Department
for Education

## National Curriculum for mathematics Key Stages 1 and 2 - Draft

National Curriculum review
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## Purpose of Study

A high quality mathematics education provides a foundation for understanding the world, the ability to reason mathematically, and a sense of excitement and curiosity about the subject. It is essential to everyday life, critical to science, technology and engineering, and necessary in most forms of employment. Mathematics is a creative and highly inter-connected discipline that has been developed over centuries providing the solution to some of history's most intractable problems. As pupils learn mathematics, they need to acquire fluency in procedures and develop a conceptual understanding if they are to be able to solve increasingly complex problems.

## Aims

The National Curriculum for mathematics aims to ensure all pupils:

- become fluent in the fundamentals of mathematics so that they are efficient in using and selecting the appropriate written algorithms and mental methods, underpinned by mathematical concepts
- can solve problems by applying their mathematics to a variety of problems with increasing sophistication, including in unfamiliar contexts and to model real-life scenarios
- can reason mathematically by following a line of enquiry and develop and present a justification, argument or proof using mathematical language.

Fluency requires the quick and accurate mental recall of facts that pupils have learned up to that point; precision and confidence in using mathematical concepts, properties and symbols, and the competent and flexible selection and application of methods in different contexts. Solving problems requires analysing information presented in different forms, recognising what is given information and what additional information is needed; identifying and conjecturing patterns, relationships, and generalisations; testing, inducing, deducing, and proving; and communicating ideas effectively. Mathematical reasoning requires breaking down problems into a series of simpler problems or steps; making decisions about gathering, processing and calculating to acquire new information; and showing perseverance in finding solutions.

The Programmes of Study are organised in a distinct sequence and structured into separate domains. However, mathematics is a highly inter-connected discipline. Pupils should therefore be taught to practise and then apply their mathematics to a range of problems. They should also be encouraged to make connections across mathematical procedures and concepts to ensure fluency, mathematical reasoning and competence in solving problems. They should also apply their mathematical knowledge in science and other subjects.

## Spoken language

The National Curriculum for mathematics reflects the importance of spoken language in pupils' development - linguistically, cognitively and socially - across the whole curriculum. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure pupils build secure foundations by using discussion to probe and remedy their misconceptions.

## School curriculum

Each Programme of Study is set out year-by-year in mathematics. All maintained schools are only required to teach the Programme of Study by the end of each key stage. Within each key stage, maintained schools therefore have the flexibility to introduce content earlier or later than set out in the Programme of Study. In addition, schools can introduce key stage content during an earlier key stage if appropriate. All schools are also required to set out their school curriculum for mathematics on a yearly basis and make this available online.

## Inclusion

Teachers should set high expectations for all pupils and should also be aware of the requirements of the equal opportunities legislation that covers gender, race and disability. A minority of pupils will have particular requirements that arise as a consequence of Special Educational Needs, disability or learning English as an additional language. Teachers must take account of these requirements and make provision, where necessary, to support this diverse group of pupils. During end of key stage assessments, teachers should bear in mind that special arrangements are available to support individual pupils.

## Use of information and communication technology (ICT)

Teachers need to consider how ICT can best be used to support the teaching of mathematics. Calculators should not be used as a substitute for pupils having poor written and mental arithmetic. Calculators should therefore only be introduced near the end of primary, and only for those pupils who are secure in written and mental arithmetic to allow them to explore more complex problems. In both primary and secondary, a wider range of new technology should be considered, including teaching through the use of the graphing, dynamic geometry, spread sheet and simulation software available. Many ICT tools allow pupils to use different mathematical representations (e.g. number, algebra, graphs) to aid conceptual development. As technology changes, teachers need to assess what the latest innovations offer in teaching mathematics.

## Attainment Targets

By the end of each Key Stage, pupils are expected to have the knowledge, skills and understanding of the matters taught in the relevant Programme of Study.

## Mathematics Programme of Study: Key Stage 1

The teaching of mathematics in Key Stage 1 should ensure pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources (concrete objects, measuring tools, etc.). At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. By the end of Year 2, pupils should know the number bonds to 20 and be precise in their use and understanding of place value. An emphasis on practice at this early stage will aid fluency

Ensure pupils read and write mathematical vocabulary, consistent with their increasing phonic knowledge at Key Stage 1.

\section*{| Year 1 Programme of Study |
| :--- |
| NUMBER |}

## Number and place value

Pupils should be taught to:

- identify using objects and pictorial representations and use the vocabulary of: equal to; more than; less than (fewer); most; least
- count from 0 to and across 100, forward and backwards, beginning with 0 or 1 , and from any given number
- count, read and write numbers to 100 in numerals, count in different multiples including ones, twos, fives and tens
- given a number, identify one more and one less
- recognise odd and even numbers


## Notes and Guidance

NUMBER

## Number and place value

Ensure pupils practise counting in ones, twos, fives and tens from different multiples to develop their recognition of patterns in the number system.

Ensure pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100.

Ensure pupils are taught when and how to use numbers for ordering (e.g. first, second, third), for counting $(1,2,3)$ or to indicate a quantity (e.g. three apples, 2 centimetres). Exclude the terms "ordinal" and "cardinal".

## Year 1 Programme of Study

Pupils should be taught to

- read, interpret and practise writing mathematical statements involving addition (+), subtraction (-) and equals (=) signs accurately
- add and subtract 1 -digit and 2-digit numbers to $20(9+9,18-9)$, including zero
- add three 1-digit numbers
- recall and use number bonds and related subtraction facts within 20
- solve simple word problems that involve addition and subtraction.


