

Engaging mathematics for all learners

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Qualifications and
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Foreword

If we want young people to do well in mathematics, it helps if they enjoy the subject. They need to see that the subject is fascinating and exhilarating, to see the way it affects everyday life and helps to change the world in which we live.

We have to strike a balance between the challenge of incremental steps in understanding, knowledge and skills, and the joy, wonder and curiosity of learning. It is not about 'basics' and 'enrichment', all children should have a rich experience.

'Engaging mathematics for all learners' draws together the experience of teachers and their learners as an inspiration to other teachers of mathematics as they implement the new programmes of study. The rich variety of content, from using recreational activities as a starting point for classroom mathematics in a residential special school to using fashion design in a mathematics and art initiative in a selective girls' school, illustrate some of the possibilities. Working with other subject disciplines, exploiting ICT and making sense of mathematics and statistics in the media are some of the innovative approaches adopted by schools to enhance the learning of mathematics.



Being willing to take risks, identifying the mathematics potential of rich tasks, giving learners the opportunity to tackle something different and unusual ... These are all themes to be exploited and used.

Enjoy this publication, exploit the ideas and enjoy mathematics with youngsters.

Mick Waters

Director of Curriculum

Qualifications and Curriculum Authority

Introduction

The new secondary curriculum aims to inspire all young people to become successful learners, confident individuals and responsible citizens. As a result, the mathematics programmes of study have changed significantly.

Every learner is entitled to experience mathematics as worthwhile, enjoyable and challenging, regardless of their prior attainment and background.

Learners need to know

- how mathematics is used in society
- where mathematics comes from
- how aspects of mathematics link together

and experience

- using mathematics to solve problems
- modelling with mathematics
- working collaboratively on mathematical tasks
- using a range of resources and technology to work on mathematics
- mathematics beyond the classroom.

One way of realising this is to make 'enrichment and enhancement tasks' available to all learners. These tasks focus on collaborative mathematical problem-solving using a wide range of contexts, ranging from the purely mathematical to historical, cultural and social. They allow learners to experience mathematics as a worthwhile and enjoyable pursuit, thereby increasing participation and motivation.

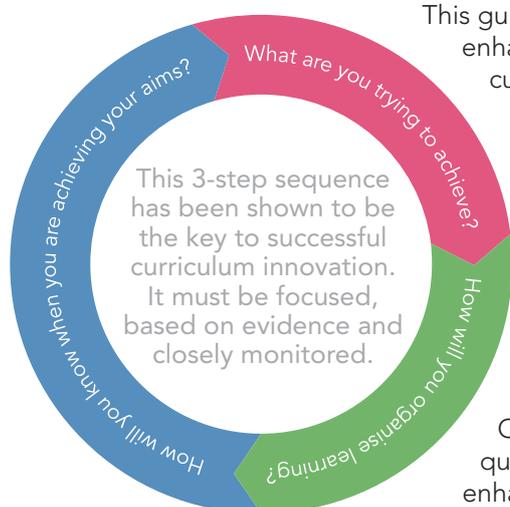
This guide will help you to reflect on mathematics enrichment and enhancement in your curriculum and suggest ways to build them into your curriculum.

There are some practical activities to help you stimulate a conversation about the mathematics curriculum in your school and decide on ways forward.

The activities focus on three key questions:

- **What are you trying to achieve?**
- **How will you organise learning?**
- **How will you know that you are achieving your aims?**

Case studies show how different schools have answered these three questions. They also offer examples of mathematics enrichment and enhancement in action, and should help you to make decisions about what you might do in your school. The Resources section gives sources of support. This publication can be downloaded as a pdf file, using the search function at www.qca.org.uk.



'Disciplined curriculum innovation: making a difference to learners.'
The National Curriculum

What are you trying to achieve?

The importance of mathematics (National Curriculum, 2007)

Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance; and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy.

Mathematics equips students with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all students when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections. Students who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, and can reason, solve problems and assess risk.

Mathematics is a creative discipline. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised. Mathematics has developed over time as a means of solving problems and also for its own sake.

The development of school mathematics provision needs to build on things that currently work well by systematically identifying priorities for change that will improve outcomes for all learners. These need to be consistent with whole-school priorities such as meeting the 'Every Child Matters' agenda, developing personal, learning and thinking skills, developing assessment for learning or embedding cross-curriculum dimensions.

When considering how you want to move your learners forward, you might want to take into account their current views about mathematics.

The Ofsted report *Mathematics – understanding the score* says:

Some schools have begun to collect pupils' views of their experience of learning mathematics. This is a positive move, and action to respond to them has the potential to make learning mathematics more fun for pupils and their teachers.

One student told Ofsted that: 'Maths makes you think – your mind grows intellectually. But sometimes you learn more from your friends than your teacher. Explaining builds up our confidence.'

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Schools are responsible for developing a coherent curriculum that will enable young people to:

- be successful learners, confident individuals and responsible citizens
- achieve higher standards and make better progress in subjects and sector-related learning
- have and be able to use high-quality personal, learning and thinking skills
- have and be able to use high-quality functional skills
- be more engaged, motivated and committed to their learning
- engage with learning to the age of 19 and beyond.

Learning and doing activities in mathematics contributes to the achievement of the curriculum aims.

Advice from teachers to other colleagues

'Take risks, if there is an area you don't feel comfortable then go into that area. Go try new things.'

Advice from teachers to other colleagues

'Be brave, try it! If it doesn't work it doesn't matter, it is one lesson, but if it works, it may be something you may want to implement generally and bring it to other colleagues.'

How will you organise learning?

Planning a compelling learning experience

A real and relevant context for learning will enable young people to recognise, for themselves, the importance of learning to their lives, both now and in the future. Learning becomes compelling when young people actively engage with their own learning and take responsibility for it. A compelling learning experience:

- gives learners a sense of autonomy to think critically, make decisions, take responsibility and manage risks
- offers opportunities for cooperation and collaboration
- broadens horizons and raises aspirations, offering contexts that challenge learners to explore the unknown
- is real and relevant, connecting learning at school to the world outside the classroom
- has a clear sense of audience and purpose
- provides contexts that draw together different aspects of learning, connecting different subject disciplines, or focusing on a specific subject, or linking through cross-curriculum dimensions or the development of personal, learning and thinking skills
- has clear learning outcomes relating to what learners need to know and understand, the skills they will acquire and areas of personal development.

Ofsted (2008) found that in good or outstanding teaching of mathematics:

Non-routine problems, open-ended tasks and investigations are used often by all pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.

Getting started – what are rich mathematical tasks?

A rich mathematical task:

- should engage everyone's interest from the start
- allows further challenges and is extendable
- invites learners to make decisions about how to tackle the activity and what mathematics to use
- involves learners in speculating, hypothesis making and testing, proving or explaining, reflecting, interpreting
- promotes discussion and communication
- encourages originality and invention
- may contain an element of surprise
- is enjoyable
- allows learners to develop new mathematical understandings

Adapted from *Better mathematics* (Ahmed and Williams, 1987) and *Even better mathematics* (2007)

Tasks with these characteristics address key elements of 'Every Child Matters', in particular 'enjoy and achieve' and 'make a positive contribution'. The National Strategies have produced guidance on embedding rich tasks in schemes of work. NRICH has many sources of rich mathematical tasks and advice and guidance on how to use them. See also NCETM's Mathematics Matters. (See Resources).

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Some strategies for devising and working with rich mathematical activities

- Use simple starting points, then ask learners how they might be varied or what questions they could think up to answer next. Collect together suggestions and invite them to choose one to work on.
- Adapt tasks and questions from textbooks, tests or examinations so that learners can make decisions, look for patterns and relationships, and test conjectures.
- Encourage learners to look for connections between old and new situations, ideas and skills, and to ask themselves whether something they've done before might be useful.
- When a student comes up with something that is not relevant at the time, avoid implying that this is the case and bank the idea for later.

Adapted from *The raising achievement in mathematics project report* (1992) by Ahmed and Williams.

Some examples of different types of rich mathematical tasks that have developed using these strategies include the following.

Ask open questions

'How many quadrilaterals can be made on a 9-pin geoboard?': This can lead to work on a wide variety of other geometric issues, including properties and names of the quadrilaterals, congruence, transformations, classifying quadrilaterals according to properties, including symmetry, calculating area, calculating (by measuring or symbolising) perimeter, calculating and/or measuring angle. This activity was used with mixed-ability groups in year 9 and students were encouraged to work in a variety of ways: individually, in pairs, in small groups and taking part in whole-class discussions. Although there are online geoboards that could be used, they do not replace the hands-on practical experience of using the actual boards and the physical movements that are possible, such as rotating them to see if the shapes are congruent. The teachers made ongoing assessments, which determined the kinds of questions they posed to individuals and to groups to develop the work further.

'Normally we have kind of a black and white view about maths, so books and whatever. But now we can see that maths is involved in a lot of things like building and so on. So it just expands our view of maths.'

Learner

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There are a variety of ways in which this task could be extended for further challenge. How do you classify shapes? What is the purpose of classification? How can you prove that you have found all the shapes?

The **NRICH website** (nrich.maths.org) has examples of 'How many ...' activities that schools have used, such as:

- M, M and M: How many different sets of five whole numbers can you find with a mean of 4, a median of 3 and a mode of 3?
- Semi-regular Tessellations: How many semi-regular tessellations can you make?

Explore an examination or textbook question

Questions in textbooks and examination papers can be a good starting point for mathematical exploration. One way of doing this is to ask an open question such as, 'How many ways can you find to solve ...?'

This changes the emphasis from finding the solution to finding and comparing different methods. It is particularly suitable as collaborative groupwork for learners in mixed-attainment groups.

For example, take an equation such as the quadratic equation used to find the golden ratio (a year 10 project) and ask: 'How many different ways can you find to solve this equation?'

This could be expressed in words, to make it more accessible, as, A number squared is equal to the original number plus one. What is the number?

