SPRINGBOARD 6 LESSONS FOR USE IN BOOSTER CLASSES

This Springboard 6 file contains the third set of eight 45-minute lessons for use in booster classes. You should already have the first two sets, which contain lessons 1–22.

You may order further copies of all three sets of materials. To do this, you need to:

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department for
education and skills

SPRINGBOARD 6
Problem solving

Contents

INTRODUCTION

These eight Springboard 6 lessons, like the previous 22 lessons, are designed to provide additional support for booster classes. The focus of these eight lessons is on problem solving: three of the lessons incorporate the use of calculators in problem solving.

Unlike the other 22 lessons, these lessons are 45-minute lessons. Supported with additional materials, and additional problems for the children to solve, each lesson could form the basis of a daily mathematics lesson. Alternatively, without the oral and mental starter, and with adjustments to the plenary, the lessons could fit in with the half-hour model of the other 22 lessons.

All the general principles of effective mathematics teaching apply to these eight lessons. Some elements are especially important for booster classes:

- a step-by-step approach;
- built-in consolidation and summaries;
- the use of direct questions and discussion about ideas and methods;
- the expression of the same mathematical ideas in a variety of ways;
- the use of demonstration by the teacher and the modelling of ideas and methods to help children to visualise the processes involved;
- the reinforcement of key mathematical vocabulary;
- the exemplification of ways to record methods and calculations, and to annotate and interpret diagrams, graphs and charts;
- the encouragement of children to articulate their mathematical thinking, orally and in writing.

You will need to adjust the lessons to take account of your children's current attainment levels, their progress and the responses they make.

PROBLEM SOLVING

Problem solving is a key skill in the 'using and applying mathematics' strand of the National Curriculum. These eight lessons are designed to help children to:

- select and apply strategies to solve problems set in different contexts;
- describe their methods of solutions and justify their decisions and conclusions;
- reason and make deductions that they can explain, present and record.

To help you plan how to make best use of these lessons, the lessons are designed to support the teaching of those problem-solving skills associated with three types of problems:

Word problems – problems that provide information in a 'story' or context from which children are to identify the required operation and do the calculations. This may involve children in one or more calculations and steps to get the required solution.

Diagram problems – problems that set out information in the form of a table, diagram or graph and require children to interpret, select and use information. Children may need to perform some calculations, sort shapes, read a scale, make decisions, show their methods and justify answers with a short explanation.

Deduction problems – problems that present a set of information, complete or partial, which children are to use to undertake calculations, draw conclusions or identify relationships. Children may be asked to explain patterns and their reasoning and support this with any necessary calculations.

The lessons addressing these problem types are shown below.

Springboard 6 – Lessons 23 to 30	**Related lessons**
Word problems	Lesson 23 (Using a calculator to solve problems 1) Lesson 25 (Using a calculator to solve problems 3)
Diagram problems	Lesson 24 (Using a calculator to solve problems 2) Lesson 28 (Problem solving 7) Lesson 30 (Problem solving 9)
Deduction problems	Lesson 26 (Problem solving 5) Lesson 27 (Problem solving 6) Lesson 29 (Problem solving 8)

HELPING CHILDREN TO REVISE

Seven effective strategies to help children revise are described below. The strategies described include ways of building revision into ongoing teaching, helping children to help themselves and to demonstrate successfully the mathematics they know, understand and can do.

1 USING PREVIOUS TEST QUESTIONS WHEN TEACHING

An effective revision strategy is to incorporate relevant test questions into the teaching of **each** unit of work. A number of schools use the QCA Testbase CD-ROM. Teachers select test items and the corresponding mark schemes from the mental and written (both calculator and non-calculator) tests to use in their lessons. Attached to each lesson are related test questions from 2002 to help you to select and use test items. The advantages for the children include:

- the opportunity to discuss and compare different approaches to the questions in order to **consolidate a strategy** that they feel confident with and can use successfully;
- a familiarisation with the different **question types**, including an understanding of what is meant by 'show your working' and 'show your method';
- an understanding of how the mark scheme works for different types of questions, including those with a 'show your working' or 'show your method' box and those worth more than one mark. Children learn how they can get 'partial credit' on these questions;
- a more systematic development of their confidence in their ability to answer test questions.

2 REFINING AND HONING THE SKILLS FOR EACH OF THE FOUR RULES OF NUMBER

Analysis of recent test papers suggests that a significant proportion of children, when answering test questions involving the four rules of number, use calculation strategies that they are not comfortable with and do not understand fully. Consequently they are unsuccessful. There is evidence that many of these children **could** have been successful if they had chosen a different method. An effective revision strategy is to review each of the four operations in turn, using a set of different question types for each operation, drawn from the QCA Testbase CD-ROM or elsewhere. For example, the attached revision worksheet concentrates on a variety of subtraction questions.

SUBTRACTION REVISION

Use an appropriate method to answer each of the following questions. You can use different methods but you need to be able to explain your method and your reasoning for each question. Remember to check your answers to be confident that they are correct.

1. 74–48

2. 175–81

3. 3000–1997

4. 1025–336

5. 28.34–17.29

6. 150–? = 27

7. Jo bought a box of cards for £6.48 and paid with a £10 note. How much change should she get?

8. Sam has a 3.5 m length of string. He cuts off 1.75 m. How much is left?

When sharing and discussing children's responses to the questions, the aim would be for the children to develop the skills and confidence to:

- use mental methods whenever appropriate, including questions on the written papers;
- examine questions and decide on the most appropriate strategy for each question, recognising that the numbers in the question often help to determine the method they feel most confident in using;
- make estimates and check their answers for reasonableness.

The role of the teacher during such revision sessions is to:

- encourage children to use mental methods as a first resort;
- show children how to record their mental and calculator methods to help with questions that require some explanation or description of the method;
- ensure that children have a secure understanding of place value, which helps them apply their methods successfully;
- enable the children to adopt the strategies they are most secure with by the time of the test;
- monitor children's methods and answers, and help children with very inefficient strategies to refine their methods as far as possible, ensuring that they understand why these are more efficient methods.

3 CHECKING ANSWERS, INCLUDING APPROPRIATE USE OF THE CALCULATOR

Children need to check their methods. It is a useful strategy to appoint a 'checker' when children are working in groups. Checking calculations also needs to be embedded in the teaching so that it becomes 'second nature' for the children. An OHP calculator can be used by the teacher to demonstrate ways of checking calculations undertaken using a calculator. In particular, the teacher can model checking strategies using inverse operations. Discussing different strategies that can be used to solve a problem should provide children with alternatives that they can use to check their own solution. This applies to both mental and written methods. For example, the answer to 'find three-quarters of 360' can be checked by finding one quarter of 360 and subtracting it from 360.

4 RELATING KEY VOCABULARY AND CONTEXT TO MATHEMATICAL OPERATIONS

In order for children to be successful in test questions such as Question 10, Test B, 2001, it is important that they can identify the **key** vocabulary and match it to the correct mathematical operation. It is important to note that the key words alone do not lead to the operation; it is also the context of the question that determines the required operation. A useful strategy is to create a display of key vocabulary and the associated operations around the classroom and then refer to this vocabulary and the context as they appear in test questions. 'How many' is often associated with addition and multiplication. In this question, because of the context, it should be linked to division.

10

This is the cost to visit the waxworks.

Adults	£8.50
Children	£4.50

On Friday morning **12 adults** and **20 children** visit the waxworks.

How much do they pay altogether?

Show your **method**. You may get a mark.

£

Guide books cost **£1.50** each.

The waxworks sells **£24** worth of **guide books**.

How many guide books is this?

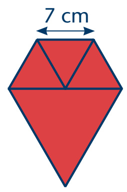
5 INTERPRETING INFORMATION AND USING ANNOTATIONS

It can be very effective to ask children to read sections of a problem and to use focused questions to establish the information that can be obtained from each section. The children should be encouraged to record this information in a way that is meaningful to them, using shorthand notes, a diagram or a flow chart. Evidence from tests suggests that children are reluctant to draw or write on published material such as test questions, apart from in the designated boxes. It is, therefore, important to encourage children to annotate and draw on test diagrams, graphs and tables, if it helps them to understand and answer the questions. For example, in attempting the question opposite, converting the diagram into seven small triangles would help many children to reach a solution.

20 Lauren has **three small equilateral triangles** and **one large equilateral triangle.**

The small triangles have sides of **7 centimetres**.

Lauren makes this shape.



Not actual size

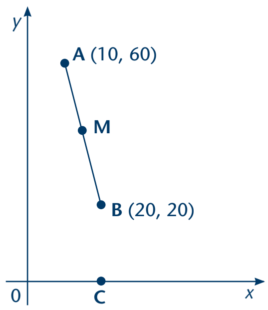
Calculate the **perimeter** of the shape.

Do **not** use a ruler.

[ ] cm

Encourage children to add information to diagrams or tables from the written information provided in the question. For example, questions such as Question 20, Test A, 2001, sometimes include graphs with no scale marked on the axes. Adding information from the question to clarify the scale or to give an indication of the magnitude of the scale often helps children to see how to answer the question.

20



A is the point **(10, 60)**
B is the point **(20, 20)**
M is the midpoint of line AB.

Write the coordinates of **M**. (,)

C is on the x-axis, directly **below B**.

Write the coordinates of **C**. (,)

6 ORDERING NUMBERS

There is evidence that many children need a more secure understanding of the relative size of numbers. Children often have difficulty answering questions that involve the ordering of negative numbers, fractions and decimals, and questions that involve the forming of equivalent fractions. A useful revision strategy is to use visual images, such as a number line, to enable children to see where negative numbers, fractions and decimals fit, and to understand why two representations, fraction or decimal, are equivalent. Remind children that they can draw their own pictures to help them see their way through to a solution.

7 PREPARING FOR THE UNFAMILIAR

Too many children stop working before they reach the end of the test, even though there is still time available for them to attempt the remaining questions. Children need to be encouraged to try questions with which they may not be familiar. If they need to draw any diagrams to help them, they should do so. Emphasise to children that it is better to do that and tackle a question than to sit and think without recording anything.

A useful revision strategy is to change the context of particular problems with the children, establish whether this has any effect on the calculation, and if so, why. Getting children to devise problems for others to answer also helps them to interpret the unfamiliar contexts.

SPRINGBOARD 6 LESSONS 23–30

Lesson	Topic	Objectives	What children should be able to do by the end of the lesson
23	Using a calculator to solve problems 1	Use a calculator to solve problems, choosing the appropriate operation Explain and record the solution to a problem	Explain the solution to a problem, identify and record the sequence of operations, and use a calculator to find and check their solution
24	Using a calculator to solve problems 2	Use a calculator to solve problems, choosing the appropriate operation Explain and record the solution to a problem	Identify the information needed to solve a multi-step problem and the calculations required at each stage Record calculations when using a calculator
25	Using a calculator to solve problems 3	Use a calculator to solve problems choosing the appropriate operation Interpret the meaning of a calculator display in the context of a word problem	Select the required sequence of operations when using a calculator to solve a problem Interpret the calculator display correctly in the context of the problem
26	Problem solving 5	Solve mathematical problems or puzzles, recognise and explain patterns Explain methods and reasoning orally and in writing	Interpret and use symbols that represent missing numbers Begin to recognise why some methods are more efficient and use them to solve problems Know what to write in the 'Show your working' box
27	Problem solving 6	Solve mathematical problems or puzzles, recognise and explain patterns Explain methods and reasoning orally and in writing	Generate and extend sequences, identify and describe patterns Begin to recognise why some methods are more efficient and use them to solve problems Know what to write in the 'Show your method' box
28	Problem solving 7	Identify and use the properties of shapes to solve problems Explain methods and reasoning orally and in writing	Calculate the perimeters of compound shapes and explain the strategies they have used
29	Problem solving 8	Solve simple problems involving ratio and proportion, and the identification of missing values in equations or calculations Explain methods and reasoning orally and in writing	Divide a length in a given ratio Use their knowledge and understanding of equality and place value to solve problems involving missing numbers and missing digits
30	Problem solving 9	Express part of a shape as a fraction Solve a problem by interpreting the scales on charts and graphs, and by extracting relevant information Explain methods and reasoning orally and in writing	Interpret the axes on bar charts and line graphs Select appropriate scales and extract the information needed to solve problems

SPRINGBOARD 6 LESSON 23
USING A CALCULATOR TO SOLVE PROBLEMS 1

TOTAL TIME 45 MINUTES

Objectives:

- Use a calculator to solve problems, choosing the appropriate operation
- Explain and record the solution to a problem

Vocabulary:

- operation
- 'Show your method'

By the end of the lesson the children should be able to:

- explain the solution to a problem, identify and record the sequence of operations, and use a calculator to find and check their solution.

Resources:

- calculators
- OHP calculator
- OHT 23.1
- Activity Sheet 23.1
- Activity Sheet 23.2

ORAL AND MENTAL STARTER

10 MINUTES

Show OHT 23.1, and give out calculators.

Explain that the OHT shows a set of prices for a range of different items. Point to each item explaining that you want the children to find out the cost of one apple, one golf ball, one metre of ribbon, etc. They can use their calculators.

Point to 6 apples.

Q: What is the cost of one apple?
Q: What operation did you use?

Invite a child to use the OHP calculator to demonstrate the calculation. Get the children to repeat the sequence on their calculators. Explain that the children are to describe their calculation in a sentence and give the cost of each item. Give as an example: **'The operation I used was divide by 6 to get an answer of 16p per apple.'**

Quickly repeat this process for one golf ball, one fence post, one spoon, and one fork.

Q: Which cost more, 4 spoons or 6 forks?

Discuss the children's methods and explore the following two methods:

Multiplying the cost of one spoon by 4 and the cost of one fork by 6.
Halving the cost of 12 forks and halving the cost of 8 spoons.

Agree that 4 spoons cost the same as 6 forks.

Q: How can we work out the cost of one metre of ribbon?

Discuss the children's answers and explore different methods.

Ask the children to decide for which of the items the cost of 'one' can be calculated without using a calculator.

