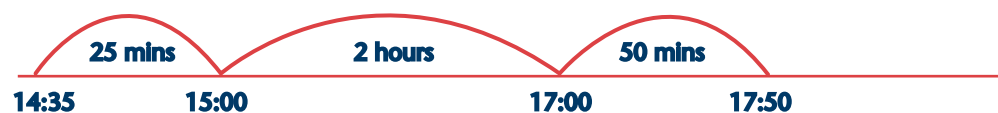
START 14:35

STOP 17:50

Explain that these are the start and stop times on a video recorder.

Q: How long was the video recording?

Get children to work through the question on their whiteboards. Encourage the use of the time line. Discuss their responses and with the children work through the answer using the time line.



Establish that the video was recording for 2 hours and 75 minutes. Refer to their earlier work to convert 75 minutes to 1 hour and 15 minutes.

Calculate the final recording time 2 hours + 1 hour 15 minutes = 3 hours and 15 minutes.

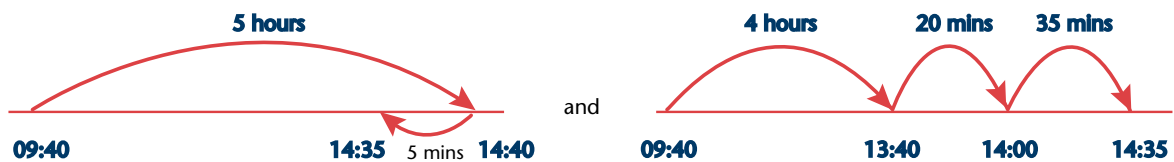
Q: If a four-hour video tape was used, how much recording time would be left on the tape?

Discuss children’s answers.

Q: A flight leaves at 09:40 and takes 4 hours and 55 minutes. When will it arrive?

Emphasise that the empty time line is very useful for finding the answer to this question. Ask the children to make a time line and to put in the steps of time they used to work out the answer.

Take children’s responses and demonstrate the different ways in which the answer can be found.



Establish that the plane arrives at 14:35 or 2:35 p.m.

PLENARY



Give out Resource Sheet 13.1.

Q: What do the figures in the table represent?

Explain that some timetables do not use a colon to separate hours and minutes.

Q: Where would we put a colon if the timetable gave a time of 0744?

Ask children to work in pairs to answer the questions on Resource Sheet 13.1.

Discuss responses and correct errors and misunderstandings.

Remember:

- When working out 24-hour times use the fact that 12:00 is 12 noon.
- 09:15 is before noon and is 9:15 a.m., 17:45 is after noon and is 5:45 p.m.
- Use a time line to help you calculate time intervals.

LESSON 13 RELATED TEST QUESTION 2000 TEST B (CALCULATOR PAPER)

8



These are the opening times at a swimming pool.

	opening times	
	am	pm
Monday	Pool closed	
Tuesday	Pool closed	
Wednesday	10:30	to 5:30
Thursday	10:30	to 8:30
Friday	10:30	to 9:00
Saturday	8:00	to 6:00
Sunday	7:00	to 4:00

(a) How many **hours** is the pool open on a **Sunday**?



hours

1 mark

(b) Which **day** has the **latest** closing time?



1 mark

(c) Habib arrives at the pool at **5:20pm** on **Saturday**.

How many **minutes** is it before the pool closes?



minutes

1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
8a	9	
8b	Friday	Accept recognisable misspellings. Accept 9:00 circled.
8c	40	

ANALYSIS OF CHILDREN'S ANSWERS

- Children found part (b) easier than part (a), and part (c) had the highest omission rates. Parts (a) and (c) each require a calculation. In part (c), children working at level 3 often treated the times as decimal numbers and performed an incorrect calculation.
- For part (a), children sometimes misread the table, confusing opening times with closing times. Similarly for part (a), children used the Saturday times to answer the question.

IMPLICATIONS FOR PLANNING

- Oral and mental starters should include activities that involve counting in time intervals using both 12-hour and 24-hour time. Activities should also include converting times from hours and minutes to minutes, and minutes to hours and minutes.
- Children should be taught how to calculate time intervals using an empty time line, particularly when finding differences in time that require them to bridge through the hour.

SPRINGBOARD 6 LESSON 14

SHAPE AND SPACE

TOTAL TIME

30
MINUTES

Objective:

- Calculate angles in a triangle and around a point

Vocabulary:

- acute, obtuse, reflex, right angle
- square, rectangle, equilateral triangle

By the end of the lesson children should be able to:

- find the missing angles in a triangle and around a point.

Resources:

- whiteboards and pens
- OHT 14.1
- OHT 14.2
- Resource Sheet 14.1
- triangle made from thin card
- OHP protractor

ORAL AND MENTAL STARTER

5
MINUTES

Write 'Target number is 180' on the board. Call out '40' and ask:

Q: What do we add to 40 to make 180?

Children respond by showing their answers on whiteboards.

Repeat for other numbers between 0 and 180, include multiples of 10 and 5.

Collect and check the children's responses.

Repeat using a target number of 360. Ask a mixture of questions using the two target numbers 180 and 360.

MAIN TEACHING ACTIVITY

15
MINUTES

Q: How many degrees are there in one complete turn?

Show the children one full turn using an OHP protractor. Remind the children that there are 360° in a full turn. Demonstrate that a straight line represents half a complete turn.

Q: How many degrees are there in half a turn?

Establish that the answer is 180° . By putting 2 right angles together show that 2 right angles are equivalent to half a turn.

Q: How many degrees are there in 1 right angle?

Establish that the answer is 90° and that it is one quarter of a turn.

Place the triangle made from thin card on the overhead projector. Refer to the image and identify the 3 angles in the triangle.

Q: What do the 3 angles in a triangle add up to?

Tear off the angles from the triangle and on the OHP demonstrate that they sum to 180° by placing them around a point to form a straight line. Tell children that they should remember that the angle sum in a triangle is 180° .

Show OHT 14.1 and demonstrate how to measure an angle in triangle A using an OHP protractor. Emphasise the key points about positioning the protractor and using the correct scale.

Invite some children to measure the other two angles in triangle A.

Q: What should the 3 angles sum to?

Add the 3 measurements and discuss how the accuracy of each measurement can affect the answer.

Say that two of the angles have been measured in triangle B. They are 90° and 24° . Ask the children to identify these angles.

Q: How can we find the size of the third angle without measuring it?

Establish that we can work out the sum of the two angles and that adding the third angle must make the total 180° .

Compare and discuss $180^\circ - 90^\circ - 24^\circ$ and $180^\circ - (90^\circ + 24^\circ)$.

Check the answer by measuring the third angle.

Remind the children of the accuracy with which we are able to measure.

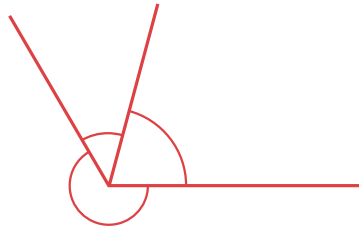
Repeat the above using triangles C and D.

Point out the obtuse angle, and demonstrate how to measure it.

Give out Resource Sheet 14.1. Children work in pairs to find the size of the missing angle in each of the triangles.

PLENARY

Draw the following diagram on the board.

**Q: How many angles are marked?**

Establish that there are 3 angles. Remind the children that two of the angles are acute and one is reflex. Say that you know that two of the angles are 220° and 60° . Invite the children to identify and label the two angles.

Q: How can we calculate the other angle?

Discuss the children's methods and correct misunderstandings. Compare different methods, $360^\circ - 220^\circ - 60^\circ$ and $360^\circ - (220^\circ + 60^\circ)$ as before.

Show OHT 14.2, and discuss the diagram.

Explain that the triangle is equilateral and that this means that the 3 sides are the same length and that the 3 angles are equal.

Q: If all the angles are equal and the angles sum to 180° , what is the size of each angle in the triangle?**Q: What other angles do we know in the diagram?**

Get the children to identify the right angles in the square and rectangle, and to annotate the diagram.

Q: How can we calculate the angle marked X on the diagram?

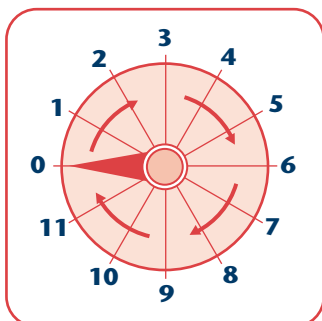
Use the angles around a point to establish that the angle marked **X** is 120° .

Remember:

- The angles in a triangle sum to 180° .
- The angles on a straight line sum to 180° .
- The angles around a point sum to 360° .
- The angles in a square and rectangle are each 90° .
- The angles in an equilateral triangle are each 60° .

**LESSON 14 RELATED TEST QUESTION
1998 TEST A (NON-CALCULATOR PAPER)**

12 Here is a dial.



The pointer on this dial turns in a **clockwise** direction.
The pointer is at 0.

(a) Which **number** does it point to after a turn of **270°**?

1 mark

The pointer moves from **10** to **11**.

(b) How many **degrees** does it turn through?

1 mark

GUIDANCE FROM MARK SCHEME

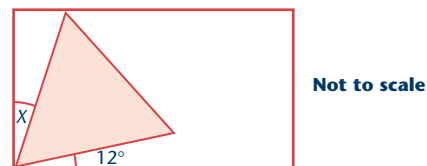
Question	Requirement	Additional Guidance
12a	9	
12b	30	

ANALYSIS OF CHILDREN'S ANSWERS

- In Question 12, children were not always clear what 'clockwise' meant and the representation of the dial was unfamiliar to some. In part (b) some children tried to divide 360° by 11, and some tried to measure the angle. Few children annotated the diagram.
- Question 26 was answered poorly and many children working at levels 3 and 4 did not attempt the question. Those who attempted the question did not know the angles of an equilateral triangle were 60° and despite the instruction, some children measured the angle.

**LESSON 14 RELATED TEST QUESTION
2001 TEST B (CALCULATOR PAPER)**

26 Here is an **equilateral triangle** inside a **rectangle**.



Calculate the value of angle **x**.
Do **not** use a protractor (angle measurer).

Show your **method**. You may get a mark.

2 marks

Question	Requirement	Additional Guidance
26	Award TWO marks for the correct answer of 18° . If the answer is incorrect, award ONE mark for evidence of an appropriate method, e.g. $90-60-12$	<i>Calculation need not be performed for the award of the mark.</i>

IMPLICATIONS FOR PLANNING

- In the oral and mental starter, include more recall involving properties of shapes with the shapes set in different contexts.
- Children should be taught how to find missing angles within combinations of shapes and how to cope with different constraints on angles. They should understand that 'calculate' does not mean 'measure'.
- Children should be shown how to annotate diagrams and be encouraged to do so. They should recognise that 'not to scale' means the shape is not drawn accurately and taking measurements is inappropriate.

SPRINGBOARD 6 LESSON 15 MEASURES 1

TOTAL TIME

 A red circular icon with the number '30' in white, and the word 'MINUTES' written in red around the bottom edge of the circle.
Objective:

- Know approximate equivalents of imperial and metric units for lb and kg, miles and km, litres and pints

Vocabulary:

- pint
- gallon
- centilitre
- capacity
- foot
- feet
- yard

By the end of the lesson children should be able to:

- identify imperial and metric units;
- order a set of imperial or metric units;
- know that 1 kg is just over 2 lb, that 1 litre is less than 2 pints and that 8 km is about 5 miles.

Resources:

- OHT 15.1
- Resource Sheet 15.1
- whiteboards and pens

ORAL AND MENTAL STARTER

 A red circular icon with the number '5' in white, and the word 'MINUTES' written in red around the bottom edge of the circle.

Show OHT 15.1.

Explain that all of the words on the OHT are units of measurement.

Q: Which of these units can we use to measure liquid?

Explain that when we measure the space taken up by liquid or air we refer to measuring capacity.

Establish that the units, centilitres, litres, pints and gallons are all units of capacity. Record these units in the table on OHT 15.1.

Ask the children to choose one of the remaining units and to say if it used to measure a length or a weight.

Agree responses and record these in the table.

Point to the column marked 'metric' or 'imperial'. Explain that some of the units are in the metric system and some of the units are in the imperial system. Work through the units asking the children to say if a unit is metric or imperial. Children respond using whiteboards.

Collect and record the correct responses on OHT 15.1.

MAIN TEACHING ACTIVITY



Children work in pairs. Give each pair a set of the cards from Resource Sheet 15.1. Ask the children to sort the cards into 3 sets of units that are used to measure length, weight, or capacity. Check responses and correct any mistakes.

Ask the children to order the set of units of length, with the shortest unit on the left and the longest unit of length on the right.

Use the following questions to help children order the lengths.

Q: Which is longer, a cm or a foot?

Q: Is a metre longer or shorter than a yard?

Q: Is a mile more than a km?

Q: Which of the units are imperial?

Invite one pair of children to attach their ordered set to the board.

Discuss the order and invite other children to suggest any changes.

Identify the imperial and metric units and underline the imperial units as shown below.

