National Research and Development Centre
for adult literacy and numeracy

Thinking Through Mathematics

Research report

Jon Swain and Malcolm Swan

Maths4Life

Published by the National Research and Development Centre for Adult Literacy and Numeracy

This document is also available in pdf and text only format from NRDC's website, **www.nrdc.org.uk**

For information on alternative formats, or to give feedback on the content and accessibility of this publication, please contact:
Publications
NRDC
Institute of Education
20 Bedford Way
London WC1H 0AL

Telephone: +44 (0)20 7612 6476
Fax: +44 (0)20 7612 6671
email: publications@nrdc.org.uk

ISBN 978-1-905188-42-0

Published by the NRDC.

Maths4Life, a Skills for Life programme funded by the Department for Education and Skills, was based at NRDC between July 2004 and March 2007. On 1st April 2007 the work to take forward and develop Maths4Life transferred to the National Centre for Excellence in Teaching Mathematics (NCETM). NRDC remains a key partner in NCETM's further development of the post-16 maths and numeracy work started by Maths4Life.

For further details see ncetm.org.uk and maths4life.org. The Maths4Life website will be live and maintained until the end of March 2008 when it will transfer to ncetm.org.uk

NRDC is a consortium of partners led by the Institute of Education, University of London (see back cover for a list of members) and is part of the Bedford Group for Lifecourse and Statistical Studies at the IoE. **www.ioe.ac.uk/bedfordgroup**

Design: Chapman Design Limited
Print: Redlin Print Management

Contents

Tables

Figures

The Research Team

Most of the analysis of the data was carried out by Malcolm Swan at the University of Nottingham, and Jon Swain at the Institute of Education, University of London.

The project employed 11 observers who attended workshops and training days, and who carried out classroom observations, interviewed teachers and learners, and wrote case studies of individual teachers. They were Fiona Allen, Jane Annets, Dave Baker, Diana Coben, Oonagh Gormley, Graham Griffiths, Helen Jarvis, Tina Lawton, Sue Niedusynska, Jon Swain and Alison Tomlin.

The project manager was Oonagh Gormley.

Maths4Life website

Copies of questionnaires, observation forms and interview schedules used in the research appear on the Maths4Life website **www.maths4life.org.**

Contact details of the research team are also available.

Key to transcripts

[*text*] background information

[...] extracts edited out of transcript for sake of clarity

... pause.


291

1 Executive summary

1.1 Outline of project

Thinking Through Mathematics (TTM) is part of a larger three-year project called *Maths4Life*, which aims to stimulate a positive approach to the teaching and learning of mathematics in the *Skills for Life (SfL)* sector. *TTM* builds on many years of research in the FE sector, and draws on the work of an earlier project commissioned by the DfES Standards Unit in 2005, *Improving Learning in Mathematics*.

TTM is a design-based research project[1] that attempts to transform educational practices in numeracy/mathematics classrooms within the *SfL* sector by helping teachers to develop more 'connected' and 'challenging' teaching methods that enable learners to develop more active orientations towards their learning. The focus of the research is to study the feasibility and potential impact of these teaching and learning approaches. In particular, the project examines the interpretations that teachers have of the underlying principles, the different obstacles they face as they try to implement them, what happens in their classrooms, and the resulting impact on their beliefs and practices. This leads us to consider the lessons that may be learned for the future design of professional development.

The project developed and trialled 30 activity-based sessions built on eight mathematical principles[2] (Swan, 2005b). These were introduced to 24 teachers from 12 organisations across the *SfL* sector in two phases between October 2005 and June 2006.

Teachers' professional experience ranged from less than one year to 29 years (mean 6.5 years). Three held a degree in mathematics as their highest mathematical qualification, while four had not achieved a mathematics qualification at GCSE/'O' level at age 16. Eleven had gained a Level 4 subject-specific teaching qualification in numeracy. Nine worked full-time and the remainder were employed part-time, on an hourly or fractional basis.

The sessions were used with over 200 learners designated to be working at Entry Level 1 to Level 1, and these were recorded and analysed by 11 observers. Each teacher was observed between four and six times. In total, 110 classroom observations were carried out and 75 interviews conducted with teachers and learners. The project also filmed three teachers working with their classes.

Throughout this process, research and design were intertwined – teaching approaches and resources were iteratively modified and developed in the light of arising issues and emerging findings, and the revised versions were observed being used to generate new research findings. The project used both quantitative and qualitative methods to carry out the fieldwork and analyse the data. The main methods of data collection were: through classroom observation; questionnaires for teachers and learners; interviews with teachers and learners; and recordings of structured discussions and oral feedback sessions.

1.2 Main findings

Teachers

Understandings and expectations

Teachers' comprehension of the underlying principles evolved gradually and, particularly during the early stages, some teachers appeared to interpret them in a partial or superficial way. At

1 A fuller explanation of what design-based research is can be found in section 2.

2 Details of the eight principles are described in section 3.1.

least six of the teachers claimed that they were already using teaching approaches that were compatible with some of these principles before the project began, and two teachers retained this view at the end. Alternative interpretations of terms such as: 'discussion' and 'talk'; 'working collaboratively' and 'group work'; 'mistakes' and 'misconceptions', meant that some teachers thought they were using the principles when in our view they were not.

A few teachers had low expectations of their learners and a 'protective' attitude towards them. This meant that some did not always challenge learners in the way the teaching approaches intended.

Teachers' practices and integration of the approaches
Whereas teachers generally rated their practice before the project began as being learner-centred, their own learners tended to see them as being more teacher-centred.

Teachers' own perceptions of change in their practice were broadly consistent with the classroom observations. Change in teachers' practices was evaluated by observers/researchers in terms of changes observed before they were introduced to the teaching approaches (or near the beginning of the project) compared with teachers' practices at the end.

Teachers and observers were in broad agreement that 18 of the 24 teachers had changed their practice towards becoming more learner-centred and seven of these had introduced changes of a substantive and wide-ranging nature. (It should be noted that some teachers were already working in learner-centred ways and one would not expect their practices to have been so noticeably affected.)

Observers/researchers judged that the main ways teachers' practice changed was in terms of their organisation (with more group work), classroom ethos (where learners were relaxed and felt less worried about making mistakes), and learners' practices (where learners were given more choices and encouraged to ask questions).

The principles that the teachers found the easiest to introduce into their practice were 'rich, collaborative tasks', 'cooperative small group work' and 'asking probing questions to assess what learners know and how they think'. Teachers found the following principles more difficult to apply: 'exposing and discussing common misconceptions', 'creating connections between topics', 'building on knowledge learners already have', 'encourage reasoning rather than "answer getting"' and the 'use of technology'.

The principles that the teachers considered to be the most important at the end of the project were not the same ones that were observed being used successfully during fieldwork. For example, whereas teachers claimed that 'exposing and discussing misconceptions' was an important principle, observers did not see this being used effectively and on a consistent basis.

One factor that hindered the implementation of the principles for some teachers was the feeling of pressure from senior management to prepare learners for accredited tests, and to map learning outcomes to particular content areas.

Almost all of the teachers reported that there had been pressures and constraints that prevented them from using the approaches in the best possible ways.

Teachers' beliefs
Teachers' self-report questionnaires showed that they reported a significant movement away from transmission orientations, and a significant increase in connectionist orientations over the course of the project[3]. The data suggests that the same beliefs develop from transmission to connectionist via the discovery orientation.

Teachers' knowledge
We categorise teachers' knowledge into three areas: *general pedagogical* knowledge (skills in classroom organisation and management); *subject-specific-pedagogical* knowledge (knowing 'how' mathematics is taught and learned); and *mathematical* knowledge (understanding the subject).

3 Broadly speaking, the transmission orientation views mathematics as a series of 'rules and truths' that must be conveyed to learners and teaching as 'chalk and talk' followed by individual practice until fluency is attained. The connectionist orientation views mathematics as a network of ideas that the teacher and learner construct together through collaborative discussion. These terms are explained more fully in the main section of the report.

Teachers' general pedagogical knowledge varied considerably, but the great majority felt that the approaches had challenged their classroom organisational and management skills. Some formative assessment techniques were new to most teachers, such as inviting learners to describe what they already knew about a topic at the beginning of a session.

Successful use of the discussion activities was directly related to teachers' knowledge of subject-specific pedagogy. This included anticipating learners' questions, and adopting a more flexible approach by being able to respond to learners' needs. We noted that some teachers had significant gaps in their deeper understanding of basic mathematical concepts, in knowing how to introduce alternative approaches to guide further learning and in detecting and recognising learners' misconceptions.

In general the project did not make new demands on teachers' mathematical knowledge. (This is not surprising, considering the project was designed for learners working at Level 1 and below.)

Learners

Background and experience

The learners were a heterogeneous group in many ways, but were predominantly female, and the vast majority were white British. One-third of the classes were composed almost exclusively of 16–19-year-olds. Class sizes ranged from two to 16 learners, and the average number of learners attending classes was eight.

Many learners had a negative experience of learning mathematics at school. However, learners' attitudes towards learning mathematics were generally very positive and, in general, they saw mathematics as a useful subject that they enjoyed doing and worked hard at.

Some learners were working at relatively low levels of mathematics. Some had difficult lives outside the classroom, and a number also had either physical or mental health problems.

Some teachers reported difficulties when it came to learners discussing mathematics in collaborative work, and attributed this to their underdeveloped language skills (one class was comprised of English for Speakers of Other Languages learners). Some teachers also cited problems when learners came to complete relatively lengthy self-report questionnaires and associated this with poor literacy skills.

The reasons learners gave for attending classes were predominantly instrumental: to improve their mathematical skills to get higher test scores, leading to higher qualifications and greater opportunities in employment.

Learners' responses

The teaching approaches also required learners (as well as teachers) to change their behaviour and practices. Learners come to classes with clear expectations of the teacher, the mathematics and the ways in which they would be expected to learn. Some found it harder, and took longer than others, to adapt to working in new ways.

Most learners noticed a major change in their teacher's practices, and by the end of the project the vast majority seemed very supportive towards the project and embraced the approaches. Overall, the majority of learners enjoyed the sessions and felt they had worked quite hard, although perceptions of the difficulty of the activities and the amount they had learned were more equivocal.

Many learners enjoyed group work: they generally liked working with others and felt less threatened and more relaxed when they worked towards making a group decision. They also felt that they learned from each other, particularly when they needed to explain their thinking.

Materials

Many teachers saw the teaching sessions and the resources as a 'bolt-on' to their normal activities, and used them rarely, while others sought to integrate the approaches and materials more fully into their normal way of working. Although some teachers used the sessions up to nine times, teachers only used the *TTM* sessions four times, on average.

1.3 Implications and recommendations for policy and practice

Below is a summary of key points that we recommend be addressed by policy-makers, management, teachers and teacher educators[4]. We recommend that the National Centre for Excellence in the Teaching of Mathematics (NCETM) should have a major role to play in taking these recommendations forward, and help ensure that the TTM approaches and the materials pack are communicated as effectively, and as widely, as possible. We feel that it is critical that management actively support teaching staff in their implementation.

For policy-makers and managers

- Address the issue of teachers' workload and build in adequate time for teachers to be able to prepare for sessions.
- Give teachers sufficient time to plan and evaluate learning with Learning Support Assistants.
- Dedicate whole-day sessions throughout the year in which teachers have an entitlement to continuing professional development that has a direct effect on their pedagogical knowledge and their classroom practices.
- Adopt an iterative model of professional development where teachers trial ideas, strategies and teaching approaches in the classroom, return as a group to report back, reflect on and evaluate them, then return to the classroom to retrial in the light of these new perceptions/experiences.

For teachers and teacher-educators in CPD and initial teacher training

Professional development should:

- include a significant element on the teaching of basic mathematical concepts, particularly aimed towards learners working at Entry Levels 1 to 3
- include a module on subject-specific pedagogy (how to teach numeracy/mathematics), so that teachers are able to compile a repertoire of activities, and are able to provide learners with a rich variety of learning opportunities geared to their level of experience and area of interest;
- include a module about the theories, strategies and techniques of formative assessment, where teachers make effective use of higher-order questioning, and listen to, and respond flexibly to learners' needs;
- tackle the issue of differentiation and the organisational consequences;
- advise teachers on different managerial and organisational strategies, such as structuring groups, to enable them to cope with the diverse range of learners found in the different contexts across the *SfL* sector;
- seek to increase teachers' awareness of learners' most common misconceptions, and develop a repertoire of ways of making these explicit in the classroom, creating cognitive conflict and managing discussions that assist their resolution. A superficial 'diagnose and correct' approach should be resisted;
- exemplify and develop the concepts of 'mathematical discussion' and 'collaborative learning' so that they have a shared meaning among teachers. Teachers need to be able to recognise and be able to stimulate exploratory talk where learners elaborate on each other's reasoning;
- prioritise the development of teachers' awareness that learners, like teachers, need induction and guidance in techniques of collaboration and working together.

1.4 Further research

The *TTM* project may be seen as one iteration of an ongoing process of theory-driven design, trial and evaluation, and this report should therefore be regarded as a stimulus for further research and development. It is a relatively small-scale study in a limited range of contexts and the generalisability of the findings need to be further studied. In *TTM* we were not able to measure the learning effects of the approaches used. It was difficult to devise an assessment instrument that was sensitive enough to deal with the variety

4 We are aware that some of these issues may be addressed in the forthcoming LLUK *New overarching standards for teachers, tutors and trainers in the lifelong learning sector (Application to Professional Standards, for Teachers of Mathematics (Numeracy))*.

of programmes, the wide range of levels taught, and the fluctuating population of learners, many of whom were enrolled on 'roll-on, roll-off' courses. Moreover, we found that the timescale of the project meant that there was an insufficient amount of teaching hours for an assessment instrument to be administered at two time points.

We further recommend that research is carried out into the professional development effects that the design-based research process can stimulate. We note the powerful teacher-learning effects that may be obtained when teachers reflect on the outcomes of novel interventions in their own classrooms. The long-term effects of doing this need to be studied and, in particular, the ways in which the teaching approaches are integrated, and session designs are adapted and extended, by teachers in different contexts, and with varying degrees of support.