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There are a number of possible methods, so this activity is suitable for a wide range of learners, including those at advanced level:

- Numerical – different strategies for systematic trial and improvement, including using calculators and spreadsheets.
- Graphical – using different combinations of graphs, possibly with the aid of graphing technology.
- Algebraic – researching how to find solutions by completing the square or using the formula for quadratic equations.
- Analytic – trying different rearrangements of the original equation to generate iterative formulae and exploring convergence and divergence of the resultant sequences.

This type of activity generates valuable assessment evidence: Which methods are learners confident with? What can they do with prompts and encouragement?

Use practical equipment and information and communication technology (ICT) to access challenging problems

This strategy was used with the geoboards activity and can also be used with examination and textbook questions. For example:

‘Find the dimensions for the maximum volume open tray that can be made from a fixed rectangle or square of paper (‘Max box’).’

This activity is on the NRICH website as ‘Cuboid Challenge’.

This is a typical AS examination question that expects use of differential calculus. However, it is accessible to a much wider range of learners if they can experiment with paper and use systematic trial and improvement. The use of ICT such as spreadsheets and graphing technology can make the activity more accessible. By generating many examples, students move more quickly to higher level thinking. It also provides opportunities for comparing the efficiency of different methods of solution. This activity was used with an entire year 11 cohort. Posing questions and using prompts to challenge students generated rich evidence for assessment of progress.

‘I have learned how to find the maximum volume of the cuboid and what volume actually means. I believe I have gained more of a mathematical understanding of shapes and improved my accuracy in measuring.’

Learner

Use and design games

Games can be an enjoyable way to practise specific mathematical skills. They can form the basis of an activity where learners design their own games with a specified aim. Not only can this be more enjoyable than routine textbook exercises, but peer evaluation of learners’ own games can lead to fruitful discussion of errors and misconceptions.

One teacher asked her group to design a card-matching game to practise solving equations. After establishing criteria for the game, one lesson was spent on design and a second on trialling the game and evaluating it against criteria decided by the class. The best design was laminated and added to the resource bank for use with other classes.

Another type of card-matching game is for learners to create their own set of ‘Loop cards’ (www.adri.pinel.btinternet.co.uk). Each card in the set, apart from the first and last, has both a response and then a further question.

The Standards Unit materials (see Resources) suggest domino-like activities to match pairs of mathematical objects with equivalent meanings. (The materials include software for teachers to produce game sets for this.) Another type of matching game groups together multiple representations of mathematical concepts, for example algebraic expressions in words, symbols and diagrams. One school used this to start students working in an active, rather than a passive, way. Students can design their own sets of cards.

An activity in one year 7 scheme of work was 'Design an algebra board game'. A sample game was given where players moved around a board according to the number thrown on a die. The rules were in both words and symbols.

The NRICH website has a 'Transformation Game', which consists of a board containing a number of congruent right-angled triangles in different orientations and a pack of cards with details of transformations.

In 'Guess my quadrilateral', one student makes a quadrilateral on a geoboard and the others try to identify it by asking questions that have yes/no answers.

Card-matching and other games can be used equally well in advanced mathematics courses, for example routine practice of more complicated techniques in algebra or calculus, or matching equivalent representations.

Keep something fixed

Maximising and minimising activities where one aspect is kept fixed have the potential to be used with a wide range of learners at all levels. One example already used is 'Max box'.

Other possibilities are

- fixed perimeter – maximise the area (eg the greatest area you can enclose with 60cm of string)
- fixed area – minimise the perimeter
- the historical challenge posed to Queen Dido, who was offered the area that could be encompassed by an ox-hide to settle the city of Carthage
- fixed volume – minimise the surface area (eg a container to hold 24 cubes / 500g of breakfast cereal / a litre of liquid)
- fixed area – maximise the volume (eg a popcorn cone made from a sector cut from a given circle).

All these activities create a need for students to explore mathematical relationships in a meaningful context. For some learners this may include manipulating formulae.

Changing one variable at a time: Some activities can develop valuable skills for both mathematics and scientific investigation. They encourage learners to develop strategies to find underlying relationships by systematically investigating the effects of changing one variable at a time.

Some of the *Bowland maths* (see Resources) activities are particularly suitable for this, for example 'Crash test' and 'Speed cameras'. One teacher reported sustained interest by her low-attaining year 7 students in 'Crash test', where they systematically explored the effect of different variables in a simulation of crash testing cars.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This type of strategy is also useful for activities such as 'T totals', used with an entire year 10 cohort. Initially students were asked to find rules to predict the total of the numbers in a particular shape (T in this case, but any shape could be used) on a 1 to 100 square. They were then encouraged to suggest their own ways of changing the problem. This involved identifying possible variables in the original problem and suggesting ones that could be changed systematically, such as the shape, its size or the size of the grid.

Use ready-made resources

The effectiveness of a resource depends almost entirely on how it is used. Textbooks can be used as a starting point, as a reference to check a particular method or to find out how to do something. The Resources section contains details of many sources of rich activities.

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Finding rich contexts for mathematics

A rich context for an activity will support the development of key concepts and processes as well as covering mathematical content and linking different aspects of mathematics together. Some may also have potential for linking with other subject areas and going beyond school.

Design activities

Design a container to ...

The maximising and minimising activities can be developed further, as in the following examples.

Design a container to hold 24 cubes / 500g of cereal / a litre of liquid / 18 sweets (as in the *Bowland* professional development materials) then negotiate with learners criteria for assessing their designs.

Designing a container such as a barrel to hold a fixed volume has the potential for a rich advanced activity by exploring suitable curves that could be used to model a barrel shape and finding volumes of revolution. Dynamic graphing software can make this activity more accessible.

Another container design activity used successfully for an entire year 10 cohort was '**Design a container to package tennis balls**'. This has the potential for exploring many aspects of geometry and measures, including circle properties. It can also be extended to more complex problems that could be used with advanced level students such as finding the dimensions of the smallest tetrahedron to package four balls (3D trigonometry) and different ways of packing spheres to minimise the space occupied – which has links with science.

Designs for living: Design activities that involve considering measurements of the human form can be rich sources of learning as not only do they tend to have links with science and design and technology, they can also incorporate a wide range of mathematical content from number, geometry and statistics.

Examples include:

- Design an emergency shelter/tent – what dimensions would it need to be to fit two average people sleeping/sitting/standing?
- Design a bench for students to use in the school grounds.
- Garment design.

A different activity that is based on building design considerations is '4 cube homes' where students are asked to find all the possible homes that could be made with four cubes and then choose the best design considering costs of painting, roofing, ground rent, and so on. Real-life examples of this exist in 'Cube homes in Rotterdam' and homes built from disused containers in London's docklands.

Visits to buildings can provide inspiration for mathematics work, for example students who visited modern buildings in London. In another school students constructed geodesic domes and used them for calculations in 3D.

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Using real-life data

It is now easier than ever to access real data thanks to the internet and other sources. There is tremendous potential for linking mathematics with other areas of the curriculum, the curriculum dimensions and exploring current issues. Learners need to develop a critical and questioning approach to the data they find and the way it is presented: Does this concur with other data I've found? What other conclusions might be drawn? What other questions could be asked about this data? Learners can develop the key process skills of representing, analysing, interpreting, evaluating, communicating and reflecting by asking their own questions and finding the data that enables them to answer them. Here are some examples.

Nutritional data: Use nutritional data to explore healthy lifestyle issues by comparing the calorie, fat, sugar and salt content of different types of meals at a fast food outlet, or looking at the information on food packaging.

Historical data: Investigate patterns in deaths during a plague, using the data for the plague in the village of Eyam in 1665.

Global data: The United Nations (data.un.org) and CensusAtSchool (censusatschool.ntu.ac.uk) provide huge databases that students can use to find answers to questions about global citizenship and sustainable development. For example: How close are we to achieving the Millennium Development Goals? (eight goals were set in 2000 for 2015). The Resources section contains a number of activity references and sources of world data.

Some other possibilities are:

- sports data
- weather data
- data from newspapers including sports, finance and weather
- data from websites such as those comparing financial products
- data that can be used to estimate risk, such as accident figures or crime statistics: the *Bowland maths* activity 'How risky is life?' looks at this and has produced interesting reactions from learners.

Mathematics outside the classroom

The school grounds and facilities close to school are valuable contexts for engaging activities. Here are some examples.

Recreational activities: Darts and snooker can be used as starting points for mathematics.

Outdoor trigonometry: The school grounds are a resource for trigonometry activities (for example Teachers' TV – see Resources).

How many matches can you make from a tree?: The lyrics of a popular song inspired this activity. Students used their own ideas for modelling the shapes of trees mathematically and collected data in the school grounds.

Playground project: A local playground can be used as a starting point for mathematical activity. Images can be used to investigate gradients, using slides, and lengths of arcs, using swings.

Along with playgrounds, fairgrounds and theme park rides are also rich contexts for mathematical activities. All can be extended to advanced level.

Uses of mathematics in society

There are many ways in which learners' understanding of the place of mathematics in society can be enhanced.

Use aspects of their everyday life where learners might be using mathematical reasoning, for example working out:

- the best mobile phone tariff for your particular needs
- how much of your daily allowance of calories, sugar and fat is in one fast food meal
- whether the two-person tent you want to buy will be big enough.

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'There was a question on the mock exam paper about arc length and the students all got it absolutely correct. When they came out of the exam they told me, 'There was a question about the swing on the paper!'

Teacher

Use learners as a resource, encouraging them to become aware of when they or their families might be using mathematics in their daily lives, possibly using their own experiences as a source of learning activities. This might include finding out where they used mathematics on work experience or in part-time jobs. Parents/guardians/carers may provide examples; for example a year 7 student who had been doing a project on codes brought in a semaphore manual and a pair of flags that were used by her father, a signaller in the Royal Navy.

Use opportunities within the school such as school visits or trips. In one school, year 7 students worked in a number of different subject areas to plan out all aspects of an end-of-term trip to a theme park, including route planning, timings and costing.