Centimetre Foot Yard Metre Kilometre Mile

Ensure that the imperial units are in the correct order and that the metric units are in the correct order.

Remind children that the plural for foot is feet, and 3 feet is 1 yard.

Establish that there are about 30 cm in a foot, that a metre is just over a yard, that a kilometre is just over half a mile and that 8 km is about 5 miles.

Repeat the above for weight. Remind children that 1000 g = 1 kg and that 1 kg is about 2.2 lb.

Repeat for capacity. Remind children that $100 \text{ cl} = 1 \text{ litre}$ and establish that 1 litre is about 1.75 pints and that 1 gallon is about 5 litres.

Say there are exactly 8 pints in 1 gallon.

Q: About how many litres are there in 32 pints?

Get the children to convert 32 pints to gallons and multiply the answer by 5 to convert to litres.

PLENARY



Shuffle a pack of the measurement cards from Resource Sheet 15.1. Show a measurement card (1 litre, say).

Q: Is this an imperial or metric measurement?

Ask the children to show one of their measurement cards that is less than 1 litre, e.g. pint, and more than 1 litre, e.g. gallon.

Repeat this for other measurements.

Remember:

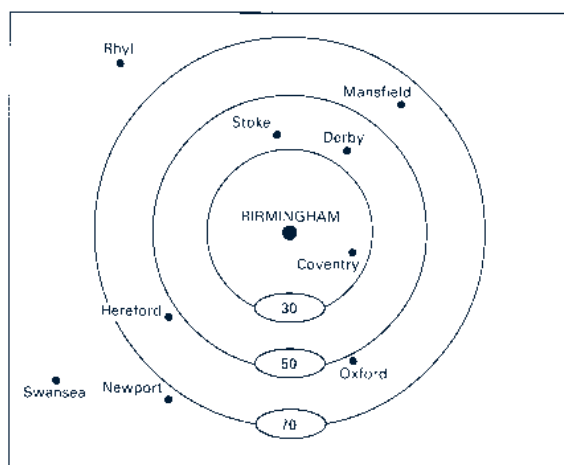
- The metric system uses multiples of 10, 100 and 1000.
- Kilo means 'thousand' and centi means 'hundredth'.
- The imperial system's main units are:
 - miles (5 miles is about 8 km);
 - pints (1.75 pints is about 1 litre);
 - pounds (2.2 pounds is about 1 kg).

LESSON 15 RELATED TEST QUESTION

1999 TEST B (CALCULATOR PAPER)

10

This diagram shows the distances in miles of different towns from Birmingham.



- (a) Write the name of a town which is **between 30 and 50 miles** from Birmingham.

.....
1 mark

- (b) Use the diagram to estimate the distance in **miles** from **Birmingham to Mansfield**.

.....
1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
10a	Derby OR Stoke	Accept recognisable misspelling OR unambiguous indications on the diagram.
10b	Answer in the range 60 to 65 inclusive.	

ANALYSIS OF CHILDREN'S ANSWERS

- Most children working at level 4 correctly identified Stoke or Derby; the most common error was to identify Hereford. These children had not understood the 'bulls-eye' presentation.
- Estimating the distance from Birmingham to Mansfield was a problem for nearly three-quarters of children working at level 3 and over one-third of children working at level 4. Incorrect answers were more often over-estimations.

IMPLICATIONS FOR PLANNING

- Plan activities that introduce children to scales that are represented in unusual forms. Children should be taught how to convert one form to another, for example, converting the 'bulls-eye' scale used in Question 10 to a linear scale with Birmingham positioned at zero.
- In the oral and mental starter children should be given more opportunity to estimate numbers and quantities and be taught how to use the information included on a scale to inform and refine their estimates.

SPRINGBOARD 6 LESSON 16

MEASURES 2

TOTAL TIME

30
MINUTES

Objective:

- Record estimates and readings from scales used to measure length, mass and capacity

Vocabulary:

- interval
- scale
- greater than (>); less than (<)

By the end of the lesson children should be able to:

- read scales and identify on the scale the interval that contains a given value.

Resources:

- OHT 16.1
- OHT 16.2
- OHT 16.3
- metre ruler
- length of string approximately 32 cm long

ORAL AND MENTAL STARTER

10
MINUTES

Show OHT 16.1. Explain that each line represents a scale from 0 to 100. Cover up all but the first scale. Establish that the middle division represents 50 and write in the 50. Ask a child to locate 23 on the scale.

Q: What interval on the scale does 23 lie within?

Establish that 23 lies between 0 and 50 and is closer to 0 than to 50.

Show the second line. Establish that it is marked in intervals of 25.

Ask a child to locate 70.

Q: What interval on the scale does 70 lie within?

Establish that 70 lies between 50 and 75 and is closer to 75.

Repeat using the other lines. Emphasise that the more refined the scale, the smaller the interval.

Explain that we are going to estimate the length of a piece of string and that we want to find an interval that contains the length of the string.

Hold up the piece of string, then hold up a metre rule and discuss the units on the metre rule.

Start by saying that this piece of string is more than 10 cm but less than 100 cm and write on the board:

$10 \text{ cm} < \text{string} < 100 \text{ cm}$.

Q: How long do you think this piece of string is?

Agree an estimate, e.g. 40 cm, and get children to locate this estimate on each of the scales on OHT 16.1.

For each scale identify an interval which contains the estimate. Record as:

$0 \text{ cm} < \text{string} < 50 \text{ cm}$, etc.

Ask a child to measure the exact length of the string.

Mark this length on each of the scales on OHT 16.1.

Q: On which scales are our estimate and our measurement in the same interval?

Discuss the scales in which the estimate and the exact answer are in the same interval and determine which of the statements made earlier about the string are true.

Q: On which scales are the estimate and our measurement in different intervals?

Determine which of the statements made earlier about the string are not true.

MAIN TEACHING ACTIVITY



Show OHT 16.2. Say that this represents a weighing scale. Point to the 0 and write '140 kg' in the box.

Superimpose the pointer on the face of the scale pointing at zero. Explain that the pointer starts at 0 and rotates clockwise.

Rotate the pointer to the first interval.

Q: Does this represent 30 kg?

Take responses. Move round the scale, counting up in 30s at each interval, until the interval with the box is reached. Establish that 30 kg is incorrect as the pointer should be at 210 kg and we are pointing at 140 kg.

Q: What does each interval represent?

Establish that each interval represents 20 kg.

Rotate the pointer to different positions on the scale, such as 86 kg, and ask the children to give an estimate of the weight by first stating the interval that contains the weight.

Q: What weight is represented when the pointer moves all the way round back to 0 kg?

Establish that this is 160 kg.

Change the scale. Write 35 kg, 105 kg or 700 g in the box and repeat the above.

PLENARY

Show OHT 16.3.

Explain that this cylinder measures the amount of liquid or capacity. Say that each interval represents 5 cl.

Q: What is the most liquid we can measure?

Establish by counting up in 5 cl steps that 45 cl is the most we can measure in one go.

Q: How would you measure 1 litre of liquid using this cylinder?

Establish that 1 litre is 100 cl and that one possible way would be to measure 45 cl, 45 cl and 10 cl. Discuss alternatives.

Draw a line on the scale to represent 27 cl of liquid.

Q: How much liquid is in the cylinder?

Establish that the amount of liquid is greater than 25 cl and less than 30 cl.

Q: Which would be a better estimate, 25 cl or 30 cl?

Establish that 27 cl is nearer to 25 cl. Repeat using other estimates.

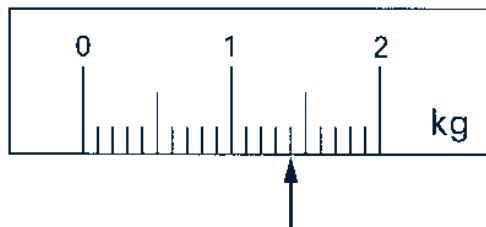
Remember:

- When reading a scale, decide what each interval is worth so that you can label the divisions on the scale.
- When estimating, decide which interval contains the measurement and then which side of the interval the measurement is nearer to.
- The larger the intervals on the scale the less accurately we are able to measure.

LESSON 16 RELATED TEST QUESTION
2001 TEST B (CALCULATOR PAPER)

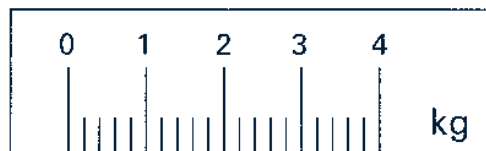
17

On this scale, the arrow (↑) shows the weight of this pineapple.



Here is a **different** scale.

Mark with an arrow (↑) the weight of the **same** pineapple.

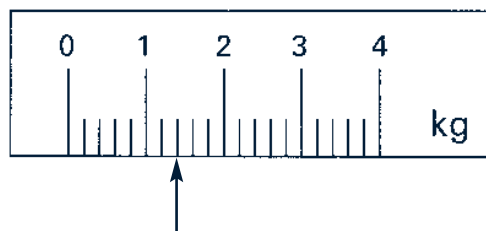


1 mark

GUIDANCE FROM MARK SCHEME

Question Requirement

17 Arrow marked on scale as shown:



Additional Guidance

Accept slight inaccuracies, provided the intention is clear.
 Accept alternative unambiguous indications, e.g. cross on scale.
DO NOT accept the number '1.4' alone.

ANALYSIS OF CHILDREN'S ANSWERS

- Most children attempted the question. The most common error for children working at levels 3 and 4 was to copy the arrow directly below the given arrow, taking no account of the differences in the scales.
- Children who answered the question incorrectly had not read the information provided and had misinterpreted the use of the words 'different' and 'same' in the question.

IMPLICATIONS FOR PLANNING

- When starting activities that involve reading scales, children should be taught how to identify the values of the divisions on the scales, and to annotate the scales to help them read and transfer values. Children need to understand that even when the gaps on the two scales are the same size, the value these intervals represent may be different.

SPRINGBOARD 6 LESSON 17 MEASURES 3

TOTAL TIME

Objective:

- Find perimeters of simple shapes and their areas by counting squares, and begin to use the formula in words for the area of a rectangle

Vocabulary:

- area
- length
- breadth
- perimeter
- square centimetres (cm²)

By the end of the lesson children should be able to:

- find the perimeter and area of a rectangle;
- find the area of other shapes and compare the areas of these shapes.

Resources:

- OHT 17.1
- Resource Sheet 17.1
- whiteboards and pens

ORAL AND MENTAL STARTER


Show the children OHT 17.1. Explain that the grid is in centimetres.

Q: What is the name of shape A?

Q: How do we find the perimeter and area of this rectangle?

Establish that the perimeter is the distance around the shape in cm and that the area is the space inside the shape measured in centimetre squares, square centimetres or cm².

Ask the children to find the perimeter and area of rectangles A, B and C and to write their answers on their whiteboards.

Collect and discuss their answers for the perimeters. Record the answers on the board.

Perimeter of A is $6 + 4 + 6 + 4 = 20$ cm.

Perimeter of B is $2 + 12 + 2 + 12 = 28$ cm.

Perimeter of C is $3 + 8 + 3 + 8 = 22$ cm.

Establish that another way to find the perimeter of a rectangle is to add together the length and breadth, and then multiply this answer by 2. Record this on the board next to the above.

$$2 \times (6 + 4) = 20 \text{ cm}$$

$$2 \times (2 + 12) = 28 \text{ cm}$$

$$2 \times (3 + 8) = 22 \text{ cm}$$

Collect and discuss the children's answers for the areas.

Establish that:

$$\text{area of A is } 6 \times 4 = 4 \times 6 = 24 \text{ cm}^2;$$

$$\text{area of B is } 12 \times 2 = 2 \times 12 = 24 \text{ cm}^2;$$

$$\text{area of C is } 3 \times 8 = 8 \times 3 = 24 \text{ cm}^2.$$

Point out that the areas of rectangles A, B and C are the same but that their perimeters are all different.

Q: What is shape D?

Q: What are its area and perimeter?

Take responses. Point out that this square has the same perimeter as rectangle A: $2 \times (5 + 5) = 20$ cm, but it has a larger area. Write: 'Area of D is $5 \times 5 = 25 \text{ cm}^2$ '.

Q: What is shape E?

Q: How do we find its area?

Establish that the area of this triangle is half the area of a 4 cm by 4 cm square. Write: 'Area of E is $\frac{1}{2}$ of $16 \text{ cm}^2 = 8 \text{ cm}^2$ '.

MAIN TEACHING ACTIVITY



Emphasise that to find the area of a rectangle we need to know the length and breadth of the rectangle.

Write on the board: 'Area of rectangle = length \times breadth'.

Give out Resource Sheet 17.1.

Explain that the grid is in cm and ask the children to find the area of the rectangle.

Encourage them to calculate the area by multiplying the length by the breadth, $2 \times 5 = 10 \text{ cm}^2$, and emphasise that the units are centimetre squares.

Q: How can we work out the areas of the triangles?

Encourage the children to draw rectangles around the triangles and use the fact that the area of the triangle is half the area of the rectangle.

Ask children to find the areas of the other shapes.