## Multiplication and division

Pupils should be taught to:

- recognise and write the multiplication symbol ( $x$ ) and the division symbol $(\div)$ in mathematical statements, calculating the answer with the teacher using concrete objects
- solve word problems involving simple multiplication and division, with teacher support.


## Fractions

Pupils should be taught to

- recognise, name and write $1 / 2$ as one of two equal parts of an object, shape or quantity
- recognise, name and write $1 / 4$ and $3 / 4$ as parts of an object, shape or quantity
- find $1 / 2,1 / 4$ and $3 / 4$ of a shape or quantity.


## Notes and Guidance

Addition and subtraction
Ensure pupils practise reading and writing mathematical statements regularly so that they become fluent.

## Multiplication and division

Ensure pupils are introduced to the multiplication (x) and division ( $\div$ ) symbols so that they can recognise and write them accurately. They should distinguish them from addition and subtraction. This prepares them for Year 2 when they are taught how to multiply and divide using two 1 digit numbers with concrete objects and then using numbers within the multiplication tables.

## Fractions

Ensure pupils are taught $1 / 2,1 / 4$ and $3 / 4$ as objects and then as operators on discrete and continuous quantities. As objects, pupils recognise and combine $1 / 2,1 / 4$ and $3 / 4$ as parts of a whole. As operators, pupils recognise and find, for example, half of a length, container, set of objects and shapes.

## Year 1 Programme of Study

## Properties of shapes

Pupils should be taught to:

- recognise and name common 3-D and 2-D shapes, including:
- 2-D shapes (e.g. square, rectangle, circle and triangle)
- 3-D shapes (e.g. cube, pyramid and sphere).


## Position, direction, motion

Pupils should be taught to:

- describe position, directions and movements including half, quarter and threequarter turns.


## Notes and Guidance

GEOMETRY AND MEASURES

## Properties of shapes

Ensure pupils practise regularly using and naming common 2-D and 3-D shapes and related everyday objects, so that they recognise the properties irrespective of their orientation or size.

## Position, direction, motion

Ensure pupils are taught to use the vocabulary of position, direction and motion, including: left and right, top, middle and bottom, on top of, in front of, above, between, around, near, close and far, up and down, forwards and backwards, inside and outside.

Ensure pupils practise making turns to show they can understand half, quarter and three-quarter turns and routinely make these turns in a clockwise direction. This will prepare them later when they are taught about angles.

## Year 1 Programme of Study

Pupils should be taught to

- compare, measure and record the following using standard units for:
- lengths and heights (e.g. long/short, longer/shorter, tall/short,
double/half)
- lengths and heights (metres, centimetres)
- mass (grams, kilograms)
- capacity and volume (litres)
- time (hours, minutes, seconds)
- compare, describe and solve practical problems for:
- lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half)
- mass (e.g. heavy/light, heavier than, lighter than)
- capacity and volume (full/empty, more than, less than, quarter, three quarters full or empty)
- time (quicker, slower, earlier, later)
- recognise and use pounds ( $£$ ) and pence ( p ) with different denominations of money, including coins and notes
- tell the time to the hour and half past the hour
- sequence events in chronological order using common terms such as: before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening
- recognise and use the language of dates, including days of the week, weeks, months and years.


## Notes and Guidance

Measures
Ensure pupils become proficient in using measuring tools, such as a ruler, weighing scales and containers to become familiar with standard measures.

Ensure pupils regularly practise reading and writing units of measure accurately and become familiar with the correct terminology and related symbols. This supports work in Year 2 when pupils are taught to calculate sums and differences with these units.

Ensure pupils regularly practise the vocabulary of time, including telling the time throughout the day, first using "o'clock" and then "half past". [38]

## Year 2 Programme of Study <br> NUMBER

## Number and place value

Pupils should be taught to:

- read and write numbers to at least 100 in numerals and in words
- recognise the place value of each digit in a 2-digit number (tens, ones)
- count in steps of 2, 3, 5 and 10, count in tens from any number, and give 10 more or less than a given number to 100
- compare and order numbers from 0 up to 100 ; use $<$, > and = signs
- arrange, read and write numbers in increasing and decreasing order
- solve word problems using place value and number facts with increasing precision.


## Addition and subtraction

Pupils should be taught to:

- rapidly recall and use addition and subtraction facts to 20
- add and subtract numbers with up to two 2-digits including using column addition without carrying and column subtraction without borrowing
- add and subtract numbers mentally including:
- a 2-digit number and ones
- a 2-digit number and tens
two 2-digit numbers
- use subtraction in 'take away' and 'find the difference' problems
- recognise and show that addition can be done in any order (commutative) and subtraction cannot
- recognise and use addition and subtraction as inverse operations including to check calculations
- solve word problems with addition and subtraction of numbers with up to 2-digits.


## Notes and Guidance

NUMBER

## Number and place value

Ensure pupils practise counting, reading, writing and comparing numbers to at least 100 to develop mental fluency, extend their concept of place value and solve related word problems. They should be introduced to counting in multiples of 3 to support their understanding of a third.

Ensure pupils continue to compare and order increasingly large numbers. They should also practise related vocabulary while using <, > and =. Pupils should be introduced to larger numbers as they become more confident with numbers up to 100 to further develop their recognition of patterns within the number system.

Ensure pupils are fluent and apply their knowledge of larger numbers to discuss and solve problems that emphasise the value of each digit in 2digit numbers. For example, they should read 46 as 'forty and six' and solve addition and subtraction mentally such as, $36-6=30$ and $50+6=$ 56.
[47]

## Addition and subtraction

Ensure pupils practise addition and subtraction of number bonds to 20 so that they become fluent in recalling them. This includes using related facts to perform calculations (e.g. using $3+7=10,10-7=3$ and $10-3=7$ to calculate $30+70=100,100-70=30$ and $100-30=70$ ).