Raising awareness of how others use mathematics in their daily lives

Teachers: Some teachers may have experience of other workplaces or may have particular interests where they make use of mathematics. For example, one teacher formerly ran a travel agency, another worked for a City finance house and a third was employed as a statistician. Others may have particular interests where they might make use of mathematics, such as origami, patchwork, photography or astronomy.

Other school contacts: Relatives, teachers' spouses, governors, representatives of local firms who provide work experience placements and others connected with school may have roles that use mathematics and may be willing to talk to learners about this. Recent school leavers can provide good role models for current students. In one school, an ex-student now studying A levels in mathematics and science was employed to support an after-hours event linking mathematics and dance. In another school, a talk for mathematics students was arranged by an ex-student who is now a rocket scientist with the European Space Agency.

Experts from outside the school: They can stimulate discussion and debate or work with students in various ways. One source of experts is the science, technology, engineering and mathematics (STEM) directories (www.stemdirectories.org.uk). The Institute of Mathematics and its Applications, the London Mathematical Society, the Royal Statistical Society and the Royal Institution have lists of mathematicians who can visit schools.

Current local, national or international events such as the Olympic Games: One school planned a series of events for a full day of mathematics with an Olympics theme. This included estimating activities related to Olympic records, and problem-solving and thinking skills activities.

Current events or issues in the news.

Newspaper and media articles or advertisements: These can be used to assess the validity of media stories based on quantitative data and identify misrepresentation and potential sources of bias.

Visits to museums, local sites of interest, hands-on science locations, theme parks, and so on: Many places have mathematics trails, or students could devise their own for other students in the school.

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Using ICT to support teachers and learners in engaging mathematics

Ofsted (2008) comments that

'Several years ago, inspection evidence showed that most pupils had some opportunities to use ICT as a tool to solve or explore mathematical problems. This is no longer the case; mathematics makes a relatively limited contribution to developing pupils' ICT skills. Moreover, despite technological advances, the potential of ICT to enhance the learning of mathematics is too rarely realised.'

There are six major opportunities for learners to benefit from the use of ICT in mathematics:

- learning from feedback
- observing patterns
- seeing connections
- developing visual imagery
- exploring data
- 'teaching' the computer.

Some examples of how these opportunities can be realised include the following.

Developing visual imagery: Digital images or video clips can be imported into dynamic geometry software or video analysis software to develop mathematical models for objects and motion. Three-dimensional dynamic geometry software can be used to create models that can be stretched and manipulated.

Exploring data: A spreadsheet can be used to help analyse and represent data to test hypotheses about nutritional data. Graphing software could also be used for this.

Teaching the computer: Logo programming (freely available software that learners can access on home computers) was used in an after-school club to produce loci relating to work in mathematics and design technology with Magic Mathworks. This is a travelling hands-on

mathematics exhibition for schools that enables learners to experience a multi-sensory approach to mathematics (see STEM directory).

Providing students with hands-on access to ICT during normal mathematics lesson is a challenge for many schools. These are some of the ways that schools make ICT available:

- a mathematics classroom with a number of networked PCs around the edge of the room on which small groups of students access a range of software
- a set of laptops for use by pairs of students
- a set of graphing calculators or other hand-held devices that students use alone or in pairs to explore sets of data, plot graphs, etc
- a wireless network linking a set of hand-held devices to the teacher's PC.

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Linking mathematics with other areas of the curriculum

Some of the activities already mentioned have the potential for linking with other subject areas, such as design and technology for the design projects (for example 'design a bench', see Case studies), or food technology and science for the nutrition data, or history for the project using plague data. Specialist status can be exploited when developing rich cross-curriculum activities. For example:

Performing arts: Mathematics teachers in schools with performing arts specialism have made links between mathematics and dance.

Sports and PE: A project linking design and technology, science and PE looked at projectile motion using basketball shots and a table tennis ball launcher to provide a context for work in mathematics on quadratic functions. (Teachers' TV programme, see Resources)

STEM – science, technology, engineering and mathematics

A modern technological society needs a well-educated workforce comfortable with all aspects of STEM. The government has identified STEM as a national priority. STEM developments need to take account of all learners and be part of their curriculum entitlement.

Making STEM integral to the curriculum provides contexts for rich learning experiences across the curriculum, and the potential to enhance teaching and learning in individual subjects. This approach increases awareness of the importance of STEM in modern society and the wide range of career opportunities that use STEM subjects (see Resources).

Cross-curriculum dimensions

To achieve the aims of the curriculum, learners need to experience opportunities to understand themselves and the world in which they live. Cross-curriculum dimensions provide important unifying areas of learning that help young people make sense of the world and give education relevance and authenticity. They reflect the major ideas and challenges that face individuals and society.

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Dimensions can add a richness and relevance to the curriculum experience of learners. They can provide a focus for work within and between subjects and across the curriculum as a whole.

The cross-curriculum dimensions are

- identity and cultural diversity
- healthy lifestyles
- community participation
- enterprise
- global dimension and sustainable development
- technology and the media
- creativity and critical thinking.

Examples of mathematics using these dimensions include the following.

Enterprise: 'Bath bombs' is an enterprise activity that uses a lot of mathematics (Teachers' TV).

Global dimension and sustainable development: Areas of social concern are a good source of applications. One school used the *Bowland maths* professional development activity 'Build a school with bottles in Honduras' where empty plastic water bottles are put to good use. Another planned work around designing emergency shelters for use after a natural disaster. These activities also address identity and cultural diversity.

Technology and the media: In one school, students created a video about their work that was made available to students, parents/guardians/carers and teachers through the school's virtual learning environment (VLE).

Creativity and critical thinking: Creativity involves the use of imagination and intellect to generate ideas, insights and solutions to problems and challenges. Creativity and critical thinking that involves evaluative reasoning are vital skills that learners need in order to be successful with any rich mathematical activity.

How will you know that you are achieving your aims?

When making changes you need to be clear why they are being made and to identify the evidence that will indicate how successful the innovation has been. You may want to involve learners in the evaluation.

Recently Ofsted found that in many schools, learners have little opportunity to:

- use and apply mathematics
- make connections across different areas of the subject
- extend their reasoning
- use ICT
- be challenged in lessons
- make links with other subjects
- develop mathematical understanding
- make connections with earlier learning and other topics
- investigate open-ended problems
- choose which approach to adopt
- reason and generalise
- work collaboratively and discuss their ideas.

Although students wanted to do well in mathematics and knew that it was important, they lacked confidence and understanding. They saw mathematics as 'boring', difficult and something to be memorised.

This contrasts strongly with learners' responses to the engaging mathematics activities in this publication. One school asked students to post comments about the 'Playground project' (see Case studies) on the school's VLE: some of these are given below.

- The playground project allowed us to be free of the classroom and to try something new which was good as sometimes maths can seem like a terrible subject and a little bit 'samey' and 'boring'.
- This project was good because it helped us remember areas of maths more and is better for revision such as Pythagoras theorem, triangles, quadrilaterals and circles. Also linking with parks which is an everyday location makes the skills we have learnt stay in our memory because it is more original.
- This project shines a new light on maths and shows that technology is possible in every subject. I think that the project was very exciting and challenging and it also gave us something different to do which means that we were not in the same routine and we were able to remember the information easier. We learned many maths techniques such as angles, shapes and lengths and are now more familiar with working using geometry software.
- We both liked the project because we were working with our friends and also using the pictures to measure the equipment because it showed how it applied to real life situations.
- I enjoyed using the computers while doing maths work as it makes the topic a little more fun. I like using the graphics to measure as it was our own sources. It probably improved my use of tools and using dynamic geometry. There was nothing I really disliked and I learned a lot.

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Below are some comments from year 8 students in a school that is developing a scheme of work based on rich tasks (see Case studies).

What have you enjoyed in mathematics this year? Why?

Working with a partner

- Because you can discuss things and be able to answer a question from a different point of view.
- Because it gives me more confidence.

Doing different activities

- Because it is much better than doing work from boring exercise books that don't help explain anything.

Do you feel you have developed any of these skills?

Having the confidence to work by yourself

- Yes because I always used to ask the person next to me.
- Yes because what matters most is at least I have tried.
- I don't feel embarrassed to speak any more.
- I have gained a skill not to copy others but to do your own work by yourself.

Think creatively

- Last year I needed help, this year I can think more for myself.

Work as part of a team

- We work better as a team because two brains are better than one.
- I have developed confidence working in a team because I never used to share my ideas but I do now.
- I like working together and having different jobs to do.
- I like to work as a team because the work gets easier when everyone is thinking about it and everyone can join in.

Coping with challenge

- We have been talking more about the questions and doing less writing.
- Yes you feel proud and it also makes you feel you can take on the challenge.
- I enjoy problem solving and investigating things.
- It is alright because I want to learn more about it.

What have you struggled with? Why?

- I struggled with algebra because I found it hard to calculate a sum but I didn't know what it meant. Now I understand and it is completely easy.

Case studies

A clear message ... is that, in most schools, mathematics does not contribute sufficiently to the five outcomes of the Every Child Matters agenda.

Ofsted (2008)

Case study 1: Every Child Matters – using recreational activities to engage learners

Background

At a residential school for boys with extreme behaviour, emotional and social difficulties, many of whom also have other learning difficulties and/or disabilities, opportunities for students to engage in a wide range of activities beyond the classroom (for example evening games of table tennis, snooker, darts, dominoes and a floor-standing 'Connect four') were used as starting points for mathematics.

What were they trying to achieve?

The head of mathematics wanted to utilise recreational, out-of-classroom games as learning contexts for using and applying mathematics in lessons. He wanted to find ways of working with data that was 'real' to the students as a way of motivating them to see how mathematics could relate to life beyond the classroom.

How did they organise learning?

Teachers discussed different types of activities and games that students played outside lessons. They generated ideas about how these might be used:

- Snooker: learners could look at the ratio of the number of points compared to the number of shots.
- Darts: they could record the scores and investigate patterns. Are any scores impossible? Which scores are easier to get?
- 'Connect four': the winning line could be written as a set of coordinates, especially if a pair of axes was marked on the board.