Encourage the children to draw in rectangles and find the areas of any rectangles and triangles that they can then combine to find the area of the original shape. Collect answers. Discuss the methods the children used and correct any errors and misunderstandings.

PLENARY

Ask the children to draw a rectangle on the resource sheet. Say that this rectangle is to have an area of 36 cm^2 . Ask the children to work out the perimeter of their rectangle.

Record the children's responses in a table on the board.

Area of rectangle = 36 cm^2		
Length	Breadth	Perimeter
4 cm	9 cm	26 cm

Q: Which rectangle has the smallest perimeter?

Identify the square, 6 cm by 6 cm. Establish that this has a perimeter $2 \times (6 + 6) = 24 \text{ cm}$. Remind the children that a square is a special rectangle and this is the shape that has the smallest perimeter of any rectangle with a given area.

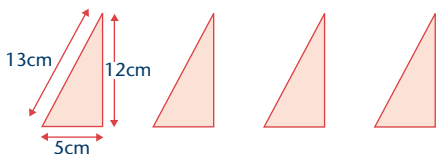
Remember:

- Perimeter is the distance around the shape and is measured in units of length such as cm.
- Area is the space inside the shape and is measured in square units such as square centimetres, written as cm^2 .
- To find the area of shapes it is helpful to find the areas of rectangles or triangles to add and subtract.

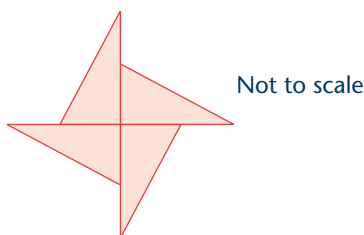
**LESSON 17 RELATED TEST QUESTION
1999 TEST B (CALCULATOR PAPER)**

20

Lindy has 4 triangles, all the same size.



She uses them to make a star.



Calculate the **perimeter** of the star.

Show your method. You may get a mark.

cm

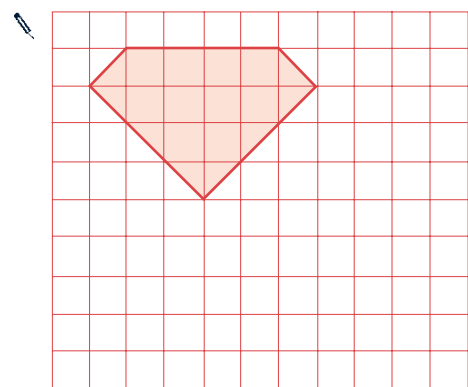
2 marks

**LESSON 17 RELATED TEST QUESTION
2000 TEST B (CALCULATOR PAPER)**

11

On the grid, draw a **rectangle** which has the **same area** as this shaded pentagon.

Use a ruler.

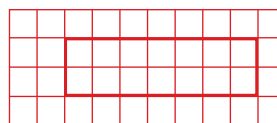


1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
20	80	If the answer is incorrect, award ONE mark for evidence of an appropriate method, e.g. $(13 + 12 - 5) \times 4$. Calculation need not be performed for the award of the mark.

Question	Requirement	Additional Guidance
11	Any rectangle with an area of 14 squares, e.g. 14×1 rectangle.	Rectangle need not be shaded or coincident with the grid lines. Accept extensions to the grid to allow, for example, a 14×1 rectangle. Accept slight inaccuracies in drawing provided the intention is clear.



ANALYSIS OF CHILDREN'S ANSWERS

- Question 11 was accessible to most children. Children working at levels 3 and 4 who answered the question incorrectly, either counted the half squares as whole squares or ignored the half squares. Consequently their rectangles were too large or too small.
- Although many children attempted Question 20, few children working at levels 3 and 4 answered it correctly. The most common error was to estimate that half the 12 cm side was in contact with the other triangle, and to give the answer as 76 cm. Very few children gained the method mark alone, and too few children annotated the diagram with numbers to help them find the perimeter.

IMPLICATIONS FOR PLANNING

- There should be planned teaching activities where children create compound shapes using familiar geometric shapes, with the opportunity to explore and discuss the properties of these compound shapes.
- Children should be taught how to annotate and label diagrams when finding areas and perimeters. Activities that are likely to lead to children making incorrect assumptions about shapes are good assessment activities for a plenary.

SPRINGBOARD 6 LESSON 18 PROBLEM SOLVING 3

TOTAL TIME**Objective:**

- Read scales to a suitable degree of accuracy and extract and interpret data from tables and charts to solve problems

Vocabulary:

- frequency
- survey
- sector

By the end of the lesson children should be able to:

- read scales from graphs and understand how data is represented on bar charts and pie charts.

Resources:

- counting stick
- OHT 18.1
- OHT 18.2
- Resource Sheet 18.1
- Resource Sheet 18.2

ORAL AND MENTAL STARTER

Show children a counting stick. Say that the stick represents the numbers from 0 to 100.

Q: What steps might we count up in from 0 to 100?

Count up and back in tens, fives, twenties, twenty-fives and fifties.

Demonstrate that the counting stick can be held horizontally or vertically to represent either one of the axes of a graph.

Show OHT 18.1. Explain that the lines represent 4 different scales. Point to a division on the first scale.

Q: What number would be here? How did you work this out?

Discuss the children's responses. Repeat this process using each scale. Ask the children to identify points that are on and between the divisions.

MAIN TEACHING ACTIVITY

Give out Resource Sheet 18.1.

Q: What information does this bar chart show?

Discuss responses and establish that the children can explain the meaning of the bar chart by referring to the labels and scales. Explain that this data is collected from page 12 of a book.

Use the following questions to develop children's understanding of the bar chart and the scale used.

Q: How many Es are on page 12? How many more Is are there than Us?**Q: How many of the vowels on page 12 are not As?**

Refer children to the table below the bar chart and explain that the table is another way of representing the information in the bar chart but that it has not been completed.

Q: What do we need to add to the table so that it represents the same information as the bar chart?

Establish that the heading 'frequency' means 'how many' there are of each letter.

Get the children to work in pairs to complete the table and collect their answers.

Tell the children that the other table on Resource Sheet 18.1 gives information about the number of some other letters on page 13 of the same book.

Q: How many Ts are there on page 13? How many more Ss than Ns are there?**Q: How can we represent this information as a bar chart?**

Establish that the bar chart would need to be completed as in the first example.

Q: What will go on each of the axes?

Establish that we need to show bars ranging in height from 9 to 23.

Q: Would a scale from 0 to 100 be sensible?

Agree that this would be too big and discuss alternative scales.

Ask the children to work in pairs to agree their scale and to draw the bar chart. Compare different bar charts and discuss the different scales used.

Give out Resource Sheet 18.2. Say that Sam and Jo carry out a survey of TV programmes that people watched.

Sam asked 120 people what TV programmes they watched and made a pie chart to show this information. Jo asked 72 people and recorded her information in a pie chart. Say the parts of the pie chart are called sectors. Get the children to estimate the fractional value represented by each sector of the pie charts. Use these estimates to find out how many people watched each programme. Ask the children to record their answers in the tables.

Q: What must the numbers total in Sam's and Jo's tables?

Agree that there were 120 people in Sam's survey and 72 people in Jo's survey.

Q: In which survey did more people watch soaps?

Establish that in Sam's survey more people watch soaps. Emphasise that even though the sector in Jo's pie chart is bigger, Sam asked more people.

PLENARY



Show OHT 18.2.

Q: What is missing from the bar chart?

Establish that the bar chart needs a title and a scale and that each of the bars needs a label.

Label one of the bars 'Crisps'.

Q: What could the bar chart be representing now?

Discuss possible labels for the other bars, the scale and the title of the bar chart. Agree on the names of the other bars, e.g. chocolate bars, cans of cola, etc. and that a title for the bar chart could be 'Number of items sold at the school fair'.

Say that the pie chart represents the same information.

Use the information above to discuss the sectors of the pie chart.

Q: What will the largest sector of the pie chart represent?

Q: What will the smallest sector of the pie chart represent?

Remember:

- A bar chart shows the frequency, or how many of each item there are by the length of the bars.
- Make sure you understand the scale on the bar chart.
- The sectors of a pie chart show how the total is divided up into different parts.

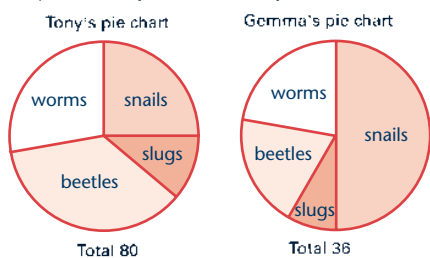
**LESSON 18 RELATED TEST QUESTION
2000 TEST A (NON-CALCULATOR PAPER)**

17

Tony and Gemma looked for snails, worms, slugs and beetles in their gardens.



They each made a pie chart of what they found.



(a) Estimate the number of worms that Tony found.

1 mark

Who found more snails?
Circle Tony or Gemma

Tony / Gemma

(b) Explain how you know.

.....

.....

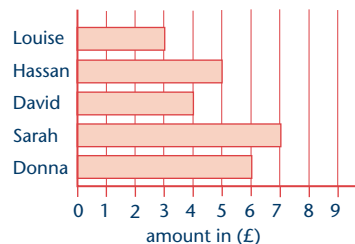
1 mark

**LESSON 18 RELATED TEST QUESTION
1999 TEST A (NON-CALCULATOR PAPER)**

10



Five children collect money to plant trees.
Here is a bar chart of the amounts they have raised so far.



Their target is £40 altogether.

How much more money do they need to reach the target?

Show your working. You may get a mark.

£

2 marks

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance	Question	Requirement	Additional Guidance
17a	An answer in the range 21 to 26 inclusive.	NO MARK is awarded for an answer which is not a whole number.	10	15	Accept £15.00 or £15.00p or £15 00 If the answer is incorrect, award ONE mark for evidence of appropriate working, e.g. $40 - (3 + 5 + 4 + 7 + 6) = \text{wrong answer}$. Calculation must be performed for the award of ONE mark. Accept £1500p or £1500 as evidence of appropriate working for ONE mark.
17b	An explanation which recognises that Tony's snails are a quarter of 80 and that Gemma's snails are half of 36, so that Tony found more, e.g. • 'Tony found 20 and Gemma found only 18'; • 'Quarter of 80 is more than half of 36'.	No mark is awarded for circling the correct answer of 'Tony'. DO NOT accept vague or arbitrary explanations, e.g. • 'Tony found loads more'; • 'Gemma found more but Tony's amount is bigger'. Accept a correct, unambiguous explanation even if the wrong name is circled.			

ANALYSIS OF CHILDREN'S ANSWERS

- On Question 10, children working at level 3 often read the information correctly but miscalculated or did not subtract the £25 from the £40.
- When interpreting the pie charts on Question 17, children's estimates too often fell outside the range allowed. Half the children working at level 3 and 4 who answered the second part of the question chose Gemma, explaining that the sector on her pie chart was bigger. These children ignored the totals the two pie charts represented.

IMPLICATIONS FOR PLANNING

- Test questions, introduced into the main teaching activity, can be used to generate further questions that require multistep responses. For example, 'How much more did Louise and Hassan collect than Donna?'
- The interpretation of charts and tables should be included in the oral and mental starter and the plenary. Children need to be taught that the sector on a pie chart represents a proportion, and that the total number of items represented by the pie chart has to be known in order to calculate how many each sector represents.

**SPRINGBOARD 6 LESSON 19
PROBLEM SOLVING 4**

TOTAL TIME



Objective:

- Solve simple problems involving ratio and proportion

Vocabulary:

- proportion
- ratio
- for every
- fraction

By the end of the lesson children should be able to:

- solve simple problems involving ratio and proportion by scaling up and down, and work out the fractional part from a given ratio.

Resources:

- OHT 19.1
- multilink cubes

ORAL AND MENTAL STARTER



Give out 4 black and 6 white multilink cubes to each child.
Using the cubes, ask the children to make the start of the repeating pattern:



Say that you have made a pattern where ‘for every 2 black cubes there are 3 white cubes’.

Explain that another way of describing the pattern is to say ‘for every 3 white cubes there are 2 black cubes’.

On the board draw the following table and write in the 2 and the 3.

Black cubes	White cubes
2	3

Rearrange the pattern to look like this:



Q: What proportion of the rod is black?

Establish that $\frac{4}{10}$ or $\frac{2}{5}$ of the rod is black.

Explain that the pattern could now be described as 'for every 4 black cubes there are 6 white cubes' or 'for every 6 white cubes there are 4 black cubes'. Add 4 and 6 to the table.

Q: If the rod had 8 black cubes, how many white cubes would there be?

Add 8 and 12 to the table.

Q: If the rod had 16 black cubes, how many white cubes would there be?

Discuss children's answers and add 16 and 24 to the table.

Q: If the rod had 9 white cubes, how many black cubes would there be?

Discuss children's responses and add the answer to the table.

Tell the children that you have a bag of cubes that have come from another class. Each child in this class had a rod the same as the children are now holding. All the rods were broken up and the cubes were put in the bag. Say that you have counted 80 black cubes.