Ensure pupils practise column addition and subtraction to write numbers with precision to calculate answers. This also reinforces the concept of place value. Horizontal written methods should progress rapidly to more efficient column methods to help prepare pupils in Year 3 when they are taught column addition with carrying and subtraction with borrowing. [56]

Ensure pupils practise mental addition and subtraction of two numbers of up to 2-digits, with answers not exceeding 100. They should know how to check calculations, including by adding to check subtraction and adding numbers in a different order to check addition; for example $5+2+1=1+$ $5+2=1+2+5$

Ensure pupils regularly practise how to interpret word problems to ensure addition and subtraction are firmly understood

## Year 2 Programme of Study

Pupils should be taught to

- recall multiplication and division facts for the 2,5 and 10 multiplication tables
- use the multiplication (x), division ( $\div$ ) and equals (=) signs to read and write mathematical statements
- write and calculate mathematical statements for multiplication and division within the multiplication tables
- recognise and use the inverse relationship between multiplication and division to check calculations
- ensure pupils can recognise and show that multiplication can be done in any order (commutative) and division cannot
- solve word problems involving multiplication and division.


## Fractions

Pupils should be taught to:

- recognise, name and write fractions $1 / 4,1 / 3,1 / 2,2 / 3$ and $3 / 4$ of a whole
- count in halves and quarters to ten.


## Notes and Guidance

Multiplication and division
Ensure pupils are taught multiplication and division through sharing out quantities; finding simple fractions of objects, numbers and quantities; doubling numbers and quantities; and find related halves. This also links to recognition of division as sharing and grouping.

Pupils are introduced to the multiplication tables in Year 2. Ensure pupils practise 2, 5 and 10 multiplication tables up to $x 12$ so they are fluent in recalling them. This includes using related division facts to perform written and mental calculations.

## Fractions

Ensure pupils regularly practise naming, counting and writing each fraction to develop accuracy and precision in preparation for writing them in mathematical statements in Year 3.

Ensure pupils count in fractions up to 10, starting from any number and using the $1 / 2$ and ${ }^{2} / 4$ equivalence (e.g. $1^{1 / 4}, 1^{1 / 2}, 1^{3 / 4}$, 2). This reinforces the concept of fractions as numbers and that they can add up to more than
one.

## Year 2 Programme of Study

Properties of shapes
Pupils should be taught to:

- recognise and name common 3-D and 2-D shapes
- identify and describe the properties of 2-D shapes, including the number of sides, right angles and line symmetry
- identify and describe the properties of polygons and non-polygons
- identify and describe the properties of 3-D shapes including the number of edges, vertices and faces
- identify 2-D shapes on the surface of 3-D shapes, for example rectangle and square on a cuboid, circle on a cylinder, triangle on a pyramid
- compare and sort common 2-D and 3-D shapes and everyday objects.


## Position, direction, motion

Pupils should be taught to:

- use mathematical vocabulary to describe position, direction and movement, including rotation as a turn and in terms of right angles for quarter and half turns (clock-wise and anti-clockwise), and movement in a straight line.


## Measures

Pupils should be taught to:

- choose and use appropriate standard units to estimate and measure length/height in any direction ( $\mathrm{m} / \mathrm{cm} / \mathrm{mm}$ ); mass ( $\mathrm{kg} / \mathrm{g}$ ); temperature $\left({ }^{\circ} \mathrm{C}\right)$; volume and capacity (litres $/ \mathrm{ml}$ ) to the nearest appropriate unit using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =
- read relevant scales to the nearest numbered unit
- tell and write the time to 5 minutes including quarter past/to the hour and draw hands on a clock face to show these times
- recognise and use symbols for pounds ( $£$ ) and pence (p); recognise coins and notes of different values; combine amounts to make a particular value and match different combinations of coins to equal the same amounts of money; add and subtract money of the same unit.


## Notes and Guidance

## GEOMETRY AND MEASURES

## Properties of shapes

Ensure pupils continue to handle and name common 2-D and 3-D shapes including: quadrilaterals and cuboid, right triangular prism, cone and polygon to ensure that they recognise the properties of a shape (number of sides, number of faces etc.).Pupils should identify, compare and sort shapes on the basis of their properties and use the vocabulary precisely.

Ensure pupils read, write and accurately name 2-D and 3-D shapes and practise using a ruler to draw polygons accurately.

## Position, direction, motion

Ensure pupils are taught the concept and vocabulary of angles by applying rotations, including in practical contexts; e.g. pupils themselves moving in turns, giving instructions to other pupils or programming robots.

## Measures

Ensure pupils continue to practise using standard units of measurement to develop increasing accuracy. They should use the vocabulary and write associated symbols accurately.

Ensure pupils regularly practise telling and writing the time.
Ensure pupils regularly practise counting and recognising coins. They should accurately use the symbols ' $£$ ' and ' $p$ ', including combinations, and say the amounts of money with confidence. Pupils should also regularly practise addition and subtraction of money in the same unit, including giving change.

Pupils should be taught to

- construct and interpret pictograms, tables and simple graphs.

Ensure pupils have regular practice in interpreting data diagrams so they become proficient in extracting information. They should apply their

## Mathematics Programme of Study: Lower Key Stage 2

The teaching of mathematics in Lower Key Stage 2 should ensure pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure they can use measuring instruments with accuracy and make connections between measure and number. By the end of Year 4, pupils should have memorised their times tables up to and including 12 times table and show precision and fluency in their work.