2 Introduction

2.1 Background to *Thinking Through Mathematics*

In this section, we describe the aims, rationale, research questions and underlying methodology of this design-based research project. Unlike research projects which seek to analyse a given state of affairs, a design-based research project sets out to *transform* a situation through the implementation of a novel design, analyses the effects and outcomes, and develops new theories and designs as practical outcomes.

This project was transformative in a similar sense. It sought to improve the quality of teaching and learning in mathematics with adult learners through the design of teaching materials and professional development resources that are based on a set of principles derived from earlier research. The title of the project, *Thinking Through Mathematics (TTM)*, attempts to capture something of the reflective nature of this process. We reflected on the nature of mathematics learning and pedagogy, and attempted to create resources that would enhance the depth of mathematical reasoning that occurs with adult learners in typical 'numeracy' classrooms across the *SfL* sector for both adults and 16–19-year-old learners.

Although there is no definitive picture of numeracy practices in England, recent research from the NRDC (Coben et al., 2007) suggests that the dominant mode of teaching numeracy to adults remains one of transmission where teachers show learners procedures, break concepts down into smaller parts and demonstrate examples. The most common forms of organisation are whole class and learners working individually through worksheets. Teachers tend to ask few higher-order questions, and there is little group or collaborative work, and little use of practical resources or ICT. These practices may be contrasted with the 'best practices' such as those advocated by Ofsted, for example:

- The best teaching gave a strong sense of the coherence of mathematical ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning. Careful questioning identified misconceptions and helped to resolve them, and positive use was made of incorrect answers to develop understanding and to encourage students to contribute. Students were challenged to think for themselves, encouraged to discuss problems and to work collaboratively. Effective use was made of ICT. (Ofsted, 2006)

TTM was designed to promote the practices advocated by this Ofsted report. It was based on a previous design-based research study *Improving learning in mathematics* (DfES, 2005), which in turn was based on earlier studies into teaching and learning in GCSE retake classes in further education colleges (Swan, 2005a, 2006a). This previous work described principles for the design of teaching and then put these principles 'to work' through the development of different generic types (or 'genres') of learning activity. These types of activity were found to promote learning, particularly when they were used in learner-centred ways. In addition, as teachers used and reflected upon the outcomes, they reported that their practices and beliefs were significantly changed from teacher-centred, 'transmission' orientations towards more learner-centred, 'connectionist' orientations[5].

5 Broadly speaking, the transmission orientation views mathematics as a series of 'rules and truths' that must be conveyed to learners and teaching as 'chalk and talk' followed by individual practice until fluency is attained. The connectionist orientation views mathematics as a network of ideas that the teacher and learner construct together through collaborative discussion.

In *TTM,* we took the same set of principles used in the development of *Improving Learning in Mathematics* and sought to discover how they might apply in a different context, with adult learners designated to be working as between Entry Level 1 and Level 1[6]. We worked with a group of 24 teachers drawn from 12 organisations to design teaching activities based on these principles, observed them in use and analysed the extent to which teachers were able to successfully integrate the principles into their own teaching. As in the previous research we also set out to study the effects of our intervention on teachers and learners.

2.2 The research questions

This research is centred around six research questions. The first two are intended to provide more information on and analysis of the nature of the context and the interventions envisioned:

1. ***What do we know about the context?***
   How might we characterise the colleges, teachers and classes for Entry to Level 1 learners?
2. ***How do we design appropriate activities for this context?***
   How can the principles be developed into appropriate classroom activities?

The next two research questions concern the impact that the interventions had on teachers and learners:

3. ***What is the impact on teachers?***
   How do teachers interpret and implement the principles?
   What is the resulting impact on the beliefs and practices of teachers?
4. ***What is the impact on learners?***
   How do learners respond to the new teaching approaches?
   What is the impact on learners' attitudes, motivation and learning?

The last two questions relate to the implications of the research:

5. ***What are the implications for the future design of resources?***
6. ***What are the implications for the professional development of teachers?***

However, it became apparent that it was not going to be possible to tackle all of these aims in the project. For example, one finding of the project was that obtaining pre and post written evidence from learners is difficult, if not impossible, for several reasons. Our data on the fourth research question is therefore extremely limited.

We decided not to try to correlate the effect of the teaching and learning approaches with gains in learners' attainment. Three main factors militated against this idea. First, learners were working on different programmes across a wide range of levels: this made the design of valid and reliable assessment instruments, which also had to be trialled, almost impossible given the time and resources available. Second, the fluctuating population of learners meant that learners spent varying amounts of time on the course: some were on 'roll-on, roll-off' courses and left while the project was still running. Third, we did not feel that there was enough teaching time between an assessment administered at two time points to be able to measure teaching effects in any meaningful way: for instance, in the Pilot phase, in some cases there were only about 10 weeks of teaching, and some of the classes ran for only one hour.

2.3 The research methodology: design-based research

The methodology employed in this project may be termed 'design-based research'. This approach to research has arisen from a desire to make research more relevant by using research-based methods to attempt to transform educational practices in real educational settings (Kelly, 2003; Swan, 2006a; van den Akker et al., 2006). It is distinct from research that attempts to explain existing causal connections between dependent and independent variables and from research that attempts to understand and explain a given state of affairs; rather it considers how education may evolve to meet given standards or ideals (NCTM, 1988). This requires an interventionist and visionary approach (Bereiter, 2002). By challenging the status quo, it is possible to discover the difficulties and elements that resist change.

6 In the *Skills for Life* literature much is made of the National Qualifications Framework in which certain 'levels' are supposedly meant to correspond to standards in compulsory schooling and higher education. In this framework Level 1 for adults is seen as the equivalent of a GCSE D-G; Entry Level 3 corresponds to a level expected of an average 11-year-old, Entry Level 2 is compatible with standards of the average 7-year-old, and Entry Level 1 is at the level of the average 5-year-old. The authors' view is that this framework is potentially insulting, and that, as the majority of learners in the project were clearly not children, it should be regarded as a rough guide only.

This in turn helps us to understand the system more fully.

It is only recently that design-based research has emerged as a recognised paradigm for the study of learning through the systematic design of teaching strategies and tools. The beginnings of this movement are often attributed to Brown (1992) and Collins (1992), though we would contend that rigorous evidence-based design and development has been around for some time under many different names and guises.

Design-based research may be characterised as:

- ***Interventionist***. The research designs an intervention for a real world educational setting and watches it at work.
- ***Iterative***. Development and research take place through cycles of design, trialling, analysis, and redesign.
- ***Theory oriented***. The designs are built on theories of learning and the testing of the designs leads to the development of new educational theories.
- ***Process oriented***. The research tries to explain how the design functions in authentic settings. It must not only document success or failure but also try to understand how and why the interventions behave as they do.
- ***Utility oriented***. The quality of a design rests ultimately in how well it works, its practicality and usefulness in the hands of the intended users.
  (Cobb et al., 2003; DBRC, 2003; Kelly, 2003; van den Akker et al., 2006.)

Design-based research raises important methodological issues. First, the context in which the designs will be used must be taken seriously. The designs must be tested in the target contexts with 'ordinary', busy teachers. (Too often designs are created by 'enthusiasts' remote from such contexts.) Second, the researcher's role may need to evolve as the research progresses. Initially, the researcher may need to intervene in order to closely study the key issues; later the researcher may need to 'stand back' and see how the design works on its own. Third, the research needs to account for the ways in which the intentions of the design 'mutate' in the hands of teachers. When designs are used, teachers interpret them in ways that the designer did not intend. Rather than viewing these mutations negatively, designs and theories need to evolve and try to explain these mutations.

3 The design of the resources

In this section we discuss the principles underlying the teaching approaches; the different types of activity and resources that were used with the learners; and the design of the professional development programme. It should be emphasised that we are not simply evaluating the effects of teaching materials here. We are also considering the manner in which these materials were introduced to teachers and the effect of these teachers being given opportunity to reflect on their design and the outcomes observed.

3.1 The aims and principles of the teaching approaches

The aims of the approaches are identical to those outlined in the *Improving Learning in Mathematics* project (see Swan, 2005a). The teaching approaches have two related aims. The first is to help learners adopt **more active approaches towards their learning**. Many adult learners appear to view mathematics as a series of disconnected procedures and techniques that must be learned by rote. Instead, we wanted learners to engage in discussing and explaining ideas, challenging and teaching one another,

Table 1: Principles guiding the development of the resources

Teaching is more effective when it…	
• builds on the knowledge learners already have	This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs (Black and Wiliam, 1998).
• exposes and discusses common misconceptions	Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies, and allow opportunities for resolution through discussion (Askew and Wiliam, 1995).
• uses higher-order questions	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall (Askew and Wiliam, 1995).
• uses cooperative small group work	Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance (Mercer, 2000). Shared goals and group accountability are important (Askew and Wiliam, 1995).
• encourages reasoning rather than 'answer getting'	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.
• uses rich, collaborative tasks	The tasks we use should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not?' questions (Ahmed, 1987).
• creates connections between topics	Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas (Askew et al., 1997).
• uses technology	Computers and interactive whiteboards allow us to present concepts in visually dynamic and exciting ways that motivate learners.

creating and solving each other's questions and working collaboratively to share methods and results.

The second aim was to develop **more 'connected' and 'challenging' teaching methods**. Traditional, 'transmission' approaches involve simplifying ideas and methods by explaining them to learners one step at a time. Questions are posed to lead learners in a particular direction or to check they are following a taught procedure. Learners then practise, practise, practise. There is plenty of evidence (e.g., Swan, 2006a; Ofsted, 2006) to show that this approach does not promote robust, transferable learning that endures over time or that may be used in non-routine situations. It can also demotivate and undermine learners' confidence. In contrast, the model of teaching we planned to adopt was 'connected' in that it emphasises the interconnected nature of the subject and it is 'challenging' in the sense that it seeks to confront common conceptual difficulties head on. For example, we reverse traditional practices by allowing learners opportunities to tackle problems *before* offering them guidance and support. This encourages them to apply pre-existing knowledge and allows us to assess and then help them build on that knowledge.

The principles that we sought to adopt in the design of pedagogical approaches for *TTM* are summarised in Table 1.

3.2 The different types of learning activity

Principles are easy to state, but 'engineering' them so that they work in a wide variety of typical learning situations is difficult. Over the course of the project, we worked with the teachers to develop and trial 30 activity-based sessions, at levels Entry 1 to Level 1, that exemplify the above principles. The activities can be categorised into the following 'types' that encourage distinct ways of thinking and learning:

- *Classifying* mathematical objects
- *Evaluating* mathematical statements
- *Interpreting* multiple representations
- *Creating and solving* problems
- *Analysing* reasoning and solutions.

3.2.1 Classifying mathematical objects

In these activities, learners devise their own classifications for mathematical objects, and/or apply classifications devised by others. In doing this, they learn to discriminate carefully and recognise the properties of objects. They also develop mathematical language and definitions. The objects varied from shapes to arithmetic problems. For example, learners might be asked to place cards showing geometric shapes into two-way attribute grids such as the following:

Figure 1: Two-way attribute grid

	Large area	Small area
Large perimeter		
Small perimeter		

3.2.2 Interpreting multiple representations

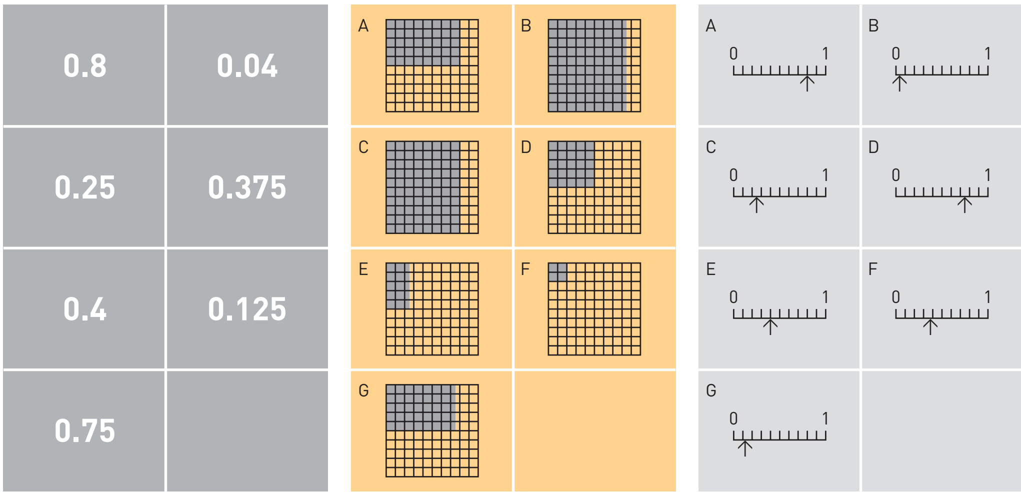
In these activities, learners work together matching cards that show different representations of the same mathematical idea. They draw links between representations and develop new mental images for concepts.

For example, in one activity learners were asked to write these numbers in order of size, from smallest to largest: 0.75, 0.4, 0.375, 0.25, 0.125, 0.04, 0.8.

This usually revealed some misconceptions, such as:

0.375 0.125 0.75 0.25 0.04 0.4 0.8

Figure 2: Multiple representation cards



I know this because they work like fractions, 0.4 is like a quarter.

They were given three sets of cards, one containing the same decimal numbers as above, one showing area and one showing number line representations of these numbers. Learners were asked to work together to group corresponding cards and then place the groups in order of size. These multiple representations gave learners new ways of visualising the numbers, and they were then asked to reflect back on their original ordering and discuss the nature of the errors that had been made. This example also illustrates how cognitive conflict could arise, as learners were confronted with answers that contradicted their initial intuitive expectations.

3.2.3 Evaluating mathematical statements

In these activities, learners decide upon the validity of given statements such as those in Figure 3, and are encouraged to develop rigorous mathematical arguments and justifications, and give counterexamples to defend their reasoning. In Figure 3 in the fractions example, they were asked to decide whether each statement is true or false and give an explanation. In the number operations example, they were asked to decide whether each statement is *always, sometimes* or *never true,* and to test their answers by substituting different numbers for the variables. One can again see how such tasks confront common misconceptions.