Q: How can we work out the cost of one cucumber?

Q: How can we work out the cost of one minute of talk time?

Discuss the children's methods and establish how they can recognise when it is sensible not to use a calculator.

MAIN TEACHING ACTIVITY

25 MINUTES

Give out Activity Sheet 23.1. Ask the children to read the question carefully.

Q: What information can we enter in to the table?

Tell the children to work in pairs to enter the information from the question on to the table. Confirm that the whole first column, the first two cells of the second column, and the last cell in the end column can all be completed.

Use the following questions to help the children to complete the table. Invite the children to explain where they should record the answer to each question in the table. Say that for each question, even though the children can use their calculator, they should show their method of calculation in the box.

Q: How many calories did Sajit burn by running?

Q: How many calories did Sajit burn by cycling?

Q: How many calories did Sajit burn by rowing?

Discuss the children's methods and the calculation they used for each question. Establish that Sajit burned 132 calories by rowing.

Q: How many minutes did Sajit row for if she burned 132 calories?

Establish that Sajit rowed for 12 minutes and that the calculation the children should record in the 'Show your method' box is 132 ÷ 11.

Q: Where should we record the answer to this question?

Identify the box with a picture of a pencil next to it. Explain that this is the answer box and only the answer should be put here. Tell the children that they should record all their calculations in the larger 'Show your method' box.

Agree that 12 should go in the answer box and that this answer tells us that Sajit rowed for 12 minutes.

Give out Activity Sheet 23.2.

Explain that the children are to plan an exercise routine lasting for 25 minutes. The calories they burn each minute is the same as for Sajit. Say that they can choose to exercise for as long as they wish on each activity but they must each do some rowing, running, and cycling.

Tell the children to use the table to record how long they will exercise on each activity.

Get the children to work out how many calories they will burn for their exercise programme, recording their calculations in the 'Show your method' box.

PLENARY

10 MINUTES

Discuss the children's programmes and compare the number of calories burned.

Q: Who burned the most calories?

Q: Does the child who runs for the longest time burn the most calories?

Identify the child who has the longest exercise time for running and compare this with the other programmes.

Remember:

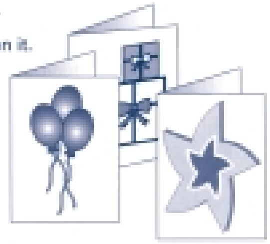
- Some calculations can be done easily without a calculator.
- Use the 'Show your method' boxes to record your calculations.
- Questions with tables and charts need reading carefully.

LESSON 23 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

6 A shop sells greetings cards.

Each card has a price code on it.

These are the codes.



code	price
AA	75p
BB	£1.15
CC	£1.55
DD	£1.70
EE	£1.99

Tina buys two cards.

One card has code **AA** on it.
The other card has code **DD** on it.

How much does Tina pay? £

Omar buys a card. He pays with a £2 coin.

He gets 45p change.

What is the **code** on his card? ....

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
6a	£2.45	*Accept £2.45* ***OR*** *£2 45* ***Do not*** *accept £245* ***OR*** *£245p*
6b	CC	*Accept 'C'* ***Do not*** *accept £1.55*

LESSON 23 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

6

A box of four balls costs **£2.96**

How much does each ball cost?

Dean and Alex buy **3 boxes** of balls between them.

Dean pays **£4.50**

How much must Alex pay?

Show your **method**. You may get a mark.

£

Question	Requirement	Additional Guidance
6a	74p **OR** £0.74	*Accept 74* ***OR*** *0.74* ***OR*** *£0.74p* ***OR*** *0 74 or £.74* ***OR*** *£.74p* ***OR*** *£0 74* ***OR*** *.74* ***Do not*** *accept £74p* ***OR*** *£74* ***OR*** *0.74p*
6b	Award **TWO** marks for the correct answer of £4.38 If the answer is incorrect, award **ONE** mark for evidence of an appropriate method, e.g. 2.95 × 3 = 8.88 8.88 – 4.50	*Accept for* ***TWO*** *marks £4.38p* ***OR*** *£4 38* *Accept for* ***ONE*** *mark £438* ***OR*** *£438p as evidence of an appropriate method. Answer need not be obtained for the award of the mark.*

ANALYSIS OF CHILDREN'S ANSWERS

- Although question 6 on Test A was generally answered well, many children, particularly those working at level 3 used £.p notation unconventionally. For part (b), the most common incorrect answer given by level 3 children was BB. Too many of these children did not read the table correctly or could not work out the difference between £2 and 45p.
- Part (a) of question 6 on Test B was answered well. Only one quarter of children working at level 3 were awarded both part (b) marks. Children used a variety of methods, including written column methods for multiplication and subtraction. Many children did not use a calculator method for part (b); they were confused by the instruction 'show your method'.

IMPLICATIONS FOR PLANNING

- Lessons should be planned to include the interpretation of information presented in tables. Children should be taught how to identify and highlight the information they need to answer the questions.
- Children should be taught how to use the correct conventions for notation such as £.p.
- Children should be presented with word problems that include more information than is necessary for solving the problem.
- There should be planned teaching activities that involve the use of calculators, with children being taught how to record their calculations and methods of solution.

SPRINGBOARD 6 LESSON 24
USING A CALCULATOR TO SOLVE PROBLEMS 2

TOTAL TIME

45 MINUTES

Objectives:

- Use a calculator to solve problems, choosing the appropriate operation
- Explain and record the solution to a problem

Vocabulary:

- perimeter
- area
- scale

By the end of the lesson the children should be able to:

- identify the information needed to solve a multi-step problem and the calculations required at each stage;
- record calculations when using a calculator.

Resources:

- calculators
- OHP calculator
- OHT 24.1
- Resource Sheet 24.1
- Activity Sheet 24.1

ORAL AND MENTAL STARTER

10 MINUTES

Give out Resource Sheet 24.1. Explain that this is a floor plan of a flat. Discuss the diagram and the meaning of 'Not to scale'.

Q: How many doors and windows are there in the flat?

Discuss the children's responses and explanations for the position of doors and windows.

Say that you are going to give the children some information about the flat and about some of the items in it. Tell the children that you want them to record this information on the Resource Sheet.

The dimensions of the flat are 16m by 9m.
The kitchen cost £3 500 to equip.
There are 3 plug sockets in each room, two in the hall and none in the bathroom.
The size of the dining room is 4m by 3m.
The carpet in the dining room cost £360.

The bathroom cost £2 100 to equip.
The size of the bathroom is 3m by 2m.
There is a radiator in each room, apart from the kitchen and hallway.

Explain that you want them to use their Resource Sheet to answer the following questions:

Q: How much did the kitchen and bathroom cost altogether?
Q: How many radiators are in the flat?
Q: How many plug sockets are in the flat?
Q: What is the perimeter of the flat?
Q: What is the area of the dining room?

Collect and discuss the answers.

MAIN TEACHING ACTIVITY

25 MINUTES

Show OHT 24.1 and give out Activity Sheet 24.1. Explain that not all of the information is on the Activity Sheet.

Ask a child to read question A.

Q: What information do you think is missing?

Establish that one piece of missing information is the amount of money Jason is paid for one hour's work.
Say that Jason is paid £5.60 each hour and ask the children to record this information on the Activity Sheet.

Q: Can we answer the question now?

Establish that we still need to know how many hours Jason worked during the week.
Tell the children that the working out boxes may give us some clues to the number of hours worked by Jason.

Q: How many hours did Jason work on Monday?

Agree that the number of hours worked by Jason on Monday was 8 hours.

Q: How many hours did Jason work on Tuesday?

Agree that the number of hours worked by Jason on Tuesday was 4 hours.

Q: Over the week how many hours did Jason work?

Agree that the number of hours worked by Jason was 31 hours.

Q: What calculation should we carry out in the first working out box?

Establish that the calculation should be 5.60 × 8. Get the children to record this calculation in the working out box and then carry out the calculation using their calculators. Confirm their answer using the OHP calculator. Ensure that children recognise that entering 5.60×8 and 5.6×8 on the calculator give the same answers. Establish that the answer 44.8 on the calculator display is interpreted as £44.80 in the context of this problem.

Get the children to work in pairs to work out how much Jason was paid each day. Say that they should record their calculation in the working out boxes.

Q: How much was Jason paid for his week's work?

Agree that Jason was paid £173.60.

Q: Where should we record this answer?

Agree that it should go in the box at the bottom of the page.

Q: Is there a way that we can check this answer using a different calculation?

Remind the children that Jason has worked for 31 hours at an hourly rate of £5.60 and that the answer could be obtained by calculating £5.60 $\times$ 31.

Explain to the children that this Activity Sheet is designed to help them set out the solution to a problem. Say that you now want them to work on a similar problem using the box below question B to record their calculations. Point out that this box is not set out in the same way and that the children will have to decide how they are going to set out their work.

Ask the children to read question B and get them to work on the question in pairs. Encourage them to set out their working in the box by breaking down the problem into steps, as in the previous problem. Use the following questions to support their working.

Q: How do you work out the cost of 8 rolls of wallpaper?
Q: What is the cost of one of the rolls sold at half price?

Discuss the children's recording of the problem and the steps they have developed. Establish that the calculation needed is $4.80 \times 8 + 2.40 \times 3$. Emphasise that they should always record their calculations even when they are using a calculator.

PLENARY

10 MINUTES

Return to the problem of Jason and the number of hours he worked. Explain that during one week he worked at the weekend. Explain that on a Saturday Jason is paid $1\frac{1}{2}$ hours' wages for every hour he works and that on a Sunday he is paid 2 hours' wages for every hour that he works.

Q: How much would Jason be paid if he worked 5 hours on Saturday and 6 hours on Sunday?

Agree that the answer would be £109.20 and discuss the children's methods.

Say that one weekend Jason was paid 20 hours' wages.

Q: How many hours could Jason have actually worked on Saturday and Sunday?

Establish that there are many solutions to this problem. For example, he could have worked 4 hours on Saturday and 7 hours on Sunday. Discuss the children's responses.

Remember:

- Read the question carefully to make sure you understand what calculations to do.
- Word problems often require several steps, always write the calculation down for each step.
- Show all your steps in the 'Show your method' box.

LESSON 24 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

4 Asif, Vicky and Nita go to town by bus.

This is what they pay.

Asif	BUS TICKET 75p
Vicky	BUS TICKET £1.35
Nita	BUS TICKET £1.55

How much **more** does **Nita** pay than **Asif**?

Vicky then takes **another** bus from town to visit her auntie.

She pays **90p** on this bus.

How much has Vicky paid **altogether** for her two bus tickets?

LESSON 24 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

14

The table shows the cost of coach tickets to different cities.

		Hull	York	Leeds
Adult	single	£12.50	£15.60	£10.25
	return	£23.75	£28.50	£19.30
Child	single	£8.50	£10.80	£8.25
	return	£14.90	£17.90	£14.75

What is the total cost for a **return** journey to York for one adult and two children?

£

How much **more** does it cost for two adults to make a **single** journey to Hull than to Leeds?

£

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
4a	80p **OR** £0.80	*Accept £0.80p* ***OR*** *0.80* ***OR*** *80* ***OR*** *£.80* ***OR*** *£.80p* ***OR*** *£0 80* ***OR*** *.80* ***OR*** *0 80* ***Do not*** *accept £80p* ***OR*** *£80* ***OR*** *£0.8* ***OR*** *0.80p*
4b	£2.25 **OR** 225p	*Accept £2.25p* ***OR*** *2.25* ***OR*** *225* ***OR*** *£2 25* ***Do not*** *accept £225p* ***OR*** *£225*

Question	Requirement	Additional Guidance
14a	£64.30	*Accept £64.30p* ***OR*** *£64 30* ***Do not*** *accept £6430* ***OR*** *£6430p* ***OR*** *£64.3*
14b	£4.50	*Accept £4.50* ***OR*** *£4 50* ***Do not*** *accept £450* ***OR*** *£450p OR £4.5* *If the final '0' is missing from both answers, ie answers given are £64.3 and £4.5 respectively, award* ***ONE*** *mark only in 14b.*

ANALYSIS OF CHILDREN'S ANSWERS

- Nearly half the children working at level 3 answered question 4, part (a), correctly. A common wrong answer was 60p, the difference between the costs of Asif's and Vicky's tickets. Another was 90p, suggesting an incorrect calculation. Part (b) had more correct answers but there was further evidence that the wrong information was used by children.
- A significant proportion of children at all levels used a written method to answer both parts of question 14. A common error on part (a) was to calculate the fare for one child, not two. Children were less successful when answering part (b). A common error was to find the difference in cost for one adult. A high proportion of children working at level 3 gave no answer to part (b).

IMPLICATIONS FOR PLANNING

- Lessons should be planned to develop children's skills at reading and re-reading questions carefully and interpreting the information accurately. Word problems provide the opportunity to introduce children to unfamiliar vocabulary.
- Children should be presented with questions that have incomplete or missing information to focus their attention on what is needed and what can be discarded.
- Children should be encouraged to highlight or underline key words in questions.
- Children should be taught how to record the calculations needed for each step when using a calculator to solve a multi-step problem.

SPRINGBOARD 6 LESSON 25
USING A CALCULATOR TO SOLVE PROBLEMS 3

TOTAL TIME

45 MINUTES

Objectives:

- Use a calculator to solve problems choosing the appropriate operation
- Interpret the meaning of a calculator display in the context of a word problem

Vocabulary:

- unit
- decimal number
- litres

By the end of the lesson the children should be able to:

- select the required sequence of operations when using a calculator to solve a problem;
- interpret the calculator display correctly in the context of the problem.

Resources:

- calculators
- OHP calculator
- Activity Sheet 25.1
- whiteboards and pens

ORAL AND MENTAL STARTER

10 MINUTES

Write on the board 339 ÷ 12.

Q: What is 339 ÷ 12?

Use the OHP calculator to show that the answer on the display is 28.25.

Present the following problem.
339 goldfish are to be divided equally between 12 ponds. How many goldfish will be in each pond?