Ensure pupils read and spell mathematical vocabulary correctly and with confidence, using their growing knowledge of spelling patterns and rules.

| Year 3 Programme of Study |
| :--- |
| NUMBER |
| Number, place value and rounding |
| Pupils should be taught to: |
| - read and write numbers to at least 1000 in numerals and in words |
| - recognise the place value of each digit in a 3-digit number (hundreds, |
| tens, ones) |
| - compare and order numbers up to 1000 |

tens, ones)

- compare and order numbers up to 1000
- count in multiples of $2,3,4,5,8,10,50$ and 100 from 0 ; give 10 or 100 more or less than a given number.


## Addition and subtraction

Pupils should be taught to:

- add and subtract numbers with up to 3 digits, including using columna addition and subtraction
- accurately add and subtract numbers mentally including: pairs of one- and 2digit numbers; 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds
- solve word problems including missing number problems, using number facts, place value, and more complex addition and subtraction.


## Year 3 Programme of Study Multiplication and division

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- write and calculate mathematical statements for multiplication and division within the multiplication tables; and for 2-digit numbers $\times 1$-digit numbers, using mental and written methods
- solve word problems involving the four operations, including missing number problems.


## Fractions

Pupils should be taught to:

- identify, name and write unit fractions up to $1 / 12$
- compare and order unit fractions and fractions with the same denominators
- recognise fractions which are equivalent to 1 and pairs of fractions that add up to 1
- perform calculations with addition and subtraction of fractions with the same denominator within one whole (e.g. ${ }^{5} / 7+1 / 7=6 / 7$ )
- count up and down in tenths; recognise that tenths arise in dividing an object into tenths and in dividing single digit numbers or quantities by ten


## Notes and Guidance

Multiplication and division
Ensure pupils continue to practise regularly the mental recall of multiplication tables when they are calculating mathematical statements until they are confident to use them.

Ensure pupils develop efficient mental methods. For example, pupils should use commutativity (e.g. $4 \times 12 \times 5=4 \times 5 \times 12=20 \times 12=240$ ) and multiplication and division facts (e.g. using $3 \times 2=6,6 \div 3=2$ and $2=6 \div 3$ to calculate $30 \times 2=60,60 \div 3=20$ and $20=60 \div 3$ ).

Ensure pupils develop reliable written methods for multiplication and division, starting with calculations with 2 -digit by 1 -digit numbers and progressing to formal written methods. This helps prepare pupils for long multiplication from Year 4 and short and long division in Years 5 and 6.
[106]

## Fractions

Ensure pupils develop an increasing fluency with fractions, for example they should continue to practise naming, reading and writing fractions so they can write mathematical statements accurately.

Ensure pupils continue to recognise fractions in the context of: parts of a whole, numbers, measurements, equal parts of a shape, or as a division of a quantity.

## Year 3 Programme of Study

 GEOMETRY AND MEASURES
## Properties of shapes

Pupils should be taught to:

- make 2-D and 3-D shapes; recognise in different orientations; and describe with increasing accuracy
[114]
- recognise angles as a property of shape and associate angle as an amount of turning
- identify right angles, recognise that two right-angles make a half-turn and four a complete turn; identify whether angles are greater or less than a right angle
- identify horizontal, vertical, perpendicular, parallel and curved lines
- use a compass to draw circles and arcs with a given radius.


## Measures

Pupils should be taught to:

- recognise and use full names and abbreviations for metric units of measure
[122]
- measure, compare, add and subtract: lengths ( $\mathrm{m} / \mathrm{cm} / \mathrm{mm}$ ); mass ( $\mathrm{kg} / \mathrm{g}$ ); volume/capacity ( $1 / \mathrm{ml}$ ); and time (hours/minutes/seconds)
- measure the perimeter of simple 2-D shapes
- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12 hour and 24 hour digital clocks
[125]
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as am/pm, morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events, for example to calculate the time taken up by particular events or tasks
- add and subtract amounts of money to give change, using both $£$ and p. [129]


## Notes and Guidance

GEOMETRY AND MEASURES

## Properties of shapes

In Year 3, teachers should extend pupils' knowledge of the properties of shapes, using more precise mathematical vocabulary including polygon, non-polygon and polyhedron.

Ensure pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate vocabulary, including acute and obtuse angles, turns and lines.

Ensure pupils practise measuring and drawing straight lines in centimetres and millimetres, and circles of different sizes with a given radius using a compass. Ensure they also understand the terms horizontal and vertical lines.

## Measures

Ensure pupils continue to practise measuring using the appropriate tools and units of measure. They should progress to using a wider range of measures, including comparing and using mixed units accurately (e.g. 1 kg and 200 g ) and simple comparisons of mixed units (e.g. $5 \mathrm{~m}=500 \mathrm{~cm}$ )

Ensure pupils use both analogue and digital clocks throughout the day so that they become fluent in telling the time.