3.2.4 Creating problems

These activities offer learners the opportunity to devise their own problems for other learners to solve. When the 'solver' becomes stuck, the problem 'creators' take on the role of teacher and explainer. We often find that, in the process of explaining, learners come to understand the ideas more deeply. In these activities, the 'doing' and 'undoing' processes of mathematics are vividly exemplified. See Figure 4.

3.2.5 Analysing reasoning and solutions

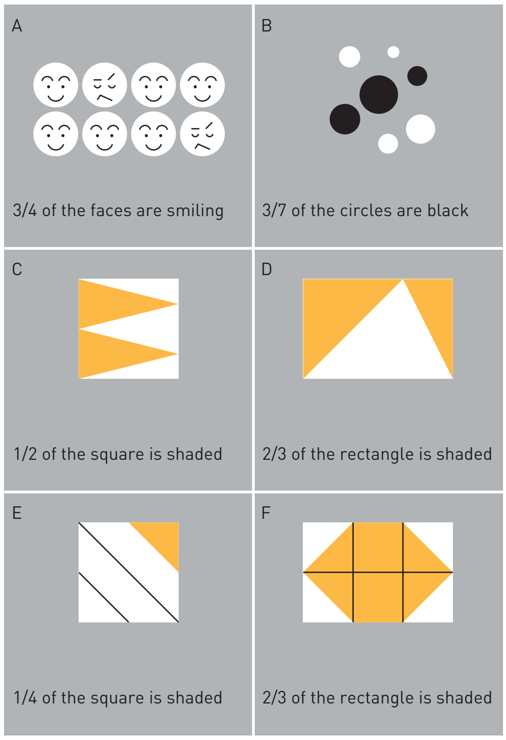
In these activities, learners compare different methods to handle a problem, organise solutions and/or diagnose the causes of errors in solutions. They begin to recognise that there are alternative pathways through a problem, and develop their own chains of reasoning. An activity of this type, for example, may involve giving learners a full explanation that has been cut up into parts. Learners then have to assemble the parts into a logical order. Alternatively, learners may be given some incorrect reasoning. They then have to correct errors in the reasoning and write advice to the person who made them. See Figure 5.

Resources for learning

In addition to the activities, we also

Figure 3: **Evaluating mathematical statements**

a + b = b + a It doesn't matter which way round you add, you get the same answer	**a - b = b - a** It doesn't matter which way round you subtract, you get the same answer
a + 10 > a If you add 10 to a number, your answer will be greater than the number	**a - 10 > a** If you take 10 away from a number, your answer will be greater than the number
a x b = b x a It doesn't matter which way round you multiply, you get the same answer	**a ÷ b = b ÷ a** It doesn't matter which way round you divide, you get the same answer
10a > 10 If you multiply 10 by a number, your answer will be greater than 10	**10a > a** If you multiply 10 by a number, your answer will be greater than the number
a ÷ 10 < a If you divide a number by 10, your answer will be less than the number	**10 ÷ a < 10** If you divide 10 by a number, your answer will be less than 10



considered generic ways in which resources such as posters, mini-whiteboards, and even 'washing lines' can enhance the quality of learning. These artefacts facilitate collaborative learning partly because they are 'larger than life'. Learners can clearly see what is being produced and are thus able to contribute.

Posters are often used in schools and colleges to display the finished, polished work of learners. In our work, however, we use them to promote collaborative thinking. The posters are not produced *at the end* of the learning activity; they *are* the learning activity and they show all the thinking that is taking place, 'warts and all'. We often ask learners to solve a problem in two different ways on the poster and then display the results for other learners to comment on.

Hand-held mini-whiteboards have rapidly become an indispensable aid to whole class discussion for several reasons:

- During whole class discussion they allow the teacher to ask new *kinds* of question (typically beginning: 'Show me…').
- When learners hold their ideas up to the teacher it is possible to see at a glance what *every* learner thinks.
- They allow learners to simultaneously present a range of written and/or drawn responses to the teacher and to each other.
- They encourage learners to use private, rough working that may be quickly erased.

Examples of a range of 'Show me …' questions are given in Figure 6. Notice that most of these are 'open questions' that allow a range of responses. It is worth encouraging a range of such responses with instructions like: 'Show me a really *different* example'; 'Show me a complicated example'; 'Show me an example that is different from everyone else on your table'.

'Washing lines'

Occasionally, activities that involve some form of 'ordering' (e.g., placing decimals or fractions in order of size) may be facilitated by hanging a 'washing line' across the room and asking learners to peg large cards

Figure 4: Creating and solving problems

DOING: the problem poser...	UNDOING: the problem solver...
• draws a rectangle and calculates its area and perimeter	• tries to draw a rectangle with the given area and perimeter
• calculates the cost of some cups of tea and a given number of cups of coffee	• finds how many cups of tea and coffee can be bought for the given sum
• rounds numbers off to the nearest hundred	• finds numbers that would be rounded to a given number of hundreds
• writes down five numbers and finds their mean, median, range	• tries to find numbers with the given mean, median, range
• thinks of a number and says the number which is half of it	• finds the original number
• thinks of a number and adds, say, 27 to it	• finds the original number
• generates an equation step-by-step, doing the same to both sides	• solves the resulting equation

Figure 5: Analysing reasoning and solutions

A learner has been asked to write down how she would solve six problems.

Do you agree with her answers?

What mistakes has she made?

Can you correct her mistakes?

Can you see why she has made these mistakes?

1	A car travels 120 miles in 3 hours at a steady speed. How far does it go in 1 hour?	120 ÷ 3
2	A snail moving at a steady speed travels 0.8 miles in 40 hours. How far does it go in 1 hour?	40 ÷ 0.8
3	I buy some apples which cost £1.50 per kilogram. I spend £3.50. What weight do I buy?	3.50 ÷ 1.50
4	I buy some tomatoes which cost £0.90 per kilogram. I spend 30 pence. What weight do I buy?	90 ÷ 30
5	Sam's motorbike does 60 miles per litre. How far can she go on 3 litres?	60 x 3
6	Clive's car does 20 miles per litre. He only has 0.4 litres left in the tank. How far will he travel before he runs out of petrol?	20 ÷ 0.4

A second learner is trying to explain how to do the last problem.
If you can't decide whether a problem is multiply or divide, then try changing the numbers to easier ones.

Just change the 20 to 6 and the 0.4 to 3. Then it is easy to see that the question should be multiply.

The first learner replies:

I don't think your method works. If you change the numbers, you might change the operation.

Who is right? Why? Write a reply to these learners.

Figure 6: An example of 'show me' questions

Typical 'show me' open questions
Show me:
Two fractions that add to 1 ... Now show me a different pair.
A number between 0.6 and 0.7 ... Now between 0.6 and 0.61.
A number between $^1/_3$ and $^1/_4$ ... Now between $^1/_3$ and $^2/_7$.
The name of something that weighs about 1kg ... 0.1kg.
A hexagon with two reflex angles ... A pentagon with four right angles.
A shape with an area of 12 square units ... and a perimeter of 16 units.
A set of 5 numbers with a range of 6 ... and a mean of 10.

containing the numbers on the line.

3.3 The design of the professional development programme

The professional development for *TTM* was carried out over a 9-month period between October 2005 and June 2006. The project involved 24 teachers in two phases: 12 teachers took part in the Trial phase between October and December, and they were joined by a further 12 for the Pilot phase between January and June. Teaching sessions were developed by Malcolm Swan and Susan Wall, with additional material from Teresa Kent[7]. A small number of additional sessions was written during the Pilot phase of the project. Throughout, research and design were intertwined – teaching approaches and resources were iteratively modified and developed in the light of arising issues and emerging findings, and the revised versions were observed in use to generate new research findings. Revised and extended versions were prepared before the start of the Pilot phase, and at the end of the project.

Participation in the Trial phase was by invitation. Five colleges of further education (FE) and a drug rehabilitation centre agreed to take part in the project, each providing two teachers. This approach was used in order to include organisations which were known to represent a range of approaches to teaching mathematics and numeracy, without the variations arising from selecting organisations across the full range of providers. Since approximately 80% of *SfL* teaching happens in FE colleges, the approach in the Trial phase of the project reflected current provision. In the Pilot phase we chose an additional six organisations from across the wider adult numeracy sector (e.g. adult and community learning, Job Centre related organisations). Seventeen organisations applied for the Pilot phase. There are more details about the sample in section 5.

Over the course of nine months the teachers met together to discuss the resources, then went back to their classes to try out ideas. They then met together again to report on what had happened.

The Trial teachers first met for a two-day residential workshop in October 2005. They spent the first half-day reflecting on their existing contexts for working, their beliefs about mathematics, teaching and learning, and on their classroom pedagogy. The Trial teachers met again for one day in November to reflect on their experiences of the project to date, and also to consider the issue of formative assessment.

The Pilot teachers attended a two-day residential meeting in January 2006, where they were joined by the Trial teachers at the end of the first day. On the second day the whole group discussed some new issues[8] which had arisen, and explored some new and revised teaching activities that had been prepared by the team mentioned above.

Two further one-day follow-up meetings were held in April and June, to which everyone was invited. Again, approximately one half of the time was devoted to teachers discussing and reflecting on their experiences through questionnaires and oral reporting sessions. For example, during the April meeting, we considered how learners who found discussion difficult might be encouraged to communicate in the classroom. We also discussed the role of Learning Support Assistants and how the aims of the project might be introduced to them. As a result of these discussions Session A – *Developing communication, discussion and collaboration* and Session LS1 – *Support in our classrooms* were written.

We encouraged the teachers to run professional development sessions for their own colleagues using the resources we had provided.

7 Teresa was also one of the teachers in the Trial phase of the project.

8 Many of these issues will become apparent and be discussed in later sections of the report. See, in particular, section 8.

4 Research methods

This section describes the methods of data collection, and the ethical issues involved.

4.1 The methods of data collection

The fieldwork for *Thinking Through Mathematics* was carried out at the same time as the strand of professional development over the 9-month period between October 2005 and June 2006.

By 'methods' we mean the tools, techniques and procedures that were used in the data-gathering process to explain how using the approaches affected the teachers' beliefs and practices in the classrooms over the course of the project. We chose a mixed method approach and we believe that the combination of quantitative and qualitative datasets is a particular strength of the project. Over the two phases, the project employed 11 observers to conduct the fieldwork, which included gathering first-hand observational data from classrooms. These people were experienced researchers and/or teachers of adult numeracy [9].

The main methods of data collection were through classroom observations, questionnaires for teachers and learners, interviews with teachers and learners, and recordings of structured discussions and oral feedback sessions. We also filmed three teachers in their classes. A breakdown and schedule for the data collection is shown in Table 2.

Teachers filled in two identical self-report questionnaires at the beginning and the end of the project so that researchers could analyse any perceived changes that had occurred in their beliefs and practices during the project's lifetime. As well as providing us with research data, these occasions provided a model of professional development.

Teachers also completed a questionnaire at the end of the project, which asked them to comment on whether they thought their practice had changed and, if so, in what ways. Due to the absence of some teachers from the last project meeting, we only succeeded in obtaining complete data from 17 of the 24 teachers, and partial data from the remainder.

We asked teachers to administer questionnaires to learners at the start and end of the project to discover their views on mathematics and their approaches to learning mathematics. We also sought to gather data from learners on the teaching styles normally used by their teachers. We were able to collect data from 146 learners on the questionnaire at the beginning of the project, but were only able to collect data again from 22 of these learners (drawn from six classes) at the end. This discrepancy was due to: 'drop-outs' and irregular attendance of learners; the changing learner population due to 'roll-on, roll-off' courses; limited literacy skills among some learners; and the unwillingness of teachers to administer similar questionnaires twice, as they felt the first occasion had taken a disproportionate amount of time out of the session, and some learners had needed a lot of persuasion to get them to finish:

➜ The questionnaires given out to students were too difficult for them to complete – their enthusiasm about the project was dampened as they felt inferior. It took a while for me to persuade them to carry on. (AA)

Learners were also asked to complete a short evaluation form at the end of each session making judgements on: the level of difficulty of the materials; their enjoyment of

9 One of the observers also took on the role of researcher (carrying out some of the analysis) and is co-author of this report.

Table 2: Schedule of data collection

Instrument	When completed
Biographical data on teachers*	October, 2005 (Trial teachers) January, 2006 (Pilot teachers)
Teachers' views on their practice and on maths before the project began*	October (Trial teachers) January (Pilot teachers)
Learners' views on maths and their teachers' practice before the project began*	October (Trial learners) January (Pilot learners)
Observations of teachers' sessions before the project began (Trial teachers only)	October
Observations of teachers' sessions during the project	Mostly 6 observations per Trial teacher (October to May); 4 observations per Pilot teacher (February to May)
Learners' evaluation of individual sessions*	At the end of each session
Interviews with teachers	Usually immediately following a particular session
Interviews with learners	Usually immediately following a particular session
Teachers' evaluations on the materials and how the project was progressing	At the meetings in November 2005, January 2006, April 2006, June 2006; and after each teaching session
Observers' views on the materials and the methods of data collection being employed	At the meetings in November 2005, January 2006, April 2006, June 2006; and after each teaching session
Learners views on their teachers' practice at the end of the project*	May 2006
Teachers' views on issues identified by observers during the project	June 2006
Teacher evaluations of the project	June 2006
Teachers' views on their practice and on maths at the end of the project	June 2006
Group interviews with teachers at review days	June 2006
24 case studies of all teachers written by observers based on data gathered over research period	June 2006
DVD produced of three teachers using the approaches	Summer to autumn 2006

* Copies of blank forms/questionnaires/schedules of the methods marked with an asterisk can be found on the website (www.maths4life.org).

Figure 7: An excerpt from an observer's narrative observation sheet

10.50-11.07

The teacher now gains the attention of the class and says that they will check their answers.

To the first example, one learner says '2' [for 0.2], the teacher responds with 'not 2 but ...' to which the same learner responds 0.2.

The teacher now says 'let's check the first one. Some of you will be very surprised.'

One learner [R] points out to another that the sequence goes 0.6, 0.8 and then 1 not 0.10.

The teacher shows an enlarged number line and shows the sequence as a leaping along the line.

[At this time two learners are talking about one of them becoming a nurse and whether the maths is relevant.]