Write on the board 28.25. Explain that this answer means that 28 fish would be in each pond with some fish left over. Write this information below 28.25 as shown below:

28.25
28 goldfish in each pond . some goldfish left over

Ask the children to work in pairs to think of a word problem involving a quantity such as sweets, stamps, etc. for which this calculation could be the answer. For each question offered, write the answer under the number 28.25 as for the example above.

28.25
28 goldfish in each pond . some goldfish left over
28 merit marks for each child . some merit marks left over
28 eggs in each basket . some eggs left over

Q: If the problem was £339 divided between 12 people what would the answer 28.25 represent?

Confirm that because we are dividing money the two numbers after the decimal point represent pence and the answer 28.25 means that the 12 people will each get £28.25.

MAIN TEACHING ACTIVITY

20 MINUTES

On the board write $17 + 365 \times 11$.

Q: How do we do this on a calculator?

Establish that children can use the calculator correctly to work this out. Write $365 \times 11 + 17$ on the board and confirm that this represents the same calculation. Explain why it is safer to work out this calculation on a calculator.

Explain that the answer to this calculation is the age of a child in days who is 11 years and 17 days old. There are 365 days in 1 year (except leap years which we will ignore).

Q: What calculation would we carry out to calculate the age in days of a child who is 13 years and 27 days?

Get the children to record the calculation on their whiteboards. Establish that the calculation would be $365 \times 13 + 27$. Confirm that the answer is in days.

Q: What would the calculation be for a child who is 4 days away from their 12th birthday?

Get the children to discuss the calculation in pairs. Share the different methods presented by the children.

Give out Activity Sheet 25.1.

Ask the children to read the first question.

Q: About how many years old do you think Mary is?

Discuss the children's estimates and explanations. Establish why Mary must be less than 10 years old.

Q: What calculation should we do to work out how old Mary is in years and days?

Remind the children that there are 365 days in each year. Establish that the required calculation would be 2000 ÷ 365 and get the children to record the calculation and the calculator display in the 'Show your method' box.

Q: What does the answer 5.47945205 mean?

Explain that this answer tells us how old Mary is in years.

Q: Which number in the calculation tells us Mary's age in whole years?

Establish that the children understand that the number 5 represents the number of years and that the decimal represents part of a year.

Q: Is Mary closer to 5 years of age or 6 years of age?

Establish that the decimal part of the number is less than one half so Mary is closer to 5 years of age.

Q: How can we work out how many days the decimal part of the number represents?

Take the children's responses and discuss their approaches.

Q: How many days old would Mary be if she was exactly 5 years old?

Establish that the answer is 1825 days and that the calculation that gives this answer is 365 × 5.
Get the children to record this information in the 'Show your method' box. Discuss what the children have recorded and remind them that Mary is 2000 days old.

Q: How can we work out how old Mary is in years and days?

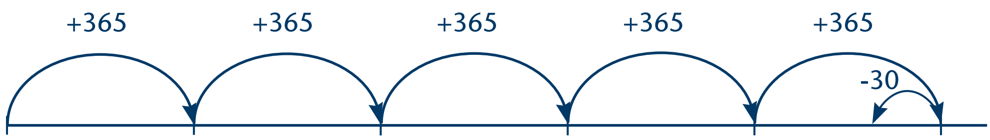
Agree that Mary is 5 years and 175 days old and that we calculate the 175 by subtracting 1825 from 2000.

Say that there is another way to work out the number of days. On the OHP calculator subtract 5 from the display to get 0.47945205. Multiply this answer by 365. Explain that this answer (175) is the number of days left over when 2000 is divided by 365.

Ask the children to read the second word problem. Invite them to ask questions about the problem in order to develop their understanding. For example, they may need to ask how many days there are in April.

Q: Can we draw a diagram to help us to work out how many days old David is?

Suggest that a number time line might give a good visual image of the problem. Invite a child to draw one on the board and get the children to draw their own line in the 'Show your method' box.



Q: What calculation should we do to answer this question?

Get the children to record their calculation in the 'Show your method' box. Establish that the required calculation would be 365 × 5 – 30. Relate this to the method shown on the number line. Ask the children to do this calculation on their calculators.

PLENARY

15 MINUTES

Remind children of the goldfish problem in the oral and mental starter.

Explain that the decimal part of the answer is telling us that we would have to divide a number of goldfish less than 12 between the 12 ponds.

Q: How can we work out the number of goldfish left over?

Explain that we can put 28 goldfish in each of the 12 ponds.

Q: If we put 28 goldfish in each of the 12 ponds how many goldfish is that?

Get the children to work out the answer on their calculators and confirm that the answer is 336 goldfish.

Q: How many goldfish are left over?

Agree that the answer is 339 – 336 = 3 goldfish.

Present the following problem:

246 goldfish are to be put into 11 ponds. How many goldfish will be in each pond?

Q: What calculation should we do to answer this question?

Establish that the required calculation will be 246 ÷ 11. Get the children to do this calculation on their calculators and confirm that the answer is 22.36363636.

Q: What does the 22 mean in the answer?

Establish that this means we could put 22 goldfish in each pond.

Q: How can we calculate how many goldfish will be left over?

Discuss the children's responses and suggestions.

Confirm that we are placing 22 goldfish in 11 ponds. This means that there are 246 – 22 × 11 goldfish left over.
Ask children to work this calculation out on their calculators. Ensure that children can do this correctly.

Remember:

- The decimal part of a number is less than 1.
- The remainder from a division calculation carried out on a calculator is expressed as a decimal.
- For calculations such as 246 – 22 × 11 it is safer to do the calculation in two steps when using a calculator.

LESSON 25 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

11

185 people go to the school concert.

They pay £1.35 each.

How much ticket money is collected?

£

Programmes cost 15p each.

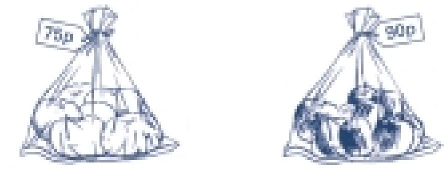
Selling programmes raises £12.30

How many **programmes** are sold?

Show your **method**. You may get a mark.

LESSON 25 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

18



6 green apples for 75p

10 red apples for 90p

Jason bought some bags of green apples and some bags of red apples.

He spent £4.20

How many **bags** of each type of apple did he buy?

Show your **working**. You may get a mark.

☐ **bags** of **green** apples ☐ **bags** of **red** apples

Nika and Hassan bought some bags of apples.

Nika says,

'I bought more apples than Hassan, but I spent less money.'

Explain how this is possible.

. . . .

. . . .

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
11a	£249.75	*Accept £249.75p **OR** £249 75* ***Do not** accept £24975p **OR** £24975*
11b	Award **TWO** marks for the correct answer of 82	
	If the answer is incorrect, award ONE mark for evidence of an appropriate method, e.g. 1230 ÷ 15 **OR** 12.30 ÷ 0.15	*Accept for **ONE** mark £82 **OR** 82p as evidence of an appropriate method.* *Do not accept 12.30 ÷ 15 as evidence of an appropriate method.* *Answers need not be obtained for the award of the mark.*

Question	Requirement	Additional Guidance
18a	award TWO marks for correct answer as shown: **2** **bags** of **green** apples **3** **bags** of **red** apples	***Both** numbers must be correct for the award of marks.*
	If the answer is incorrect, award **ONE** mark for evidence of appropriate working, e.g. Listing of cost of apples: 75 90 150 180 225 270	*Calculation must be performed for the award of **ONE** mark.*
18b	An explanation that shows how it is possible to buy more apples but spend less money, e.g. ■ 'Nika buys 2 bags of red apples, giving 20 apples for £1.80, and Hassan buys 3 bags of green apples, giving 18 apples for £2.25'.	***Do not** accept vague or arbitrary explanations, e.g.* ■ *'She got bigger bags than he did';* ■ *'She bought a lot of small ones'.* *Ignore slight errors in arithmetic that do not contradict the explanation.*

ANALYSIS OF CHILDREN'S ANSWERS

- Some two-thirds of children working at level 3 answered part (a) of question 11 correctly. A significant proportion of children, however, lost the mark through inappropriate use of units, often recording the calculator-displayed number without taking account of the context. For part (b), very few children were awarded only the mark for method. A significant proportion of children incorrectly multiplied the given values together. The most commonly recorded method was a standard short or long division, as children interpreted 'show your method' as 'do not use a calculator'.
- A significant proportion of levels 3 and 4 children omitted question 18; only one-fifth of level 3 children were awarded the two marks for part (a). Those who answered it correctly often showed no working. Almost no children working at level 3 or 4 gave a credit-worthy explanation for part (b). Attempts at an explanation were generally about the bags of apples, not the quantity or cost of apples.

IMPLICATIONS FOR PLANNING

- There should be planned teaching activities that involve children in interpreting their calculator displays to take account of the context of the problem.
- Children should be given questions that ask 'How much?' or 'How many?' and that require each of the four operations, not just multiplication.
- Children should be taught that using a calculator is not the method by which they solved the problem, but that their calculations were their methods of solution.
- In the main teaching activity, children should discuss their solutions, compare one another's explanations, and be asked to write a brief explanation that includes a calculation.

SPRINGBOARD 6 LESSON 26
PROBLEM SOLVING 5

TOTAL TIME

45 MINUTES

Objectives:

- Solve mathematical problems or puzzles, recognise and explain patterns
- Explain methods and reasoning orally and in writing

Vocabulary:

- complement
- symbol
- represent
- efficient
- 'Show your working'

By the end of the lesson the children should be able to:

- interpret and use symbols that represent missing numbers;
- begin to recognise why some methods are more efficient and use them to solve problems;
- know what to write in the 'Show your working' box.

Resources:

- Activity Sheet 26.1
- whiteboards and pens

ORAL AND MENTAL STARTER

10 MINUTES

On the board write: 60.

Ask the children questions which involve 60 and repeat each question once. The children must write their answers on their whiteboards while you count to ten. Tell the children to show you their answers when you say: 'show me'.

- Ask ten questions such as:

Q: What is 25 more than 60?
Q: What is 60 multiplied by 100?
Q: How many more than 48 is 60?
Q: How many less than 117 is 60?
Q: What is 60 divided by 20?
Q: What is half of 60?
Q: What is 10% of 60?

Q: What is 60 centimetres in metres?
Q: I buy shoes costing £49.99, what change will I get from £60?
Q: How many seconds in 15 minutes?

After each question collect and discuss the children's answers. Invite individuals to explain their methods. Correct any errors or misunderstandings.

MAIN TEACHING ACTIVITY

25 MINUTES

Write on the board:

□ + △ = 13

Say that □ and △ are missing positive whole numbers.

Q: What could these numbers be?

Ask the children to write appropriate pairs of numbers on their whiteboards and show their answers.

Discuss the answers and write some pairs on the board in a random order.

Q: How can we organise these pairs of numbers to help us get all the answers?

Take suggestions. Ask the children to work in pairs and organise the numbers on their whiteboards adding any extra pairs to their list.

Q: How many answers are there? How do we know we have them all?

Establish that to make a total of 13, if the number in □ can be 1 to 12 then that in △ will go from 12 to 1 to make a total of 13. There are 12 possible pairs.

Write on the board:

□ – △ = 5

Explain that this is additional information about the missing numbers in □ and △.

Q: Can we find the two numbers now?

Ask children to look at their list for □ and △ and see if they can find an answer.

Confirm that the answer is □ = 9 and △ = 4.
With the children check by writing 9 + 4 = 13, 9 – 4 = 5.

Give out Activity Sheet 26.1.

Ask the children to read the first question.

Q: How is it different from the problem we have just solved?

Establish that it is the same type of problem without the box and triangle.

Q: What could we write using □ and △ to make the problems look the same?

Get the children to write statements on the board using □ and △.

Agree that the problem refers to:

□ + △ = 57 and □ – △ = 9

Write this on the board and establish that the children understand that this represents the problem.

Q: How many pairs would we list if we recorded all the pairs that sum to 57?

Agree □ would go from 1 to 56 and △ from 56 to 1, so 56 pairs.

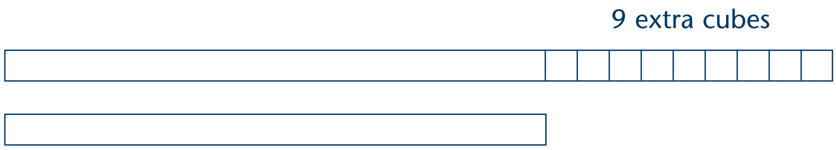
Q: Do we need to list all 56 pairs?

Agree it is not necessary nor is it efficient. Refer back to the two statements.

Q: Which of the numbers represented by the two symbols is larger?

Establish it is □, and □ is 9 more than △.

Ask the children to imagine that they have 57 cubes. They make two rows of cubes, one of which is 9 cubes longer than the other. Draw this on the board:



Q: Does this represent the problem? Which row of cubes represents the missing number in □?

Agree it represents the problem and the top row is □, as it is 9 longer.

Cover up the 9 extra cubes shown on the diagram.

Q: If we remove these 9 cubes, altogether how many cubes are left now?

Collect answers and confirm there are 57 – 9 = 48 cubes left.

Q: How long are each of the two rows of cubes now?

Agree they are the same length and both have △ cubes.

On the board write:

$2 \times \triangle = 48$

Discuss this and refer to the diagram on the board.

Q: What number does △ represent?

Establish △ = 24 as 24 + 24 = 48, or 2 x 24 = 48.

Q: What number does □ represent?

Agree that it is 9 more, so □ = 24 + 9 = 33.
With the children check by writing 33 + 24 = 57, 33 – 24 = 9.

Clear the board. Ask the children to work through question 1 on their own, recording their method and calculations in the 'Show your working' box on the Activity Sheet.

PLENARY

10 MINUTES

Ask the children to read through question 2 on the sheet.

Q: What is different about this problem?

Collect answers. Establish that the numbers are bigger but it is still about the sum and difference of two positive whole numbers.