Ensure pupils continue to practise recognising the value of coins, addition and subtraction of amounts, including compound units, and giving change using manageable amounts.

| Year 3 Programme of Study |  | Notes and Guidance |
| :--- | :--- | :--- |
| Data |  | Data |
| Pupils should be taught to: |  | Ensure pupils use both horizontal and vertical representations as well as <br> scales for pictograms, for example, where each picture represents 10 <br> - read, interpret and present data using pictograms and bar charts <br> with scales |
| [135] <br> solve problems using information presented in pictograms, bar charts <br> and tables. | [134] |  |


| Year 4 Programme of Study | Notes and Guidance |
| :---: | :---: |
| NUMBER <br> Number, place value and rounding <br> Pupils should be taught to: <br> - read and write numbers to at least 10,000 <br> - recognise the place value of each digit in a 4-digit number (thousands, hundreds, tens, and ones) <br> - order and compare numbers up to 10,000 <br> - count in multiples of $2,3,4,5,6,7,8,9,10,25,50,100$ and 1000 from any given number, and 10 or 100 more or less than a given number <br> - round any number to the nearest 10 or 100 <br> - read and write negative numbers; order, count forwards and backwards with positive and negative whole numbers through zero <br> - read Roman numerals to 100 and understand how Hindu-Arabic numerals included the concept of zero and place value <br> - solve word problems that involve negative and increasingly large positive numbers. | NUMBER <br> Number, place value and rounding <br> Ensure pupils continue to practise counting regularly so that they become fluent in the order and place value of numbers beyond 1000 and include regular practice counting in tens and hundreds. <br> Ensure pupils read and write 4-digit numbers accurately, including the use of zero as a place holder. <br> Ensure pupils are applying their mathematics, including completing number sequences and finding the difference. <br> Roman numerals should be put in their historical context so pupils understand that there were different ways to write whole number and that Hindu-Arabic numerals introduced the important concept of zero and place value. |
| Addition and subtraction <br> Pupils should be taught to: <br> - add and subtract numbers using formal written methods with up to 4 digits <br> - accurately add and subtract numbers mentally including two 2-digit numbers <br> - estimate, within a range, the answer to a calculation and use inverse operations to check answers. | Addition and subtraction <br> Ensure pupils continue practising formal written methods and mental methods with increasingly large numbers, and include the terms 'sum' and 'difference'. For mental calculations, include increasingly large numbers, for example, $12,462-2,400=10,062$ or $12,462+600=13,062$. <br> Ensure pupils say and write the numbers correctly and with precision, so that they are clear about place value and confident when working with mental calculations. This will prepare them for Year 5, when pupils are taught to calculate the sum and difference of two decimal numbers (up to 2 decimal places). |

## Year 4 Programme of Study Multiplication and division

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to $12 \times 12$
- mentally perform multiplication and division calculations quickly and accurately, including multiplying by 0 and dividing by 1
- multiply or divide 2-digit and 3-digit numbers by a 1-digit number using written methods; interpret remainders appropriately as integers
- recognise and use factor pairs within 144
- solve word problems involving the four operations.


## Fractions

Pupils should be taught to:

- identify and name equivalent fractions of a given fraction with denominator not greater than 12
- write the equivalent fraction of a fraction given the denominator or the numerator
- reduce fractions to their simplest form
- add and subtract two fractions with common denominators within one whole.


## Decimals

Pupils should be taught to:

- compare numbers with the same number of decimal places up to 2 decimal places
[165]
- find the effect of dividing a 2-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths
- recognise and write decimal equivalents to $1 / 4,1 / 2,3 / 4$ and any number of tenths and hundredths.


## Notes and Guidance

Multiplication and division
Ensure pupils continue to practise recalling and using multiplication tables and related division facts on a regular basis until they are confident using them mentally.

Ensure pupils continue to practise mental methods and extend this to 3digit numbers to derive facts, for example $300 \times 2=600$ into $600 \div 3=$ 200. Pupils should also use the distributive law to derive facts, for example, $30 \times 7+9 \times 7=39 \times 7$.

## Fractions

Ensure pupils continue practising to add and subtract like fractions within one whole and extend this to equivalent fractions. Ensure pupils practise counting as often as possible using simple fractions and decimal fractions both forwards and backwards.

## Decimals

Ensure pupils are taught decimal notation and vocabulary, including in the context of measurements. Ensure pupils are taught to make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places.
[168]
Ensure pupils' understanding of decimal place value is extended to tenths and then hundredths. This will prepare them for Year 5 when they are
taught how to relate the decimal notation to division of 2-digit numbers by 10 and later 100, and to the groups of fractions for $1 / 10$ and later $1 / 100$.

## Year 4 Programme of Study

## Properties of shapes

Pupils should be taught to:

- identify lines of symmetry in 2-D shapes presented in different orientations
- compare and classify geometric shapes, including squares, rectangles and triangles based on their properties and sizes
- identify acute and obtuse angles and compare the size of different angles.


## Position, direction, motion

Pupils should be taught to:

- describe positions, and movements between positions, on a 2-D grid, and as coordinates in the first quadrant
- plot specified points and draw sides to complete a given polygon
- recognise a symmetric figure and complete a symmetric figure with respect to a specific line of symmetry.
[176]


## Measures

Pupils should be taught to:

- convert between different units of measure, for example: kilometre to metre; metre to centimetre; centimetre to millimetre; kilogram to gram; litre to millilitre; hour to minute; minute to second; year to month; week to day
- measure and calculate the perimeter of a rectilinear figure, where each side is labelled in centimetres and metres
- find the area of squares and rectangles and related composite shapes
- read and convert time between analogue and digital 12- and 24- hour clocks
- estimate, compare and calculate different measures, including money in pounds and pence.


## Notes and Guidance

 GEOMETRY AND MEASURES
## Properties of shapes

Ensure pupils continue to classify shapes, extending to classifying different rectangles and triangles. Ensure pupils continue to practise drawing circles with a compass and use the related vocabulary.

## Position, direction, motion

Ensure pupils draw a pair of labelled axes in one quadrant and regularly read, write and use pairs of coordinates, e.g. $(2,5)$.
[177]
Ensure pupils regularly practise recognising line symmetry in a variety of diagrams. Exclude rotational symmetry.
[178]

## Measures

Ensure pupils continue, from Year 3, to practise calculating the perimeter of rectilinear and related composite shapes (rectangles and squares), including where one or more lengths have to be deduced using properties of the shape.