Key to the success of the project was teachers' work with evening support staff to explain how they could encourage students to gather information from the games they play, without this being perceived as an imposition on the students' social time.

How well are they achieving their aims?

The school routinely gathers detailed information on students' social, emotional and academic development. Gathering information from the students' activities was an important step, both for summative purposes and for periodic assessment, which informed students of their achievements and suggested next steps for learning and teaching. The use of out-of-classroom activities has already had a positive impact on students and staff. The head of department recognised that the activities used so far are only the 'tip of an iceberg' – the key issue is to enable learners to see that mathematics is not confined to the classroom.

What does the school plan to do next?

The next stage is to sustain the developments so that out-of-classroom activities are used even more. They intend to:

- begin using *Bowland maths* materials
- consider other out-of-classroom activities
- gather data from learners for use in the mathematics classroom.

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Case study 2: Every Child Matters – working inclusively with all ability groups

Background

An 11–18 mixed community school with specialist technology college status, teaching mathematics in mixed-ability tutor groups, used 9-pin geoboards.

What were they trying to achieve?

Teachers wanted learners to take greater responsibility for their work and develop their mathematical thinking skills. The mathematics department wanted to develop a wider range of investigative approaches in lessons and build these into schemes of work. They looked to find ways of engaging with the key concepts and processes of the new mathematics curriculum.

How did they organise learning?

The department chose to develop a single specific resource, the square 9-pin geoboard. Teachers in the department were highly aware of the need to use ideas and resources that enable different students to achieve different outcomes, as they work inclusively with all-ability tutor groups. This could be working on more complex ideas as well as working to different depths of understanding.

Students worked on the problem: 'How many quadrilaterals can be made on a 9-pin geoboard?'

Students were encouraged to work in a variety of ways: individually, in pairs, in small groups and taking part in whole-class discussions.

'It's a social thing. All students feel they are doing the same task and are not left out.'

Teacher

How well are they achieving their aims?

The teachers assessed continuously and this determined the questions for individuals and groups. Students used and developed geometric understanding about quadrilaterals, congruence, angle, perimeter and area.

Lesson outcomes were used to adapt subsequent lesson plans. Learner evaluations were collected from the year 9 pilot and these were used to inform developments for using geoboards with year 7 students the following year. In turn these year 7 student evaluations shaped further developments for later work with other year 7 groups.

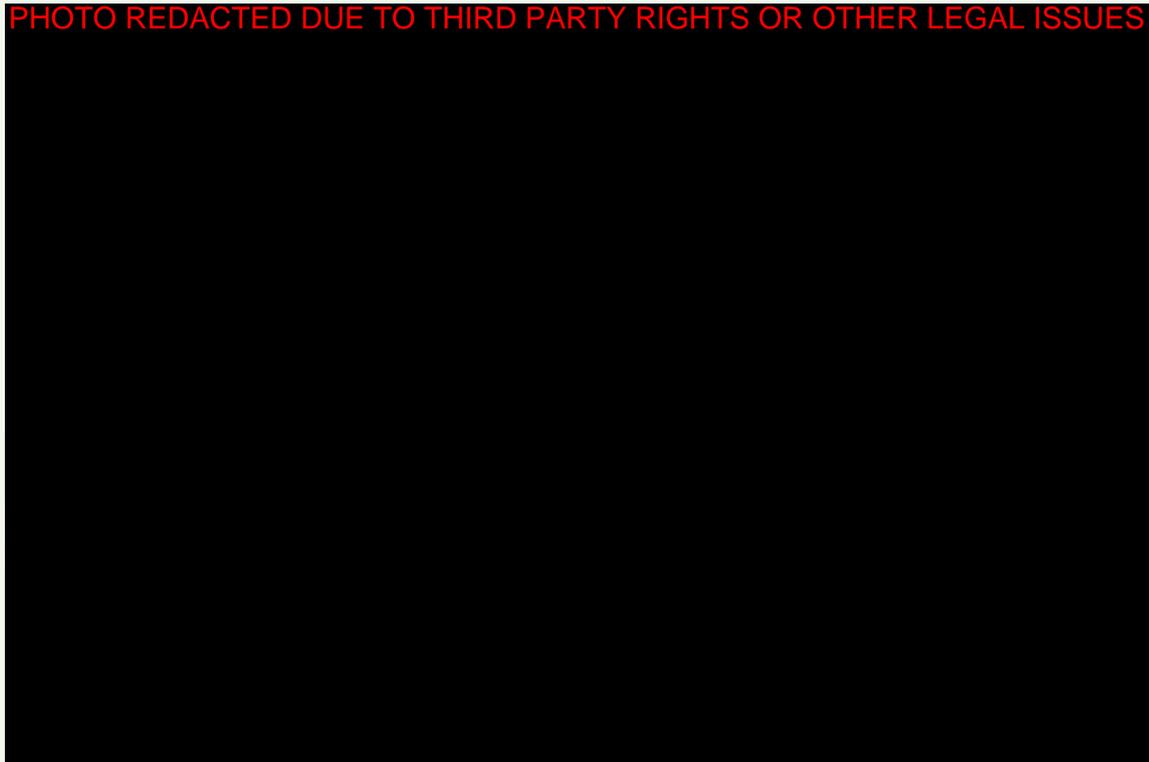
Teachers felt that there was a high level of engagement by students, and the quality of discussion led to them achieving greater clarity of understanding.

What does the school plan to do next?

The approaches and the strategies used in this project might form one model for developing further modules in years 7, 8 and 9. Professional development will focus on finding a number of other 'rich' starting points that enable learners to engage with mathematics.

Case study 3: Historical and cultural roots of mathematics

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Background

A boys comprehensive school with a coeducational sixth form used 'The power of two', a Royal Institution mathematics masterclass based on a historical episode, with year 9.

The Josephus Problem: given a circle of people where every alternate living person is killed in succession, where should one stand in order to be the last person alive (the survivor)? The problem provides an accessible introduction to number systems and to the notion and importance of proof.

What were they trying to achieve?

Mathematics teachers wanted learners to work collaboratively in mixed-ability groups.

They chose 'The power of two' to engage a wide range of students. There were multiple mathematical goals:

- to experience the interconnectedness of mathematical ideas
- to help learners gain a deeper understanding of number systems
- to give an accessible and engaging introduction to proof.

How did they organise learning?

Seventy-five year 9 boys worked in mixed-ability groups of eight or nine in the school hall. There were additional teachers on hand to support the groups.

In session one, learners were introduced to the problem. Through exploration they found ways of identifying who would be the survivor for any group size.

In session two, they were introduced to binary counting and asked to find the solution to the Josephus Problem by working in binary. Most boys were able to use binary to justify their findings in session one.

How well are they achieving their aims?

Teachers were delighted by learners' engagement and the way the activity reinforced understanding of place value. However, they felt that group work could have been more effective if learners had more experience of group work and more time.

Students said that they enjoyed working in a different environment and in groups, and being made to think. They wanted to know when something like that might happen again. They also enjoyed encountering a different number system and, in follow-up lessons, explored addition and subtraction in binary and other number bases.

What does the school plan to do next?

The department will use similar learning experiences again. Ensuring that students know how to work collaboratively and allowing more time will help everyone to get even more out of such experiences. They plan to make use of school 'activity days'. They hope to share their experience with other schools in the same local authority.

'It was very interesting when he introduced the suggestion of Josephus Flavius and it got me thinking.

The group work was fun and entertaining and it was easier to work because we could discuss it.

The group work was good and it gave different opinions about the same problem

I learned that a problem starts small and builds up to make a bigger problem.

The solution was hard but not too hard, just challenging.'

Learner

Case study 4: Historical and cultural roots of mathematics – the golden ratio

Background

An 11–19 comprehensive coeducational specialist language community school wanted to motivate and engage year 10 learners.

What were they trying to achieve?

The mathematics department wanted to improve motivation and engagement during the period between an end of GCSE unit examination and the end of the summer term. They decided to trial an enrichment and enhancement project with the entire year 10 cohort.

How did they organise learning?

Two teachers who shared responsibility for key stage 4 discussed ideas for a package of materials on the golden ratio and Fibonacci to be used for a two-week period. They prepared for more lessons than the time available so that staff could choose activities appropriate to their classes. They presented different aspects of the materials to colleagues at a key stage 4 department meeting. Working together on some of the activities helped teachers to build a shared vision.

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The lessons aimed to revise, reinforce and extend existing mathematical and ICT skills, and gave learners opportunities to:

- do their own research (eg art, architecture, Fibonacci in nature)
- construct geometrical objects in two and three dimensions
- test hypotheses such as 'girls are more golden than boys', using body measurements.

How well are they achieving their aims?

Students' reactions were very positive. They were intrigued by the patterns and the appearance of ϕ or of the Fibonacci sequence in many different places, and easily identified things they wanted to investigate. Students from other classes were fascinated by displays of the work.

'I found it particularly interesting finding out all the natural things that have the golden ratio such as sunflowers and shells, and also growing. Also how different things divided by each other equal to the golden ratio.'

Learner

Staff noted the students' enthusiasm and engagement across the entire cohort. The project had a lasting effect as students were proud of their work, regarding it as 'real maths', and were more willing to try different approaches when working on something new. The following term, some students were able to make links with the project when developing methods for solving quadratic equations and revising geometrical constructions.

What does the school plan to do next?

The activity will be repeated at the same time next year. Teachers will develop and incorporate other compelling learning experiences in the departmental schemes of work.

Case study 5: Modelling with mathematics

Background

An 11–16 mathematics and computing specialist college tackled a 'real-life' problem in year 9.

What were they trying to achieve?

Teachers wanted learners to work on a 'real-life' problem to develop their mathematical modelling skills. Learners needed to make and justify assumptions, and develop strategies for solving the problem, including breaking it down into more manageable tasks.

How did they organise learning?