Q: How many children are in this class? How many white cubes will be in the bag?

Establish that there are 20 children and 120 white cubes in the bag. Discuss the different ways the children worked out their answer and add the correct answer to the table.

Emphasise that all the numbers under 'Black cubes' are multiples of 2 and all the numbers under 'White cubes' are multiples of 3. Explain that the ratio of the black cubes to the white cubes is 2 to 3.

MAIN TEACHING ACTIVITY



Show OHT 19.1.

Tell the children that we are going to work out how much of each ingredient will be needed to make a potion for 15 wizards.

Q: How many frogs' eyes are needed to make a potion for 15 wizards?

Establish that 3 eyes would be needed for 1 wizard, so 15×3 eyes = 45 eyes will be needed for 15 wizards.

Q: What weight of dragons' hooves are needed for 15 wizards?

Establish that each wizard needs $\frac{1}{2}$ kg so 15 wizards will need $15 \times \frac{1}{2}$ kg = $7\frac{1}{2}$ kg.

Q: How can we use the same method to find how much sheep's liver will be needed for 15 wizards?

Explain that we can find how much is needed for 1 wizard by dividing 240 g by 6 and that for 15 wizards the answer will be:

$$(240 \div 6) \times 15 = 40 \times 15 = 600 \text{ g.}$$

Children work in pairs to answer the following question:

Q: For 40 wizards, how much of each ingredient will be needed?

Collect and discuss solutions. Discuss the children's methods and work through the method of dividing each quantity by 6 and then multiplying up by 40.

PLENARY

Pose the following problem:

Kim shares out 12 sweets. She gives Ashley 1 sweet for every 3 sweets she takes. How many sweets do they each get?

Discuss the children's answers. Establish that Ashley gets 3 sweets and Kim gets 9.

Q: What fraction of the sweets do they each get?

Establish that Ashley gets $\frac{3}{12}$ or $\frac{1}{4}$ and Kim gets $\frac{9}{12}$ of the sweets.

Pose the problem:

100 g of sweets cost 60p. Kim spends £1.50 on sweets. What quantity of sweets does Kim buy?

Discuss children's answers. Establish that Kim will get 50 g of sweets for 30p and 25 g for 15p. For £1.50 (150p) she will get 250 g.

Remember:

- When changing a potion or recipe, work out the quantities for 1 person, and then increase the quantities by multiplying by the number of people involved.
- 1 for every 4 means that there are units of 5 being shared out and the fractional parts are $\frac{1}{5}$ and $\frac{4}{5}$.

LESSON 19 RELATED TEST QUESTION 2001 TEST B (CALCULATOR PAPER)

19

Here is a recipe for raspberry ice cream.

<p>raspberry ice cream for 8 people</p> <p>$\frac{1}{2}$ litre of cream</p> <p>1kg raspberries</p> <p>250g sugar</p>
--



This recipe is for **8 people**.

Josie makes enough raspberry ice cream for **12 people**.

(a) How much **cream** does she use?




1 mark

Fred makes raspberry ice cream in the same way.

He uses $2\frac{1}{2}$ kg of raspberries.

(b) How much **sugar** does he use?



Show your **method**.
You may get
a mark.

2 marks

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
19a	$\frac{3}{4}$ or 0.75	Accept equivalent fractions.
19b	Award TWO marks for the correct answer of 625. If the answer is incorrect, award ONE mark for evidence of an appropriate method, e.g. 2.5×250 OR $250 + 250 + 125$	Accept for ONE mark 0.625 OR 6.25 OR 62.5 OR 6250 as evidence of appropriate method. Calculation need not be performed for the award of the mark.

ANALYSIS OF CHILDREN'S ANSWERS

- Children who answered the question incorrectly did not read the question carefully enough. Few children converted the units and annotated the question before attempting an answer.
- A common strategy was to halve the quantities in the recipe, but too often this was not followed through and answers remained incomplete. Few children who answered the second part incorrectly obtained the method mark, and children's strategies were often inefficient.

IMPLICATIONS FOR PLANNING

- Planned teaching activities that involve units of measure should ensure that time is devoted to discussing the units with the children, and to converting all the quantities to common units before any calculation takes place.
- In the main teaching activity, children should share the methods they use, compare the different strategies and decide on which strategy might be more efficient and why. Children need guidance on how to set out their methods in ways they can explain and use again.

SPRINGBOARD 6 LESSON 20

PROPERTIES OF AND REASONING ABOUT NUMBERS 1

TOTAL TIME

Objective:

- Use tests of divisibility and factors of numbers to inform and check division calculations

Vocabulary:

- multiple
- factor
- divisible by

By the end of the lesson children should be able to:

- Decide whether a number is divisible by 2, 3, 4 or 5 and by multiples of 2, 3, 4 and 5.

Resources:

- OHT 20.1
- OHT 20.2
- whiteboards and pens
- OHP calculator
- set of calculators

ORAL AND MENTAL STARTER


Show OHT 20.1.

Q: Each of these division calculations will have a remainder. Is this true or false?

For A, B and C, ask the children to write T (true) or F (false) on their whiteboards. Discuss their answers and their reasons.

Establish the tests of divisibility for 2, 3 and 5.

For A: 1347 is an odd number, so 1347 is not divisible by 2.

For B: 2196 does not end in a 5 or 0, so 2196 is not divisible by 5.

For C: the digits sum to 8 and as 8 is not divisible by 3, so 22211 is not divisible by 3.

Use an OHP calculator to confirm the above.

Remind the children that a number that divides into another number without a remainder is called a factor.

MAIN TEACHING ACTIVITY



Give out calculators. On the board, write ' $156 \div 12 = 13$ ' and get children to check this calculation with their calculators. Say that this means 12 is a factor of 156.

Q: Is 2 a factor of 156?

Establish that 156 is even so 2 is a factor of 156.

Q: Is 3 a factor of 156?

Establish that the digits of 156 sum to 12 and because 12 is divisible by 3, 156 is divisible by 3. Get children to check using their calculator.

Point to the 12 in the calculation $156 \div 12 = 13$ and ask:

Q: What are the factors of 12?

Establish that the factors of 12 are 1, 2, 3, 4, 6 and 12.

Q: Which of these factors do we know will divide exactly into 156?

Establish that we know 1, 2, 3 and 12 are factors of 156.

Q: Are the other factors of 12 also factors of 156?

Children use their calculators to confirm that 4 and 6 are also factors of 156.

Write on the board ' $198 \div 18 = 11$ '.

Get the children to list the factors of 18 on their whiteboards.

Establish that all the factors of 18 are also factors of 198.

Show OHT 20.2. On the first table partially cover all the numbers, revealing only the last two digits.

Q: What do you notice about the numbers you can see?

Establish that all the 2-digit numbers can be divided by 4.

Ask the children to think of a 2-digit number that is divisible by 4.

Write 34579 on the board. Ask the children to put their 2-digit number on to the end of 34579 and to divide this new 7-digit number by 4 using their calculators. Confirm that 4 is a factor of this number and to decide if 4 is a factor of any number we only need to know that 4 is a factor of the number made by the last 2 digits.

PLENARY

Return to OHT 20.1. Discuss D, E and F in turn.

For D:

establish that any number divisible by 8 must be even. As 4441 is odd, it will not be divisible by 8.

For E:

establish that the factors of 15 are 1, 3, 5 and 15. If 11105 is divisible by 15, it must be divisible by all the factors of 15.

Q: Is 11105 divisible by 3?

Establish that 11105 is not divisible by 3 because the digits do not sum to a multiple of 3, so 15 is not a factor of 11105.

For F:

partially cover the number revealing the last 2 digits.

Q: Is this 2-digit number (16) divisible by 4?

Establish that it is and say that because of this 26416 will be divisible by 4. Show the second table on OHT 20.2. Establish that each number is even, so is divisible by 2. Confirm that the numbers are also divisible by 3 by adding the digits. Explain that as the numbers have factors 2 and 3 they are divisible by 6.

Remember:

- Any odd number cannot be divisible by an even number.
- A number is divisible by 3 if the digits sum to a number that is divisible by 3.
- A number is divisible by 4 if its last 2-digit number is divisible by 4.
- If 24 divides into a number all the factors of 24 will divide into that number.

LESSON 20 RELATED TEST QUESTION
1999 TEST B (CALCULATOR PAPER)

4

Millie and Ryan play a number game.

What's my number?

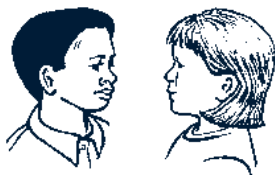


- Is it under 20? Yes
- Is it a multiple of 3? Yes
- Is it a multiple of 5? Yes

(a) What is the number?

1 mark

They play the game again.



- Is it under 20? No
- Is it under 25? Yes
- Is it odd? Yes
- Is it a prime number? Yes

(b) What is the number?

1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
4a	15	
4b	23	

ANALYSIS OF CHILDREN'S ANSWERS

- The response rate to the first part was high. The most common incorrect answer to the question was '10', as children only took account of two conditions.
- Children had greater difficulty managing the four conditions in the second part of the question. A common misconception was to assume 21 was prime.

IMPLICATIONS FOR PLANNING

- Children need to be taught that 1 is not a prime number, that 2 is the only even prime number, and not all odd numbers are prime.
- Oral and mental starters should include the identification of numbers that satisfy given properties. The planned activities should gradually increase the number of properties, with children being taught how to manage one property at a time and being shown how to check their final answer against each property.

SPRINGBOARD 6 LESSON 21

PROPERTIES OF AND REASONING ABOUT NUMBERS 2

TOTAL TIME

Objective:

- Use a given relationship expressed in words to develop a sequence and describe in words the rule for a given sequence.

Vocabulary:

- term
- sequence

By the end of the lesson children should be able to:

- use a given rule to generate a sequence;
- describe a sequence in words and decide if a given number is in the sequence.

Resources:

- OHT 21.1
- whiteboards and pens

ORAL AND MENTAL STARTER


Show the children OHT 21.1. Write the number 2 in the first box and the number 5 in the second. Say that these numbers are the start of a sequence.

Q: What might the next number in the sequence be?

Take responses and ask the children to explain their reasons. Say that there are many answers, but for the sequence you are thinking of, 8 is the next number. Write 8 in the third box. Say these are called terms of a sequence.

Q: What might the next number in the sequence be? How would you describe the rule for this sequence?

Agree that the next number is 11 and establish that there is a difference of 3 between each term. Confirm the rule would be 'add 3'.

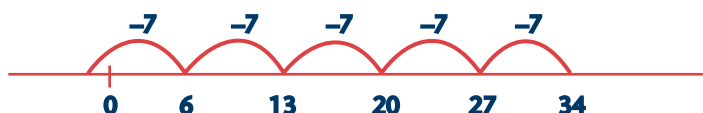
On OHT 21.1 write the sequence 34, 27, 20, 13, 6.

Q: How is this sequence different to the last sequence?

Establish that the pattern is descending and the rule is 'subtract 7'.

Q: What would be the next number in the sequence?

Use an empty number line to show that the next term would be -1 .



On OHT 21.1, write the sequence 1, 6, 11, 16.

Confirm that the next term is 21 and that the rule is 'add 5'.

Q: Will there ever be a multiple of 5 in this sequence?

Establish that each term in the sequence will always be a multiple of 5, plus 1.

MAIN TEACHING ACTIVITY



On OHT 21.1, write 3 in the first box and enter the rule 'add 5'. Ask children to develop the sequence by inviting them to enter the next term on OHT 21.1. On the board draw the pattern as dots as shown below.

Terms	3	8	13	18	23
Pattern	•••	••• •••••	••• ••••• •••••	••• ••••• ••••• •••••	••• ••••• ••••• ••••• •••••

Q: What do you notice about the pattern?

Explain that the pattern has 3 dots and rows of 5 dots.

Q: Will the number 143 be a term in the sequence?

Discuss the children's decisions and their reasons.

Q: How many rows of 5 will be in the pattern that represents 143?

Use the pattern to explain that the number of rows of 5 dots will be $(143 - 3) \div 5 = 28$.

Confirm that $3 + (28 \times 5) = 143$. As 143 can be represented by a pattern with a 3 and 28 rows of 5, it is a term in the sequence.

Q: Will 79 be in the sequence?

Use a similar method to establish that $79 - 3 = 76$ and that 76 is not a multiple of 5, so 79 is not a term in the sequence.

On OHT 21.1, write '7' in the first box and enter the rule 'add 4'.
Point to the seventh box and ask:

Q: How can we work out this term in the sequence without completing all the boxes in between?

Discuss strategies and establish that the seventh term will be 7 plus 6 rows of 4 and the term is $7 + (6 \times 4) = 31$.

Q: Will the sequence ever contain an even number?

Explain that multiples of 4 are always even, but adding 7 makes the numbers in the sequence odd.

PLENARY

Tell the children that Alan and Rose make a sequence. They start with the number 4. Alan always adds 3 and Rose always adds 2. Alan goes first.

Ask children to discuss the sequence and to write down the first 5 terms.