Ensure pupils are introduced to area, initially by counting squares (e.g. $\mathrm{cm}^{2}$ squares) and later using perimeter measurements to calculate areas.

| Year 4 Programme of Study | Notes and Guidance |
| :--- | :--- |
| Data | Data |
| Pupils should be taught to: | Ensure pupils continue to practise interpreting a variety of bar graphs so <br> that they can read, write, analyse and solve problems confidently in Year |
| - read, interpret and solve problems using information in bar graphs, including <br> reading scales on the axes. <br> [186] | . They should continue to apply their knowledge in science and other <br> subjects as appropriate. |
| [187] |  |
| Ensure pupils use horizontal and vertical representations of bar graphs so |  |
| that pupils are confidently able to interpret and write the variable on the |  |
| horizontal axis (e.g. shoe size) and the frequency on the vertical axis (e.g. |  |
| number of people). |  |
| [188] |  |

## Mathematics Programme of Study: Upper Key Stage 2

The teaching of mathematics in Upper Key Stage 2 should ensure pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections pupils make between multiplication and division with fractions, decimals, percentages and ratio. At this stage, pupils should develop their ability to solve a wider range of problems including increasingly complex properties of numbers and arithmetic and demanding efficient written and mental methods of calculation. Teaching in geometry and measures should consolidate and extend knowledge developed in number. This should also ensure pupils classify shapes with increasingly complex geometric properties and related vocabulary. By the end of Year 6 , pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Ensure pupils read and spell all mathematical vocabulary correctly.

\section*{| Year 5 Programme of Study |
| :--- |
| NUMBER | <br> NUMBER}

## Number, place value, approximation and estimation

Pupils should be taught to:

- read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
[189]
- count forwards or backwards in steps of 100, 1000 or 10,000 for any given number up to $1,000,000$
- round any number up to $1,000,000$ to the nearest $10,100,1000,10,000$ and 100,000
- estimate the answers to calculations involving addition, subtraction, multiplication and division
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals.


## Addition and subtraction

Pupils should be taught to:

- add and subtract whole numbers with up to 5 digits, including using formal written methods
- add and subtract numbers mentally with increasingly large numbers.


## Notes and Guidance

## NUMBER

## Number, place value, approximation and estimation

Ensure pupils continue to practise reading and saying regularly the place value of each digit in up to six digit numbers, including decimals.

## Addition and subtraction

Ensure pupils continue practising formal written methods with increasingly large numbers so they are fluent and precise. This will aid the introduction of adding and subtracting with decimals in this year.

Ensure pupils continue to practise fast responses for mental calculations with increasingly large numbers, for example: 12,462 $-2,300=10,162$.
[198]

## Year 5 Programme of Study

Pupils should be taught to

- identify multiples including common multiples, and factors including common factors
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall the prime numbers up to 19
- multiply numbers up to 4 -digits by a 1 or 2-digit number using a formal written method, including long multiplication
- accurately multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a 1 -digit number and 10 and interpret remainders appropriately
- multiply and divide numbers by 10, 100 and 1000
- recognise and use square numbers and square roots, and the notation for square $\left({ }^{2}\right)$ and square root $(\sqrt{ })$ [206]
- solve word problems involving addition and subtraction, multiplication and division.


## Fractions

Pupils should be taught to:

- compare and order fractions with different denominators
- recognise mixed numbers and improper fractions and convert from one form to the other
[211]
- add and subtract fractions with the same denominator and related fractions;, write mathematical statements that exceed 1 as a mixed number: (e.g. $2 / 5+4 / 5=6 / 5=1^{1} / 5$ )
- multiply proper fractions and mixed numbers by whole numbers.


## Notes and Guidance

Multiplication and division
Ensure pupils extend their use of written methods for multiplication to practise long multiplication. Also, ensure pupils continue to practise and apply all the multiplication tables and related division facts as often as possible to ensure they are committed to memory and can be used confidently to make larger calculations.

Ensure pupils record answers for non-integer division in different ways, including: with remainders, fractions, decimals or with rounding; for example, $98 \div 4=24 \mathrm{r} 2=241 / 2=24.5 \approx 25$.

## Fractions

Ensure pupils continue to develop further their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities, writing remainders as a fraction.

Ensure pupils read and write proper fractions and mixed numbers accurately and continue to practise counting forwards and backwards with mixed fractions. Pupils should extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number.

## Year 5 Programme of Study

Pupils should be taught to:

- read and write decimal numbers as fractions (e.g. $0.71={ }^{71} / 100$ )
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- read, write, order and compare numbers with up to three decimal places
- add and subtract numbers with up to three decimal places.


## Percentage

Pupils should be taught to:

- recognise the per cent symbol (\%) and understand that per cent relates to "number of parts per hundred" for example that $100 \%$ represents a whole quantity and $1 \%$ is $1 / 100,50 \%$ is $50 / 100,25 \%$ is ${ }^{25} / 100$, etc.
[224]
- write simple fractions as percentages and decimals as percentages (e.g. $1 / 2=$ $50 \%=0.5$ ).

Notes and Guidance
Decimals
Ensure pupils practise adding and subtracting decimals, initially calculating with the same number of decimal places, moving on to a mix of whole numbers and decimals with different numbers of decimal places. $\qquad$
Ensure pupils recognise and use complements of 1 using addition and subtraction facts and place value, e.g. $0.83+0.17=1$.