Ask the children to work in pairs to solve the problem. Each child must record their own working in the 'Show your working' box on their Activity Sheet.

Get the pairs to swap their Activity Sheets. With the children work through the problem getting prompts from individuals. Ask them to share what is recorded in their 'Show your working' boxes.

Establish that the numbers are 78 and 54 and check by writing 78 + 54 = 132, 78 – 54 = 24.

Ask the children to discuss in pairs quickly what they wrote in their 'Show your method' box that was helpful to them and made sense when they read someone else's work.

Collect key points from the children.

Remember:

- Making lists can be helpful. Decide where to start the list and ask if this is an efficient method.
- A quickly drawn picture can help you to understand and to sort out a problem.
- Always write any calculations you do, and draw any pictures you find helpful, in the 'Show your working' box.

LESSON 26 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

2 Write in the missing numbers.

5 × 70 = □

4 × □ = 200

LESSON 26 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

20 Write in what the missing numbers could be.

170 + □ = 220 − □

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
2a	5 × 70 = **350**	
2b	4 × **50** = 200	

Question	Requirement	Additional Guidance
20	Any pair of *numbers which total 50,* eg 30 and 20	*Accept fractions and decimals.* *Accept zero in either box.* ***Do not** accept boxes left blank.*

ANALYSIS OF CHILDREN'S ANSWERS

- Question 2, part (a), was well answered; nearly three-quarters of level 3 children answered it correctly. Success rates for part (b) dropped; a significant proportion of level 3 children did not give an answer.
- Only 5 per cent of children at level 3 gave a correct answer to question 20; one quarter of level 4 children were successful. The most common error was to write 50 in the left-hand box with an incorrect value in the right-hand box. Children misinterpreted the equals sign in the number sentence and often tried to 'balance' the two sides by putting 50 in each box. Ignoring the '+' and '–' in this way was an error made by over 20 per cent of children at levels 3 and 4.

IMPLICATIONS FOR PLANNING

- There should be planned teaching activities that involve children in finding missing numbers in number statements involving all four operations and with the missing number set in each possible position in the number statement.
- Children should be encouraged to read aloud number statements with missing numbers, saying 'a number' where there is a box representing a missing number. The emphasis should be placed on the operation to draw it to children's attention.
- Children should be taught to use the commutative properties of addition and multiplication to rearrange statements to help them find missing numbers.
- When there are two missing numbers in a number statement, children should be taught that choosing one of the numbers determines the other. The equals sign means that the answers to the calculations on both sides of the sign are identical.

SPRINGBOARD 6 LESSON 27
PROBLEM SOLVING 6

TOTAL TIME
45 MINUTES

Objectives:

- Solve mathematical problems or puzzles, recognise and explain patterns
- Explain methods and reasoning orally and in writing

Vocabulary:

- sequence
- pattern
- multiple
- efficient
- square number

By the end of the lesson the children should be able to:

- generate and extend sequences, identify and describe patterns;
- begin to recognise why some methods are more efficient and use them to solve problems;
- know what to write in the 'Show your method' box.

Resources:

- a counting stick
- Activity Sheet 27.1
- interlocking cubes
- an empty box
- calculators

ORAL AND MENTAL STARTER

5 MINUTES

Using a counting stick count forwards from 0 in steps of 4 to 40.

Repeat, but stopping at intervals, e.g. 24.

Q: What multiplication facts involving 4 and 24 can you tell me?

Collect and record $6 \times 4 = 24$ and $4 \times 6 = 24$.

Q: Now what two division facts can you tell me?

Collect and record $24 \div 4 = 6$ and $24 \div 6 = 4$.

Count in steps of 5 and 6 and collect associated multiplication and division facts.

MAIN TEACHING ACTIVITY

25 MINUTES

Show the children an empty box which you explain once held interlocking cubes. Say that when using the cubes in the box to make towers of 4 cubes, there was 1 cube left over.

Q: How many cubes could there have been in the box?

Make a tower of 4 cubes and show the 1 cube left over.
Write on the board: $1 \times 4 + 1 = 4 + 1 = 5$ cubes.

Say there were more than 5 cubes in the box.
Make two towers of 4 cubes and show the 1 cube left over.

Q: How many cubes have I got now?

Collect answers and, on the board, record: $4 + 4 + 1 = 9$ cubes.

Q: How else can we record this?

Record: $2 \times 4 + 1 = 8 + 1 = 9$ cubes under the first statement.

Say there were more than 9 cubes in the box. With the children, collect other possibilities for the number of cubes in the box and record then as shown below.

$1 \times 4 + 1 = 4 + 1 = 5$
$2 \times 4 + 1 = 8 + 1 = 9$
$3 \times 4 + 1 = 12 + 1 = 13$
$4 \times 4 + 1 = 16 + 1 = 17$
$5 \times 4 + 1 = 20 + 1 = 21$

Q: Can you see a pattern for the number of cubes?

Share suggestions.

Q: Can you describe the pattern in words?

Ask the children to write a sentence that describes the pattern. Discuss the children's sentences and agree that the pattern is the 4 times table plus 1.

Say the number of cubes in the box was between 35 and 40. With the children recite the 4 times table and add 1 to generate numbers in the sequence. Stop at 41. Establish that the number of cubes in the box was 37.

Say that in another box of cubes, when you made towers of 5 cubes there were 3 left over, and when you made towers of 6 there was 1 left over.

Q: How many cubes were there in the box?

Discuss the problem.

Q: What multiplication tables should we use?

Establish that it is the 5 and 6 times tables.

Start the sequences with the children, recording on the board:

$1 \times 5 + 3 = 5 + 3 = 8$ $\quad$ $1 \times 6 + 1 = 6 + 1 = 7$

$2 \times 5 + 3 = 10 + 3 = 13$ $\quad$ $2 \times 6 + 1 = 12 + 1 = 13$

Agree that 13 cubes is a possible answer.

Say that there were over 40 cubes in the box and ask the children to continue the sequences to find the number of cubes in the box.

Collect the children's solutions and correct any errors or misunderstandings. Agree that the answer is 43 cubes. Emphasise how listing sequences like this is a useful strategy for solving these types of problems.

Give out Activity Sheet 27.1, and calculators.

Ask the children to read the first problem.

Q: Do you recognise the problem? What strategies can you use to solve it?

Establish the problem is the same kind of problem as the cubes and towers.

Discuss the children's strategies and ask them to solve the problem, recording their work in their 'Show your method' box on the Activity Sheet.

PLENARY

15 MINUTES

Collect children's answers and correct any errors or misunderstandings.

Q: What did you write in the box?

Share contributions and the children's writing.

Q: Where did you start your sequences?

Establish that the children need not have started at:

$1 \times 5 + 3 = 8$ $\quad$ $1 \times 4 + 3 = 7$

$2 \times 5 + 3 = 13$ $\quad$ $2 \times 4 + 3 = 11$

Q: What other information is given?

Point out that Hameed made over 55 sweets.

Q: Where could we have started?

Discuss suggestions. Agree that starting at 10×5 and 10×4 would be sensible. With the children, extend the sequences, recording on the board:

$10 \times 5 + 3 = 50 + 3 = 53$	$10 \times 4 + 3 = 40 + 3 = 43$
$11 \times 5 + 3 = 55 + 3 = 58$	$11 \times 4 + 3 = 44 + 3 = 47$
$12 \times 5 + 3 = 60 + 3 = 63$	$12 \times 4 + 3 = 48 + 3 = 51$
$13 \times 5 + 3 = 65 + 3 = 68$	$13 \times 4 + 3 = 52 + 3 = 55$
	$14 \times 4 + 3 = 56 + 3 = 59$
	$15 \times 4 + 3 = 60 + 3 = 63$
	$16 \times 4 + 3 = 64 + 3 = 67$

Q: Which multiple of 5 is close to 55?

Establish that 55 is a multiple of 5.

Q: Which multiple of 4 is close to 55?

Establish that 52 is a multiple of 4.

On the lists on the board underline 11×5 and 13×4. Say that to save writing and to be more efficient we could have started with these calculations to form the sequences.

Q: What would we have written in the 'Show your method' box?

Discuss the children's suggestions. Emphasise that they should write down all their calculations in this box to help them answer the problem and get the marks.

Ask the children to read the second question.

Q: Is this problem similar to the others?

Establish that it is, and confirm the sequences that represent the number of cards. Ask children to solve the problem, recording their calculations in the 'Show your method' box. Remind them that 21 is not a square number. Establish that 81 is a square number.

Remember:

- Writing out the sequence can help you to see the pattern.
- Use all the information you are given to help you to decide where you might start the sequence.
- Similar problems may be described in different ways – always read the question carefully and compare it with problems you have done before.

LESSON 27 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

9 Jemma thinks of a number.

She says,

'Add 3 to my number and then multiply the result by 5
The answer is 35'

What is Jemma's number?

Riaz thinks of a number.

He says,

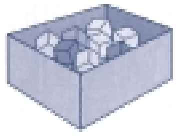
'Halve my number and then add 17
The answer is 23'

What is Riaz's number?

LESSON 27 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

13 There are 24 coloured cubes in a box.

Three-quarters of the cubes are red,
four of the cubes are blue
and the rest are green.



How many **green** cubes are in the box?

Show your **method**. You may get a mark.

One more **blue** cube is put into the box.

What fraction of the cubes in the box are **blue** now?

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
9a	4	
9b	12	

Question	Requirement	Additional Guidance
13a	Award **TWO** marks for the correct answer of 2	
	If the answer is incorrect, award **ONE** mark for evidence of an appropriate method, e.g. $\frac{3}{4}$ of 24 = 18 green = 24 – 18 – 4	*Answer need not be obtained for the award of the mark.*
13b	$\frac{1}{5}$	*Accept equivalent fractions.* ***Do not** accept '1 in 5' **OR** '1 : 5'.*

ANALYSIS OF CHILDREN'S ANSWERS

- Half of the children working at level 3 answered part (a) of question 9 correctly, though a significant proportion omitted the question. A common mistake was to give the answer 7, suggesting children knew how to complete the first stage of the calculation. Success rates for part (b) fell to 30 per cent for level 3 children. Again the most common error was to complete the first step and record the answer as 6.
- It was rare for a single mark to be awarded to children for part (a) of question 13. Just over 10 per cent of level 3 children were awarded 2 marks. Finding three-quarters of 24 was the stumbling block for many children and the methods recorded showed that children could not make effective use of their calculator to do this. Some 20 per cent of level 3 children answered part (b) correctly. A common error was 5/24. Children added one to the blue cubes but not to the total number of cubes.

IMPLICATIONS FOR PLANNING

- In the oral and mental starter, children should be asked 'think of a number' type problems involving one step then two steps, supported by discussion on the strategies that the children used. The problems should include all four operations and halving and doubling.
- Children should be taught how to check their solutions to 'think of a number' type problems by applying the rules of the problem to their answer.
- Children should be taught how to use a calculator to find fractions of quantities.
- Plan activities that help children to recognise that when increasing or decreasing part of a quantity, the whole quantity must also be increased or decreased.

SPRINGBOARD 6 LESSON 28
PROBLEM SOLVING 7

TOTAL TIME

45 MINUTES

Objectives:

- Identify and use the properties of shapes to solve problems
- Explain methods and reasoning orally and in writing

Vocabulary:

- isosceles
- equilateral
- vertices
- regular pentagon
- symmetry
- edges
- perimeter

By the end of the lesson the children should be able to:

- calculate the perimeters of compound shapes and explain the strategies they have used.

Resources:

- small equilateral triangles
- OHT 28.1
- OHT 28.2
- Activity Sheet 28.1
- Activity Sheet 28.2
- Activity Sheet 28.3
- Resource Sheet 28.1
- Resource Sheet 28.2

ORAL AND MENTAL STARTER

10 MINUTES

Show OHT 28.1. Point to the pentagon, trace your finger around the perimeter and ask:

Q: What is the name of this shape?

Establish that the shape is called a pentagon and that the perimeter of the pentagon is the distance made up by the five outside edges.

Q: Is the pentagon regular?

Remind the children that regular means all edges are equal and all angles are equal. Agree that this is a regular pentagon. Write the words 'Regular pentagon' on OHT 28.1. Explain that the vertices are identified by the letters and the pentagon is labelled ABCDE. Trace a triangle in the pentagon and ask for its label using the letters, e.g. CDE, DEC, ECD. Say that these labels represent the same triangle.

Q: How many different sizes of triangle can you see in this shape?

Give each child Activity Sheet 28.1. Get the children to work in pairs and record the different triangles on the Activity Sheet by shading the triangle. Encourage them to label the vertices of the triangle they have found.

Refer to triangle EAB on OHT 28.1.

Q: What type of triangle is triangle EAB?

Establish that the triangle EAB must be isosceles because two of its sides are equal as they are edges of the regular pentagon, which has equal edges.

Q: Which two angles must be equal?

Establish that angle AEB must be equal to angle ABE.

Q: Are any of the other triangles you found isosceles triangles?

Discuss other triangles found by the children. Identify them on the OHT and ask them to say if they think they are isosceles. Note the children's responses and say that you will look at these triangles again at the end of the lesson.

MAIN TEACHING ACTIVITY

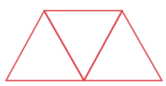
25 MINUTES

Give out 10 equilateral triangles and Activity Sheet 28.2 to each pair of children.

Q: What different shapes can you make using 3 triangles.

Ask children to use the equilateral triangles to make the shape and record these on the Activity Sheet.

Discuss the shapes that the children make. Establish that triangles must be joined edge to edge and agree that there is only one possible shape as all the others are either a reflection or rotation of this shape (isosceles trapezium):



Say that you want children to imagine that the perimeter of one equilateral triangle is 6 cm.