The leading mathematics teacher worked with two recently qualified teachers. They wanted to use rich, open-ended activities and be more adventurous in their classroom approaches. They chose the activity 'How many matchsticks can you make from a tree?'. This activity had been shared by a teacher from another school at a local authority meeting.

The activity was introduced with the chorus of the song '1000 trees' by The Stereophonics: 'One tree can make 1000 matches but it only takes one match to burn down 1000 trees.'

The learners were invited to test the hypothesis that one tree could make 1000 matches.

Three different attainment groups across year 9 worked on the activity.

How well are they achieving their aims

The learners explored questions such as: What shape is a tree? What about branches? How high do we go?

The mathematics content included measuring or estimating heights and circumferences, trigonometry and scale drawing, standard form, use of and conversion between different units.

Some groups produced their results as a poster while others chose group presentations. They were able to justify their results using their assumptions. Some went on to suggest and explore their own extensions. The teachers were very enthusiastic about the response of the learners and their results.

What does the school plan to do next?

The outcomes provide valuable evidence of learners' achievements for assessing pupils' progress. The two less-experienced teachers are more confident about incorporating open tasks into their practice. The department is looking for other accessible tasks that bring aspects of mathematics together.

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Case study 6: Mathematics in society – ‘number sense’

Background

A boys’ comprehensive specialist mathematics and computing college used ‘Number sense’, a Royal Institution mathematics masterclass, with year 10.

The tiger that isn’t: seeing through a world of numbers by Michael Blastland and Andrew Dilnot, *‘How risky is life?’* (Bowland maths) and websites such as www.understandinguncertainty.org and the Office for National Statistics (www.ons.gov.uk) are sources for understanding the use of number in modern society.

What were they trying to achieve?

Teachers wanted to empower learners to make sense of and ‘see through the world of numbers’. In order to broaden their appreciation of number and how it is used, learners needed to engage and be challenged on aspects of estimation and application in real life.

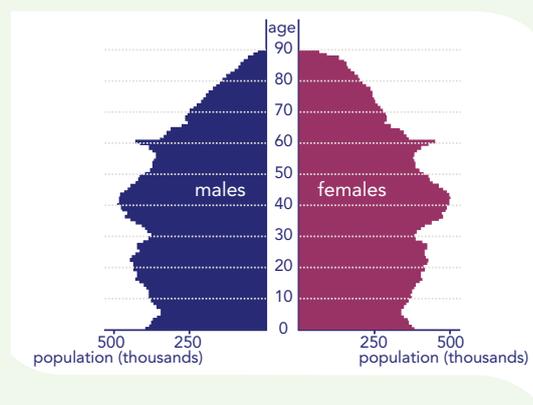
How did they organise learning?

The department decided to use the opportunity given by changes in the curriculum to incorporate approaches, processes and activities from the Royal Institution masterclass into classroom practice and schemes of work. They chose to work with high-achieving year 10 pupils, working on problems in small groups.

Students used estimation to explore examples that demonstrated the relevance and magnitude of numbers, for example the size of the NHS annual budget, the impact of speed cameras, the increase in cancer risk from consuming alcohol, school league tables, teenage pregnancy and world debt.

A simulation was used to estimate the average number of children per family and the female-to-male ratio in a hypothetical society where couples keep having a child until a male child is born, at which point the couple has no more children. Assuming that males and females were equally likely meant that a coin toss could be used to simulate the situation. The solution challenges most people’s intuition, illustrating that dramatically different populations can still have the same average family size and female-to-male ratio.

Work on assessing risk incorporated resources from *‘How risky is life?’* (Bowland maths) and data from the Office for National Statistics. Students used a population pyramid graph to analyse the different life expectancies and mortality rates for males and females.



How well are they achieving their aims?

Students engaged fully in the activities, which didn’t have ‘one right answer’ but required them to make assumptions and estimate. Asking students to probe and justify or debunk press stories or government releases, based on their own estimation skills, gave them a sense of ownership and satisfaction. Problems relating to education, disease and poverty were particularly motivating and helped learners to see the relevance of mathematics to social justice issues.

Simulation strengthened students’ understanding of probability and allowed them to generate their own examples.

What does the school plan to do next?

The department is collating a set of problems that will form a core of 'Number sense' activities to be used throughout the school. This and the *Bowland maths* activity will be incorporated into the schemes of work, with the opportunity for teachers to add their own examples or develop existing ones. Every daily newspaper has articles that are starting points for problems – students could be asked to look at these for homework. The potential for more extended pieces of work will also be explored.

Case study 7: Mathematics in society – technology and the environment



A boys 11–16 specialist technology college used the Royal Institution masterclass 'Shape of space' with year 9.

What were they trying to achieve?

The mathematics staff wanted to increase students' awareness of the mathematical world outside school and facilitate a deeper understanding of geometry (especially spatial awareness and 3D) inside the classroom.

They wanted to embrace the new curriculum with stimulating project-based work, making meaningful links between mathematics, technology and the environment.

Teacher about using perfect thirds as a starter for sixth form students

'The pupils told me: 'Sir, we should have done algebra like that!'

How did they organise learning?

Working with the Royal Institution, the students visited some modern buildings in central London, including the 'Gherkin' (30 St. Mary Axe) and City Hall. As preparation, students read the *Plus* magazine article 'Perfect buildings: the maths of modern architecture' and researched the award-winning buildings themselves: their shape and the technology and the mathematics behind the designers' decisions.

The trip culminated in a meeting with a team of architects from Foster and Partners to discuss questions that the boys had prepared in advance. The session enabled them to increase their awareness of the mathematics in the built environment and of how aesthetics,

design requirements, the logistical limitations of cost, physical limits and material properties, all need to be included and balanced in architects' work. It also raised awareness about STEM careers.

Follow-up classroom activities were practical and exploratory, for example:

- stability of 3D shapes (inspired by the NRICH problem 'Building With Solid Shapes')
- plans and elevations (using the Standards Unit materials)
- Euler's formula
- folding an origami dodecahedron and a bucky-ball (adapted from Thomas Hull's *Project Origami*)
- folding a strip into perfect thirds (*Project Origami*).

How well are they achieving their aims?

Students now better understand geometric properties and the relationships between them. The work spread to other year groups: the dodecahedral construction was taught to year 8 students by the year 9 students. Perfect thirds by folding was used with high-attaining year 10 students who were able to make links between geometry and algebra. The project increased interest and motivation throughout the school. Mathematical models have joined the art exhibition in the school reception area and the project was reported in a local newspaper.

Other teachers are keen to use more creative and practical approaches in their teaching.

What does the school plan to do next?

These activities are now in the schemes of work. Two strategies to maintain the impetus are team teaching across the department and doing mathematics together in departmental meetings. Teachers are exploring the potential of local buildings (for example the football stadium and local churches) as venues for trips and starting points for mathematics.

Getting different year groups working together will continue, as communicating mathematics can aid understanding.

Learners about peer mentoring

'We've already taught a year nine set once, so we're kind of used to it now and it's always easier if it's somebody slightly younger than you, than doing it with your peers.'

I think it does help you learn if you teach others because if you teach others it means you already have that knowledge and it's just every time you teach others it's kind of, kind of enforces that knowledge that you have.'

Teacher about the perfect thirds activity

'And even the thirds, the perfect thirds, I didn't think at first that it was very rich and I went home and I did some maths and actually involved my husband, I was sure we could prove it. That is something I think we should do ... kids don't see maths teachers doing maths, and maths teachers don't do maths, our team meeting is all about admin. We should do some mathematics.'

Case study 8: Mathematics across the curriculum – performing arts

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Background

An 11–16 comprehensive performing arts college used dance to aid mathematical understanding and increase motivation.

What were they trying to achieve?

The head of mathematics identified that middle-attaining girls appeared to be underachieving in mathematics when compared with their other subjects. He noticed that many of this group were highly motivated in dance and stayed for activities outside school hours. He wanted to build on this to improve their motivation and achievement in mathematics.

How did they organise learning?

The head of mathematics and the director of dance planned and delivered a number of mathematics-focused dance lessons for key stage 3 girls and boys. They produced picture packs 'Dance your way to maths' which they displayed in both mathematics and dance areas.

The lessons included developing dance approaches for geometric concepts, for example symmetry, location, transformation, parallel, perpendicular, angle, slope and shape. They ran a Saturday morning event entitled 'Dance, maths and pizzas'. A former student helped and was an excellent role model – good at dance and studying A level mathematics!

How well are they achieving their aims?

The 'Dance your way to maths' materials have been used with a range of students across key stage 3. The 'Dance, maths and pizzas' event was both successful and enjoyable. The Saturday mornings have been extended to revision classes for GCSE retake students, who divide their time between devising original dance routines and small group work on examination techniques for mathematics.

What does the school plan to do next?

The materials will be embedded in the schemes of work for both mathematics and dance. The head of mathematics is working with the drama department to explore possible synergies, for example role play or writing short plays with a mathematical theme.

Case study 9: Mathematics across th

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Background

A boys' comprehensive specialist mathematics and computing college used 'Calculating colours', a Royal Institution mathematics masterclass.

Starting from 'how' colour works can lead to a 'mathematisation' of colour. Colour can be quantified and measured. Colour can be used as a context for developing understanding of certain aspects of mathematics.

What were they trying to achieve?

Teachers wanted learners to have a hands-on, accessible and engaging experience that would help them to understand important mathematical principles. They wanted learners to appreciate that mathematics is connected to other aspects of the curriculum.

How did they organise learning?

They planned for pupils to work on 'Calculating colours', as this provides an accessible and novel experience. Year 7 students undertook a practical activity mixing red and blue liquids to give them a tangible representation of fractions, percentages and ratios using colour. Given the experimental approach, students could access the activities at a variety of levels. This provided a context for discussing multiple representations of numbers and appreciating that there are infinite possibilities for numbers, yet just a finite spread in this practical context given the limitations of our senses and technology.

Mixed-ability year 9 students used spinning tops and mixed light with data projectors. They then designed and constructed their own tops. They were introduced to hexadecimal representation of numbers and the use of these for representing colour on the internet and in other computer-facilitated technologies (RGB monitors, CMYK printers).