Agree on the sequence 4, 7, 9, 12, 14.

Q: After another 2 goes each, what will be the last term in their sequence?

Establish that together Alan and Rose add 5. This time they add 10 to get to 24.

Q: If the starting number is 11 and Alan and Rose each have 12 turns, what will be the last term in the sequence?

Children show their answers on whiteboards. Collect responses and discuss their strategies.

Remember:

- Decide if the rule means you add or subtract a number each time.
- To decide whether a given number is a term in the sequence, check whether the gap between the first number and the given number is a multiple of the number in the rule.

**LESSON 21 RELATED TEST QUESTION
2001 TEST B (CALCULATOR PAPER)**

22 The rule for this sequence of numbers is 'add 3 each time'.

1 4 7 10 13 16 ...

The sequence continues in the same way.

Mary says,

'No matter how far you go there will never be a multiple of 3 in the sequence'.

Is she correct?
Circle Yes or No.

 Yes / No

Explain how you know.



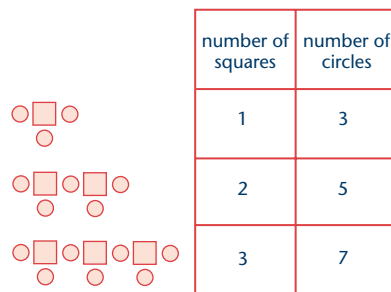
.....

.....

1 mark


**LESSON 21 RELATED TEST QUESTION
2001 TEST A (NON-CALCULATOR PAPER)**

23 Here is a sequence of patterns made from squares and circles.



The sequence continues in the same way.

Calculate how many **squares** there will be in the pattern which has **25 circles**.

 Show your working. You may get a mark.

2 marks

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
22	Explanation which recognises that each number is one more than a multiple of 3, e.g. <ul style="list-style-type: none"> • 'It starts at 1 and keeps adding 3 so it misses all the multiples of 3'. • 'Multiples of 3 are all 1 less than the numbers'. 	No mark is awarded for circling 'Yes' alone. DO NOT accept vague or arbitrary explanations such as: <ul style="list-style-type: none"> • 'They're too big'; • 'It doesn't go far enough'; • 'It is adding 3 all the time'. If 'No' is circled but a correct unambiguous explanation is given then award the mark.

Question	Requirement	Additional Guidance
23	Award two marks for the correct answer of 12. If the answer is incorrect, award ONE mark for evidence of appropriate working, e.g. $25 - 1 = 24$ $24 \div 2 =$ wrong answer	Calculation must be performed for the award of ONE mark.

ANALYSIS OF CHILDREN'S ANSWERS

- Few children working at levels 3 or 4 answered these questions successfully. On Question 22, children had difficulty providing a clear explanation; they often referred to the three-times table but not to the numbers in the sequence to explain why they were different.
- Children had difficulty extending and generalising the sequence in Question 23. Children who answered the question successfully used the diagram to help them and generated the pattern from the picture rather than the numbers.

IMPLICATIONS FOR PLANNING

- Children need to be taught how to build up a sequence arithmetically, to identify the rule, and to use this to extend the pattern.
- The planning for the main part of the lesson should give children the opportunity to explain and justify their decisions, both orally and in writing, and to try to convince a 'sceptical friend'. The plenary also provides an opportunity to assess how well children can use mathematical language to articulate their thoughts.

SPRINGBOARD 6 LESSON 22

PROPERTIES OF AND REASONING ABOUT NUMBERS 3

TOTAL TIME

 A circular icon with the number 30 in the center and the word MINUTES written around the bottom edge.
Objective:

- Use knowledge of number facts and properties of numbers to solve number problems

Vocabulary:

- inverse operation
- units digit
- product

By the end of the lesson children should be able to:

- Use a calculator and known number facts to solve simple number problems.

Resources:

- OHT 22.1
- set of calculators
- OHP calculator
- whiteboards and pens

ORAL AND MENTAL STARTER

 A circular icon with the number 10 in the center and the word MINUTES written around the bottom edge.

Tell children you are thinking of a number. Say:

Q: I multiply my number by 38 and the product is 532. What is my number?

Ask the children to write on their whiteboards a number statement that represents this question.

Collect responses and on the board write: ' $\square \times 38 = 532$ '. Emphasise that 'product' refers to multiplication.

Q: Using a calculator, what strategies would you use to solve this problem?

Remind children how they have used inverse operations in the past. Establish that

$$\square \times 38 = 532 \text{ means } \square = 532 \div 38.$$

Use an OHP calculator to find the answer. Check the answer by confirming that $14 \times 38 = 532$.

Write on the board;

$$\square \div 36 = 24$$

$$36 \times \square = 1368$$

$$1296 \div \square = 54$$

Ask the children to find the missing numbers using a calculator. Collect responses and ask the children to explain their methods and to talk through their calculations. Emphasise the use of the inverse operations.

MAIN TEACHING ACTIVITY



Write ' $\square \square \times 6 \square = 6272$ ' on the board. Explain that each box represents a digit and that we are trying to find two 2-digit numbers that multiply together to make 6272. Point to the 6 and say that one of the numbers must be 'sixty something'.

Q: How could we begin to solve this problem?

Ask children what the digit after the 6 could be.

Discuss their responses and show OHT 22.1.

Point to the statements involving 60 and 65.

Q: The answer cannot involve either of these two numbers. Can you explain why?

Explain that 6272 is not a multiple of 60 or 65 because the units digit in 6272 is not a 0 or a 5. Cross out the statements involving 60 and 65 on OHT 22.1.

Q: Could 61 be the number?

Using your OHP calculator, try $6272 \div 61$. Point to the display of 102.81967...

Q: What does this answer mean?

Establish that the answer to the problem cannot be the number 61 because 61 is not a factor of 6272. Cross out the statement on OHT 22.1 involving 61.

Remind the children how to use the calculator's memory and use this procedure to test the other statements, beginning with 62.

Step 1: Clear memory.

Step 2: Enter 6272.

Step 3: Press M+ (this puts 6272 into the memory).

Step 4: Clear the display by pressing the C/CE button once (there will be a 0 and M in the display).

Step 5: Press the recall memory key RCM (6272 will reappear).

Step 6: Enter $\div 62$ and press = for the answer.

Step 7: Interpret the display. Is it a whole number?

If not, press clear and repeat Steps 4 to 7 with the remaining numbers.

Establish that $98 \times 64 = 6272$.

Children work in pairs to solve $\square \square \times \square 3 = 1764$ using a procedure similar to the one above.

Collect responses and discuss the strategies the children used.

PLENARY



Write $\square \times \square \times \square = 4913$ on the board.

Explain that each box represents the same number.

Q: Is the number greater than 10?

Get the children to multiply 10s in their heads, and agree the answer to $10 \times 10 \times 10 = 1000$ means that the number in the box must be greater than 10.

Q: Is the number greater than 20?

Establish that the number must be between 10 and 20.

Q: Why would 15 be a good number to try next?

Establish that choosing the mid-value will cut the number of possibilities by half.

Children work in pairs to find the correct answer.

Remember:

- When finding missing numbers use the inverse operation to rewrite the number statement.
- Look at the last digit to see if you can use your knowledge of number facts to eliminate possible values.
- Start by using values that you are able to work out quickly in your head.

LESSON 22 RELATED TEST QUESTION 2001 TEST B (CALCULATOR PAPER)

21 Write in the missing number.

$$404.09 \div \boxed{} = 8.5$$

1 mark

LESSON 22 RELATED TEST QUESTION 2000 TEST B (CALCULATOR PAPER)

12 Nadia is working with **whole** numbers.

She says,

'If you add a two-digit number to a two-digit number you cannot get a four-digit number'.

Is she correct? Circle Yes or No.

Yes / No

Explain why.

.....

.....

.....

1 mark

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance	Question	Requirement	Additional Guidance
21	$404.09 \div \boxed{47.54} = 8.5$		12	Explanation which recognises that the largest two-digit number (99) added to itself only gives a three-digit number (198) e.g. 'Because if you do $99 + 99$ you only get a three-digit number'; 'if you add any 2 two-digit numbers, you will get a three-digit number or a two-digit number'.	No mark is awarded for circling 'Yes' alone. DO NOT accept vague or arbitrary explanations such as: <ul style="list-style-type: none"> • 'The numbers aren't big enough'; • 'It doesn't work'; If 'No' is circled but a correct unambiguous explanation is given then award the mark.

ANALYSIS OF CHILDREN'S ANSWERS

- Children working at levels 3 and 4 found both questions difficult, and on Question 12, omission rates were unusually high. Children who answered the question usually agreed with Nadia but many could not explain why they agreed.
- Children who answered Question 21 too often relied on trial and improvement, which resulted in strange answers with no evident rationale. Few children working at levels 3 or 4 wrote down a calculation involving an inverse operation they could use on their calculators.

IMPLICATIONS FOR PLANNING

- In the main part of the lesson children should be asked to decide whether a statement is true or false and be expected to justify their choice. The plenary should provide an opportunity for children to ask questions and discuss different methods to help them to use appropriate language to support their mathematical reasoning.
- Work on identifying inverse operations should include the use of a calculator. Finding inverses when undertaking simple calculations needs to be firmly established so children can apply their understanding to more complex calculations.

LESSON 12 ADDITION AND SUBTRACTION 1**RESOURCE SHEET 12.1**

School	Year 6 children
Subtraction Street Primary	28
Fewer Than Field Primary	46
Parallelogram Park Junior	67
Roundup Road Middle	93
Greater Than Grove Primary	33
Addition Avenue Primary	29
Cylinder Crescent Junior	58
Number Line Lane Primary	41
Trapezium Town Junior	57
Integer Enclosure Primary	16
Total Terrace Primary	76
Double Drive Middle	128

LESSON 12 ADDITION AND SUBTRACTION 1**OHT 12.1**

How many ...

Subtract ...

Add ...

More than ...

Sum ...

Difference ...

Take away ...

Altogether ...

Minus ...

Total ...

Less than ...

LESSON 12 ADDITION AND SUBTRACTION 1

RESOURCE SHEET 12.2

Question 1

ZOO

1470 visitors this week
Up by 174 on last week

How many visitors were there last week?

Question 2

YOUTH CLUB

£5740 collected so far.
Need another £6985.

How much money does the youth club hope to collect?

Question 3

0 0 4 2 6 8

Remember to service your car every 6000 miles.

How many miles to go before the first service?

Question 4

Cinema ticket
Adult £2.75

Cinema ticket
Child £1.80

Ice cream 75p

Popcorn £1.10

Cola 60p

Crisps 50p

Chocolate bars 80p

Two adults and three children visit the cinema. The adults each have an ice cream. The children share a bag of popcorn and have a cola each. How much change will there be from £20?

LESSON 13 ADDITION AND SUBTRACTION 2

RESOURCE SHEET 13.1

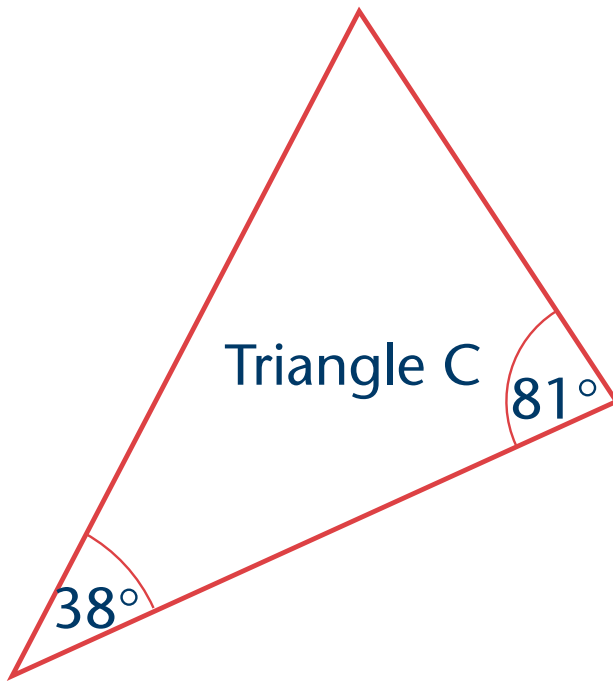
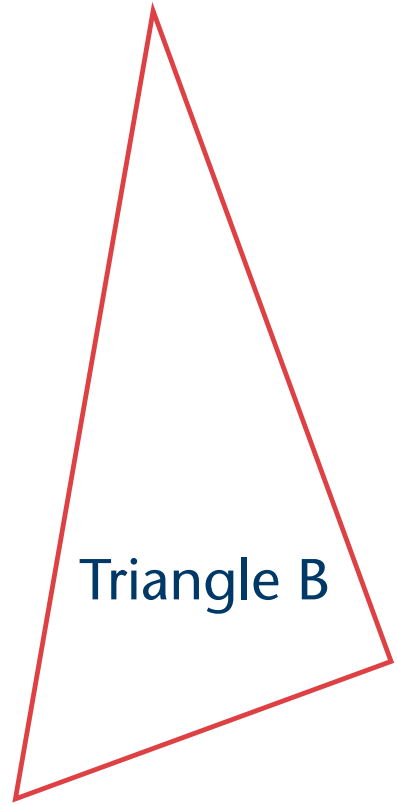
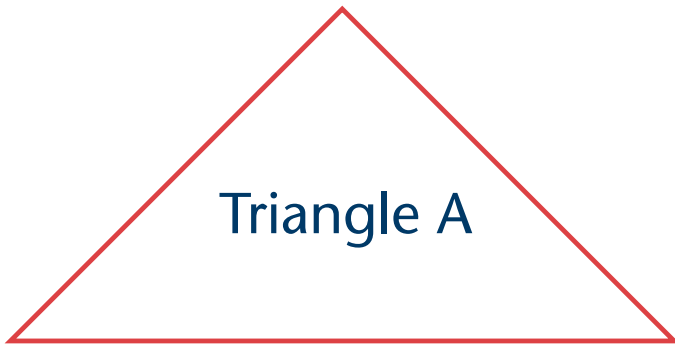
Timetable showing the train times from York to London