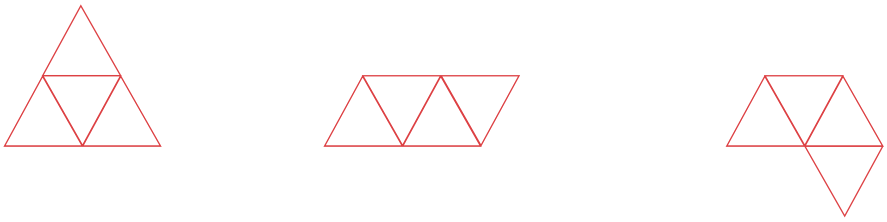
Q: What is the perimeter of this new shape?

Discuss the children's responses and reasons and establish that the side length of each equilateral triangle is 2 cm, so the new shape has a perimeter of 10 cm.

Q: How many different shapes can you make with 4 equilateral triangles?

Ask children to record these on the Activity Sheet.

Discuss the different shapes made and agree that 3 different shapes can be made, one of which is a larger equilateral triangle:



Q: What is the perimeter of this larger triangle?

Establish that the sides of this large triangle are 4 cm and that the perimeter is 12 cm.

Give out Activity Sheet 28.3.

Q: Can we answer the question on the activity sheet?

Establish that we need more information about the size of some of the shapes. Explain that the perimeter of a small equilateral triangle is 21 cm and get the children to record this on their Activity Sheet.

Q: How many small equilateral triangles are in Maria's shape?

Take the children's responses and get them to justify their reasons.

Q: How can we show that the large triangle is made up of 4 small equilateral triangles?

Explain that we can draw in the other triangles on the Activity Sheet. Get the children to do so.

Ask the children to work in pairs to find the perimeter of the shape. Explain that the children should show all their working in the box provided.

Take the children's responses and discuss the solution.

Give out Resource Sheet 28.1.

Q: Is this question the same as the one we have been working on?

Agree that it is the same question but that there is no 'Show your working' box.

Q: What should we do if there is no 'Show your working' box?

Emphasise that the children should draw on the diagram to add information given in the question and use any spaces on the paper to record their working out.

PLENARY

10 MINUTES

Show OHT 28.1 again, and ask children to look at Activity Sheet 28.1.

Remind the children of the different triangles they found and the way the letters were used to label the triangles.

Ask the children to work in pairs and decide:

Q: Which triangle has the largest perimeter?
Q: Which triangle has the smallest perimeter?

Discuss children's answers and, on OHT 28.1, refer to the triangles, using the letters as labels.

Show OHT 28.2 with the six triangles.

Get the children to place the triangles in order of size of perimeter.

Discuss the children's responses and reasons. Cut out the pentagon on Resource Sheet 28.2. Use folding to show that the triangles in pictures 1 and 6 have the same perimeter.

Q: Which of these triangles are isosceles?

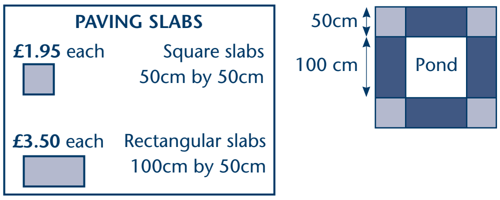
Create lines of symmetry by folding to establish that all the triangles are isosceles.

Remember:

- Isosceles triangles have two equal sides, two equal angles and one line of symmetry.
- Record any information in questions on the diagrams if it helps you.
- Write down your working out even if there is no 'Show your working' box.

LESSON 28 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

11 Mr Singh buys paving slabs to go around his pond.



He buys 4 rectangular slabs and 4 square slabs.

What is the total cost of the slabs he buys?

Mr Singh says,

'It would cost more to use square slabs all the way round.'

Explain why he is correct.

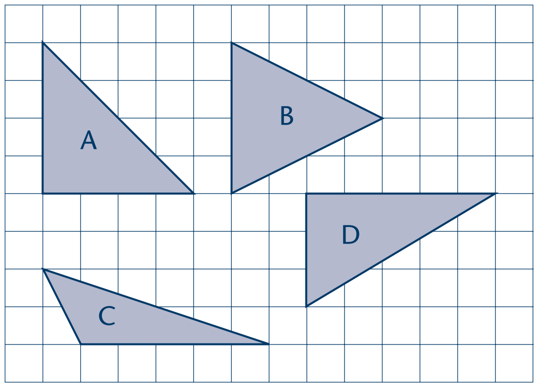
....

....

....

LESSON 28 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

12 Here are four triangles drawn on a square grid.



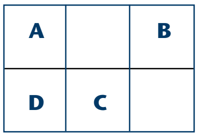
Write the letter for each triangle in the correct region of the sorting diagram.

One has been done for you.

	has a **right** angle	has an **obtuse** angle	has 3 **acute** angles
is isoceles	A		
is **not** isoceles			

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
11a	Award **TWO** marks for the correct answer of £21.80	*Accept £21.80p **OR** £21 80*
	If the answer is incorrect, award **ONE** mark for evidence of appropriate working, e.g. 3.50 × 4 = 14.00 1.95 × 4 = 7.80 14.00 + 7.80 = wrong answer	*Accept for **ONE** mark £2180p **OR** £2180 **OR** £21.8 as evidence of appropriate working.* *Calculation must be performed for the award of **ONE** mark*
11b	An explanation which recognises that each square slab costs more than half a rectangular slab or equivalent, e.g. ■ 'Half of £3.50 is £1.75, which is less than £1.95'; ■ 'Two square slabs cost more than one rectangular slab'; ■ 'Because 12 squares cost £23.40'; ■ 'Because it would cost £1.60 more'.	***Do not** accept vague or arbitrary explanations, e.g.* ■ *'Because he would need more slabs';* ■ *'Because square slabs are cheaper than rectangular slabs';* ■ *'Because it costs more';* ■ *'He is right because the square slabs are £1.95 each and the rectangular slabs are £3.50 each'.*

Question	Requirement	Additional Guidance
12	Award **TWO** marks for three letters in the correct regions of the sorting diagram, as shown:	***Do not** accept letters that are written in more than one region.*
		*Accept alternative indications such as lines drawn from the shapes to the appropriate regions of the sorting diagrams.*
	Award **ONE** mark for two letters in the correct regions of the sorting diagram.	

ANALYSIS OF CHILDREN'S ANSWERS

- Nearly 30 per cent of children working at level 3 gained both marks to part (a) of question 11; a further third were awarded 1 mark. As many level 3 children used a standard method for multiplication and addition as those who used an informal method. Informal methods were less common with level 4 or 5 children. None of the level 3 children was awarded the mark for part (b); only one-fifth of level 4 children gained the mark. The most common error was to say that using square slabs cost more without any comparison with other costs.
- Children working at level 3 were much less successful at answering this question correctly than children at level 4. Many children did not recognise that triangle B was isosceles, while 20 per cent of levels 3 and 4 children thought triangle C was isosceles. Only 5 percent of level 3 children were awarded 2 marks; 15 per cent gained 1 mark.

IMPLICATIONS FOR PLANNING

- Children working at the level 3 to 4 borderline who can use informal methods confidently and accurately, should be encouraged to continue to use them.
- There should be planned activities where children are asked to justify a statement, first orally, then in writing. Children should be taught to recognise when an explanation is clear and why it offers a complete justification.
- Children should be presented with lines and shapes drawn onto a grid and taught how to determine whether two lines or the edges of shapes are the same length.
- There should be planned activities where children identify shapes in various orientations and within compound shapes.

SPRINGBOARD 6 LESSON 29
PROBLEM SOLVING 8

TOTAL TIME

45 MINUTES

Objectives:

- Solve simple problems involving ratio and proportion, and the identification of missing values in equations or calculations
- Explain methods and reasoning orally and in writing

Vocabulary:

- not to scale
- mid-point
- equals
- equation
- digit

By the end of the lesson the children should be able to:

- divide a length in a given ratio;
- use their knowledge and understanding of equality and place value to solve problems involving missing numbers and missing digits.

Resources:

- whiteboards and pens
- Activity Sheet 29.1
- length of ribbon
- pegs

ORAL AND MENTAL STARTER

10 MINUTES

On the board draw a straight line. Mark the ends P and Q. Explain that the distance from P to Q is 60 centimetres and add this to the diagram.

Say that the diagram is not to scale. Explain that this means it is a picture not an accurate length.

Ask the children to draw a picture of the line PQ on their whiteboards.

Add a point R on the middle of the line. Tell the children that R is the mid-point so that PR is the same distance as RQ.

Q: What is the distance from P to R?

Establish it is 30 cm and that the children understand why.

Show the children the ribbon held vertically. Say that the ribbon represents the line PQ, which is 60 cm long. Fold the ribbon in half and agree that each half is 30 cm. Put a peg at the mid-point, and say that the peg represents the point R.

Say that you are going to move the point R. Say that you want to put R on the line so that the distance from P to R is twice the distance from R to Q.

Q: Where does point R go on our ribbon?

Ask the children to use the peg to identify the new position of R.

Ask the children to put R on the line on their whiteboards.

Q: How can we check that the peg is in the right position?

Collect answers and establish by folding that half the distance from P to R is the same as the distance from R to Q. Ask a child to put point R on the line on the board.

Establish that there are three equal distances.

Q: How long is each of the three equal distances?

Use the ribbon to confirm the three equal distances and agree that each distance must be 20 cm as $60 \div 3 = 20$. Record this calculation on the board.

Q: How long is the distance from P to R?

Establish it is 40 cm as $20 \times 2 = 40$. Record this calculation on the board.

Move R again. Say that this time the distance from P to R is three times as far as from R to Q. Ask the children to represent this on their whiteboards. Use the ribbon and peg to represent R.

Q: What markers can we draw on the line to help us?

Encourage the children to divide PR into three, and use pegs to represent these divisions on the ribbon.

Q: How many equal distances are there on our line?

Agree there are four.

Q: How long is each of these distances?

Establish they are each 15 cm. Ask the children for the calculation involved.

Record 60 ÷ 4 = 15 on the board.

Q: How long is the distance PR?

Confirm PR is 45 cm as 15 × 3 = 45 and record this on the board.

MAIN TEACHING ACTIVITY

30 MINUTES

Write on the board:

40 + □ = 60

Say that this is an equation. Because of the equals sign both sides represent the same values. Say that the box represents a missing number.

Q: What is the missing number?

Emphasise that we need to add 40 and 20 to get 60 to ensure both sides of the equals sign have the same value, 60. Say that this is the solution to the equation.

Write 20 in the box.

Write on the board:

50 + □ = 70 – □

Explain that the boxes represent two different positive whole numbers.

Q: What could the missing numbers be?

Write 30 in the left-hand box.

Q: What does the left-hand side of the equation add up to?

Agree that it is 80.

Q: Can we find a positive whole number to make 80 on the other side?

Agree that this is not possible. Remove the 30.

Write 30 in the right-hand box.

Q: What is the answer to the right-hand side of the equation?

Agree it is 40.

Q: Can we find a positive whole number to make 40 on the other side?

Agree that this is not possible. Remove the 30.

Q: What identical totals can we make on both sides of the equals sign?

Collect answers.

Q: Can we make totals of 50 on both sides?

Establish that this is possible with 0 and 20 in the boxes, but 0 is not a positive whole number.

Q: Can we make totals of 55 on both sides?

Confirm the children's answers and correct any errors or misunderstandings.

Write on the board:

1		5	+	1	4		=	2	7	3

Explain that this time the empty boxes have missing digits, i.e. single numbers between 0 to 9.

Say that you are going to rewrite this question:

	1		5
+	1	4	
	2	7	3

Q: How should we start this calculation?

Discuss suggestions. Emphasise that it is better to start with the units, then work with the tens and then the hundreds.

Q: What can we add to 5 to get a 3 in the units column?

Establish that 5 + 8 = 13. Write 8 in the units box.

Q: What do we do with the 13?

Emphasise that as 5 + 8 = 13 the ten from the 13 needs to be recorded. Write a 1 above the empty box in the tens column. Remind the children that now we want to make 70.

Q: How many tens do we have so far?

Identify the 40 and the 10, so we have 50.

Q: What do we add to 50 to make 70?

Establish that 50 + 20 = 70. Write 2 in the empty box.

Q: Is the answer in the hundreds column correct?

Agree it is.

Write on the board:

$$\square 0 \times \square 0 = 1\,0\,0\,0$$

Explain that again the empty boxes represent missing digits.

Write the digit 1 in each box.

Q: Is this correct?

Establish that the left-hand side is $10 \times 10 = 100$ but the right-hand side is 1000. Emphasise that whatever digits we write in the box the numbers will both be multiples of ten. Remove the two ones from the boxes.

Q: When we multiply together two tens numbers, how many zeros do we get in the answer?

Work through some examples, 40×20, 30×40, etc. Establish that there are always two zeros. Draw a vertical line between the 10 and 00 on the right-hand side and explain that there are 10 hundreds represented in 1000.

Q: What two digits multiply together to give 10?

Agree that $2 \times 5 = 10$ or $5 \times 2 = 10$.

Write 2 and 5 in the boxes.

Q: Does $20 \times 50 = 1000$?

Confirm this is correct.

Give out Activity Sheet 29.1. Ask children to work in pairs and identify the missing numbers and the missing digits.

Collect all the answers and discuss the methods the children used.

PLENARY

5 MINUTES

Draw line AB on the board. Say it is 100 cm long but it is not drawn to scale.

Mark C on the line and say that the distance from A to C is four times as far as from C to B.

Q: How long is the distance from A to C?

Collect answers. Remind the children that A to C is four times C to B.

Q: How many equal lengths did you divide AC into?

Use the ribbon and pegs to confirm that there should be four lengths, so all together there are five lengths. Each length is 20 cm, as $100 \div 5 = 20$.

The distance from A to C is 80 cm as $20 \times 4 = 80$.

Remember:

- Empty boxes may represent a missing number or a missing digit.
- When finding solutions to an equation with missing numbers on both sides of the equals sign, to get started it may be helpful to try some numbers to see if you can make both sides have the same value.
- When working out what the missing digits are in a calculation, it is usually best to start from the units and work up to the tens and hundreds.
- Be prepared to draw on diagrams or draw your own.