The teacher points out that it goes to 1.

One learner says 'that is weird'.

[A late learner now comes in and settles down with others.]

K says 0.1 is the same as 0.10.

R now says that 0.10 is bigger than 0.1 even though he had already checked the results and noticed that 0.10 is not 0.8 + 0.2.

Teacher asks some of the class to point out where they would put the answer on the number line (in order to make them realise that they would be putting their 0.ten on 1). The teacher points out that there is nowhere for it to go except on 1.

the session; how hard they had worked; and how much they had learned.

A series of interviews with teachers and learners was conducted between October 2005 and May 2006. Forty-nine interviews were with teachers and 26 interviews were with learners (involving 41 learners altogether). The interviews were semi-structured around a series of questions with a choice of routes, but observers also used directed questions to confirm their interpretation of classroom events. Some observers were unable to carry out interviews for practical reasons, for example when teachers and learners had other classes that they had to attend immediately afterwards, or when learners needed to leave after the class to pick up children.

We believe that one of the strengths of *TTM* was that we did not rely on teachers' and learners' own interpretations and perceptions on how the approaches had affected their beliefs and practices. We also observed teachers' evolving practices over the course of the project to see for ourselves what was happening. In the case of the Trial teachers, observations of their practice were also conducted before teachers had received any input about the approaches. The 12 Trial teachers were generally observed on six occasions, and the 12 Pilot teachers on four occasions.

In total, 67 classroom observations were carried out with the Trial teachers and 43 with the Pilot teachers (making a total of 110 observations). Although the observations involved an element of passive observation, with observers sitting to one side of the room, they were also participatory where observers interacted with small groups of learners in order to try to find out more about what was happening.

A session observation form was completed during each of the visits. This was in two parts: a *descriptive,* narrative, part and a retrospective *evaluative* part. In the descriptive part, observers recorded objective data, including the length of the session, the number of learners present, and a summary of how the class had been organised. They then wrote a narrative of the session in progress, which was divided up into episodes where there was a change in direction or content in the session. Each observer was instructed to be as objective as possible by *describing* rather than *interpreting* events. They were also asked to exemplify any general assertions by including, where possible, verbatim descriptions of classroom talk. A sample extract from a narrative sheet is shown in Figure 7.

After each session teachers and observers met to fill in the *evaluative* part of the form together. This dialogue included sections on their overall impressions of how the session had gone, the response of learners, the effectiveness of the principles being used, and suggestions on how the materials might be further developed. Observers were able to act in the capacity of an advisory role if individual teachers requested it, for example on general matters of pedagogy, or more specific matters relating to the individual principles. Each observer was asked to produce a written case study of each teaching site, including a summary of the apparent impact of the project on teachers' beliefs and practices.

4.2 Some ethical issues

The research team followed the ethical guidelines laid down by BERA (2004). We wanted both teachers and learners to take part in the project on the basis of knowledgeable consent where voluntary choice is exercised by individuals who are competent or able to choose freely. All interviews were conducted on a voluntary and confidential basis, and all respondents were guaranteed that their names would be anonymised in written reports[10]. However, although observers explained the purposes of the project, and learners were given an explanatory, introductory sheet, we also recognise that, particularly at the beginning, some participants may have only partially understood the aims and nature of the research project. Moreover, it transpired that many of the teachers had been 'conscripted' onto the project by their senior management colleagues. We feared that this might negatively affect their involvement and commitment. In the event, teachers reported that their initial reservations and concerns had been quickly dispelled and they willingly agreed to take part.

Once the teachers were committed, it was more difficult for learners to withdraw freely from certain aspects of the project. For example, while learners were free to choose whether or not to be interviewed or filmed, they were unable to withdraw from general session observations without leaving the classroom. Again, we found that the learners were quite happy to be involved and they had few concerns.

10 Teachers' initials have been changed throughout to preserve anonymity.

5 The context

In this section we characterise the contexts in which the research took place. We begin by providing more details on how the 12 organisations were selected, and more information on the numeracy courses that were taught, including the physical resources and Learning Support Assistants. We then provide a profile of the 24 teachers, discussing their status and identities, and some of the pressures and constraints they operate under, including the issue of time. After looking at an example of good planning, the chapter considers teachers' practice at the beginning of the project, both from the teachers' and their own learners' points of view. The next section on teachers' knowledge is categorised into three areas: pedagogical, mathematical and subject-specific-pedagogical. The chapter ends with a profile of the learners and their classes, including their attitudes towards mathematics, and their reasons for attending classes.

5.1 The selection of the organisations for the research

The project involved 24 teachers from 12 organisations. Although one class was a job-share, one teacher in another institution taught two programmes to two different groups of learners meaning that 24 classes were researched in total. The sites were urban and rural, metropolitan and regional, and across different *SfL* sectors, with most of the research taking place in further education colleges (FE):

FE college	7
Re-integration (drugs)	1
Private training company (linked to Jobcentre Plus)	1
Sixth form college	1
Local Authority Adult Education	2

Seven (58%) of the 12 institutions were FE colleges, which compare to the 73% of all *Skills for Life* numeracy learners (n = 266,000) that were in FE nationally in 2003–2004 (LSC, 2005).

When selecting the institutions, it was felt important that the senior management of the organisations was able to offer support to the participating teachers. This would allow teachers time off to attend meetings, and give them the flexibility to adapt their schemes of work to include the *TTM* approaches. The project made a contribution to participating organisations of £2,500, to contribute to the costs of teachers attending meetings, briefings and workshops, and for other incidental expenses, such as equipment costs and travel expenses. The project covered all costs of residential events, including accommodation.

When we reviewed the sample of learners at the end of the Trial phase, we found that a sizeable proportion of classes contained learners aged 16–19 years old, and so, at the beginning of the Pilot phase we stipulated that we wanted to conduct the research with classes consisting of post-19 learners only. We also, again, emphasised that we wanted learners working between Entry Level 1 and Level 1. However, it soon became apparent that some of the Pilot organisations had supplied us with classes containing learners who did not fulfil these criteria. Over both phases we found that between one-third and a half of all learners on roll were aged 16–19, and, while some classes included a minority of learners who were working above Level 1, at least two classes from the Pilot phase had a *majority* of learners working at Level 2 and above and so were, technically, outside the project's remit. In two other classes, observers had to find another group of learners, because they found during the first visit that the class chosen to act as a research site contained too many Level 2 learners. In general, however, most organisations were supportive and committed to the project, and those that were less supportive were not obstructive.

5.2 The courses taught

The majority of courses catered for more than one level of the NQF, and over half contained learners working between Entry Level 2 and Levels 1 or 2, which provided teachers with the challenge of planning differentiated activities and different materials: only two classes were designated for one specific level. The length of courses ranged between three months and nine months (September to June), and classes lasted between 45 minutes and three hours per session (see Table 3)

Table 3: The length of course and classes

Duration of course	Number of classes
3 months	1
6 months	1
9 months	19
Variable	3
Duration of class	
Under 1 hour	1
1 hour	3
1.5 hours	8
2 hours	7
2.5 hours	4
3 hours	1

Nineteen of the classes took place in the daytime, four were held in the evening and the remaining class was scheduled at irregular times. While almost all of the numeracy provision was discrete and stand-alone, two classes were embedded into other courses such as 'Land-based studies in Animal Care' or 'Administration', and this may have had an effect on some learners who were unaware that numeracy was an 'added extra' to their course. It also meant that some of the 16–19-year-old learners' ultimate qualification did not depend on their passing a numeracy examination.

Over one-third of the programmes were held on a 'roll-on, roll-off' basis, with learners joining and leaving the course at different points. This caused problems of continuity, particularly when teachers planned to explore a mathematical topic in depth over several sessions. Some learners also arrived at classes at different times, particularly during the early part of the session. This may have been because they had childcare arrangements to make or had other work commitments. This created difficulties when using the project resources. One teacher told us:

> ➜ I work on a roll-on, roll-off workshop, which does make it difficult as people come in at different stages and what have you. So trying to make groups, like I've got people still joining me, on Wednesday I've got a new learner coming in to one group so it can be quite difficult to get them in. And that I think is the main problem with the approach, to try and get people on board when they're all at different stages. (BB, oral)

However, another teacher commented that the non-linear 'rich' activities included in the project resources were more amenable to 'roll-on, roll-off' programmes. It also suggests that many of the difficulties and issues that some teachers expressed were not insurmountable:

> ➜ You've mentioned the problem of students coming in on roll-on, roll-off, I have that as well. Funnily enough, I find it easier with the sort of materials we're doing. I don't think you have to start at one end and go right through to the other now, with these sort of materials, they can drop in almost anywhere. (CC, oral)

5.3 The physical resources and Learning Support Assistants

The learning environment in the organisations was generally adequate, although many of the rooms used were not dedicated mathematics rooms, and so teachers had to carry their resources with them. For example, one teacher (DD) worked on two different sites, and the room that she mostly taught in was too cramped. (On one occasion she arrived to find her usual teaching room being decorated, and she and the researcher had to clear tables and set up a whiteboard before the session could begin.) Although learners were generally able to use calculators, there was little access to computers in the majority of classes, and this had an effect on teachers' ability to integrate the use of technology into their sessions.

At least one-third of the classes had Learning Support Assistants (LSAs), and some classes had more than one extra adult when they contained several learners with specified difficulties. While the majority of LSAs were supportive of the project, at least three teachers reported that their LSAs did not understand the project aims and unintentionally undermined them, for example by discouraging discussion, showing learners short cuts, or by supplying answers and thereby removing the need for learners to think for themselves. This was an important example of a more general need for teachers to plan time to share their teaching approaches with colleagues; usually, however, LSAs would arrive at a session at the same time as the learners (sometimes this was because they had been working with learners in the previous lesson) and so there was no time to discuss appropriate ways of working for that session[11].

Lack of administrative support meant that many teachers had to prepare materials themselves, and this was particularly difficult for part-time teachers.

5.4 The sample of teachers

The 24 teachers (6 male, 18 female) in the sample came from a variety of backgrounds, and had a wide range of experience and qualifications. Nine worked full-time and the remainder were employed on part-time or fractional basis (contact hours for those on hourly-paid contracts can vary significantly from term to term). The mean number of years of professional experience amongst the teachers was about 6.5 years, ranging from under a year to 29 years. The distribution of their ages and highest mathematical qualifications are shown in Tables 4 and 5 below.

5.4.1 Mathematics qualifications

Four teachers did not achieve a mathematics qualification at GCSE or 'O' level at age 16. Of these, one took 'A' level mathematics as an adult, and one took a Level 3 Number Skills qualification (both of these appear in

Table 5: Highest qualifications in mathematics

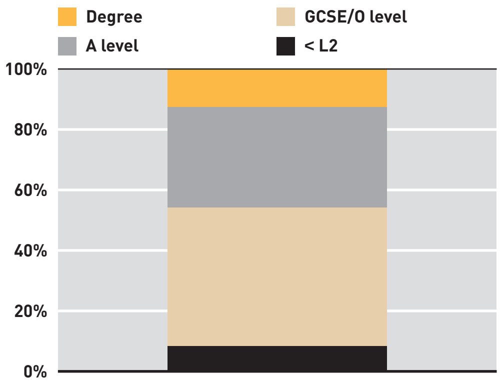
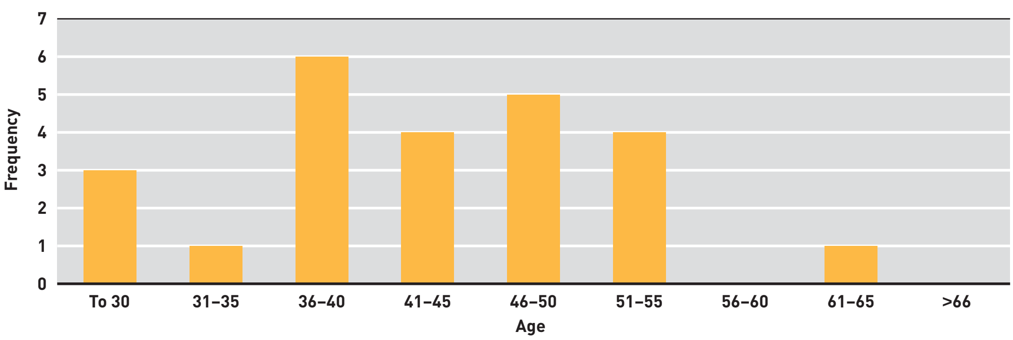


Table 4: Age distribution of teachers



11 We are pleased to see that some of these issues are to be addressed in new proposed standards for learning support staff. See http://www.lifelonglearninguk.org/documents/nrp/164_ls_standards_revision_v9_feb07.doc. There is also a section in the Application to Professional Standards, for Teachers of Mathematics (Numeracy) on 'working with colleagues', which includes LSAs.

Table 6: Teaching qualifications

	L4 only	PGCE	Cert. Ed.	B.Ed.	None
Have gained new Level 4 qualification as well as...	2	3*	5	1	
Have not gained new Level 4 qualification, but have ...		4**	4		4
Unknown					1

* 1 in Adult Basic Skills, 1 in History, 1 in Secondary Mathematics
** 2 in Secondary Mathematics, 2 in Basic Skills

the 'A' level' category in Table 6). Eight of the teachers in the sample have an 'A' level in mathematics (or equivalent), and three have a degree in mathematics. Some teachers also have qualifications such as Open University modules in statistics, and degrees in mathematically related subjects such as engineering or psychology.

5.4.2 Teaching qualifications

At the time of the project, 11 of the teachers had gained a Level 4 subject-specific teaching qualification in numeracy; five held a Certificate in Education; three had a PGCE and one a B.Ed. Of those without the Level 4, while eight teachers attained at least one teaching qualification, four teachers had no formal qualification to teach in the *SfL* sector. See Table 6.

5.5 Status and identities; pressures and constraints

Teachers working in the *SfL* sector of adult basic skills are paid relatively poorly in comparison with mathematics teachers in schools. The concept of identity work is seen as of growing importance in contemporary research (see, for example Mendick, 2005; Swain, 2005) and it is pertinent to note that some of the teachers in the project did not regard themselves as being 'real' or 'proper' mathematics teachers[12]. Moreover, some had teaching backgrounds in other subjects such as ESOL or ICT.