How well are they achieving their aims?

Students enjoyed experimenting in mathematics and relating fractions and ratios in a hands-on way. Many of the year 7 students were able to appreciate that many 'colour' fractions are indistinguishable by eye and were disappointed when the colour mixing lesson finished. This activity was also used with year 5 and year 6 learners from local primary schools. The year 9 students enjoyed the challenge.

What does the school plan to do next?

Teachers will incorporate these practical experiences into the schemes of work, and are looking at how they might work more closely with art and science on colour. They are looking for other experiments they can incorporate into their schemes of work. For year 9 there are further experiments available, in particular a statistical one where students mix different amounts of RGB to create the colours of various fruits and vegetables and then study statistical attributes such as average and range (Experiments at school, see Resources). This will provide the ideal opportunity to consolidate ideas in colour mixing and statistics simultaneously.

Colour	Hexadecimal	Colour	Hexadecimal	C
aqua	00FFFF	green	008000	na
black	000000	gray	808080	ol
blue	0000FF	lime	00FF00	pi
fuchsia	FF00FF	maroon	800000	re

Case study 10: Mathematics across the curriculum – STEM and PE

Background

An 11–16 comprehensive specialist sports college used video analysis software for capturing data from PE.

What were they trying to achieve?

Through cross-curriculum work with ICT, science and PE, the mathematics department aimed to make the subject more relevant, engaging and challenging for all learners. They also wanted learners to develop a better understanding of how mathematics can be used to model real life phenomena.

How did they organise learning?

Students videotaped free throws in basketball. The videos were edited during ICT lessons to produce an image of the ball's trajectory. This was used as a background in a dynamic geometry package where students superimposed graphs of different quadratic functions in order to find a mathematical model of the ball's trajectory. The video clips were also used in science to help understand the principles of projectile motion.

How well are they achieving their aims?

The students can see the relevance of mathematics to other areas of the curriculum. The mathematics department has built the video analysis approach to quadratics into its scheme of work for all students and has extended the approach for use with year 10 students on transforming functions, where they model their trajectories, with confidence, using functions of the form $f(x) = p + q(x - r)^2$.

The approach has been extended to design and technology, where students design and make a table tennis ball launcher.

'Results for Maths, in particular, were outstanding and beyond expectations. From September 2008, Maths is to be a lead department, linked to our specialism, raising standards in education throughout the college.'

2008 School prospectus

What does the school plan to do next?

Teachers are trialling other activities based on trajectories of projectiles, including one using air-based and water-based rockets. Their use of video analysis software will be shared through local authority and other networks.

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'If you have a carpenter measuring something, he is not thinking 'today I am doing measurement', he is just using it to do his particular job. The idea is getting them [the learners] to understand that maths is a tool that we can use and is not just some abstract thing that we use in the classroom.'

Teacher

'I like the fact our teachers bother to put maths into some sort of context. It makes it a lot easier to learn.'

Learner

Case study 11: Mathematics and curriculum dimensions – healthy lifestyles

Background

An 11–19 girls school used nutritional data from a fast food restaurant chain.

What were they trying to achieve?

The teacher wanted to introduce project-based work to students who were unused to working in this way, to use real data as a basis for work on the handling data cycle and to encourage students to think about issues relating to healthy eating.

How did they organise learning?

A teacher collected data from a fast food chain and made it available to year 9 students who were asked to work in groups and develop their own questions for exploration. Some of the hypotheses suggested by students were that salads have fewer calories than sandwiches, breakfasts contain more fat than sandwiches, and the more calories a meal has, the more fat it has.

How well are they achieving their aims?

Students responded positively; they liked working in pairs or groups and working on what they considered to be a real problem with real data. They recognised that they had made use of a range of statistical techniques and understood them better as a result. Their awareness and understanding of food and healthy eating was also raised.

What does the school plan to do next?

More extended activities will be used with the students in the future. The potential for making links with other subjects such as science and food technology will also be explored.

Case study 12: Mathematics and curriculum dimensions – technology and the media

Background

An 11–16 coeducational comprehensive arts college used the playground as a starting point for year 10.

What were they trying to achieve?

The head of department wanted to develop learners' ICT skills using dynamic geometry and video analysis software, and to introduce them to working in a 'real world' context. She wanted students to model real situations with mathematics.

How did they organise learning?

In the year 10 'Playground maths' project, the students used photographs and video clips of roundabouts, swings and slides as contexts for work on circle theorems, locus, triangles, trigonometry, gradients, equations and mensuration. The photographs and video clips were available on the school's VLE and could be manipulated to form backgrounds for work with dynamic geometry software.

Distances, angles and gradients were measured using photographs of the slide in dynamic geometry software. A video clip of the swing in action was used as a starting point for work on locus and calculating arc lengths.

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How well are they achieving their aims?

The students produced a video, with the aid of the school technician, showing how they could extract various aspects of mathematics from playground equipment – swing, slide and roundabout. They gave instructions to the technician on editing the video and on which parts they wanted included.

'I've been to that park 20 times and I've never seen the maths I do now.'

Learner

The following term, the teacher used the *Bowland maths* continuing professional development (CPD) activity 'Build a school with bottles in Honduras'. She found that the students' confidence in using the software to assist their work was considerably increased and that they were really competent in using the dynamic geometry software to help them simplify and represent the problem.

What does the school plan to do next?

Other teachers are beginning to try similar activities. They are also experimenting with less structure, for example: 'What mathematics can you find in the playground?'

Case study 13: Mathematics and curriculum dimensions – creativity and critical thinking

Background

A girls' grammar school used 'Geometry in fashion', a Royal Institution mathematics masterclass delivered by fashion designer Julian Roberts about his 'subtraction cutting' technique.

'Stretching the comfort zone' used the masterclass as a starting point and as a vehicle for exploring mathematics in a creative way and making links with art and design and technology.

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What were they trying to achieve?

Teachers wanted to challenge high achievers and offer a practical and novel experience. They also wanted to help students to deepen their understanding of geometry, strengthen their visualisation skills and appreciate the relevance of mathematics in other disciplines.

How did they organise learning?

Mathematics and art teachers worked with the Royal Institution to develop a project about fashion design and garment construction. This provided an appealing context to explore intriguing mathematical concepts through hands-on tasks. Starting with a subtraction cutting masterclass by fashion designer Julian Roberts, at the school, teachers then worked with sixth-form students to help year 9 girls learn Julian's technique. This uses straightforward ideas of perimeter, area and volume; understanding the approach and outcomes requires abstract concepts such as the topology of surfaces, curvature and space-filling curves.

'Later on in the course when we talk about area and perimeter of even more complex shapes, we can refer back to that and it is an obvious starting point that they will remember.'

Teacher

This was followed by a range of related activities that aimed to consolidate geometric and spatial understanding, drawn from plans and elevations, area and perimeter, making new shapes by edge identification (for example sewing or gluing), space-filling curves, the Möbius band, the torus and the Klein bottle.

How well are they achieving their aims?

Julian's visit was celebrated with displays in the mathematics department. These displays help students to make connections between architectural views and garment construction, and this 'association' is now part of the students' mathematical toolkit. Students' visualisation skills have developed considerably and they are now able to view mistakes as something to be dealt with positively and constructively.

What does the school plan to do next?

The activities will be embedded in the schemes of work for both mathematics and art. In art, year 10 students are learning Julian's technique to produce garments using silk-screen fabric that they have designed and printed. The school plans to continue developing inspirational learning experiences.

Case study 14: Using timetable opportunities for engaging mathematical activities 1

Background

A large rural school with media arts status used NRICH activities in a school event.

What were they trying to achieve?

Teachers wanted to develop students' problem-solving and group working skills through the use of rich tasks.

How did they organise learning?

As a first step they used a school-wide one-off event to work with year 10 sets in mixed-attainment groups. Teachers worked together on the activities and planned a common experience for both students and teachers. The two selected tasks were:

- 'Zin Obelisk', a problem-solving activity where a variety of information has to be analysed and synthesised to find a solution
- 'Doesn't Add Up', a geometric problem with two diagrams that appear to contain the same shapes but have different areas.

'They seem to be getting on really well and helping each other without too much input from me.

I love it. It's the way I love to teach and if they learn things almost by accident then they're having to do the learning and they take it on board more.'

Teacher

'It's more working with other pupils and more problem solving than normal mathematics lessons.

It's more of a 'real life' situation.

We're working in groups and it's a bigger challenge.'

Learner

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How well is the school achieving its aims?

The first stage of the project has been a success – all members of the department were involved in teaching the lesson involving the two tasks. Teachers felt that the groups had worked well and students had surprised them with what they had managed to do.

At the end of the day, students commented on the benefits of working with people they wouldn't normally work with, how specified roles meant that everyone in each group had to be involved, learning to cooperate and compromise, and looking at things differently by considering other people's points of view.

What does the school plan to do next?

Having seen the benefits of using rich tasks, teachers are keen to embed them into their everyday practice, enriching learners' mathematical experience. As teachers become more familiar with the key concepts and key processes, they will draw students' attention to the important mathematical behaviours that they want to encourage. The curriculum mapping documents on the NRICH website are helping them to identify further activities to consider for their scheme of work.

Case study 15: Using timetable opportunities for engaging mathematical activities 2

Background

A 12–16 girls' technology college used fortnightly flexidays for engaging mathematics.

What were they trying to achieve?

The school introduced fortnightly flexidays where half year groups worked with their usual teacher on a specific subject for the day. Each pupil had three mathematics flexidays per year. Teachers wanted to exploit these opportunities for learners, and identified sustaining interest and motivation throughout the day as a priority. They also wanted to make mathematics more enjoyable, relevant and real, use cross-curriculum dimensions and nurture the development of personal, learning and thinking skills.

How did they organise learning?