LESSON 29 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

12 Write in the missing digits.

4 [ ] 4 + 3 8 [ ] = 8 5 1

19 Write in the **two** missing digits.

[ ] 0 × [ ] 0 = 3 0 0 0

LESSON 29 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

21

Not drawn to scale

The distance from **A** to **B** is three times as far as from **B** to **C**.

The distance from **A** to **C** is **60 centimetres**.

Calculate the distance from **A** to **B**.

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
12	Digits written in boxes as shown: 4 **6** 4 + 38 **7** = 851	
19	5 and 6 written in the boxes in either order as shown: **5** 0 × **6** 0 = 3 0 0 0 **OR** **6** 0 × **5** 0 = 3 0 0 0	

Question	Requirement	Additional Guidance
21	Award **TWO** marks for the correct answer of 45 cm	
	If the answer is incorrect, award **ONE** mark for evidence of an appropriate method, e.g. 60 ÷ 4 × 3	*Answer need not be obtained for the award of the mark.*

ANALYSIS OF CHILDREN'S ANSWERS

- Just over 20 per cent of level 3 children answered question 12 correctly. A common error was to get the units digits correct but to write an incorrect digit, often 7, in the tens box as they were unable to bridge across 10.
- Only 10 per cent of level 3 children answered question 19 correctly. A common error was to use the digits 1 and 3 without considering place value and the product. Some 20 per cent of level 3 children put 3 in both the boxes, with no attempt at checking if this made sense.
- It was rare for a single mark to be awarded for answers to question 21. Over a quarter of level 3 children omitted the question; only 5 per cent gained both marks. The most common mistake was to divide 60 by 3 and double to get 40, or just give an answer of 20. Some 10 per cent of level 3 children annotated the diagram while 30 per cent attempted the question with no working recorded. Many level 4 children misinterpreted the instruction 'Show your method' to mean they should not use a calculator.

IMPLICATIONS FOR PLANNING

- Lessons should be planned to include calculations with missing digits that involve bridging across 10 and across 100. These questions should be presented horizontally and in column format and involve addition and subtraction.
- Children should be taught how to use their knowledge of place value, such as knowing that a tens number multiplied by a tens number is a hundreds number and 3000 can be read as 30 hundred.
- Children should be presented with pictures and diagrams that are labelled with letters and be taught how to read and interpret this labelling.
- Children should be taught how to annotate a diagram using the information they are given and that 'not drawn to scale' means that the diagram is not drawn accurately, so taking measurements will not help them.

SPRINGBOARD 6 LESSON 30
PROBLEM SOLVING 9

TOTAL TIME

45 MINUTES

Objectives:

- Express part of a shape as a fraction
- Solve a problem by interpreting the scales on charts and graphs, and by extracting relevant information
- Explain methods and reasoning orally and in writing

Vocabulary:

- bar chart
- line graph
- horizontal
- vertical
- axes

By the end of the lesson the children should be able to:

- interpret the axes on bar charts and line graphs;
- select appropriate scales and extract the information needed to solve problems.

Resources:

- OHT 30.1
- OHT 30.2
- OHT 30.3
- counters

ORAL AND MENTAL STARTER

10 MINUTES

Show the large square on OHT 30.1.

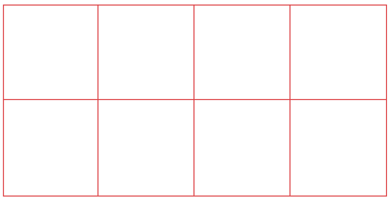
Explain that there are four identical smaller squares in the large square. Place a counter on a small square.

Q: What fraction of the large square is the square with the counter on it?

Agree it is one quarter of the large square. Record the fraction $\frac{1}{4}$ on the board.

Add another counter and establish that the two squares represent two quarters or one half. Record $\frac{2}{4} = \frac{1}{2}$. Extend using 3 or 4 counters. Establish that $\frac{4}{4} = 1$, the whole square.

Remove the counters, and using two copies of OHT 30.1, combine two large squares.



Q: What is the name of this shape?

Agree it is a rectangle. Put a counter on one small square.

Q: What fraction of the rectangle is this square?

Agree it is one eighth. Record $\frac{1}{8}$ on the board.

Add another counter and establish that the two squares represent two eighths or one quarter.

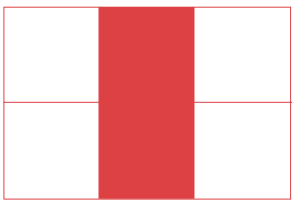
Record on the board: $\frac{2}{8} = \frac{1}{4}$

Q: Does it matter which two small squares the counters are on? Will these squares always form one quarter of the rectangle?

Agree that this is true.

Add more counters to generate the eighths and emphasise that $\frac{4}{8} = \frac{1}{2}$, $\frac{6}{8} = \frac{3}{4}$ and $\frac{8}{8} = 1$

Remove the counters and overlap the two squares to make a smaller rectangle.

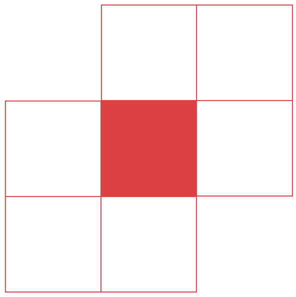


Put counters on the two overlapping small squares.

Q: What fraction of this new, smaller rectangle do these squares represent?

Collect answers. Agree that there are six small squares and two have counters on. Confirm the fraction is $\frac{2}{6}$ or $\frac{1}{3}$ and record the fraction on the board.

Remove the counters and recombine the two large squares.



Put a counter on the overlapping square.

Q: What fraction of this new shape is the square with the counter?

Agree that it is one seventh of the square. Quickly identify $\frac{2}{7}$, $\frac{3}{7}$, etc.

Emphasise that even though we started with two large squares, we were identifying the fractions of the new shapes we made by combining the squares.

MAIN TEACHING ACTIVITY

30 MINUTES

Show the bar chart on OHT 30.2.

Q: What is this type of graph called?

Confirm it is a bar chart.

Q: What is missing from the bar chart?

Establish that the axes are not identified.

Q: What could such a bar chart be showing?

Collect suggestions and develop some of the children's ideas.

Suppose the horizontal axis represented days of the week.

Q: What could the vertical axis show?

Collect responses and establish that it will show how many or how much.

Write the following list on the board:

Press-ups, Sunshine, Rainfall, Spending on sweets.

Say that these are the titles of bar charts.

Take each in turn and identify what units would appear on the vertical axis:

Frequency or number of: press-ups; hours of sunshine; centimetres of rainfall; pounds and/or pence spent.

Say that the bar chart shows the number of visitors to a certain museum each day of the week. Say that the greatest number of visitors was just over 70 people.

Q: What numbers should we put on the vertical scale?

Identify the highest bar, and use it to establish the axis would be in steps of 10. Label the axes and ask the children to identify the number of visitors each day.

Show the line graph on OHT 30.2.

Q: What is the name of this type of graph?

Confirm it is a line graph.

Say that the line graph shows the cost of running a large heater for a period of up to 20 minutes.

Write on the board:

Cost for 10 minutes is 16p.

Q: How should we label the axes?

Establish that the horizontal axis represents the time in minutes and the vertical axis the cost in pence.

Q: What will each interval be on the horizontal axis if the heater runs for 20 minutes?

Take suggestions and with the children count along the axis in the steps size suggested. Establish that the intervals are each worth 2 minutes and ask a child to label the axis.

Q: What else do we know about the cost?

Remind the children that the cost for 10 minutes is 16p.

Q: What is the cost for 20 minutes?

Agree 20 minutes cost 32p.

Q Where will this information appear in the graph?

Invite children to show the class, and confirm the top right-hand point represents 20 minutes costs 32p.

Q What are the intervals worth on the vertical axis?

Establish that there are 8 intervals for the 32p so each interval represents 4p. With the class count up the axis to check, and label the axis.

Use the graph to ask the children a series of questions about the cost for different times, e.g.

Q: How much does it cost to have the heater on for 12 minutes, 7 minutes?

Show the children how to use the graph to read the values, by drawing in lines from the horizontal axis to the line, then along to the vertical axis.

Ask questions about how long the heater could be left on for a given sum of money.

Q: For 20p how many minutes would the heater be on? How long for 15p?

Ensure that children recognise how to transfer from one axis to another via the graph.

PLENARY

5 MINUTES

Show OHT 30.3. Explain it is a rectangle made of identical squares. Place a counter on a small square.

Q: What fraction of the rectangle is this square?

Agree it represents one fifth of the rectangle.

Remove the counter, and using two copies of OHT 30.3 overlap one square from each rectangle.

Put a counter on the overlapping square.

Q: What fraction of the new rectangle is this?

Agree it is one ninth, and that this represents the overlapping square in the new rectangle.

Overlap two squares.

Put counters on the two overlapping squares.

Q: What fraction of the new shape is this?

Agree it is two eighths or one quarter, and this represents the overlap in the new shape.

Continue to overlap three or four squares and ask the children for the fraction that represents the overlap in the new shape.

Ask the children to record in writing a sentence that explains their solution to three overlapping squares. Share these with the class and agree a sentence that represents their thinking.

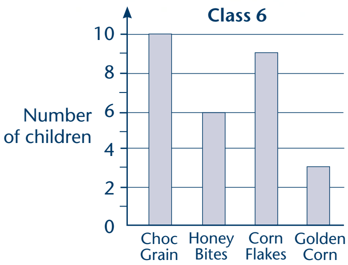
Remember:

- Bar charts show how many there are or how much there is by the length of bars.
- Make sure you understand the scales on the axes before you decide how to answer the questions.
- There are two scales on a line graph and the line shows you the relationship between values on one scale and values on the other scale.

LESSON 30 RELATED TEST QUESTION 2002 TEST B (CALCULATOR PAPER)

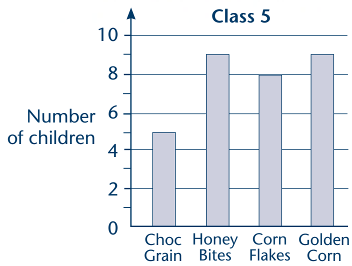
3 Tom does a survey of children's favourite breakfast cereals.

These are the results for Class 6



How many **more** children in Class 6 prefer **Choc Grain** than **Golden Corn**?

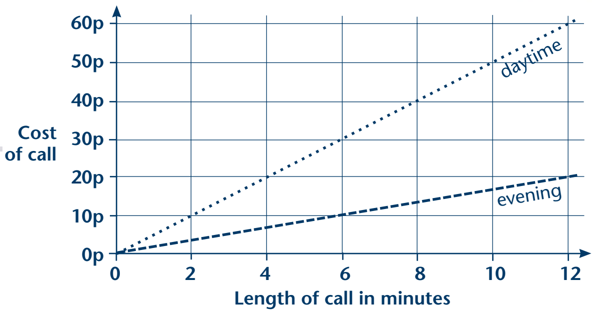
These are the results for Class 5



How many children in **both** classes like **Honey Bites** best?

LESSON 30 RELATED TEST QUESTION 2002 TEST A (NON-CALCULATOR PAPER)

10 This graph shows the cost of phone calls in the daytime and in the evening.



How much does it cost to make a **9 minute** call in the **daytime**?

p

How much **more** does it cost to make a **6 minute** call in the **daytime** than in the **evening**?

p

GUIDANCE FROM MARK SCHEME

Question	Requirement	Additional Guidance
3a	7	
3b	15	*Accept '9 and 6' or similar.*

Question	Requirement	Additional Guidance
10a	Answer in the range 44p to 46p inclusive.	
10b	20p	*Accept £0.20p* ***OR*** *£0 20* ***Do not*** *accept 0.20p* ***OR*** *£20p*

ANALYSIS OF CHILDREN'S ANSWERS

- Over 60 per cent of level 3 children answered part (a) of question 3 correctly, and 70 per cent gained the mark for part (b). The most common error was to misread the question and use the wrong data or give an incomplete answer.
- About two-thirds of level 3 children gained the 2 marks for question 10. The most common error to part (a) for levels 3 and 4 children was to give an answer outside the accepted range, often 41p or 49p, not identifying and using the half-way values involved. For part (b), level 3 or 4 children who gave the correct values for the costs of the evening and daytime calls did not always go on to find the difference between the two amounts.

IMPLICATIONS FOR PLANNING

- Lessons should be planned to develop children's skills at reading information from charts and graphs and using this information to solve multi-step problems.
- Children should be taught to annotate diagrams and to draw horizontal and vertical lines on charts and graphs to help them to find and read appropriate values on the scales.
- When reading frequencies from bar charts, children should be encouraged to write these frequencies on the tops of the appropriate bars. They should reread the questions to make sure they select the right information.
- Children should be taught that adding or subtracting one to a value on the horizontal axis does not mean that one will be added to or subtracted from the corresponding value on the vertical axis.

LESSON 23 USING A CALCULATOR TO SOLVE PROBLEMS 1

OHT 23.1

6 apples 96p	6 minutes of talk time £1.20
Half a cucumber 47p	11 golf balls £17.60
35 fence posts £157.50	12 forks £7.20
8 spoons £7.20	One and a half metres of ribbon £1.05

LESSON 23 USING A CALCULATOR TO SOLVE PROBLEMS 1

ACTIVITY SHEET 23.1

Running burns 14 calories each minute. Cycling burns 12 calories each minute and rowing burns 11 calories each minute. Sajit ran for 7 minutes, cycled for 8 minutes and then finished her training programme by rowing. Altogether she burned 326 calories. For how long did Sajit row?

Activity	Calories burned each minute	Number of minutes exercising	Calories burned during exercise
Running			
Cycling			
Rowing			
	TOTALS		

Show your method

Minutes

LESSON 23 USING A CALCULATOR TO SOLVE PROBLEMS 1

ACTIVITY SHEET 23.2

You are going to exercise for 25 minutes. Decide how long you will row, cycle and run. Put this information in the table and use the method box to show how you calculate how many calories you will have burned in 25 minutes.