➜ We are all admin tutors, or in my case, IT. It's just that we like maths. (EE)

One teacher told researchers how she sometimes felt out of her depth with the level of mathematics involved on the initial training days:

➜ There were times that we did feel very beyond our... out of our depth ... purely because a lot of it didn't seem to apply to us, in our circumstances – we're vocational teachers first, mathematics teachers second. At that point, that was the lowest point that I got, I thought, what am I doing here? I don't know what I'm doing here. (EE)

Post-16 education in England has recently become the focus for reform and massive change, which has affected the practice of teachers working in the sector (Lucas (2004). Since 2001, teachers have been expected to work with a standardised core curriculum (ANCC) which is divided into five levels (ranging from Entry Levels 1, 2 and 3 to Levels 1 and 2). Each level has a prescribed, discrete body of knowledge and level of skill, which teachers are generally expected to 'cover' and 'map' onto learners' Individualised Learning Plans (ILPs)[13]. This means that there is a tendency for some teachers to both see teaching as an individual, rather than as a social activity, and also to view mathematics as a set of discrete and disconnected skills.

In our pre-questionnaire, almost all teachers stated that they did not teach in ways that were consistent with their personal beliefs about teaching and learning. Reasons given were varied: the 'corporate' assessment-driven culture in colleges; their own lack of subject knowledge; the diverse and individual learning needs of learners; a lack of suitable resources; the need for syllabus coverage; the lack of time for preparation and 'delivery'; and the pedagogical expectations of learners. Typical comments were:

12 One of these teachers held an 'A' Level in mathematics

13 Not all teachers follow the ANCC prescriptively, and many regard the curriculum as a framework.

- I need to cover specification/syllabus, get through a course, get students a qualification, cheat the system. (FF)

  Many of the objectives and the thrust towards 'understanding and learning' (which I support and agree with) sit uncomfortably with a results-orientated culture. (GG)

  Things that prevent me from teaching as I would wish: lack of good resources, space, the environment isn't very stimulating as it is used for lots of different subjects. (AA)

  Time is a constraint … as they need a particular qualification. (CC)

  (I am) frequently let down by my own skills and understanding. The other factor that makes it difficult is the wide range of ability/previous knowledge in my classes – it's a challenge! (AC)

Most teachers believed that their learners were capable of learning mathematics, although a few had low expectations of learners:

- It is important to remember that they may never grasp certain concepts and for some learners we are talking about maintaining skills rather than making progress. (JJ)

The need to prepare learners for accredited tests, linked to funding, occasionally obstructed the progress of the project. Observers sometimes found it difficult to arrange visits because learners were preparing for examinations and, in some classes, the 'exam' permeated the teacher's interactions with the class:

- KK constantly referred to 'the exam' in the first session observed. 'In the exam you may need to rotate your paper'. 'In the exam you will see questions like. …' Other evidence is provided by the workbooks that [the teacher] produces for her learners. They give examples of the types of problems learners will meet, with solutions exemplified. Learners are then expected to mimic the solutions to solve similar problems. (Observation of KK)

At the final review day, almost all of the teachers reported that there had been pressures and constraints that prevented them from using the approaches in the best possible ways. Again lack of time was a central factor.

5.6 The issue of time and an example of good planning

As the project progressed, the issue of time began to emerge as a recurring and important issue. Teachers told us that their time was taken up with an increasing amount of paperwork. In the first instance, the materials needed time to prepare (cutting, laminating etc.) and reproduce and, in some cases, there were also reproduction costs. As we have already mentioned, very few *SfL* teachers have access to administrative staff and have to find the time to create the resources themselves. This not only affected the nine teachers in our project who were contracted to work full-time, but also the other 15 who were employed on a part-time or fractional basis, and who generally had poorer access to photocopying machines and laminators.

- I've got to be very, very resourceful and it's all done in my own time at home […] all the things like photocopying work. I bought my own laminator but that's the only thing. I've had to do it in my own time, so it's been very time-consuming for me on a personal level to prepare all the materials. (DD)

Another issue connected to time concerned planning. In order to do the approaches/materials justice, teachers needed to give themselves time to absorb and acquaint themselves with the session plans and guidance so that they could anticipate problems, potential misconceptions, learners' questions and so on. Not every teacher was able to do this, and occasionally a teacher was caught out because they had not had enough time to think about the session and prepare adequately. The extract below comes from an observer's narrative early on in the project:

➜ The number line was not a success and was rapidly abandoned. There was no line to hang numbers – they were put in order along the top of the table which was too short and which most could not see. There was no challenge for any in this, all could order whole numbers. [The teacher] agreed that she should have been better prepared and used harder numbers. (This is about DD)

In contrast, other teachers appeared to plan thoroughly. Observers found that the key to one teacher's successful integration of the approaches was his thorough familiarisation with the session plans. He wrote:

➜ When preparing for a specific session I begin, a few days before the lesson, by looking at the Maths4Life lesson template to study its contents, and then begin to formulate how I can use the materials effectively. I will then have another look, and begin to synthesise these materials into a lesson plan. When I feel comfortable about the format, I write the lesson plan, probably on the same day as the session [...] I will go through the lesson template and highlight the key words. This will again help me to visualise the lesson format/flow. I need to be sure that I will cover the key learning areas in the time allowed. I also look through the lesson text to ensure any words used will be understood by the learners: for example, 'digit'. Have I used it before? Will I need to explain the term? (LL)

He also wrote down the main learning areas that he thought were particularly important as an aide-memoire or checklist.

- Discussion/thinking
- Link topics – patterns
- Challenge
- Conversation style – discussion
- Must have beginning and end – GO WITH FLOW
- Don't give answers, get learners to think
- Solve problems together
- Peer learning/exchanging ideas
- Learners not afraid to make mistakes
- Everyone has different way of learning
- If it isn't broke, don't fix it. Whatever works for you as a learner
- I will only suggest different strategies to unlock the door
- Help some learners to help their kids – positive motivation

One of the key phrases seems to be the teacher's remark that he tries to *visualise* the session in his mind. He also told us how he tried to anticipate learners' questions so that he could think about his own responses.

Not all the teachers were able to spend so much time preparing for each session. Although managers may be inclined to say that, if one teacher can prepare like this, the others should be able to do the same, this is too simplistic. Each context is different and has its own structural factors (such as teachers having no breaks between teaching classes), which produces its own constraints. Lack of teacher time is generally acknowledged to be one of the greatest problems facing teachers across all sectors of education, and needs to be addressed principally by managers and policy-makers.

5.7 Teaching practices at the beginning of the project

As already noted in section 2.1, recent research from the NRDC (Coben et al., 2007) suggests that the dominant mode of teaching numeracy to adults remains very transmission oriented, where '**ex**planation', 'worked **ex**ample' then 'practice **ex**ercise' dominates, and which one observer has categorised as, the 'Triple X' (XXX) approach to teaching.

In our preliminary questionnaire (see Swan 2006a; Swan, 2006b), teachers and learners were independently asked to rate the relative frequencies of 16 teaching behaviours using a 5-point scale (1= almost never, 2 = occasionally, 3 = half the time, 4 = most of the time, 5 = almost always). The wording of the questionnaires was slightly different in each case (see Table 7). The

Table 7: Comparison of wording on two questionnaires

Learner's wording		Teacher's wording	
1	The teacher asked us to work through practice exercises	1	Learners learn through doing exercises
2	The teacher expected us to work mostly on our own, asking a neighbour from time to time	2	Learners work on their own, consulting a neighbour from time to time
3	The teacher showed us which method to use, asked us to use it	3	Learners use only the methods I teach them
4	The teacher let us choose which questions we do	5	Learners choose which questions they tackle
5	The teacher asked us to compare different methods for doing questions	7	Learners compare different methods for doing questions
6	The teacher showed us how topics link together	11	I draw links between topics and move back and forth between topics
7	The teacher tried to prevent us from making mistakes by explaining things carefully first	13	I avoid learners making mistakes by explaining things carefully first
8	The teacher expected us to follow the textbook or worksheet closely	14	I tend to follow the textbook or worksheets closely
9	The teacher expected us to learn through discussing our ideas	15	Learners learn through discussing their ideas
10	The teacher asked us to work in pairs or small groups	16	Learners work collaboratively in pairs or small groups
11	The teacher let us invent and use our own methods	17	Learners invent their own methods
12	The teacher told us which questions to do	19	I tell learners which questions to tackle
13	The teacher showed us just one way of doing each question	21	I only go through one method for doing each question
14	The teacher taught each topic separately from other topics	25	I tend to teach each topic separately
15	The teacher encouraged us to make and discuss mistakes	27	I encourage learners to make and discuss mistakes
16	The teacher jumped between topics as the need arose	28	I jump between topics as the need arises

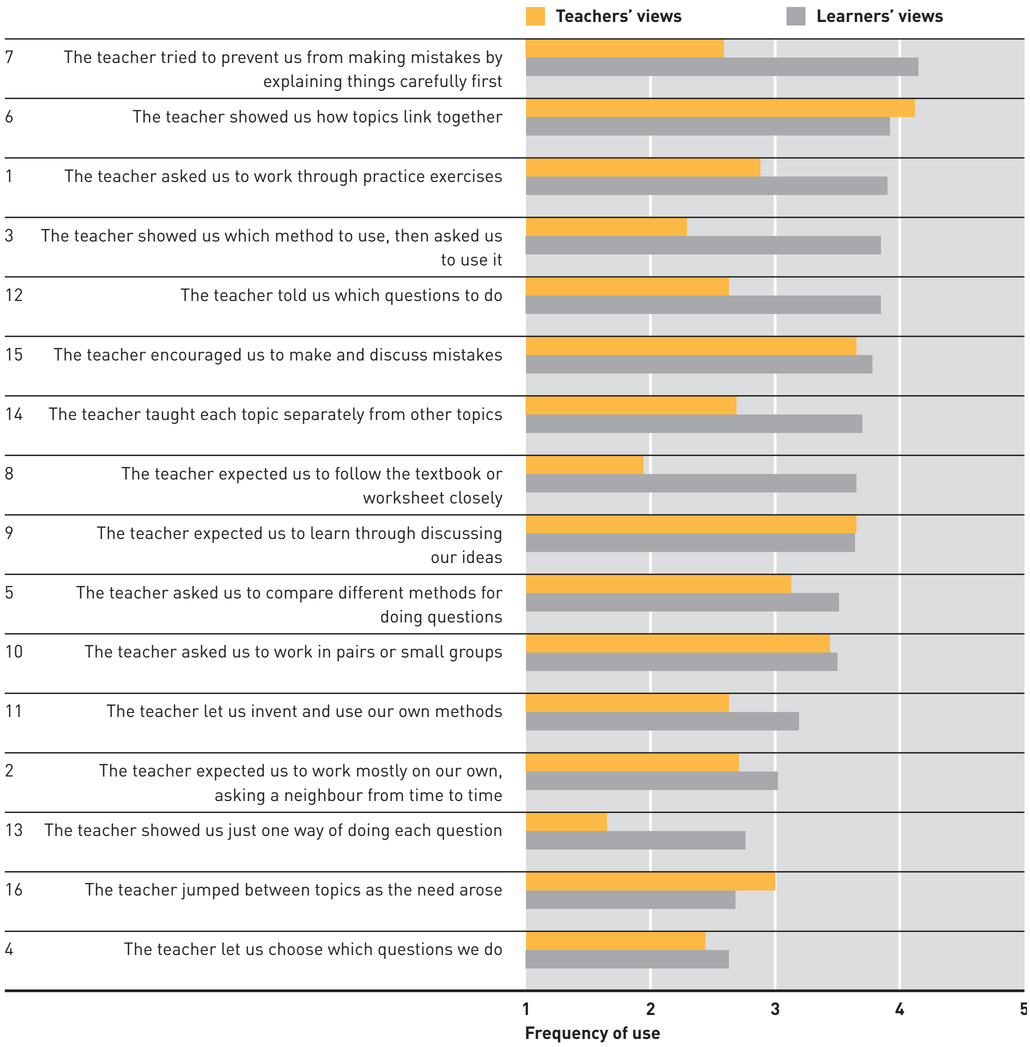
shaded behaviours may be described as more teacher-centred, while the unshaded behaviours may be termed as more learner-centred.

This provoked an interesting response. Learners viewed their experience as mostly teacher-centred, where teachers avoided mistakes arising by carefully explaining everything first, gave practice exercises, demonstrated methods, showed connections, and defined the questions to tackle. According to learners, there was less room for learner choice, or creativity.

From the teachers' point of view, however, they were much less teacher-centred than this (teachers gave similar ratings to the learners for the learner-centred behaviours, but much lower ratings to the teacher-centred behaviours). See Table 8.

Teachers may have described themselves as being more learner-centred because teachers and learners had different interpretations of the same practices. What the teacher saw as a hint, for example, the learners interpreted as an instruction.

Table 8: Ways of working in maths lessons



Learners and teachers rated frequencies of 16 behaviours 1 (= hardly ever) to 5 (= almost always). The mean ratings for each group are shown. Statements are ranked in order from most to least common practice according to learners.
Sample: 146 learners (drawn from 16 classes); 17 teachers.

In addition to the 16 behaviours described above in Table 7, teachers were also asked to rate 12 additional behaviours. The most common practices were (with the mean frequencies):

- Learners start with easy questions and work up to harder questions. (3.82)

  I teach each learner differently according to individual needs. (3.53)

  I know exactly what maths the lesson will contain. (3.18)

  I teach the whole class at once. (3.06)

These are behaviours you would expect to find in small teacher-centred classes. The teacher plans the content of the session beforehand, uses carefully graded exercises and divides their time between working with individuals and talking to the whole class. In the project we were promoting a more flexible approach than this, using richer, more challenging tasks where the mathematics covered in sessions might become less predictable as we would try to build on what learners already know.

5.8 Teachers' knowledge

Below we categorise teachers' knowledge into three areas: general, pedagogical knowledge (including skills in classroom organisation and management); mathematical knowledge (understanding the subject); and subject-specific-pedagogical knowledge (knowing 'how' mathematics is taught and learned)[14].