Teachers worked together to plan activities for flexidays, bearing in mind the need to cater for all learners. Activities included:

- 'Design a bench' – year 8 had to design a seat for use in the school grounds
- 'Greece to China' – an Olympics theme day with a circus of activities for year 9, which included the use of the computer room, an estimation activity using the mathematics corridor and the gym, a group problem-solving activity and a thinking skills activity requiring the accurate reproduction of a poster
- 'Anyone for tennis?' – year 10 had to design a package for tennis balls.

How well are they achieving their aims?

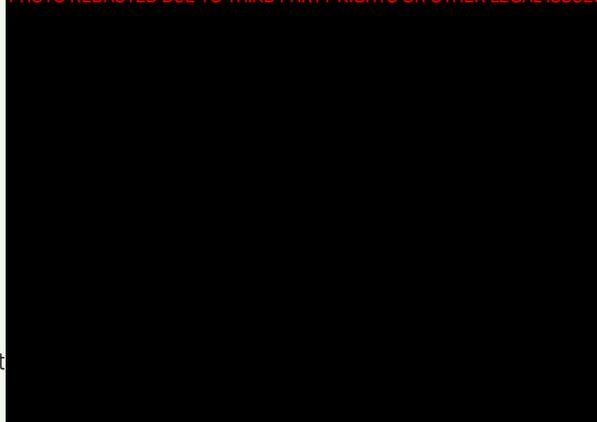
Teachers are pleased with the quality of response to many of the activities and they are developing strategies for ensuring that they have something for everyone throughout the day. Working together for the whole day has had positive benefits for staff and pupil relationships.

The bench activity was run as a competition, which proved motivating for many. However, the bench hasn't been made, which is a disappointment to some. A3 posters of the designs were displayed in classrooms and excited the interest of students from across the school.

What does the school plan to do next?

The department is building up a bank of activities and resources for flexidays and pursuing collaborative events with other subject areas, such as art and design, design and technology, and science.

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Case study 16: Working together to trial engaging mathematical activities (*Bowland maths*)

Background

An 11–18 specialist technology school used *Bowland maths*.

What were they trying to achieve?

Teachers wanted to develop students' mathematical thinking skills by using a range of contexts and teaching approaches.

How did they organise learning?

The mathematics department had been involved in the *Bowland maths* developments and recognised how these materials could support development of the key processes detailed in the new programme of study for mathematics. Prioritising time to explore the materials and plan for their use proved a challenge.

However, when one teacher shared her students' positive experiences of using the 'Organising a table tennis tournament' idea from the professional development materials, others were inspired to use the activity too. Other activities that have been used with classes from year 7 to year 10 include 'Product wars', 'Outbreak', 'Point zero', 'Crash test' and 'How risky is life?'. The latter module produced a particularly interesting response from a group of year 9 students who initially thought they were taking part in a personal, social and health education (PSHE) lesson.

How well are they achieving their aims?

The *Bowland maths* materials are being used more widely within the department and, consequently, more communication is taking place between staff about ways of working and resources that can be used. For example, greater use is being made of making posters as a way of students demonstrating achievement.

'How risky is life?' evoked the following reactions from year 9 students

- They found the work a little morbid but interesting as it related to their lives.
- They initially thought that they were taking part in a PSHE lesson – because of the presentation style they didn't realise how much mathematics they were doing.
- They felt comforted by just how safe the streets of their town are and realised how media reports probably need further investigation to assess the scale of the issue.

One member of the department has presented the materials to other teachers both locally and nationally. Taking such opportunities to disseminate the materials more widely has strengthened the resolve within the department to use *Bowland maths* materials.

What does the school plan to do next?

The department intends to use some of the *Bowland* professional development materials across the entire year 7 cohort, undertake further collaborative planning to incorporate case studies into year 8 and year 9 schemes of work, undertake further professional development work to consider whole-class questioning, developing group work approaches and using more open, exploratory work, and ensure that ICT resources are more easily accessible by working with ICT technicians in the school.

Case study 17: Working together to introduce rich tasks into the mathematics curriculum for all learners

Background

A rural 11–16 village college developed its scheme of work.

What were they trying to achieve?

Teachers wanted learners to function more mathematically, by improving the use of rich tasks.

How did they organise learning?

All the mathematics staff were familiar with the NRICH website, but few were using the activities with all their learners. Some thought that the resources were only for the highest attaining students; others used them for starter or plenary activities. Teachers spent time in each department meeting exploring the richness of a chosen problem. They worked on a problem, and discussed the opportunities it would offer for students' mathematical development and possible teaching approaches they might adopt.

Informal opportunities were also used to keep activities in the front of teachers' minds. For example:

- a 'problem of the week' was included in the department newsletter
- emails were circulated recommending particular activities, with a hyperlink to the problem on the website and any resources attached
- posters were printed from the website and displayed in every classroom
- peer observations, reflection and sharing formed a key part of the department's strategy.

'The way they talked to each other was amazing.

The ideas that came out were overwhelming and each was worth exploring.

I'm so proud of them.'

Teacher

How well are they achieving their aims?

The department is continuing to make use of the 'curriculum mapping documents' available on the NRICH website to embed rich tasks into their mathematics teaching. Teachers regularly share their experiences of using tasks and recommend problems to one another.

Students are responding very positively, and teachers are seeing evidence that students are:

- communicating and collaborating more effectively
- becoming more tenacious, resourceful and self-reliant in the face of difficulties
- working more systematically on problems
- beginning to think about mathematics in a different way, using more sophisticated mathematical reasoning and relating their findings to the context of the problem.

What does the college plan to do next?

The college will continue to allocate time in department meetings to look at, solve and discuss rich problems. Each teacher will be encouraged to take a turn in leading these sessions.

Teachers plan to use annual peer observations to give teachers the opportunity to use a problem in a lesson, and observe the same problem being used in another teacher's lesson. They will continue to share recommendations (verbally and through email) and use the 'problem of the week'.

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Working together to engage learners

The activities referred to in the section 'How will you organise learning?' and considered in more detail in the 'Case studies' section were developed by teachers working collaboratively to engage learners and enrich their learning experience.

Key elements of collaborative working include:

- exploring mathematical situations to identify some of the potential of an activity or evaluating a resource for learning mathematics
- devising and planning activities with a clear purpose
- sharing ideas and strategies for effective classroom implementation, including ways to exploit resources and overcome difficulties
- evaluating the effectiveness of jointly planned classroom initiatives.

Support networks

Some of the most successful activities or strategies developed by schools have been passed on from teacher to teacher through networks or other media and then refined or adapted to suit different needs. These include:

- local authority meetings for heads of mathematics, meetings for Specialist Schools and Academies Trust (SSAT) lead practitioners or for Advanced Skills Teachers etc
- professional development activities, including courses at masters degree level organised by universities
- national conferences such as those organised by the Association of Teachers of Mathematics (ATM) and the Mathematical Association (MA)
- the journals of the subject associations such as Mathematics Teaching, Mathematics in School and Mathematics Today
- online forums such as those available through the National Centre for Excellence in the Teaching of Mathematics (NCETM)
- Teachers' TV programmes that show ideas in action in the classroom or groups of teachers sharing and discussing ideas (see Resources)
- teacher meetings and other offerings of the United Kingdom Mathematics Trust (UKMT).

What are you trying to achieve?

Starting points for discussion with colleagues about how to improve learning and teaching in mathematics include:

- developing the personal qualities and good working habits in mathematics that you would like to encourage
- improving mathematical understanding and examination performance
- incorporating rich activities into the mathematics scheme of work
- using learners' personal interests and mathematical experiences from other areas of the curriculum and beyond school
- developing links with other subject areas
- using the local community as a source of mathematics
- developing activities for one-off timetable opportunities
- embedding whole-school priorities such as personal, learning and thinking skills or cross-curriculum dimensions

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- identifying and sharing good practice
- developing more effective use of ICT
- exploring the use of newly acquired equipment, resources or software
- involving parents/guardians/carers, eg by organising events that allow them to do mathematics with their children.

Examples from schools

For developing learner characteristics, mathematics teachers wanted:

- to motivate a group of learners, increase their engagement and, ultimately, improve their examination performance
- to develop thinking skills and cooperative group work across their year 9 cohort
- to develop learners' skills in using ICT to enhance their understanding
- their students to be more creative through tackling more open-ended problems and taking risks
- their retake GCSE students to develop confidence and acquire high-quality functional skills.

Other issues that schools chose to work on were:

- planning rich activities for use in one-off timetable opportunities
- sharing ideas and strategies for using rich activities, so that these activities could be embedded in the scheme of work
- devising engaging activities with a broad cultural appeal to cover specific mathematics content
- developing activities to encourage personal, learning and thinking skills
- looking for recreational activities in the vicinity of the school that could be used as starting points for developing mathematics
- collaborating with other subject areas to enhance learning
- identifying rich activities that could be used across an entire year group to generate assessment evidence.

How will you organise learning?

This section is about developing activities and planning compelling learning experiences.

What and where?

There are many potential sources of activities and resources, which you can adapt to meet the needs of your learners:

- engaging mathematical activities that you are familiar with (eg previous GCSE coursework tasks)
- existing resources, activities or exercises that could be made more accessible/challenging, or presented in a different way
- transition activities with feeder schools
- free resources available on the internet (eg NRIC, *Bowland maths*, the Standards Unit, the National Strategies, Subject Associations, NCETM, ... – see Resources)
- the local community
- the specialist status of your school, or particular facilities, or STEM clubs that could be exploited to generate ideas for compelling learning experiences
- national or international events such as the Olympic Games
- school trips (eg QCA curriculum website: video case studies skiing and Rome – see Resources)
- teachers' experiences, particular interests or pastimes (eg origami, ancient history).

When?

Engaging activities need to be part of the everyday mathematics curriculum. However, there may be particular opportunities within school:

- to test out activities before embedding them in the scheme of work
- to be more ambitious or adventurous
- to work in different ways – with other subjects, in mixed-age groups, in mixed-attainment groups
- to organise visits or external speakers.