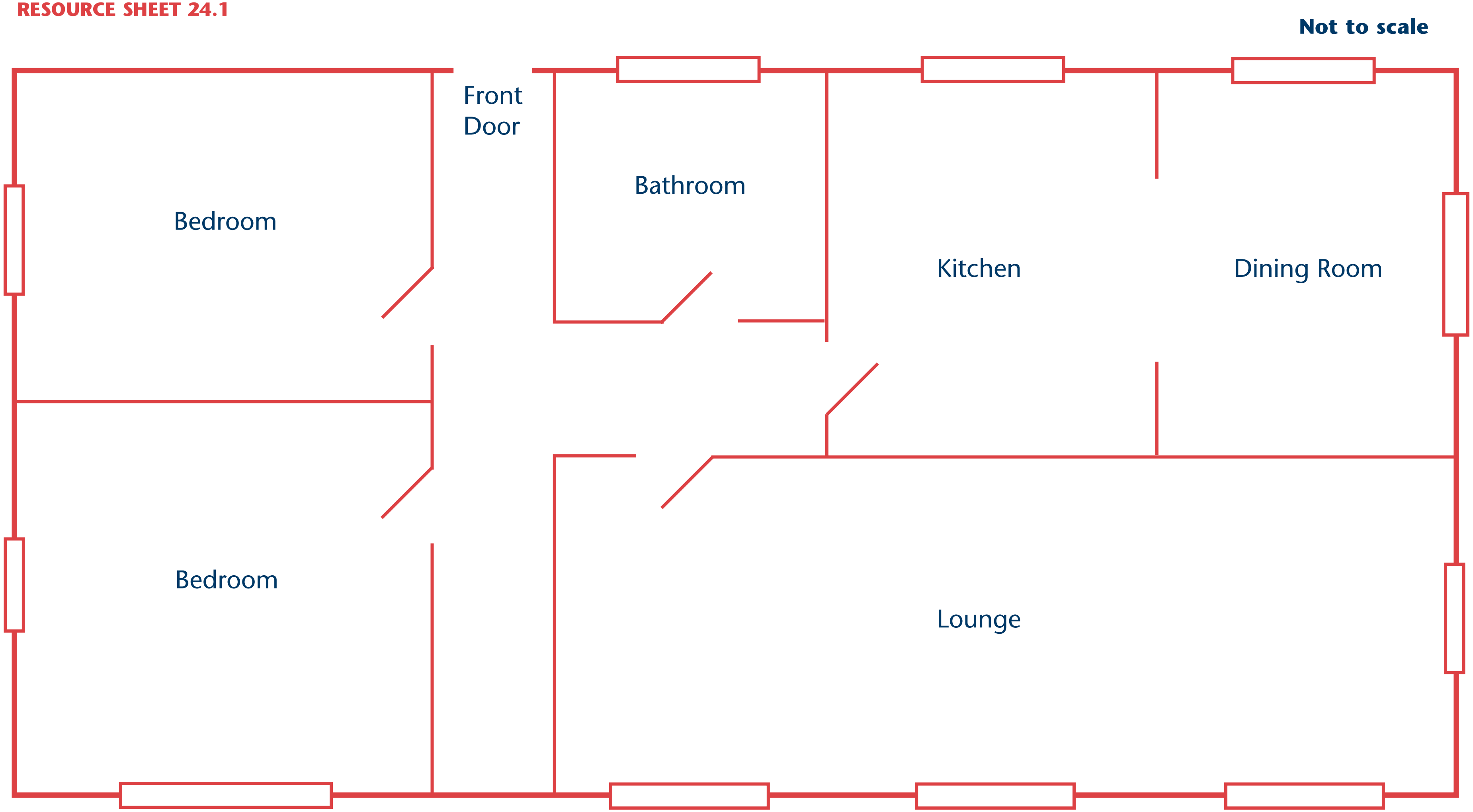
Activity	Calories burned each minute	Number of minutes exercising	Calories burned during exercise
Running			
Cycling			
Rowing			
	TOTALS		

Show your method

Calories

LESSON 24 USING A CALCULATOR TO SOLVE PROBLEMS 2

RESOURCE SHEET 24.1



LESSON 24 USING A CALCULATOR TO SOLVE PROBLEMS 2

OHT 24.1

For each hour's work Jason was paid ____.
How much money did he receive for the week's work?

Day 1

Jason was paid ____ for a full day of 8 hours.

Day 2

Jason was paid ____ for half a day.

Day 3

Jason was paid ____ for 6 hours.

Day 4

Jason was paid ____ for a full day of 8 hours.

Day 5

Jason was paid ____ for 5 hours.

£ ____

LESSON 24 USING A CALCULATOR TO SOLVE PROBLEMS 2

ACTIVITY SHEET 24.1

A. For each hour's work Jason was paid ☐.
How much money did he receive for the week's work?

Day 1

Jason was paid ☐ for a full day of 8 hours.

Day 2

Jason was paid ☐ for half a day.

Day 3

Jason was paid ☐ for 6 hours.

Day 4

Jason was paid ☐ for a full day of 8 hours.

Day 5

Jason was paid ☐ for 5 hours.

£

B. Angela buys 11 rolls of wallpaper. Eight of the rolls cost £4.80 each. The other rolls are sold at half this price. How much do the 11 rolls of wallpaper cost Angela?

Show your method

£

LESSON 25 USING A CALCULATOR TO SOLVE PROBLEMS 3

ACTIVITY SHEET 25.1

1. Mary is 2000 days old. What is Mary's age in years and days?

Show your method

2. David is 4 years and 11 months old on 1 April. How many days old is David?

Show your method

LESSON 26 PROBLEM SOLVING 5

ACTIVITY SHEET 26.1

1. Two positive whole numbers sum to 57 and their difference is 9. What are the two numbers?

Show your working

2. Heather says: 'I think of two positive whole numbers. One is 24 more than the other. Their total is 132'. What are the two numbers Heather has in mind?

Show your working

LESSON 27 PROBLEM SOLVING 6

ACTIVITY SHEET 27.1

1. Hameed made sweets for a party.
When he puts the sweets in packs of 5 he has 3 left over.
When he puts the sweets in packs of 4 he has 3 left over.
He made over 55 sweets. How many sweets did Hameed make?

Show your method

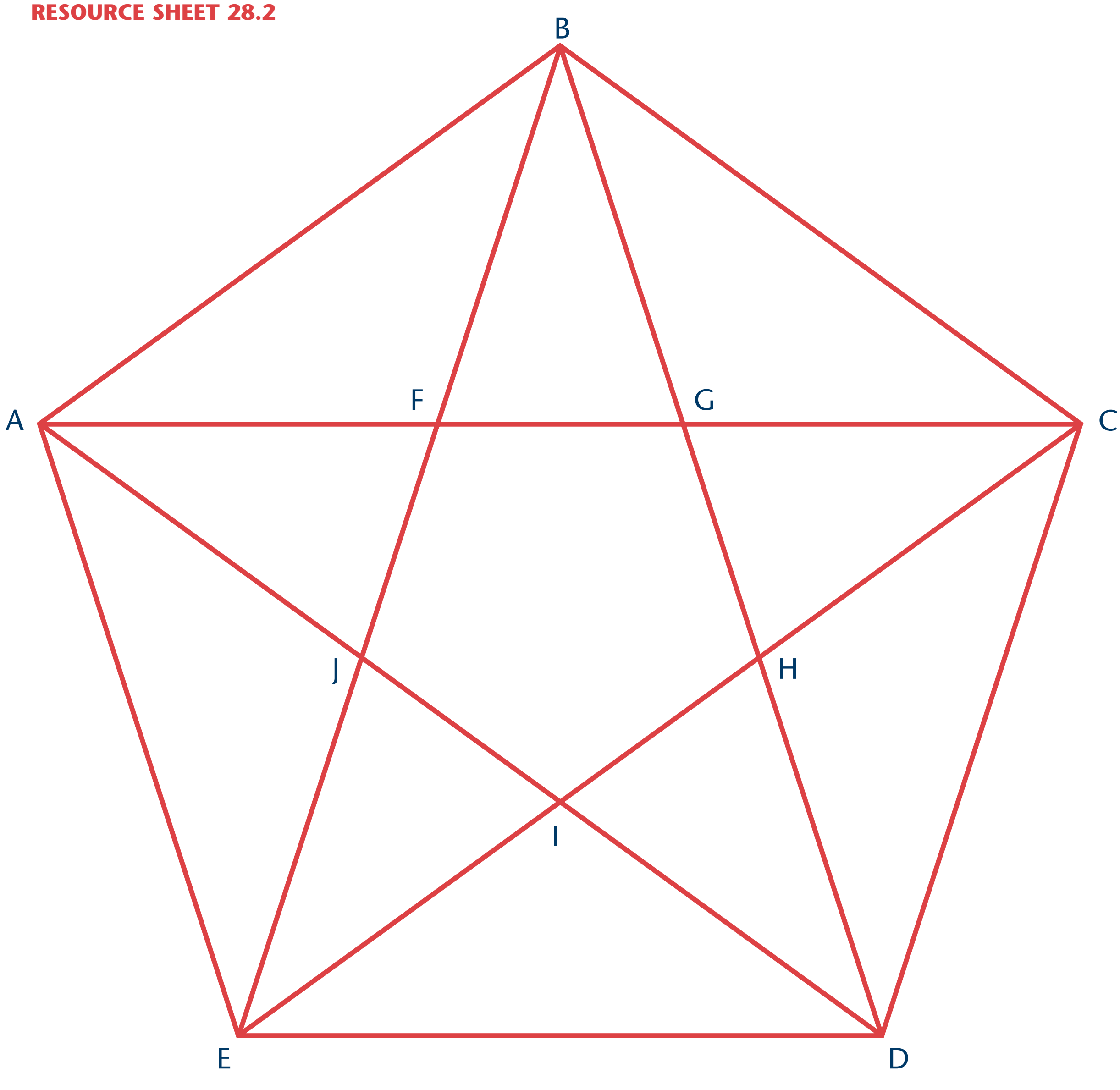
2. Hannah and Keith have the same number of picture cards.
When Hannah puts her cards into piles of 12, she has 9 left over. When Keith puts his cards into piles of 10, he has 1 left over. Hannah remembers that the number of cards she has is a square number. How many cards does she have?

Show your method

LESSON 28 PROBLEM SOLVING 6

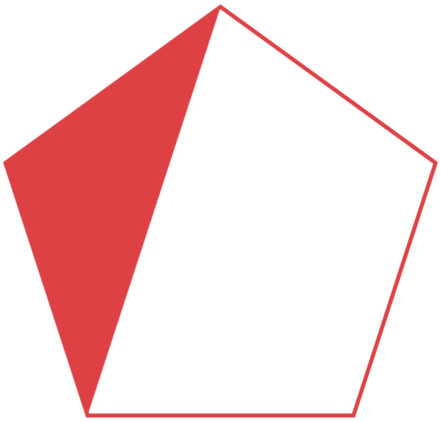
OHT 28.1

RESOURCE SHEET 28.2

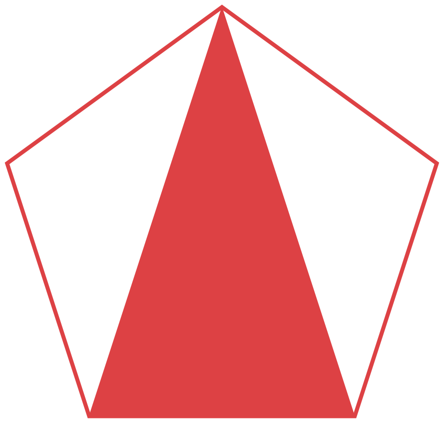


LESSON 28 PROBLEM SOLVING 7

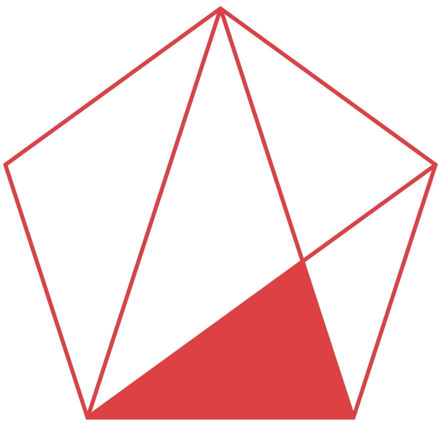
OHT 28.2



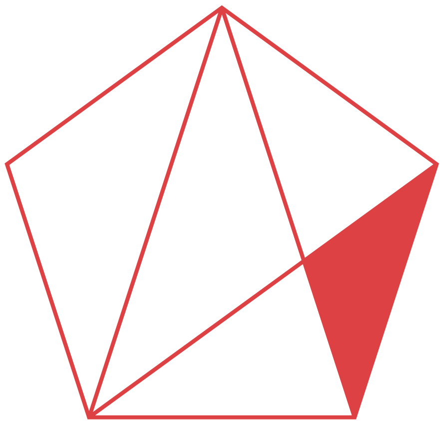
1



2



3



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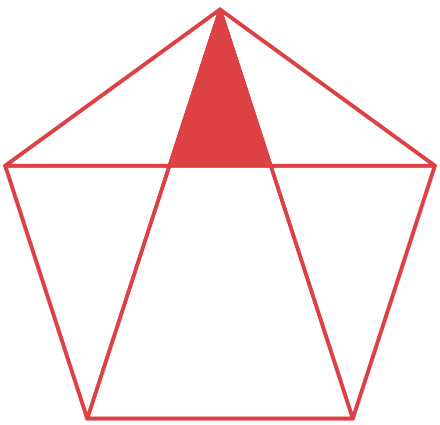