5.8.1 General pedagogical knowledge

Given the variety of teachers' backgrounds in the sample, it was not surprising that the teachers' general pedagogical knowledge also varied considerably, and this included their skills in classroom organisation and management. One teacher told us that he based his teaching practice on the way he had been taught mathematics at school:

→ I'd been taught to sit up straight and pay attention and very much chalk talk and textbook and that's the way it had been done for me and that's what I brought with me. (GG)

Nearly all of the 17 teachers present at the final review day in June felt that the suggested approaches had challenged their classroom management skills. Some teachers were more used to working with learners on an individual basis and had little previous experience of managing small group and whole class collaborative discussion. In respect of classroom organisation, this meant that some teachers' practice had further to travel than others when it came to integrating the principles required by the *TTM* approaches. As one teacher said:

→ As far as the project's concerned, I feel it has changed my practice in that teaching, most of my teaching is based in workshops, so it is actually quite difficult to do any sort of whole class teaching. (BB)

Formative assessment techniques, such as inviting learners to describe what they already knew about a topic at the beginning of a session and then building constructively on this, was new to most teachers.

5.8.2 Mathematical knowledge

We would like to build on the work of Coben et al. (2007), to make a distinction between teachers' *mathematical knowledge*, and teachers' *subject-specific pedagogical knowledge,* which refers to *how to teach* mathematics to learners, or the ways in which the teacher constructs mathematical concepts, knowledge and skills with the learners. Again, it is hardly surprising that there was wide variation in teachers' subject knowledge in mathematics. As we have already written, two teachers held qualifications at a level below GCSE (Level 2), and this had an effect on their level of confidence.

→ I was kind of given this, you're going to support the literacy and numeracy tutor role, it was thrown upon me, so I had my own barriers as well as these fears of teaching different types of learners. I then had to teach numeracy which I'd never taught before. I think, 'I don't like numeracy, I don't do numeracy'. (MM)

In general, though, the project did not make new demands on teachers' mathematical knowledge, which is not surprising, considering *TTM* was designed for learners working at Level 1 and below.

5.8.3 Subject-specific pedagogical knowledge

In contrast to their own personal knowledge of mathematics, teachers also need to know how learners might come to understand mathematics and the teaching strategies that facilitate this. Successful use of the discussion activities was directly related to teachers' knowledge of subject-specific pedagogy. Where there were gaps in this form of knowledge, opportunities were missed and learning suffered.

We noted several examples where teachers appeared unfamiliar with alternative methods for performing calculations, for example. A few teachers were unsure of the distinction between grouping and sharing in division and with the decomposition method of subtraction[15]. Some also revealed a limited knowledge of common mistakes

14 We note that Shulman (1986, 1987) has made a similar classification where he distinguishes between different forms of knowledge; knowledge of mathematics, knowledge of general pedagogy, and pedagogical content knowledge specific to the general teaching of mathematics, and also to particular individual topics.

15 Concepts such as these are not mentioned in the current teacher training requirements. However, we believe that some basic mathematical concepts may be covered in the new *'Application to Professional Standards, for Teachers of Mathematics (Numeracy)'*, due to be introduced in September 2007 by LLUK.

and misconceptions, and of strategies for responding to these.

5.9 Learners and their classes

The learners were a very heterogeneous group. Approximately 200 were involved in the project from the 24 classes. Nineteen of the 24 classes had almost exclusively white British learners, two classes were predominantly Bangladeshi and two classes had learners of mixed ethnic origin. Two-thirds of the classes were predominantly female, 22% were predominantly male and 13% had a fairly equal gender balance. Learners' ages ranged from 16 to mid-60s, although one-third of the classes were composed almost exclusively of 16–19-year-olds (see Table 9).

Class sizes ranged from two to 16 learners, and the average number of learners attending classes was eight (see Table 9). Like the majority of *SfL* learners, most of the learners in *TTM* attended classes on a voluntary basis, which meant some came intermittently. Eleven of the classes had a mean attendance of six or less. Although such small numbers are not necessarily an impediment to effective collaborative discussions (we observed good discussions between the teacher and an individual learner on occasion), one teacher felt that the 'group dynamic' might suffer. One teacher wrote in his session plan:

→ I am worried not many learners will turn up for the session. This will affect group dynamics . One of main characteristics of this group is the rapport, interaction and general 'buzz' that is created when discussing each session topic. (LL)

5.9.1 Prior learning experiences and learners with specific learning difficulties

Many of the learners in the project had a poor experience of learning mathematics/ numeracy at school. One learner wrote:

→ At school it was harder to catch up when you fell behind and I had so many other subjects that I didn't bother doing the work to catch up. It wasn't something I was interested in so I concentrated on the subjects I did enjoy.

Some of the learners lacked confidence in mathematics and told us that they perceived themselves to be failures. Sally, for example, told us:

→ Maths was never my ... I mean I enjoyed maths at school but my teacher accused me of being thick ... So because of that it totally knocked my confidence with maths.

Table 9: Usual number of learners attending each class, and age range

Teacher	Usual number of learners attending class	General age range
GG	9	16-19-year-olds
SS	9	16-19-year-olds
NN	9	16-19-year-olds
PP	10	16-19-year-olds
LL	8	Over 25
QQ	6	Over 25
AC	10	Over 25
FF	10	16-19-year-olds
JJ Class 1	13	16-19-year-olds
JJ Class 2	7	16-19-year-olds
RR	6	16 to over 50
AD	5	Mid-20s (majority)
MM	5	Mid-20s (majority)
CC	5	Over 25
TT	9	Under 25
BB	5	Over 25
KK	6	Over 25
VV	6	Under 25
DD	8	Over 25
WW	6	Over 25
XX	4	Over 25
AA	12	Over 25
YY	4	Over 25
ZZ	6	16-19-year-olds
AB	11	Over 25

Some learners were working at relatively low levels, and many of the 16–19-year-olds had a grade E or below at GCSE. For example, one observer noted that some learners used their fingers to calculate the addition of single units such as 9 + 5, and were not comfortable with adding basic number bonds. However, in contrast, and as we have already written, there were also some learners who had been designated as working at Level 2 or above.

Poor literacy skills also caused teachers problems when interpreting and discussing mathematics and when completing questionnaires. The extract below concerns a learner called Daniel (in RR's class) who has considerable difficulty with reading as well as writing, and who told the observer:

→ It does hold me back. It does hold me back when I have got to write something, holds me well back. That is what put me behind in my course work and everything. Puts me well back. [In the *Thinking Through Mathematics* work] it slows me down to think of something, because I am trying to think of what the words say and it puts me straight back. [...] I'd rather have some people with me, like helping me, in a group, so they can help me with the reading, because if I was doing it on my own I wouldn't get anywhere with it.

Some learners had difficult lives outside the classroom, and a number also had either physical or mental health problems, which was particularly noticeable in five classes, where LSAs provided specific support. Learning difficulties or disabilities, included a disability caused by head injury, controlled epilepsy and Down's syndrome. The following description comes from one class of 16–19-year-old learners:

→ Many of the group have particular difficulties, and many were excluded from school for a variety of reasons. One learner is autistic and another is epileptic. One learner has severe social problems which affects her ability to learn; one has had a car crash which has affected his short-term memory; one is in the care of the Local Authority which affected her prior learning and confidence; another is the sole carer for two siblings, one of whom is disabled.

In another class, all the learners were judged to have some condition which affects learning including ADHD, cerebral palsy, MLD and other physical conditions, although it is unclear whether these learners had been formally assessed. In the Drug Reintegration Centre, the teachers noted that many learners had changeable motivations and attitudes due to their circumstances and medication, and what might seem a small setback, could have a dramatic impact on learners' attitudes to learning.

Research (see, for example, Coben et al. (2003); Swain et al., 2005; Baxter et al. (2006) AQ13; Coben et al., 2007) also suggests that many learners have high levels of anxiety when they step into a mathematics classroom, particularly when they have been out of formal education for some time. Overall, learners tended to blame their previous lack of progress on personal factors: the difficulty they had learning mathematics and their application to learning, as well as on the anxiety they felt. This was followed by reasons that were attributable to others (their previous teachers and classes) and last came reasons attributable to personal circumstances or luck. It is noticeable that females tended to emphasise personal factors more strongly than males.

5.9.2 Learners' attitudes towards mathematics

Many of the 16–19-year-old learners were receiving Educational Maintenance Allowance (EMAs), which means, in effect, that they were being paid to attend the course. However, their attitudes towards learning mathematics were generally very positive (see Table 10) In fact, learners in general saw mathematics as a useful subject that they enjoy doing and work hard at.

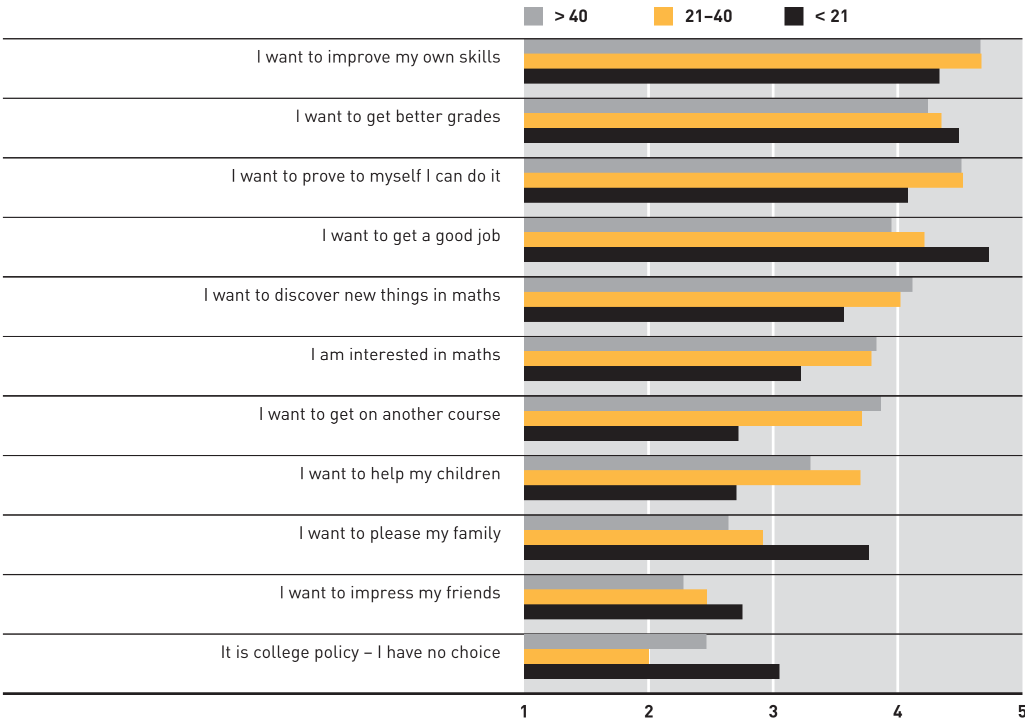
Table 10: Learners' attitudes towards learning mathematics

	Females	Males	Total (n = 130)	
	n = 74	n = 56	Mean	SD
Maths is a very useful subject in everyday life	4.57	4.31	4.45	0.66
I am sure that I can learn maths, if I work hard	4.46	4.31	4.39	0.61
I usually work hard in maths lessons	3.91	3.70	3.82	0.97
I enjoy learning maths	3.71	3.47	3.60	1.07
Maths is a very interesting subject	3.55	3.43	3.50	1.12
I like being challenged by maths problems	3.42	3.34	3.39	1.12
Even if I work hard I can't seem to learn maths	2.72	2.47	2.61	1.19
Maths problems are boring	2.38	2.89	2.60	1.11
Maths is a very dull subject	2.34	2.79	2.54	1.22
I do not enjoy learning maths	2.42	2.52	2.46	1.12
I do as little work in maths as possible	2.22	2.56	2.37	1.15
Maths is not much use in everyday life	1.78	1.70	1.75	1.14

Table 11: Learners' reasons for attending their numeracy course

	Females		Males		All		
	Mean	N	Mean	N	Mean	N	SD
I want to improve my own skills	4.66	82	4.40	63	4.54	145	0.66
I want to get better grades	4.41	80	4.33	63	4.38	143	0.89
I want to prove to myself I can do it	4.41	80	4.26	62	4.35	142	0.83
I want to get a good job	4.09	80	4.61	62	4.32	142	1.04
I want to discover new things in maths	3.99	81	3.80	61	3.91	142	0.93
I am interested in maths	3.69	80	3.53	62	3.62	142	1.03
I want to get on another course	3.68	80	3.03	59	3.40	139	1.30
I want to help my learners	3.56	55	2.84	37	3.27	92	1.48
I want to please my family	2.97	78	3.36	61	3.14	139	1.40
I want to impress my friends	2.48	80	2.49	59	2.48	139	1.28
It is college policy – I have no choice	2.13	72	2.95	56	2.48	128	1.49

Table 12: Learners' reasons for attending numeracy classes by age



5.9.3 Learners' reasons for attending numeracy classes

Recent research (see, for example, Coffield, 2000; Swain et al., 2005) has confirmed that *SfL* learners come to numeracy/mathematics classes with a variety of different motivations and needs. In Table 11, learners were asked to give a ranking from 1 to 5 for each statement. Results are rank ordered with the most popular reason for the whole sample shown first.

Table 11 shows that the main reasons learners gave for attending classes were predominantly instrumental, for their self-improvement: they wanted to improve their mathematical skills to get higher grades (or test scores), which, in turn, would lead to higher qualifications and greater opportunities in employment. However, a significant number also indicated that they wanted to prove to themselves that they could study, and succeed, in what they perceived to be a high status subject. For men, the major motivation was to improve their job prospects, and intrinsic interest in the subject was generally given a lower priority.

Not many learners gave any ranking to the statements 'I want to help my children', presumably because over a third of the sample were 16–20-year-olds, and they felt that this statement did not apply to them. When we organise the sample by age (see Table 12), we see that the under-21s appear more affected than older learners by the wishes of family and friends and by the improved job prospects. More older learners see mathematics as a stepping stone and appear to have a greater inherent interest in the subject than younger learners.