These are some examples:

- flexidays – one day per fortnight when half a year group spends the entire day on a single subject
- after examinations – the two weeks following a GCSE unit examination was identified as 'dead time' so was used to trial some innovative mathematical activities in order to improve motivation and engagement
- one-off school events – a whole day of mathematics provided the opportunity to use rich mathematical activities with an entire year, organised into mixed-attainment groups.

How?

Having identified possible activities or starting points, you need to consider how to introduce these activities to learners.

- What is the purpose of the activity – mathematically, or more broadly?
- What choices and responsibilities will learners have?
- What questions will you ask?
- What questions might learners ask?
- How will you introduce the activity or situation to learners to engage them from the outset?
- How will you organise the classroom, and what resources might be needed?
- How will you group the learners? Who will decide the groupings or the roles within the group?
- How will learners communicate their thought processes or results?
- What other strategies might you use to encourage learners to get the most out of the activity?
- What problems do you anticipate? How might you prepare for these?

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How will you know that you are achieving your aims?

Look at your aims for your learners and the activity you have chosen. Can you write down some questions for teachers or learners to answer that will show how well you are achieving your aims? This might be one broad question or more targeted questions. Some broad questions and responses from learners are given in the main section entitled 'How will you know that you are achieving your aims?'. More specific questions could relate to **key concepts** or **key processes**, or specific aspects of **personal, learning and thinking skills**.

For example, to evaluate learners' creative thinking, ask: Did the learner ...

- generate ideas?
- explore possibilities?
- ask questions to extend their thinking?

Involving learners in evaluating their progress is an important part of engaging them in the learning process. One way to do this is to share evaluation questions with them.

Questions need to be open to give learners the opportunity to show what they have achieved. Here are some examples of questions you might use:

- Were there any things you had to find out before you started work?
- What assumptions did you make?
- Did you consider more than one method? Which one did you choose? Why?
- Did you look for any patterns? What did you find?
- What predictions did you make? How did you test them out?
- How did you record your work? Why did you choose this method? Did you consider others?
- How did you reach your results? What do they mean?
- Did you get stuck? Why, and what did you do about it?
- How did you check your work?
- In what ways could you extend the work?

Making it happen

'I don't have the time to find and prepare tasks!'

Allocating time in department meetings for staff to look at, attempt to solve and then discuss one or two rich activities helps to reduce what teachers perceive to be the time burden associated with preparing to use rich activities. It also helps teachers to experience for themselves how learners might feel when approaching unfamiliar problems, and gives them an opportunity to work with their peers in a different way, becoming excited about the mathematics, and then discussing:

- why they would use the activity
- who they would use the activity with
- how they would use the activity
- which key questions they would ask
- what support they would provide.

This enables most teachers to use the activities with their classes, and may inspire them to investigate which other activities they could use.

In one school, teachers formed into collaborative working parties to plan the next two weeks' lessons, with an 'expert' leading each group. In another, teachers produced interactive whiteboard flipcharts based on NRICH problems to share with other colleagues. Collaborative planning sheets were prepared to support this activity.

'I don't feel confident in my ability to manage the classroom environment when I change the lesson structure.'

Observing colleagues managing student activity in lessons where problem-solving is the norm can be particularly helpful. This allows colleagues to discuss and identify strategies that they might use. In several schools, good use was made of enthusiasts in the department who were willing to be observed by their colleagues. This acted as a catalyst for joint planning and further peer observation. In some schools, lessons were videotaped for staff development purposes. In one school, a video was used during a department meeting to focus on questioning skills and on how to support students through activities, without doing the thinking for them.

'My results are very good anyway – why should I change my teaching style?'

Teachers have found that attainment levels do not decline when they use rich activities. Students become more enthusiastic about mathematics and more successful when tackling open-ended problems. It is hoped that this may lead to more students choosing to study mathematics post-16. Are students really achieving as well as they might? Could they do even better?

'I am worried about how parents/guardians/carers will respond to this style of learning.'

Using rich activities is just one of many possible teaching approaches. It is not a replacement for other strategies, rather an addition to the teacher's repertoire.

Parents/guardians/carers of some students expressed concern over the lack of formal written work and marking in students' exercise books. They also worried about the challenge presented by open-ended homework tasks and by the amount of time their child spent on homework (both too little and too much). Communication with parents helped to overcome this problem.

One school held a year 7 mathematics evening at which they explained to parents/guardians/ carers how and why rich activities would be used as a teaching resource. One teacher suggested time limits for investigative homework.

Preparing learners to think for themselves to solve problems will become increasingly important as examinations change to take account of the new curriculum.

'I am concerned about the reaction of senior staff and inspectors to this style of teaching.'

Problem-solving is at the heart of the new programmes of study and an essential component of learners' classroom experiences. Senior staff need to be aware that a greater emphasis on process skills will be reflected in lesson objectives and in how learners work (for example being stuck is normal in a problem-solving environment).

In some schools, a senior member of staff has attended department meetings when rich tasks are being discussed, or observed lessons when they are being used.

'Some of my students just want to be told what to do.'

Many teachers find that some students are very resistant to working on problems that are less structured. Some lack confidence and resent being asked to think for themselves. Allowing students to work in pairs or small groups encourages students to support each other. Pooling ideas about how to tackle the problem after some time for initial discussion is often helpful. An alternative approach is to use envoys, where a representative shares their group's ideas with another group.

In one school, teachers sometimes use mini-whiteboards or interactive voting as the activity is introduced. This helps to focus students' attention and indicates to the teacher who might need support.

It became apparent how important it was for teachers to develop their questioning skills (questioning the answers rather than answering the questions); good questioning was used to open up new lines of inquiry without telling learners what to do. Teaching assistants and other support staff need to be fully aware of how they can best support learners working on rich activities. Advice is available in the teachers' notes to the NRICH problems.

'My students are not very good at working collaboratively.'

Students need to learn how to work collaboratively. One teacher experimented with different group sizes. She found that groups of three worked best, especially when she allocated them particular roles (learning coordinator, scribe and presenter). This gave all students a clear purpose within their group. This model has been successfully transferred to other classes and other schools. In other schools, any member of a group must be able to answer the teacher's questions. This means that everyone is responsible for ensuring that all members of the group can explain what the group has done. One school has started to experiment with group assessment based on this approach, where the teacher selects the work of any one member of the group and uses this to assess the achievements of the whole group. In addition, teachers are beginning to assess the way the group worked and not just the product of their work.

Resources

- Bowland Maths*
www.bowlandmaths.org.uk
- Developing reasoning through algebra and geometry (QCA, 2004)
www.qca.org.uk
- Integrating ICT into the mathematics classroom (ATM)
www.atm.org.uk/buyonline/products/rea025.html
- Key stage 3 National Strategy ICT across the curriculum: ICT in mathematics
<http://publications.teachernet.gov.uk/default.aspx?PageFunction=productdetails&PageMode=publications&ProductId=DFS+0176+2004G>
- Making mathematics count (Smith, 2004)
www.mathsinquiry.org.uk/report
- Mathematics: understanding the score (Ofsted, 2008)
www.ofsted.gov.uk/Ofsted-home/Publications-and-research/Browse-all-by/Documents-by-type/Thematic-reports/Mathematics-understanding-the-score
- Oliver Byrne's 1847 edition of Euclid's Geometry where colour represents congruence
<http://sunsite.ubc.ca/DigitalMathArchive/Euclid/byrne.html>
- Plus* magazine
www.plus.maths.org/
- QCA Respect for all: Mathematics
www.qca.org.uk/qca_9580.aspx
- QCA - RSS review of handling data and statistics in GCSE mathematics
www.rsscse.org.uk/qca/resources0.htm
- QCA curriculum website
<http://curriculum.qca.org.uk/>
- Science learning centres for STEM Enrichment & Enhancement
www.sciencelearningcentres.org.uk
- STEM directories
www.stemdirectories.org.uk
- STEMNET The Science, Technology, Engineering and Mathematics Network
www.stemnet.org.uk
- Teachers' TV mathematics
 eg Hard To Teach - Secondary Maths
www.teachers.tv/video/19119
- The Association of Teachers of Mathematics (ATM)
www.atm.org.uk
- The FunMaths Roadshow
www.maths.liv.ac.uk/lms/funmaths
- The Mathematical Association (MA)
www.m-a.org.uk
- The National Centre for Excellence in the Teaching of Mathematics (NCETM)
www.ncetm.org.uk
 includes Resources; Courses and events; Research; Communities e.g. ICT in mathematics, *Bowland* forum
- The National Strategies
www.nationalstrategies.org.uk
 includes guidance, support, resources
- The NRICM Mathematics Project
<http://nrich.maths.org/public/>
 includes activities, teacher notes, curriculum mapping documents
- The Practical Support Pack: Learning and teaching using ICT
www.dcsf.gov.uk/psp
- The Royal Institution of Great Britain
www.rigb.org
- The Royal Statistical Society Centre for Statistical Education
www.rsscse.org.uk
- The Standards Unit - Improving Learning in Mathematics
www.ncetm.org.uk/Default.aspx?page=13&module=res&mode=100&resid=1442
- UK Mathematics Trust
www.ukmt.org.uk
- Video analysis software: Tracker 2 from
www.cabrillo.edu/~dbrown/tracker
- Sources for statistics**
- CensusAtSchool
www.censusatschool.ntu.ac.uk/default.asp
- Experiments at school
www.experimentsatschool.org.uk
- International Statistical Literacy Project
www.stat.auckland.ac.nz
- Understanding Uncertainty
www.understandinguncertainty.org
- Office for National Statistics
www.ons.gov.uk
- Stats4schools
www.stats4schools.gov.uk
- United Nations cyberschoolbus
cyberschoolbus.un.org
- View and compare United Nations data from Countries around the world. The State of the World's children
www.unicef.org/publications/index.html

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Who's it for

Teachers of mathematics in schools and colleges, advisers, consultants, educators, trainers.

What's it about

Making mathematics more exciting, meaningful and relevant to learners.

What's in it

Principles for engaging mathematics; case studies from a wide variety of schools.

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