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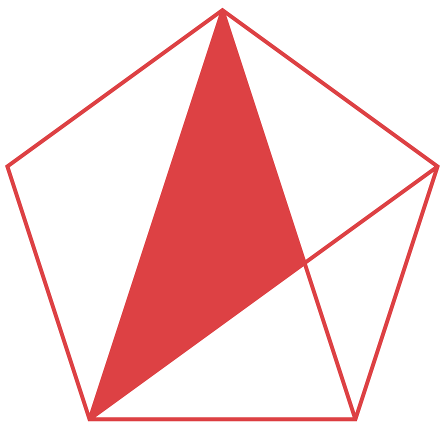
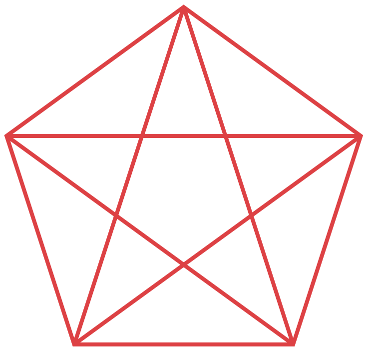
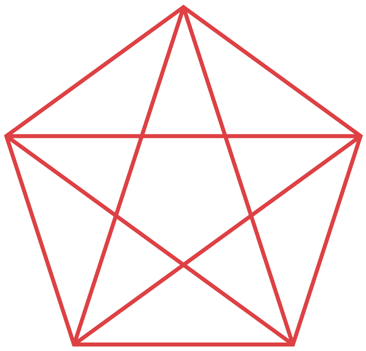
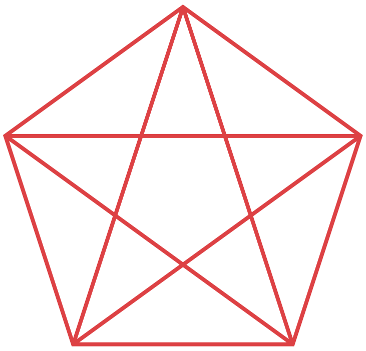
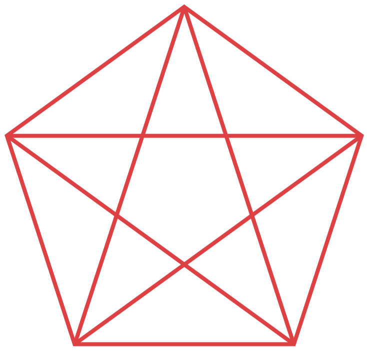
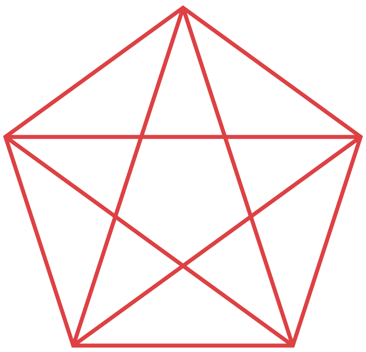
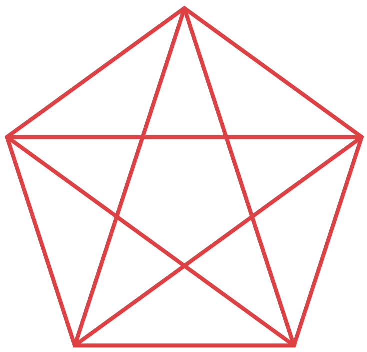
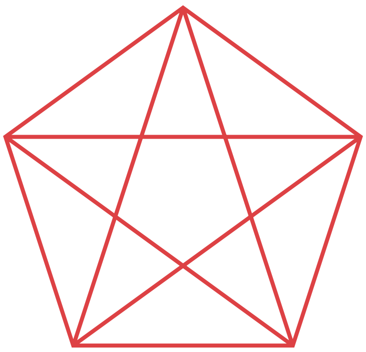
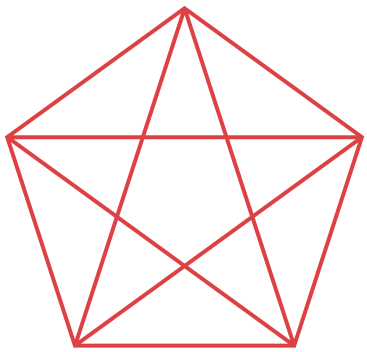


6

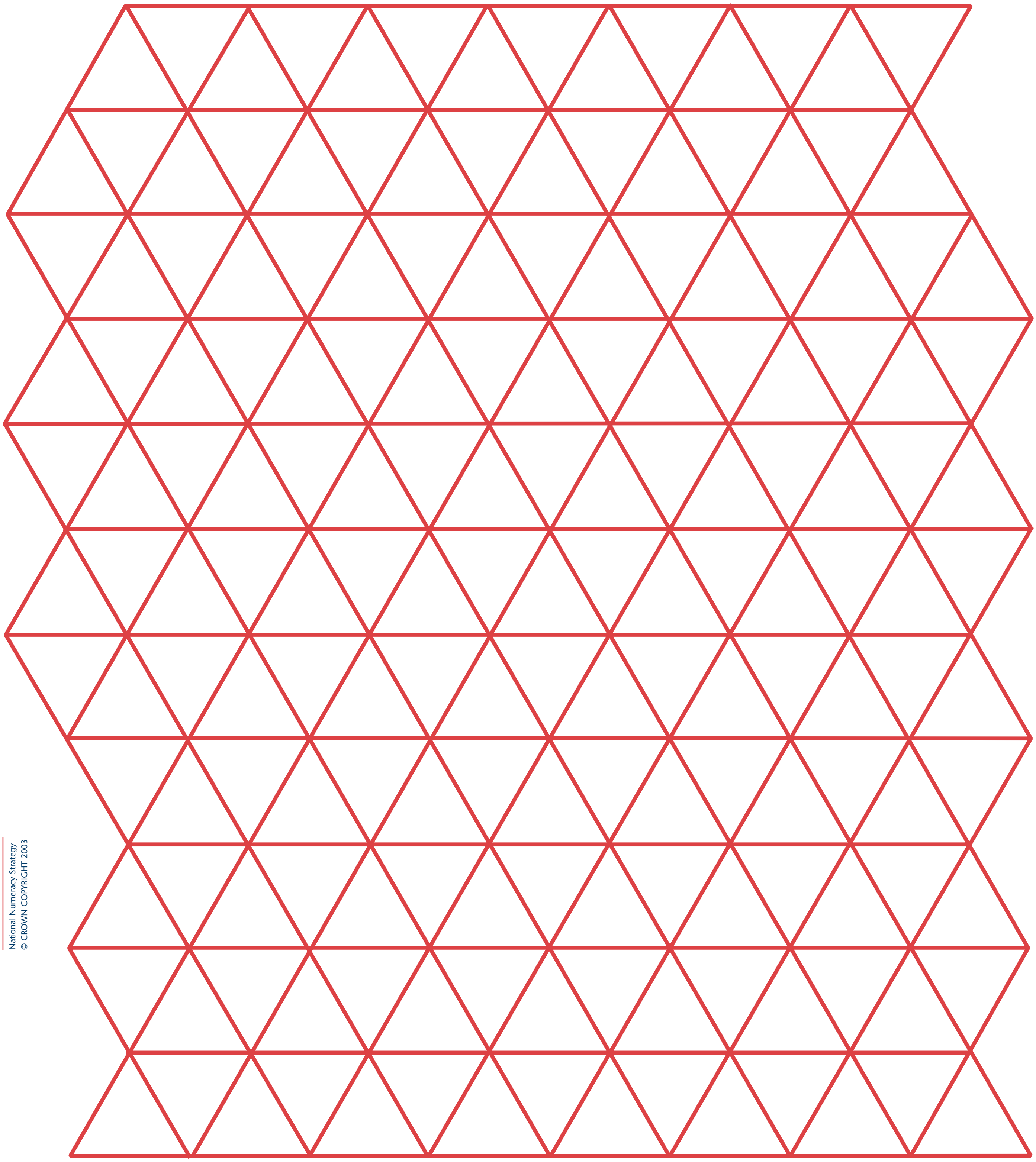
LESSON 28 PROBLEM SOLVING 7

ACTIVITY SHEET 28.1

LESSON 28 PROBLEM SOLVING 7

ACTIVITY SHEET 28.2

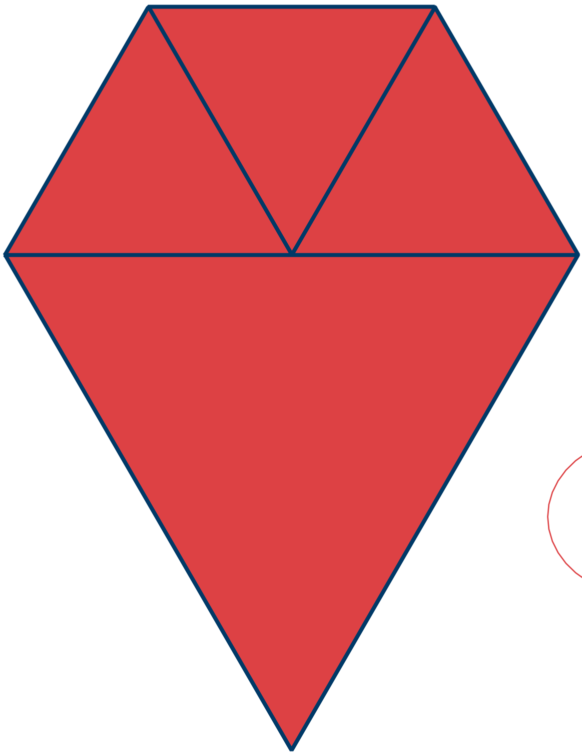


LESSON 28 PROBLEM SOLVING 7

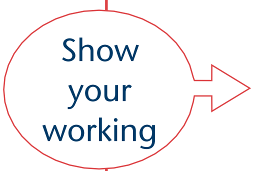
ACTIVITY SHEET 28.3

Maria has **three small equilateral triangles** and **one large equilateral triangle**.

Maria makes this shape:



Not actual size



Calculate the **perimeter** of the shape.

Do **not measure** with a ruler.

cm

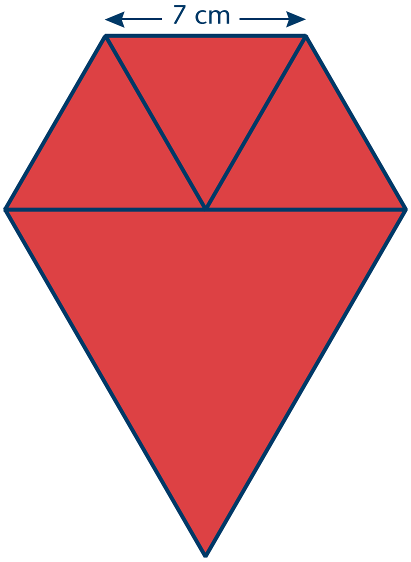
LESSON 28 PROBLEM SOLVING 7

RESOURCE SHEET 28.1

Lauren has **three small equilateral triangles** and **one large equilateral triangle**.

The small triangles have sides of **7 centimetres**.

Lauren makes this shape:



Not actual size

Calculate the **perimeter** of the shape.

Do **not** use a ruler.

cm

1 mark

LESSON 29 PROBLEM SOLVING 8

ACTIVITY SHEET 29.1

1. Write in what the missing numbers could be:

160 + ☐ = 240 – ☐

180 + ☐ = 230 – ☐

300 – ☐ = 220 – ☐

2. Write in the missing digits:

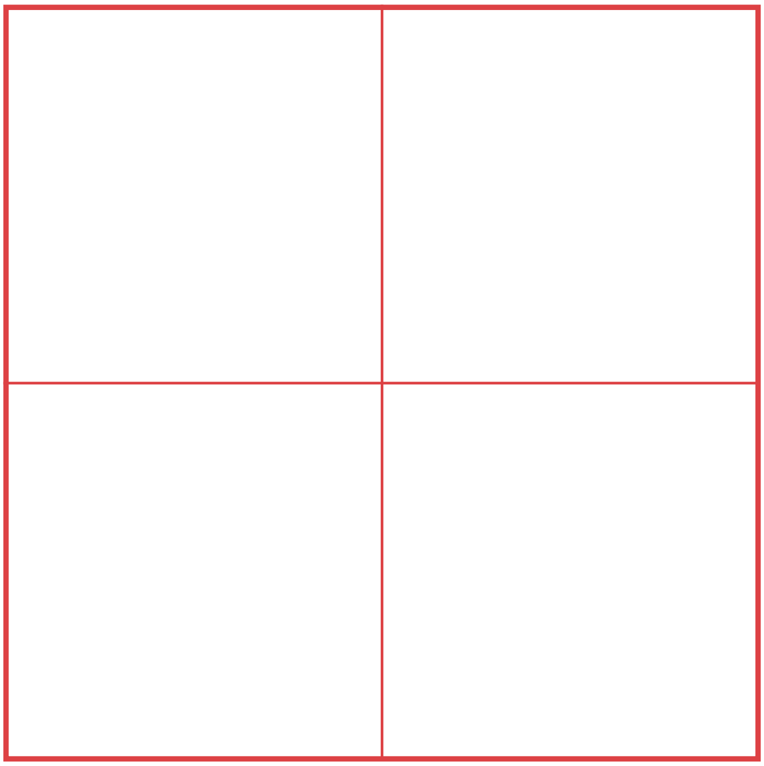
1		5	+	1	4		=	2	7	2
1	5		+	1		6	=	3	4	3
2			+	4	4	8	=	7	0	4

3. Write in the two missing digits:

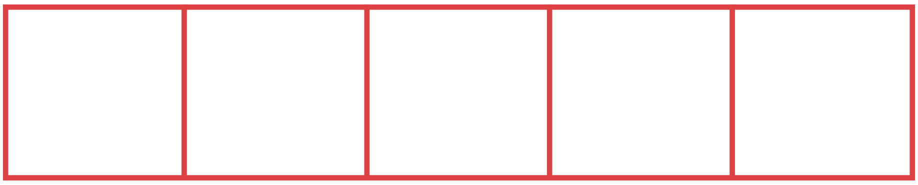
	0	×		0	=	8	0	0	
	0	×		0	=	2	0	0	0
	0	×		0	=	4	0	0	0

LESSON 30 PROBLEM SOLVING 9

OHT 30.1



OHT 30.3



LESSON 30 PROBLEM SOLVING 9

OHT 30.2