6 Interpreting and using the activities

Sections 6 and 7 form the core of this report and present the project's main findings. Before considering the impact that the project had on teachers and learners, this chapter discusses the ways in which teachers interpreted the principles that underlie the approaches and the various ways in which they were used.

The issues discussed in this section lie at the heart of the research. Many of these have produced rich and stimulating debates amongst teachers and observers/ researchers, and have helped inform the design of the professional development aspects of the project.

6.1 Alternative interpretations of aims and terminology

The principles underlying the project are complex. Teachers' comprehension of them happened only gradually and, particularly during the early stages, understanding was often partial[16]. They interpreted the principles (see Table 1, section 3.1) in different ways and with different emphases. For example, below are three examples of initial common misinterpretations. These are stated in somewhat exaggerated form, for clarity.

(i) 'The project is mainly about developing and testing resources.'

Some teachers initially interpreted the project as generating 'materials for fun activities', or simply 'adding variety' to what they saw as an otherwise dull curriculum. They saw the activities as providing 'enrichment' to existing resources that could be slotted in at appropriate points. All that was needed was some help with referencing the new activities to the curriculum specification.

- It would be more useful if the session were core curriculum referenced as I feel this would have ensured activities were at the appropriate levels. (AA)

This misinterpretation misses the underlying generic purpose of the activities – to foster different forms of reasoning about mathematical concepts.

(ii) 'The project is mainly about using groupwork.'

Early on, some teachers appeared to believe that they were adopting the approaches if they simply organised their classes into groups.

- In the first session YY provided a selection of activities based on equivalent fractions, including some taken from the project materials, and indicated that learners could select activities to complete in whatever order they felt appropriate. Most of the group were disposed around the room in pairs or working at computers. There was no introduction to the session, nor any whole class discussion. The observation report focused on the work of two of the learners who clearly found equivalent fractions challenging. Although the two (observed) learners were sat at the same table they did not work cooperatively on tasks.
(Observation of YY)

(iii) 'The project is mainly about learning by discovery.'

Some teachers believed that the approaches were about 'standing back and letting the learners discover things for themselves'. The extract below comes from an interview with a teacher after the fourth observation:

16 This was also the case with some of the observers.

> It is them [the learners] doing the learning and teaching in some ways. They are teaching each other, they are learning and exploring. It is more than just a worksheet, it is giving them a puzzle and problems and letting them get on with it and seeing where they go, and just making sure they stay on the right track [...]. It is allowing them to make discoveries for themselves rather than you writing it up on the board [...] It is their discovering, not mine; it is nothing to do with me really. I have just to keep an eye on it. (SS)

As we will see (see section 7.2), some teachers became aware of the shortcomings of transmission methods of teaching and recognised that 'telling' was not always an effective way of helping learners to understand concepts. Perhaps in reaction to this, they moved to an extreme position of 'not telling'. In contrast, the practice we sought to promote involved teachers developing a collaborative relationship with learners; at times they would allow learners to think and reason without interruption, while at others they would intervene to help learners modify their own thinking.

These three differing interpretations may also explain why at least six of the teachers believed that they were already using many principles from the approaches before the project began. This meant that, while they were happy to use new materials and resources, they felt that they had little to learn from them.

> What I've found is actually a lot of these techniques is stuff that's been used a long time in teaching ESOL [...]. These are not new ideas, they've been going around for years. [...] In terms of how much has this improved my teaching, I can't honestly say it has a lot, [but] it's nice, really nice to have some ideas that you can add to and supplement to. (ZZ)

Further difficulties arose because of alternative interpretations of terminology used in the project. Examples are:

(a) 'Discussion' and 'Talk'
Sometimes teachers claimed that learners were having discussions even when the teacher (or one learner) was taking the lead, dominating, showing, or telling other learners how to think. This contrasts with our own view that discussion is reciprocal in nature and involves shared reasoning.

(b) 'Working collaboratively' and 'Group work'
In a similar way, teachers sometimes claimed that learners were working collaboratively, when in fact they were just sitting in groups and working independently. Teachers did not always recognise a clear difference between working *in* a group and working *as* a group, and while learners might be seen to be cooperating and enjoying themselves, they were not necessarily collaborating in the sense of working jointly and reciprocally to solve problems. Real collaborative work involves 'exploratory talk' where decisions are challenged and/or justified, and alternative ideas are proffered and built upon.

(c) 'Misconceptions' and 'Mistakes'
Some teachers used these terms synonymously. The difference between a misconception and a mistake is that a misconception is always based on reasoning; misconceptions are often the result of over-generalising from a specific context. An example is when someone generalises from working with whole numbers, that 'to multiply a number by 10 you always add a zero'. Although this rule works in the domain of natural numbers, it does not when this domain is enlarged to include decimals, for example. Misconceptions are tacit in nature and difficult to observe in the course of discussion. They only become apparent when a consistent pattern of responses is observed, or when learners explicitly describe their ways of thinking.

Mistakes often occur when a learner loses concentration, is distracted or is hurried. They also occur when the learner forgets or misremembers a rule or procedure. Mistakes in arithmetic may or may not have consistent reasoning behind them. When they do, and patterns of incorrect answers are evident, then this may be symptomatic of an underlying misconception.

Some teachers tended to attribute all mistakes to misconceptions. Others did not always appreciate that the teaching resources were designed to expose misconceptions. In the early days of the project, one teacher believed that the deliberate mistakes in the resources were due to oversight on the part of the task designer.

6.2 Teachers' expectations of learners

A few teachers had low expectations of their learners and a 'protective' attitude towards them. As we have seen, many of these learners already regarded themselves as failures in mathematics and teachers, understandably, wanted to avoid further reinforcement of a poor self-image. Teachers were therefore reluctant to give learners activities that they perceived might be too demanding and intervened at the first signs of difficulty in order to 'sort them out'.

When teachers 'held back' support until after learners had been allowed opportunities to think for themselves, they reported a considerable 'feeling of achievement':

➜ Initially I could sense their [the learners'] frustration with their inability to understand the concept and I thought I was pushing them too far. However, after some support and guidance and discussion, both in pairs and as a group, they began to work it out. This gave us a tremendous feeling of achievement, and it was a watershed moment. (LL)

This teacher did not expect learners to gain conceptual understanding on their own. He recognised that learners needed 'support and guidance', but he also saw the need to allow them time to think for themselves *before* intervening. When he did intervene, he *collaborated with his learners* to resolve difficulties. The achievement was shared, not imposed.

One teacher appeared to believe that his learners were unable to discuss mathematics at all, and this became a self-fulfilling prophecy as he therefore rarely gave them opportunities to do so.

➜ These students have been doing the same thing since they were very young. They were doing 'Time' when they were five years old and they are still doing 'Time' now – they still haven't grasped it. If they haven't the ability to grasp 'Time' then they haven't got the ability to have mature mathematical discussions. (NN)

6.3 Learners' expectations of teachers and the project

Learners come to mathematics sessions with clear expectations of the teacher, the mathematics and the ways in which they will be expected to learn. Many had previously measured their success in mathematics by worksheets covered or ticks obtained, rather than by developing understanding. Collaborative approaches to learning conflicted with their previous experiences and they found it difficult to adjust. The following quote (from the teacher of a numeracy session for ESOL learners) illustrates how many learners saw mathematics as a subject to be learned through individual practice rather than collaborative discussion:

➜ I think a lot of my students find the approaches really, really alien; most of my students have just recently come to this country [...] they're used to sitting in big room, teacher at the front desk: 'This is what we do, copy it all down.' [...] They're very 'Give me a worksheet' really quite seriously, not just 'I think I'd like to do a worksheet' but like 'Why are you not giving us any? This is not proper. What am I learning?' I think people are feeling that quite strongly that they're not learning anything.

These learners do feel that the teacher is only there to give a method; in fact the teacher is not doing the job for which they are paid for if they do not do this. Maths classes are viewed not as places for talking; they are only places for listening, writing and pondering on your own. (NN)

Discussion-based approaches have less tangible outcomes than traditional practice-based approaches. Even when tangible outcomes exist, these are generated by groups and learners cannot always keep individual records of them:

➜ My students like to have work to go in their folders – at times using the approaches this wasn't always possible. I photocopied the work from posters etc. to put in folders – students still didn't feel that they had learnt anything without the evidence. (AA)

Many learners wanted *physical evidence* to show they had been working, and they gained a sense of security from 'capturing' information in written form[17]. It is almost as though productivity has displaced understanding as the primary goal for learning. We even observed instances (e.g., AA) where learners did not recognise mental calculations as doing 'work':

Jo: Are we doing work after break?

AA: Isn't this work?

Jo: It's just doing numbers ...

Some learners expected to be 'spoon-fed' information and became irritated when teachers asked them to discuss something with their peers. They could not understand why the teacher would not simply tell them the answer or show them the method. One learner discussed this issue with the observer during the first visit to CC's class:

Emma: I was really upset at the end of last lesson. ... I didn't want to come to maths anymore. ... CC didn't help me. He just kept saying 'Simona will explain' ...but I didn't understand Simona ... [unclear] a different world of maths ... I prefer it like the way it was [...].

Observer: How would it work before?

Emma: CC would explain and if you didn't get it he would come and show you again how to do it. One-to-one.

As we have already stated, a few teachers initially appeared to (mis)interpret the approaches we were advocating as essentially 'learning by discovery'. This was understandably criticised most strongly, both by teachers and by learners:

➜ I think a lot of people have the problem that they have nowhere to go. When they don't understand it, they look at it and just think – No! And when they do ask [...] they are told to go away and find it out for themselves, they just get ... the pox with it, basically. (AB)

The difficulty of the approaches we were advocating was knowing when to withdraw support and when to intervene and offer it. These are delicate decisions – if scaffolded support is withdrawn too quickly, the learners flounder, yet if it is never withdrawn they will remain unable to resolve issues without support.

Some teachers told us that, although 'holding back' was difficult at first, learners eventually began to accept and adapt to new ways of working. One teacher wrote:

➜ I found it very difficult at first not to intervene when the students came across a problem. You were saying yourself, they're asking questions, 'Hey you're the teacher, you know this, you're supposed to tell us this!' [...] 'No, what do you think?' And to get them to continue the conversation, again, works well I think. Very pleased with it. And they don't do that now. Now they'll say, 'he's not going to tell us, we've to got work this out ourselves'. (CC)

This data underlines the importance of teachers explaining to learners the purpose of the project and the approaches. Like teachers, learners need to be made aware of the reasons for working in new ways. We considered some possible ways of doing this, and a handout for discussion with learners was produced as part of the professional development package.

17 It is possible that some learners wanted to keep work as 'evidence' and also use it for revision purposes.

6.4 Teachers' use of the activities

The extent to which each of the teachers used the materials is shown in Table 13.

Table 13: Teaching sessions trialled by teachers, both observed and unobserved

Session	Total
Ordering whole numbers	14
Fractions	11
Converting times	11
Using money	3
Adding two-digit numbers	5
Writing the date	3
Sorting	1
Understanding decimal place value	9
Comparing decimals	2
Multiplying and dividing by powers of 10	3
Interpreting multiplication and division	5
Exploring the effect of number operations	2
Choosing the correct operation to perform	5
Measuring everyday quantities	9
Interpreting and ordering fractions	7
Total	**90**

The teaching sessions[18] listed in the table above were used 90 times. While many teachers saw the sessions and resources as a 'bolt-on' to their normal activities, and used them rarely, others sought to incorporate the approaches and materials more fully into their normal way of working. Two teachers used the sessions on nine occasions, while one teacher did not use any of the *TTM* sessions, although they used resources from the Standards Unit pack, *Improving Learning in Mathematics*, instead. Overall, teachers used the *TTM* sessions on average four times, with approximately three of these being observed.

The most frequently used sessions contain straightforward materials on well-known areas of the curriculum. Some were also demonstrated at workshop events, so may have caught the imagination of the teachers, who had become familiar with them.

It is more difficult to conjecture why the least used sessions were unpopular. For example, in *Exploring the effects of number operations* some teachers may have been put off by the use of algebraic expressions, believing this to be too difficult for their learners. *Comparing decimals* requires learners to articulate a method for determining the order of decimal numbers. We have already seen that some teachers believed their learners were unable to complete such tasks.

The two least-used sessions involved activities involving 'Multiple representations' and 'Sometimes, Always, Never True', and none of the most frequently used ones did. Teachers may have felt they needed more time to become confident in these possibly new styles of discovering and addressing misconceptions in their learners. Sessions involving domino or hexagonal puzzles did not seem to put the teachers off in the same way, possibly because card-matching activities are relatively easy to understand and organise, or they had experience of using similar activities before.

6.5 Teachers' adoption of the principles

Table 14 indicates the percentage of teachers that recognised each principle as being salient, and the degree to which observers judged teachers to be using this principle effectively and consistently. The term 'effectively' means, in this sense, whether the teacher was using the principle in the way it is intended to be used. To take the example of the first principle (building on knowledge that learners bring to sessions), if a teacher only asked one or two questions at the beginning, and did not integrate and build on this knowledge during the session, they would not be judged as using the principle effectively. In the case of the term 'consistently', a teacher would need to be seen using the principle in the majority of the four to six observed sessions, rather than, say, on one occasion.

At the final project meeting, we invited teachers to identify the most important 'messages' that had arisen from the project, for them personally. They were given the list of eight principles and were

18 The final pack contains 30 teaching sessions. The above table shows 15 sessions because some were split into shorter sessions.

Table 14: Teachers' perceptions of the most important principles and observers' evaluations of how teachers used these

	A	B	C
Description of principle	Percentage of teachers judging this principle to be important at the end of the project (n = 17)	Percentage of teachers judged to be using this principle effectively (n = 17)	Percentage of teachers from the whole sample judged to be using this principle effectively (n = 24)
Builds on knowledge that learners bring to sessions	71	24	33
Exposes and discusses common misconceptions (with individuals and groups of learners)	82	18	17
Uses probing questioning to assess what learners know and how they think	65	59	54
Organises cooperative small group work	82	47	50
Emphasises methods rather than answers, where learners are encouraged to explain and articulate their reasoning	53	29	33
Uses rich and collaborative tasks	59	65	63
Creates connections between mathematical topics	65	41	33
Uses technology in appropriate ways	29	18	17

encouraged to add their own ideas and amplifications underneath. The results are shown in column A of Table 14. In columns B and C, we summarise the extent to which the observers considered teachers had incorporated each principle into their teaching both effectively and consistently. Column B shows the observational data but only refers to those 17 teachers who completed the post-questionnaires, while column C shows the observational evidence of the 24 teachers in the whole sample. As we can see, the observational evidence is broadly similar for the 17 teachers as for the total sample; the only noticeable difference is in 'creates connections'.