Ensure pupils continue to practise counting forwards and backwards using decimal fractions, and mental addition and subtraction of tenths and 1 digit whole numbers and tenths.

Ensure pupils say, read and write decimal fractions and related tenths, hundredths and thousandths with accuracy and ensure pupils are confident in checking the reasonableness of answers.

## Percentage

Ensure pupils make connections between percentages, fractions and decimals. They should recognise that percentages are operators. [226]

## Year 5 Programme of Study

Properties of shapes
Pupils should be taught to:

- measure angles in degrees and draw a given angle, writing its size in degrees
- know angles are measured in degrees and identify:
- right-angles and $1 / 4$ turn (total $90^{\circ}$ )
- angles at a point on a straight line and $1 / 2$ a turn (total $180^{\circ}$ )
- angles at a point and one whole turn (total $360^{\circ}$ )
- reflex angles and compare different angles
- recognise and compare different triangles including: isosceles, equilateral and right-angled; identify and name the following: parallelogram; rhombus; trapezium
[229]
- construct shapes from given dimensions; state and use properties of a square and rectangle
- identify 3-D shapes including cubes and cuboids from 2-D representations.


## Position, direction, motion

Pupils should be taught to:

- identify, describe and represent the position of a shape following a reflection or translation using the appropriate vocabulary.


## Measures

Pupils should be taught to:

- add, subtract, multiply and divide units of measure (e.g. length, mass, volume, money) using decimal notation
- understand and use basic equivalencies between metric and common imperial units and express them in approximate terms
- measure force in Newtons (N)
- calculate, estimate and compare the area of squares, rectangles and related composite shapes using standard units, including centimetre squared ( $\mathrm{cm}^{2}$ ) and metre squared ( $\mathrm{m}^{2}$ )
- recognise volume in practical contexts, for example using sand and water, $1 \mathrm{~cm}^{3}$ blocks or interlocking cubes to build cubes and cuboids.


## Notes and Guidance

GEOMETRY AND MEASURES

## Properties of shapes

Ensure pupils continue to practise regularly drawing lines with a ruler and measuring with a protractor and become confident with using conventional markings for parallel lines and right angles.

Include the term 'diagonal' and related properties of diagonal with reference to angles and sides.

## Position, direction, motion

Ensure pupils recognise and use reflection and translation in a variety of diagrams, including continuing to use a 2-D grid and co-ordinates in the first quadrant.
[235]

## Measures

Ensure pupils' calculation of area is extended to include scale drawings in metres ( m and $\mathrm{m}^{2}$ ) but without converting between $\mathrm{cm}^{2}$ and $\mathrm{m}^{2}$. Also ensure pupils' calculation of perimeter is extended to composite shapes.
[241]

| Year 5 Programme of Study | Notes and Guidance |
| :--- | :--- |
| Data | Data |
| Pupils should be taught to: | Ensure pupils regularly practise reading and interpreting so that pupils are <br> confident in completing tables and bar graphs, and using data diagrams <br> such as tally charts. Also ensure that pupils relate pie charts to angles and <br> - <br> [243] <br> using data presented in bar graphs, tables and simple pie charts. |

## Year 6 Programme of Study

NUMBER

## Number, place value and rounding

Pupils should be taught to:

- read, write, order and compare numbers up to 10 million and determine the value of each digit
- round any number to a required degree of accuracy
- recognise binary numerals to 15 (1111) and convert between binary and decimal numerals.


## Addition, subtraction, multiplication and division

Pupils should be taught to:

- add and subtract negative integers
- multiply numbers with at least 4-digits by a 2-digit whole number using long multiplication
- divide numbers up to 4-digits by a 2-digit whole number using long division, and interpret remainders as whole number remainders, fractions, decimals or by rounding
- perform mental calculations, including with mixed operations and large numbers
- use estimation to check answers to calculations and determine, in the context of a problem, whether an answer should be rounded or written as a fraction or a decimal
- carry out combined operations involving the four operations accurately and state the order of operations
- solve word problems involving addition, subtraction, multiplication and division.


## Notes and Guidance

NUMBER

## Number, place value and rounding

Ensure pupils regularly practise saying, reading and writing numbers accurately. Binary numerals should be introduced so pupils are familiar with the concept of place value using a different base.

## Addition, subtraction, multiplication and division

Ensure pupils continue to practise calculating addition, subtraction, multiplication and division using formal written methods. Extend application of written methods to larger numbers.

Ensure pupils continue to practise fast responses for mental calculations with increasingly large numbers and more complex calculations.

Ensure pupils continue to use all the multiplication tables to calculate mathematical statements to maintain fluency.

Include rounding answers to a specified degree of accuracy.
For the order of operations include the use of brackets; for example,
$2+1 \times 3=5$ and $(2+1) \times 3=9$.

## Year 6 Programme of Study <br> \section*{Fractions}

Pupils should be taught to

- add and subtract mixed numbers and fractions with different denominators
- multiply simple unit fractions by fractions and pairs of proper fractions, writing the answer in its simplest form
- divide proper fractions by whole numbers
- associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. ${ }^{3} / 8$ )


## Decimals

Pupils should be taught to:

- identify the value of each digit to three decimal places and multiply and divide numbers up to three decimal place by 10, 100 and 1000
- multiply and divide numbers with up to two decimal places by 1-digit and 2digit whole numbers.


Pupils should be taught to:

- use percentages for comparison and calculate percentages of whole numbers or measures such as $15 \%$ of 360
- recall and use equivalences between fractions, decimals and percentages.


## Notes and Guidance

Fractions
Ensure pupils should use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity. For example, if $1 / 4$ of a length is 36 cm then the whole length is $36 \times 4=144 \mathrm{~cm}$.