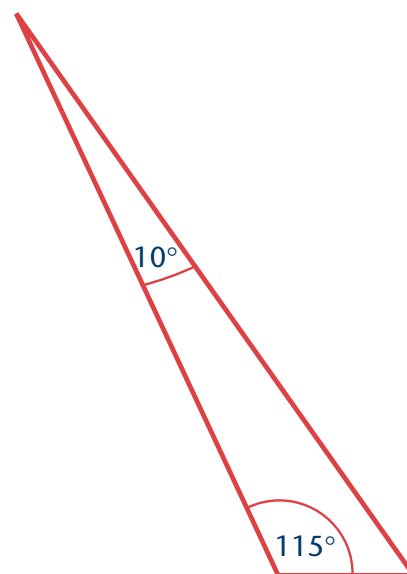
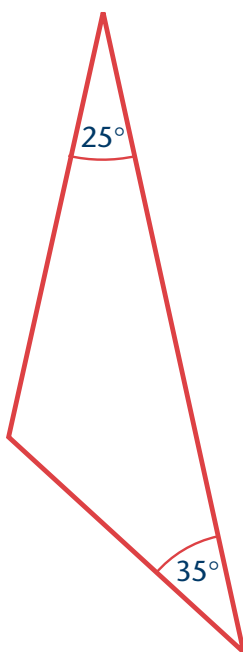
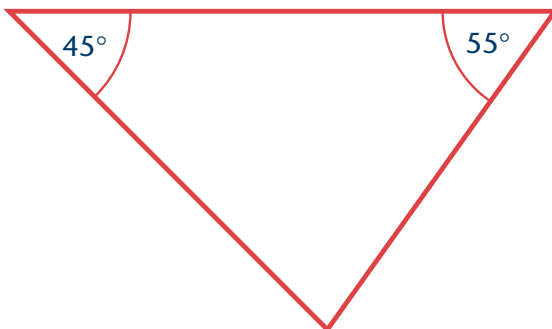
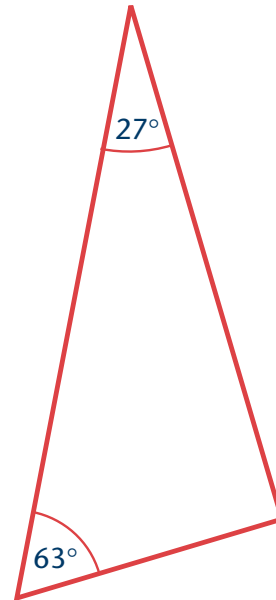
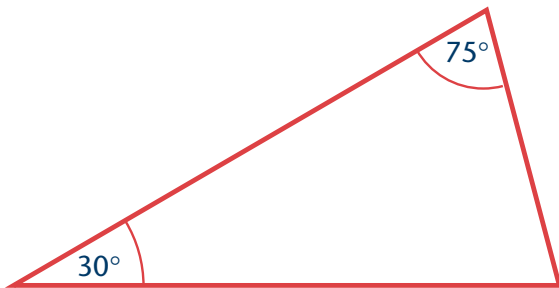
York	0744	0906	0943	1406
Leeds	0835	0938	1015	
Sheffield	0927	1027	1127	1527
Derby	1003	1103	1203	1603
Bedford	1109			1709
London	1147	1246	1342	1750

1. What time does the 1603 train from Derby arrive in London?
2. How long does this journey take?
3. I live in York and I need to be in Derby before noon. Which train do I need to catch?
4. How long is the journey from York to Derby when I catch the 0943 from York?

