Mathematics

Programme of study for Key Stage 4

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Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary in most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, and a sense of enjoyment and curiosity about the subject.

Aims

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The programmes of study are organised in a distinct sequence and structured into separate domains. Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

Attainment targets

By the end of each key stage, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

Introduction

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate and extend their numerical, algebraic, geometric and graphical understanding
- work fluently and accurately with fractions, surds, and symbolic expressions, simplifying appropriately
- apply appropriate reasoning strategies and degrees of accuracy to increasingly complex problems
- work with linear and quadratic expressions and graphs, applying appropriate reasoning strategies, to solve increasingly complex problems
- use the real number system, algebraic and graphical representations
- use technology to represent and interpret functions
- increasingly evaluate situations based on the underlying mathematical properties rather than on surface features
- increasingly understand the world of finance and apply arithmetical and graphical methods in this and other contexts.

Reason mathematically

- identify and express variables and relations algebraically and graphically, and begin to use a range of functions in their reasoning
- begin to develop and express deductive and inferential reasoning in appropriate geometrical, algebraic and statistical contexts
- select and use other forms of reasoning as appropriate: algebraic, geometric, statistical, probabilistic and logical, and know when to express their arguments informally or formally, including working directly from definitions.

Solve problems

- use mathematical knowledge to solve problems within and outside mathematics, including financial mathematics and mechanics; particularly problems that are unfamiliar in presentation and context, and that embed mathematical ideas which have not yet been formally taught
- model realistic situations mathematically within a given range of functions; express the results of their investigations using a range of formal mathematical representations
- develop mathematical knowledge, in part through problem solving and evaluating the outcomes
- increase their use of multiple representations where appropriate.

Subject content

Number: calculation and accuracy

Pupils should be taught to:

- calculate with powers and roots
- calculate with standard form A x 10^n where n is positive or negative
- use and interpret limits of accuracy, including upper and lower bounds
- calculate exactly with fractions and surds, including simplification and rationalising of denominators, and with multiples of π
- set up appropriate algorithms and iterative procedures (e.g. systematic numerical solution of equations) to solve familiar and unfamiliar problems.

Number: number theory

Pupils should be taught to:

- understand and use algebraic arguments, relying on the multiplicative structure of number.

Algebra: expressing relations

Pupils should be taught to:

- identify variables and relations in familiar and unfamiliar situations within and outside mathematics
- construct, interpret and use algebraic, graphical and function representations of abstract and contextual situations
- understand and use mathematical arguments, relying on rearrangement of equivalent expressions
- interpret and express linear, quadratic and trigonometric relationships algebraically and geometrically
- recognise, sketch and produce graphs of cubic, trigonometric, reciprocal and exponential functions of one variable, using equations in x and y and the cartesian plane
- find the inverse of familiar one-to-one functions
- express composition of two familiar functions using function notation and know which are distributive over addition
- recognise a geometric progression and find the nth term
- read and interpret rational expressions, e.g. those involving surds, sine rule and algebraic fractions
- manipulate algebraic fractions using the four operations.

Algebra: using equations and functions (includes pre-calculus)

Pupils should be taught to:

- model simple contextual and subject-based problems algebraically
- use efficient numerical and graphical methods to approximate solutions of equations in one variable
- solve algebraically: linear and quadratic equations in one variable in a variety of contexts, including equations that require rearrangement, e.g. rational; and simultaneous equations in two variables (linear/linear or linear/quadratic)
- apply methods of solving equations to solving inequalities; express solutions using set and graphical notations
- identify and interpret gradients, roots, intercepts, turning points graphically and numerically, using the full range of familiar functions
- determine algebraically the gradient and intercept of linear functions, and the roots and turning points of quadratic functions
- solve velocity and acceleration problems, such as those involving distance/time and velocity/time graphs, and mechanics problems, such as those involving collisions and momentum.

Ratio, proportion and rates of change (includes pre-calculus)

Pupils should be taught to:

- extend ratio notation to include 1: r where r is a rational number
- compare lengths, areas and volumes using ratio notation and scale factors and make links to similarity
- use and understand ratio, proportion and rates of change, including contextual and subject-based problems
- solve growth and decay problems, such as financial mathematics problems with compound interest, including using powers
- use iterative methods to solve problems such as loan repayment
- understand and use the concepts of instantaneous and average rate of change in graphical representations (chords and tangents), including with velocity and acceleration.

Geometry and measures (includes pre-calculus)

Pupils should be taught to:

- derive and use formulae, using dimensional analysis where appropriate, to calculate the area of parallelograms and trapezia; use formulae for volume and surface area of spheres, pyramids and cones
- extend the use of compound units to contexts such as momentum in science
- interpret and express linear, quadratic and trigonometric relationships algebraically and geometrically
- solve problems involving arc length, angles and areas of sectors
- apply trigonometric ratios, sine and cosine rules, and Pythagoras' Theorem in two and three dimensions
- identify, describe and construct congruent and similar shapes on coordinate axes, using rotation, reflection, translation and enlargement; identify, describe and construct shapes transformed by stretch parallel to an axis; know the invariants of each transformation
- describe a translation using a vector, add and subtract vectors, multiply vectors by a scalar, using diagrammatic and column representations
- describe the changes and invariance achieved by the combinations of simple transformations, including using vector notation with combinations of translations
- apply deductive reasoning to prove conjectures in familiar and unfamiliar two-dimensional contexts
- understand, know and use circle theorems concerning angles, radii, tangents and chords.

Probability

Pupils should be taught to:

- describe the outcomes of probability experiments in terms of relative frequency and compare to theoretical probability
- calculate the probability of independent and dependent combined events, including tree diagrams and other representations, understanding underlying assumptions
- calculate conditional probabilities through representation using tree and Venn diagrams
- understand that empirical samples tend towards theoretical probability distributions, with sufficient sample size and provided there is lack of bias.

Statistics

Pupils should be taught to:

- distinguish between using statistics to describe a population or a large data set and inferring properties of populations or distributions from a sample
- describe and compare grouped, continuous data using graphical representations, and measures of central tendency and spread, and cumulative frequency
- describe relationships in bivariate data: interpret and sketch trend lines through scatter plots, make predictions, interpolate and extrapolate trends.

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