Ensure pupils practise working with simple fractions and decimal fraction equivalents. Include listing of equivalent fractions to identify fractions with common denominators. Denominators of given fractions should not exceed 12 with the exception of 100 and 1000.

Pupils can use a calculator for a division calculation to convert a simple fraction to a decimal fraction, e.g. $3 \div 8=0.375$. For simple fractions with infinite decimal equivalents, pupils should round the decimal to three decimal places

## Decimals

Ensure pupils multiply decimals by whole numbers starting with the simplest cases, such as $0.4 \times 2=0.8$, and practical contexts, such as measures and money.

Ensure pupils are introduced to division of decimal numbers initially in practical contexts involving measures and money and by single digit whole numbers. They should recognise division calculations as the inverse of multiplication.
[270]
Ensure pupils also develop their rounding and estimation skills as a means of predicting and checking the order of magnitude of their answers to decimal calculations. Include rounding off answers to a specified degree of accuracy and checking the reasonableness of answers.

## Percentages

Ensure pupils understand that calculating a percentage of a quantity is the same as calculating a fraction of a quantity.

## Year 6 Programme of Study

Pupils should be taught to

- use ratios to show the relative sizes of two quantities
- recognise equivalent ratios and reduce a given ratio to its lowest terms
- recognise and use division in the context of fractions, percentages and ratio.


## Algebra

Pupils should be taught to:

- solve linear missing number problems, including those involving decimals and fractions, and find pairs of numbers that satisfy number sentences involving two unknowns
- use simple formulae expressed in words
- generate and describe linear number sequences, including those involving negative and decimal numbers, and proper fractions e.g. 1.4, 1.1, 0.8 .


## GEOMETRY AND MEASURES

## Properties of shapes

Pupils should be taught to:

- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference
- recognise, describe and build simple 3-D shapes, including making nets
- describe properties of cuboids and other common 3-D shapes including prisms and identify parallel planes and symmetries
- estimate the size of angles
- find unknown angles involving angles at a point, on a straight line, in a triangle $\left(180^{\circ}\right)$, in a quadrilateral $\left(360^{\circ}\right)$ and vertically opposite angles.


## Notes and Guidance

Ratio and proportion
Ensure pupils are introduced to and use the ratio notation and symbol
(a:b) in the context of comparing quantities,
sizes and scale drawings.
Ensure pupils practise solving a wide variety of problems so that pupils are taught to apply ratio and proportion flexibly.

279]

## Algebra

Ensure pupils write some known arithmetical rules algebraically, such as $a+b=b+a$, and known relations such as $p=4 s$ for the perimeter of $a$ square. They should also interpret word problems as statements about number and record as a mathematical statement.
[283]
Pupils should also write missing number problems algebraically; for example, $2 x-4=8$ therefore $2 x=12$ therefore $x=6$ or finding missing lengths in perimeters and missing angles at a point. Pupils should also find possible solutions for equations with two unknown variables, for example $x$ $+y=5$ includes solutions $x=1$ and $y=4, x=2$ and $y=3$.

## GEOMETRY AND MEASURES

## Properties of shapes

Ensure pupils practise drawing shapes and nets regularly and accurately using measuring tools and conventional markings and labels for lines and angles.
[291]
Ensure pupils also describe properties of shapes and explain how they derive unknown angles and lengths from known measurements.

## Year 6 Programme of Study Position, direction, motion

Pupils should be taught to

- describe positions on the full coordinate grid (all four quadrants)
- construct, translate and reflect simple shapes on the coordinate plane.


## Measures

Pupils should be taught to:

- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, including between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles
- recognise when it is necessary to use the formulae for area and volume of shapes
- Calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed $\left(\mathrm{cm}_{3}^{3}\right)$ and cubic metres $\left(\mathrm{m}^{3}\right)$ and extending to other units, such as $\mathrm{mm}^{3}$ and $\mathrm{km}^{3}$
- use decimal notation to three decimal places to solve problems involving calculation and conversion of measures.


## Notes and Guidance

Position, direction, motion
Ensure pupils practise drawing and labelling a pair of axes in all four quadrants and drawing pairs of axes. This extends pupils knowledge of one quadrant to all four quadrants, including the use of negative numbers.

Ensure pupils draw and label rectangles, squares, parallelograms and rhombuses, specified by coordinates in the four quadrants, and that they measure the lengths of sides and diagonals, including calculating perimeters.

## Measures

Ensure pupils use, add and subtract positive and negative integers for measures such as temperature and money.
[303]
Ensure pupils use the formula to calculate area of a triangle and a parallelogram. Include identifying the base and its corresponding height.
Exclude finding the base or height of a triangle given its area.
Pupils can be introduced to other compound units for speed such as miles per hour and apply their knowledge in science as appropriate.

## Year 6 Programme of Study <br> Data

Pupils should be taught to

- draw, read and interpret line graphs and use these to solve problems
- use and interpret averages including mean, median and mode and solve simple problems using different kinds of averages.


## Probability

Pupils should be taught to:

- use the language associated with probability such as certain, equally likely, unlikely, impossible and use this to describe the likelihood of particular events.


## Notes and Guidance

Data
Ensure pupils understand and use a range of graphs such as: temperature time, distance-time and currency conversion. They should apply their knowledge in science and other subjects as appropriate.

## Probability

Ensure pupils understand and use the notation of probability and fractions; for example, the probability of rolling a 3 on a six-sided die is $\mathrm{P}(3)=1 / 6$, with the numerator showing the number of given outcomes and the denominator the number of possible outcomes.
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