LESSON 14 SHAPE AND SPACE

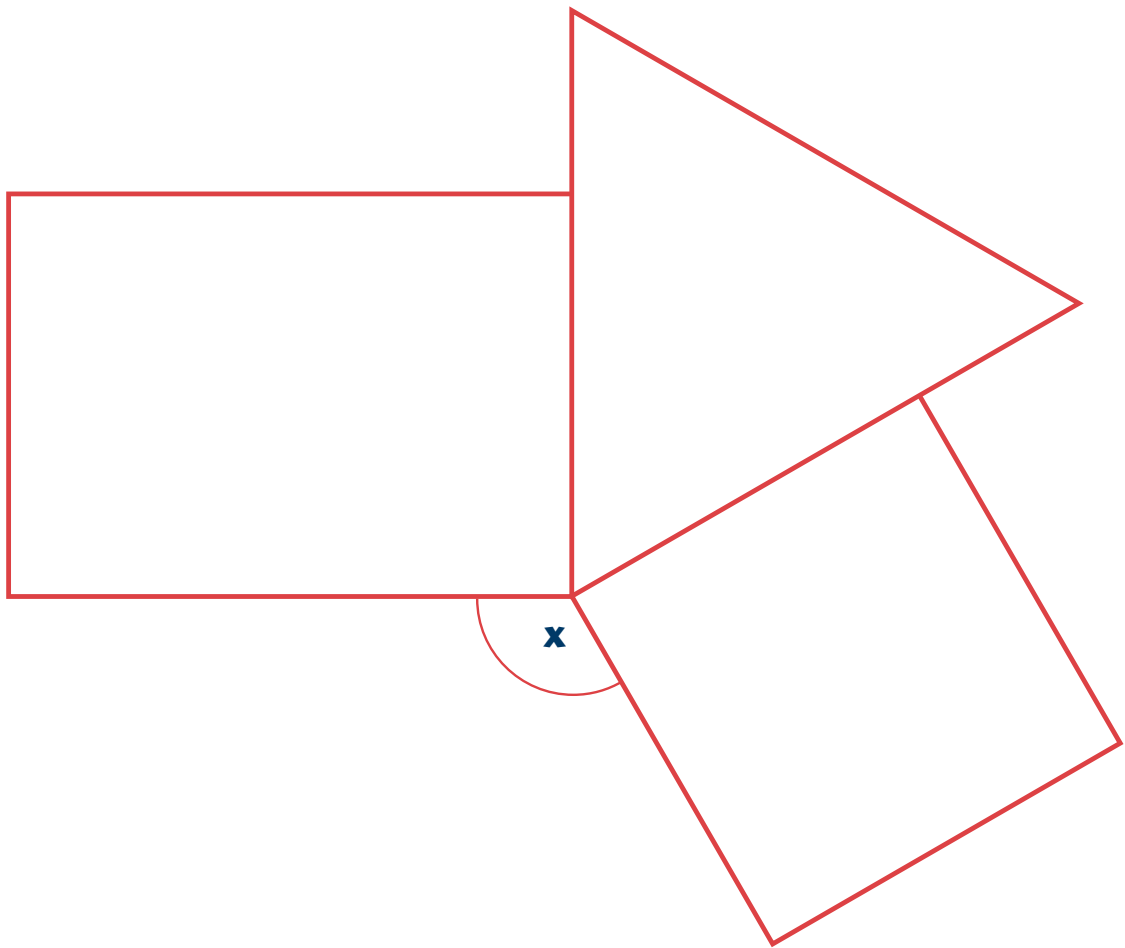
OHT 14.1



LESSON 14 SHAPE AND SPACE**RESOURCE SHEET 14.1**

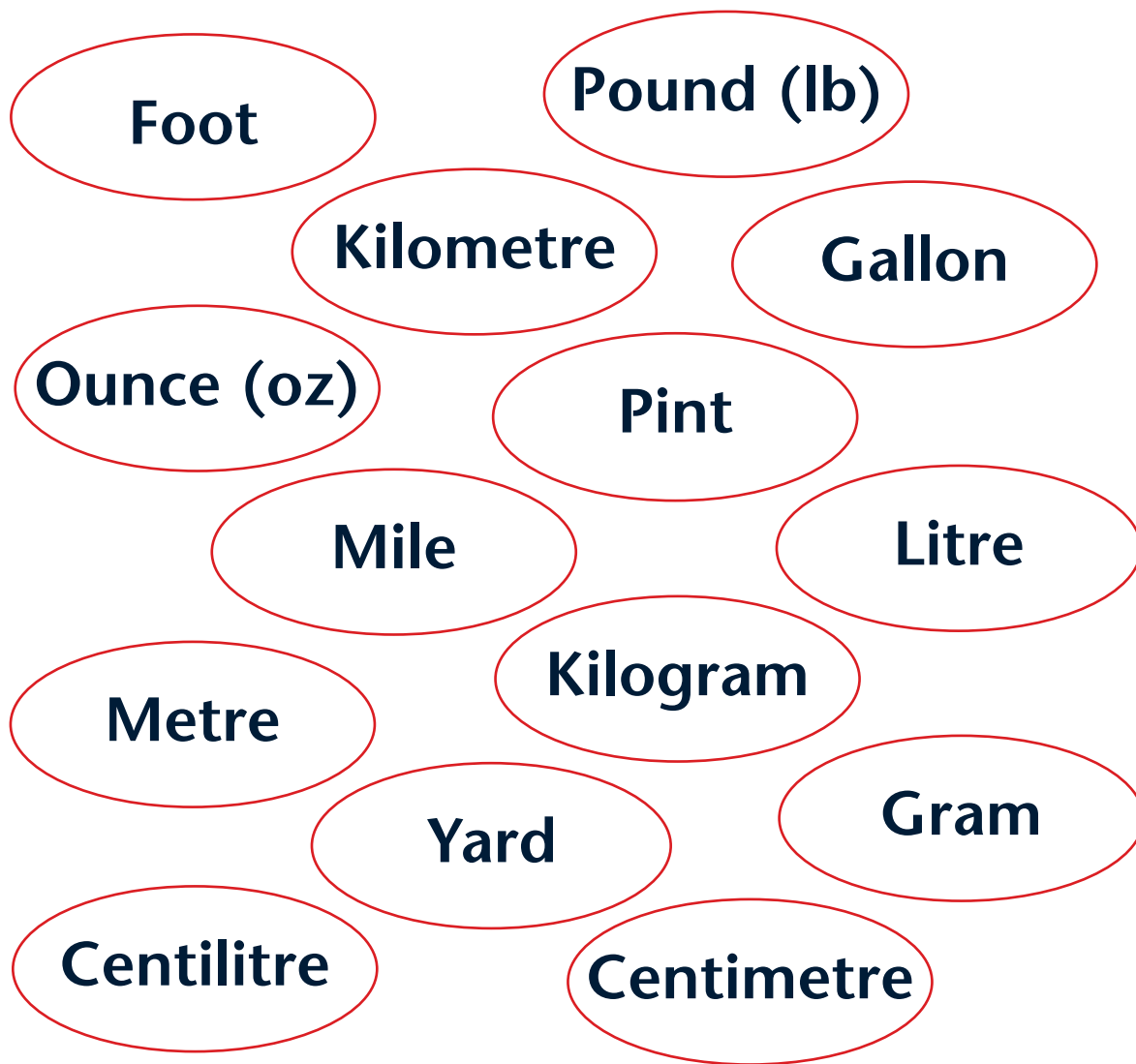
LESSON 14 SHAPE AND SPACE

OHT 14.2



LESSON 15 MEASURES 1

OHT 15.1



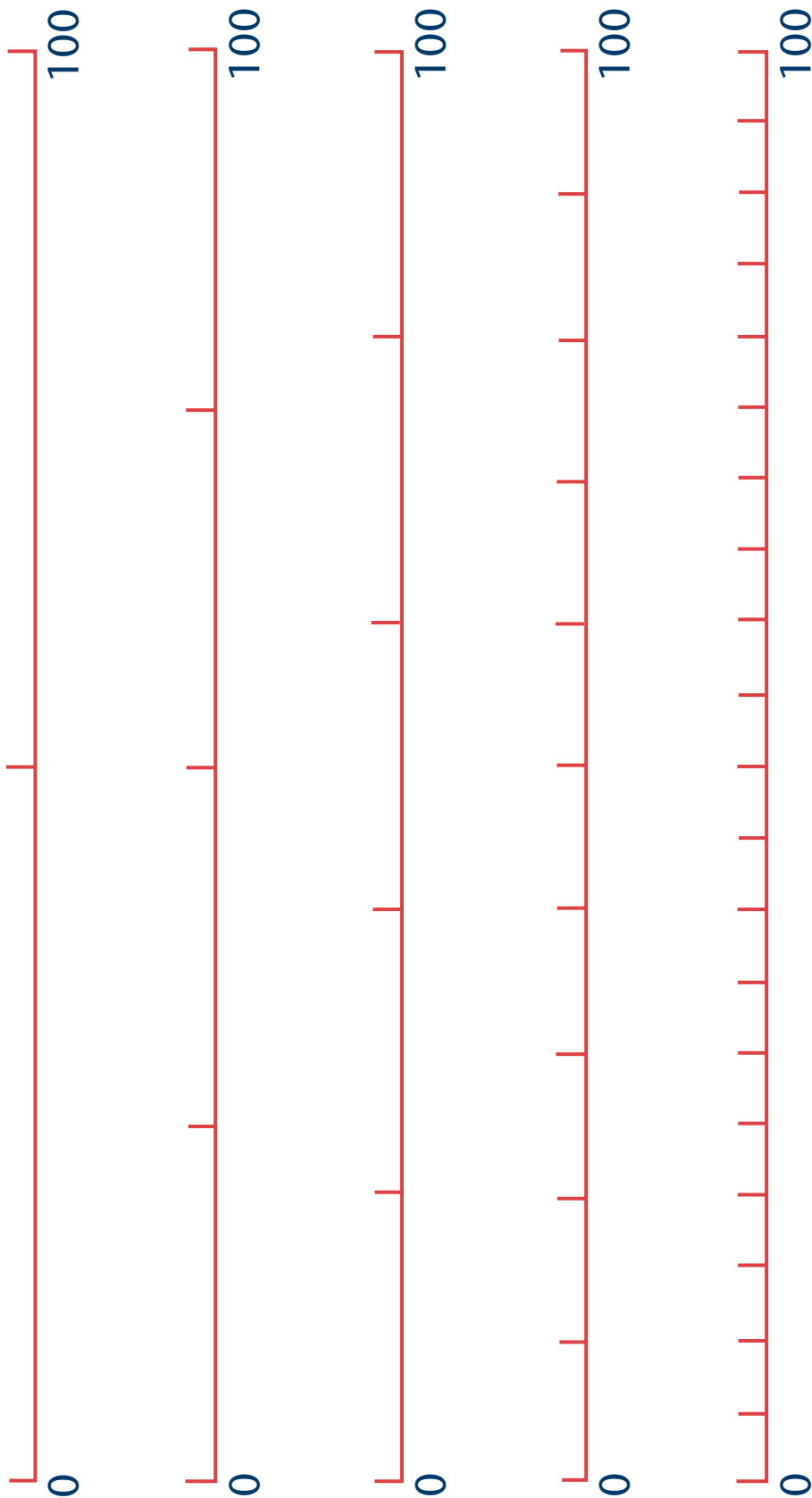
Length	Metric or Imperial	Weight	Metric or Imperial	Capacity	Metric or Imperial

LESSON 15 MEASURES 1**RESOURCE SHEET 15.1**

Metre	Kilometre
Mile	Gallon
Pint	Gram
Yard	Foot
Centimetre	Pound (lb)
Ounce (oz)	Kilogram
Litre	Centilitre

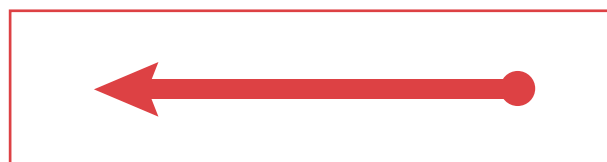
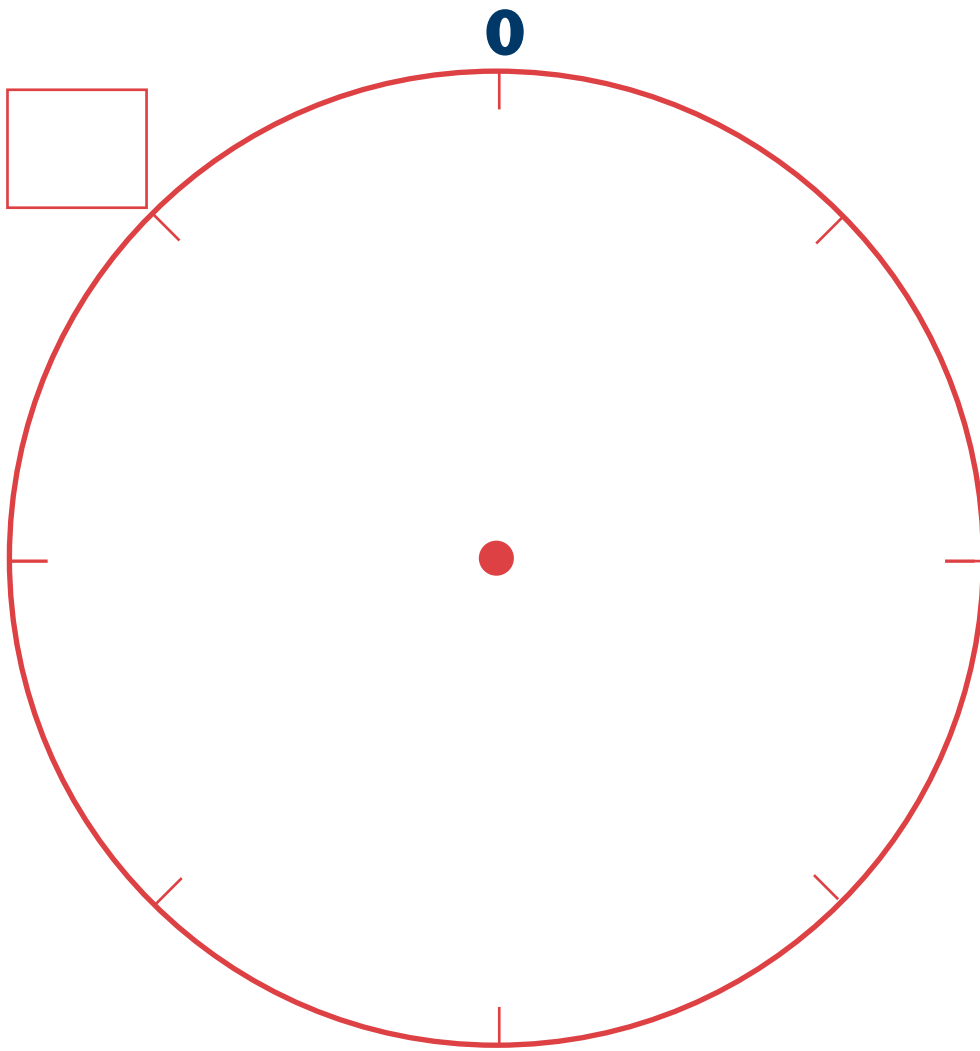
LESSON 16 MEASURES 1

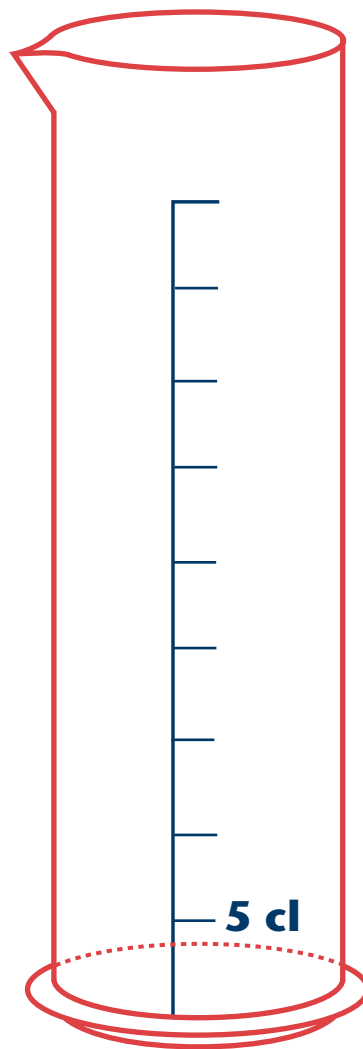
OHT 16.1



LESSON 16 MEASURES 2

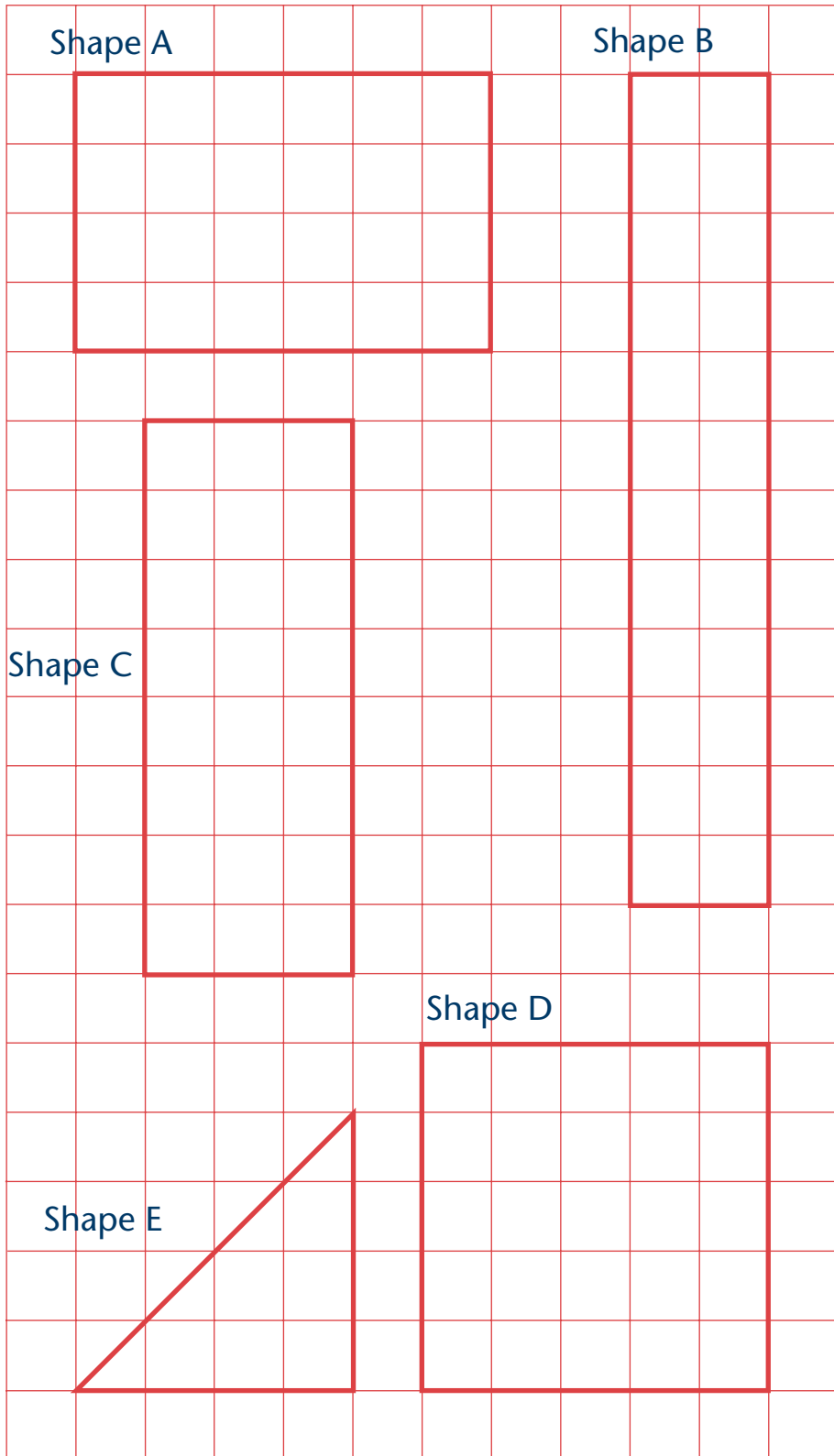
OHT 16.2



LESSON 16 MEASURES 2**OHT 16.3**

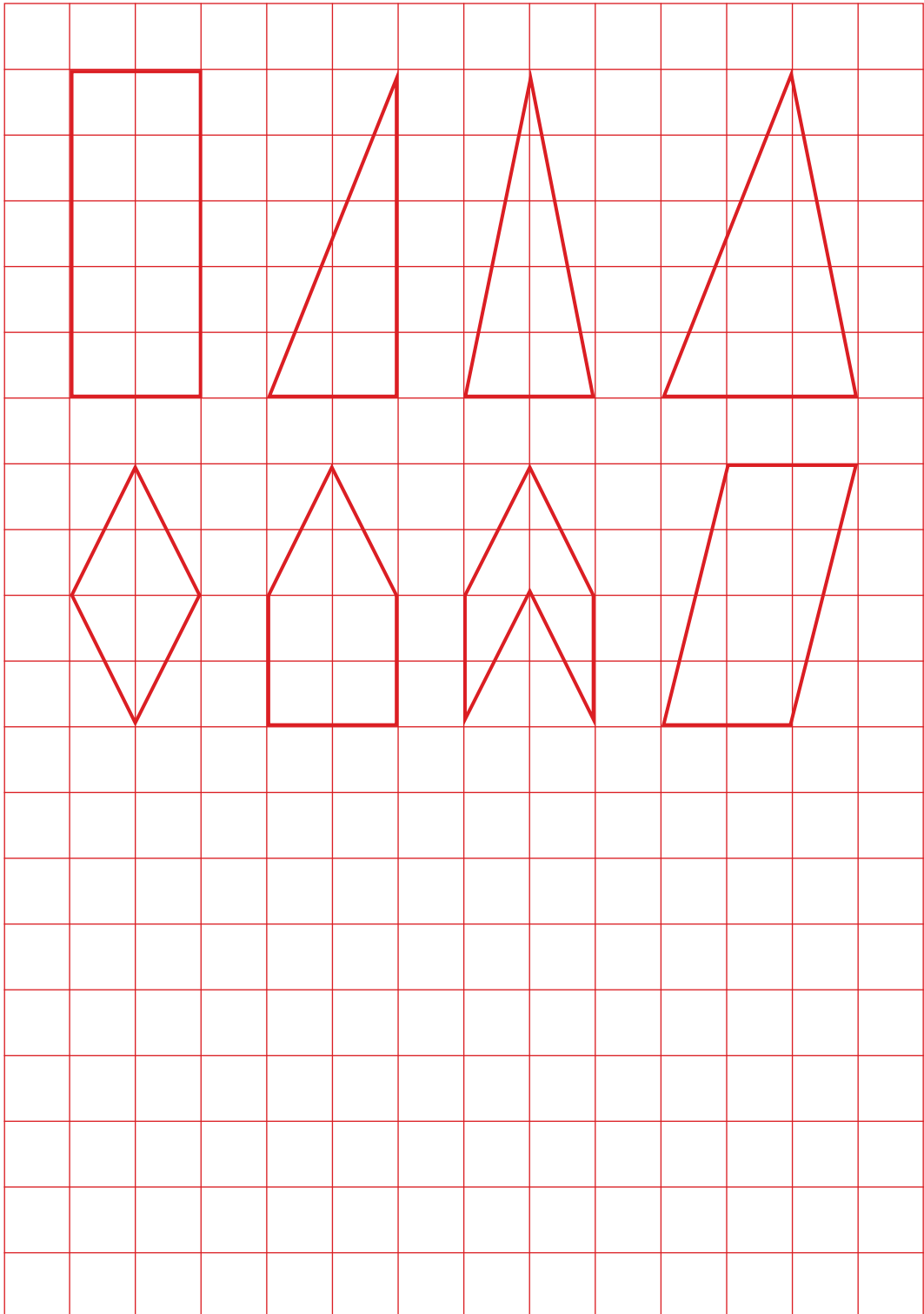
LESSON 17 MEASURES 3

OHT 17.1



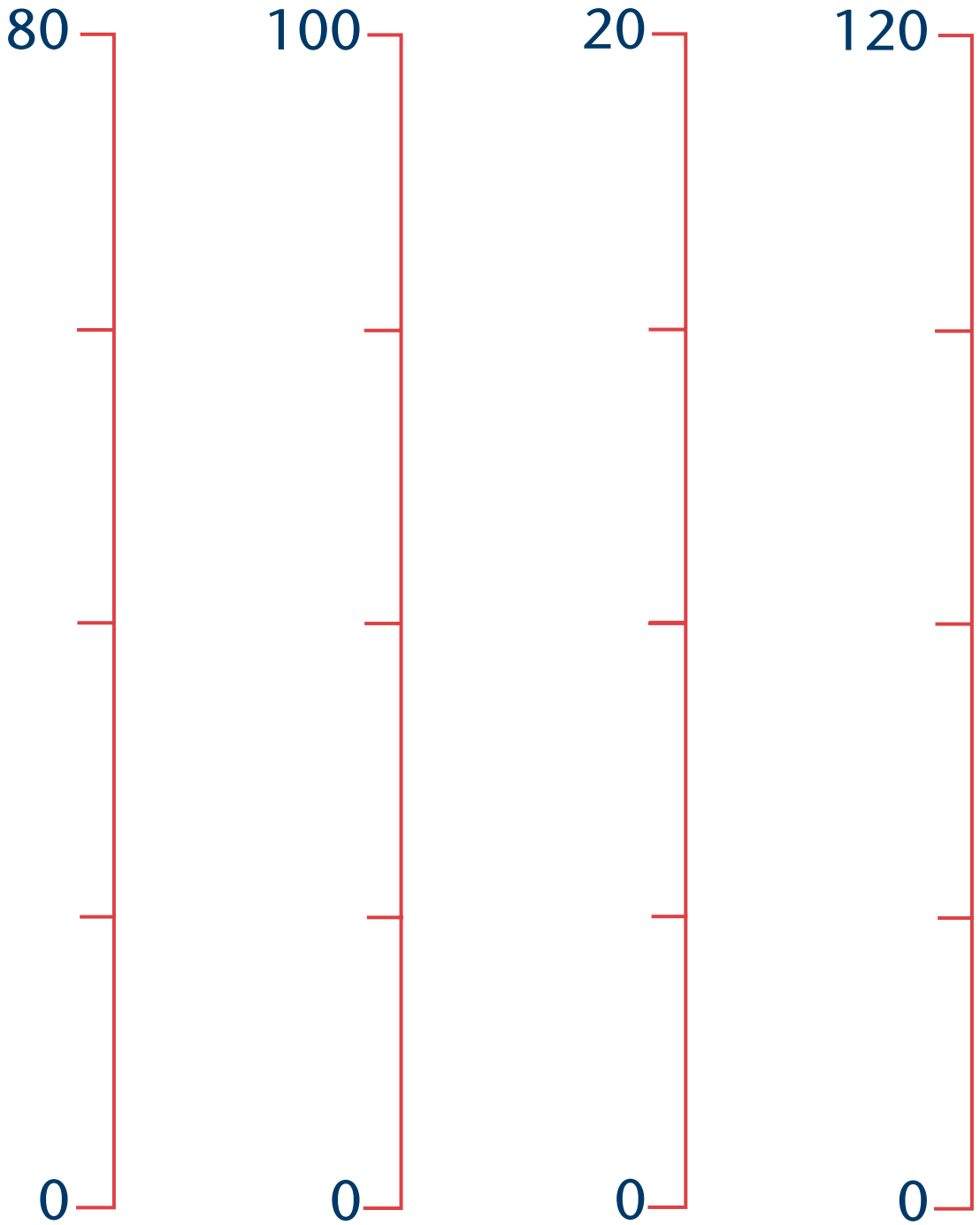
LESSON 17 MEASURES 3

RESOURCE SHEET 17.1



LESSON 18 PROBLEM SOLVING 3

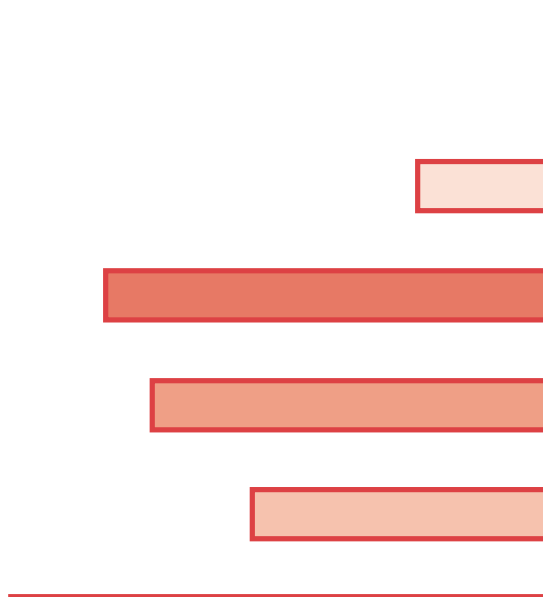
OHT 18.1



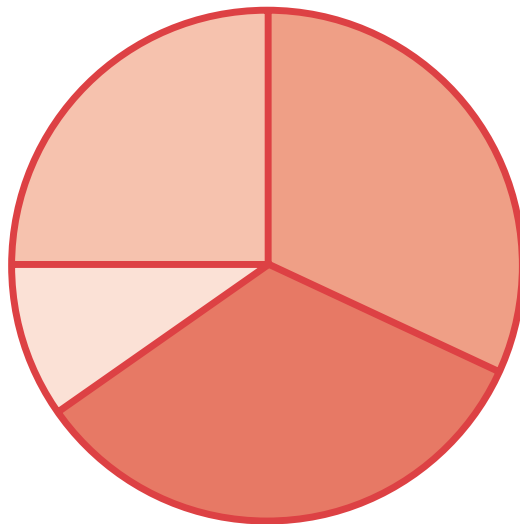
LESSON 18 PROBLEM SOLVING 3

OHT 18.2

Bar Chart



Pie Chart

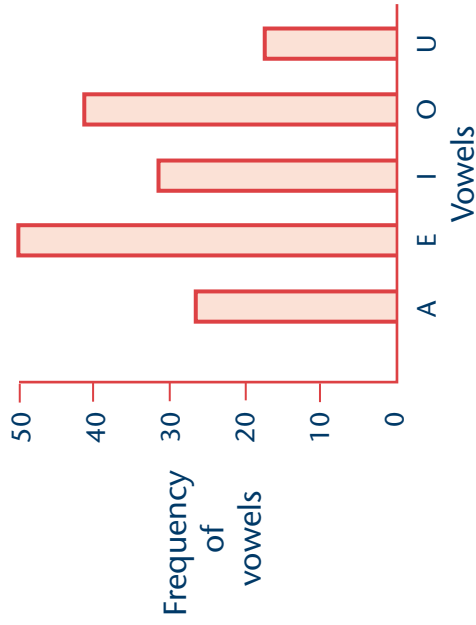


LESSON 18 PROBLEM SOLVING 3

RESOURCE SHEET 18.1

Letters in my book

Page 12



Vowel	Frequency
A	
E	
I	
O	
U	
Total	

Page 13



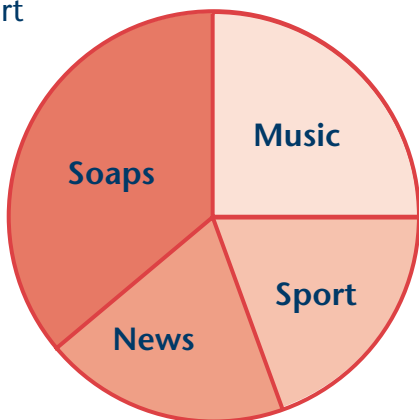
Letter	Frequency
R	11
T	18
H	16
S	23
N	9
Total	

LESSON 18 PROBLEM SOLVING 3

RESOURCE SHEET 18.2

What type of TV programme do you watch?

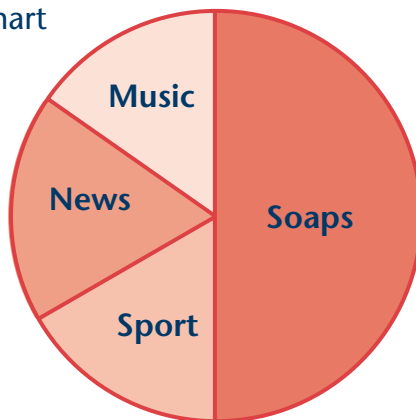
Sam's pie chart



People asked: 120

TV programme	People
Sport	
Music	
Soaps	
News	
Total	

Jo's pie chart



People asked: 72

TV programme	People
Sport	
Music	
Soaps	
News	
Total	

LESSON 19 PROBLEM SOLVING 4

OHT 19.1

Wizards' Magic Potion

For 6 wizards:

18 frogs' eyes

240 g of sheep's liver

3 kg of dragons' hooves



Make a potion for 15 wizards.

LESSON 20 PROPERTIES OF AND REASONING ABOUT NUMBERS 1**OHT 20.1**

$$A \quad 1\,347 \div 2$$

$$B \quad 2\,196 \div 5$$

$$C \quad 22\,211 \div 3$$

$$D \quad 4\,441 \div 8$$

$$E \quad 11\,105 \div 15$$

$$F \quad 26\,416 \div 4$$

LESSON 20 PROPERTIES OF AND REASONING ABOUT NUMBERS 1**OHT 20.2**

All these numbers are divisible by 4
124
128
264
304
144
1 000 024
312
636
484
234 446 344

All these numbers are divisible by 6
624
102
456
2 448
216
270

LESSON 22 PROPERTIES OF AND REASONING ABOUT NUMBERS 3**OHT 22.1**

$$\square \square \times 60 = 6272$$

$$\square \square \times 61 = 6272$$

$$\square \square \times 62 = 6272$$

$$\square \square \times 63 = 6272$$

$$\square \square \times 64 = 6272$$

$$\square \square \times 65 = 6272$$

$$\square \square \times 66 = 6272$$

$$\square \square \times 67 = 6272$$

$$\square \square \times 68 = 6272$$

$$\square \square \times 69 = 6272$$

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