The ratings in column C are based on an independent researcher's interpretations of the observers' judgements that they made in their case studies about the individual teachers. Sometimes it was difficult for the researcher to make an accurate assessment about a particular principle. When this happened the narrative sheets were used to find the data that corroborated or contradicted the assertion made. There are two other caveats concerning this data: firstly, observers did not record everything the teacher said or did, and sometimes the fact that a principle does not appear in the observers' narrative does not necessarily mean that it was not being used; secondly, some principles are more visible and therefore easier to record than others. An example of this may be in observing a teacher discussing misconceptions, and a teacher organising the class into groups. Nevertheless, we still believe the observers' and researcher's interpretations are a useful indication of which principles the teachers found the easiest and most difficult to integrate into their practice.

Table 14 shows that the principles that teachers regarded as being most important were not the same as those that were used most effectively. Teachers highlighted 'organising cooperative small group work', 'exposing and discussing misconceptions' and 'building on prior knowledge' as being the most important principles but, according to the observers, the latter two were not used consistently and effectively.

Table 14 also reveals that teachers found 'using rich and collaborative tasks', 'asking probing questions to assess what learners know and how they think' and 'organising cooperative group work' the easiest to integrate into their practice.

The open responses underline the importance for these teachers of discussing misconceptions and cooperative small group work.

➜ The most important feature has been using cooperative small group work and also exposing and discussing misconceptions. The group work has helped to motivate and engage the learners and the misconceptions approach has helped me to target areas where learners require more support more effectively. (DD)

Another important feature for me has been that these approaches have encouraged my learners to discuss and talk about maths. I am sure they had an original expectation of the old transmission methods. (LL)

It has allowed me and my learners to explore in discussion the misconceptions, myths and barriers associated with numeracy. It helped identify why these have occurred and past experiences that have impacted their/our future learning. (AD)

I have been impressed at the way students will arrive at a solution in collaboration with peers. I now use this to a very large extent. (CC)

The mixed abilities being able to work together and the learners' relationships with each other developed quicker and stronger bonds made. (TT)

Others mentioned principles underlying the design and philosophy of the activities; the focus on conceptual understanding and challenging learners with more complex tasks:

➜ The approach which underpins the resources is most important. Many of the objectives and the thrust towards 'understanding and learning' (which I support and agree with) sit uncomfortably with a results-orientated culture. (GG)

Sometimes it's just a phrase or concept that can be empowering – 'rich collaborative task' – the concept of a task being rich, the idea of challenge rather than difficulty. (FF)

It seems likely that 'using rich and collaborative tasks' and 'organising cooperative group work' were directly facilitated by the design of the materials. 'Exposing and discussing misconceptions' was also encouraged by the materials but, as we have already pointed out, the tacit nature of misconceptions and the difficulty observers had in identifying examples of this category may mean that the 17% (of the 24 teachers) is an underestimate. Moreover, and again as we have already mentioned, teachers found some difficulty in distinguishing misconceptions from other causes of error and they also found it difficult to generate and manage discussions about them. This issue is discussed more fully below. We did not offer teachers many activities incorporating technology, and many had little or no access to computers, so it is hardly surprising that this aspect did not feature strongly.

The next section looks at how teachers interpreted and integrated each of the eight principles into their practice. The data are based on the whole sample.

Builds on knowledge learners bring to sessions

Eight of the 24 teachers were judged by observers to be using this principle consistently and effectively.

The teachers that used this strategy effectively began sessions by asking the class to describe what they already knew about a topic, following this up with further questions and challenges. Their aim was one of formative assessment – to treat the beginning of a session as a 'fact-finding' exercise in order to discover both 'firm ground' and also identify learning needs that needed to be followed up. JW[19], for example, began most sessions by asking learners to tell him what they already understood and developed these understandings throughout the rest of the session.

➜ I say 'Before we start tell me what you know'. I'll put that on the whiteboard and that is my starting point. So even if things are down that are not correct, it doesn't matter, we'll put them down and then discuss them. (JW, oral)

JW reported that, on occasions, he had even abandoned session plans when he discovered that learners already knew what he was planning to teach them.

In the following session, JW draws out examples of applications of the topic and also some key terminology:

➜ *JW:* You are used to this now! What I'd like you to do just quickly, is just write down a few things what decimals mean to you. What are decimals? What does it mean 'Decimal'? A lot of the time we use these terms in maths, but what does it mean to you? ... (Five seconds pause, they all start writing on mini-whiteboards) Can you think of two or three things? Anything at all. You know we use decimals in maths but where do we use decimals every day?

Maggie: Money.

JW: OK, so that was one, Money. *(Writes 'money' on whiteboard)*

Maggie: I put temperature down. I was thinking of point something. I don't know whether that is right.

JW: If that is what it means to you, that is important. So you have seen that because you work in the medical field, don't you. Because you've seen it on thermometers and things ... like 26.5 degrees. *(Writes temperature ... 26.5°)*
Alexia: Point for the value. You know like you are saying with the decimal you have got your money, value, money and things.

JW: Right. *(He writes down 'value' next to 'money')*

Alan: Place value. And then on the right track you have got units of ten.

JW: Yes, based on ten, yes. *(Writes down 'based on ten')*

JW: Place value. Can you expand on that a little bit Alan? What do you mean by place value? *(Writes 'Place value')*

Alan: Hundreds, tens and units... and then you need the decimal point of course.

JW: Ah, of course ... but it wasn't before was it?

Maggie: No.

JW: And what next ... anything after that at all?

Alan: More zeros.

Maggie: It would be units, tenths, hundredths, thousandths.

Alan: Tens, hundreds ...

JW: Tenths, it would be tenths, hundredths ... and ... *(Writes down these fractions)*

Maggie: Thousandths.

JW then challenged the class further to put a collection of decimals in order of size.

JW: I'm going to put some decimal numbers on the whiteboard and I want you to work individually and put them in order from smallest to largest and I want you to put down the thinking about how you did it. (He writes down the following

19 The next two sections are not anonymised. Clips of these sessions with John Warburton (JW) and Joy Hallsworth (JH) can be seen on the TTM DVD.

decimals: 0.75, 0.4, 0.375, 0.25, 0.125, 0.04, 0.8). If you are struggling it doesn't matter. We are going to try ... Stick your thoughts on the mini-whiteboard.

Alan produces the correct list, but mutters to himself, 'I wish I could explain it myself. Can I understand it?' Alexia writes 0.75, 0.25 at the top of her list. She stops. Maybe she really believes that 0.75 < 0.25 (perhaps because of the denominators with fractions), but maybe it is just a slip.

In the remainder of this session, JW used a card-sorting activity to link together multiple representations of these same decimals. This enabled learners to sort them correctly and explain their sorting using different images and methods.

Other teachers did not adopt this principle as easily as JW. Some asked learners a few questions at the beginning of the session, but did not appear to know how to use the responses constructively in the remainder of the session. This requires considerable pedagogical skill coupled with a sensitive flexibility.

Exposes and discusses common misconceptions

Only four of the 24 teachers were observed to be using this principle effectively and consistently.

The results from the questionnaire show that teachers were aware that this was a key element of the approaches. We also know from observation evidence that teachers referred to misunderstandings and misconceptions when the session notes drew attention to these. This proved, however, to be one of the most difficult principles to integrate into teachers' practice, and few were able to do this effectively. Moreover, as we have noted earlier, misconceptions were not always as visible to the observers as some of the other principles may have been, particularly when the teachers may have been discussing them with a group of learners away from the observer's gaze. When we came to analyse the data, we found that many of the incidences that teachers and observers first identified as examples of *misconceptions* were actually concerned with learners' *mistakes*. That is, they did not appear to be alternative forms of conceptual reasoning, rather they were examples of simple slips or unreasoned 'guesses'.

Sometimes, teachers would fail to notice possible misconceptions or misunderstandings when they did emerge. On other occasions, teachers would intervene quickly, suggesting the correct answer or method, without exploring a learner's reasoning or seeing whether a second learner could assist in the explanation or interpretation.

The extract below comes from a session aimed at developing an understanding of multiplication and division. Learners were given a series of generalisations on cards and were asked to say whether the statements were always, sometimes or never true. In the extract below, learners were discussing the common misconception that:

If you divide a number by 10, the answer will be less than the number.

Paul: It's like take away.

Oliver: You're dividing. If you divide a number by ten, so it's what goes into ten, the answer will be less than the number.

JH: Put some numbers in.

Oliver: What goes into ten? Put two. Two divided by ten.

Paul: What does that equal?

JH: What do you think that equals Oliver?

Oliver: Two.

JH: Just check that on your calculator.

Pat: I got two.

[*Oliver presses the + key by mistake. This is pointed out and he corrects this.*]

> *Paul:* Nought point two.
>
> *JH:* What are you expecting to get Oliver?
>
> *Oliver:* If you divide a number by ten you are dividing the number into ten. So if you are dividing twos into ten goes two ... no ... twos into ten goes five.
>
> *JH:* What you have written isn't twos into ten. If you wanted to divide two into ten how would you write it?

[*Oliver writes 2 ÷ 10 = 5 and reads it as 'Two into ten equals five'.*]

> *JH:* Do that on a calculator and see if you get 5.
>
> *Paul:* 0.2
>
> ...
>
> *Oliver:* So it's the higher number first. That's where I'm going wrong. I know that five twos are ten, I know that.

This extract illustrates the value of such statements in revealing conceptual difficulties[20]. Clearly, the issue at stake here is the whole concept of division, how it is read and represented and the significance of the order in which it is written. JH recognised the value of such a discussion and devoted much of her session to it. She did find it very difficult, however, to resist the temptation to 'take over' and explain everything to the learners before they had had a chance to think through the issue for themselves.

Teachers began to welcome evidence of possible misconceptions, but did not know how to respond to these when they did arise. For example, in a session on the addition of fractions, MM saw a learner writing 1/12+1/12+1/12+1/12 = 4/48) and exclaimed:

➜ I'm glad you said that as it's false ... that's a misconception, that it's 4/48.

MM then asked a series of leading questions until the learner said '4/12'. To the learner, these questions might have appeared quite unconnected to the problem; the teacher was doing the linking and reasoning while the learner was responding to verbal cues. The teacher did not ask further questions to check that the learner had understood the explanation. This type of behaviour was common. We found that many of the sample teachers need further professional development in uncovering and resolving misconceptions or misunderstandings through careful, probing questioning. Furthermore, we found that learners were rarely given time to discuss their own interpretations and methods without teacher intervention.

To summarise, the design of the activities generated many opportunities for intense discussion of misconceptions and errors, but teachers found it very difficult to know how these discussions should be managed and resolved. The temptation to 'take over' and explain 'the correct' viewpoint before learners had been given a chance to explore the ideas was irresistible for many. When this happened, learners could not relate what was being said to their own line of thinking and so reverted to passive behaviours. In the best implementations of this principle, learners were encouraged to explore the consequences of their own misconceptions, so that a vivid cognitive conflict/surprise would arise and lead to a realisation that new ideas needed to be accommodated.

Uses probing questioning to assess what learners know and how they think

Thirteen of the teachers were judged to be using this principle effectively and consistently.

This aspect of the approaches was discussed in workshops, and by the end of the project over half of the teachers were judged to be using probing questions consistently and effectively. The majority of the teachers were asking a broader range of question types, including those that were more diagnostic. They were also increasing 'wait times' after asking questions to allow learners more time for reflection.

Research and analysis of teachers' questioning (Bills et al., 2004; Black and Wiliam, 1998; Chanda et al., 2005; Mason

20 As we have noted in section 6.4, this was one of the least used tasks from the pack of materials.

and Watson, 1998) reveals that low-level closed, factual recall questions are used much more frequently than higher-level open questions that require mathematical reasoning. In our own observations we found several examples where teachers led learners through closed questions towards pre-determined answers through a process that seemed a mystery to learners. In the following example a learner seeks to answer the question: what is 20% of £100? The teacher has a broader aim; she wants the learner to become aware that 20% is equivalent to one-fifth and this in turn requires division by 5. She tries to lead the learner to an awareness of this by referring to analogous examples (25% and dividing by four; halving and dividing by two) through what seemed a convoluted series of questions. This ends up frustrating the learners:

→ *RR:* You're saying that's 20%. What is – I want to get the best way to word this – if I said what is 25% of a hundred, would you know?

No response.

RR: Would you know what 25% is as a fraction?

Ben: a quarter.

RR: What's a quarter of a hundred?

Ben: 25.

RR: Has that gone up by 25?

Ben: No.

Alice is doing some cancelling of a fractions calculation; she gets one-fifth.

RR: How would you say that?

Alice: A fifth.

RR: If I say half of 10 ...

Alice: 5

RR: What did you do, to get a half?

Alice leans back, despairing.

RR: I'm not trying to confuse you. Did you divide it by 2?

Alice: Oh yes.

RR goes back to a quarter.

RR: So that means 1 divided by 4. If I said to you what's a quarter of a 100, you'd divide it by 4?

Both agree.

RR: So to get a fifth, divide a hundred by 5.

Over the course of the project, however, teachers began to reduce the frequency of lower-order questions and use a wider variety of higher-order 'open' questions (see Table 15).

We found frequent examples of teachers using questioning to challenge learners' perceptions and answers, as well as to encourage thought about the activity. For example, in an ordering activity some learners' had put 58, others 85

→ *AA:* Which is lowest?

Paul: 58

AA: Why?

Paul: Because it's lower.

AA: Yes, but why?

Sonia: Because that's 8 tens and 5 units

Paul: And that's 5 tens and 8 units

Teachers still found it very difficult to pause after asking questions so that learners had time to reflect and respond. There were several reasons for this: some seemed afraid of placing learners in a position of discomfort or uncertainty, others appeared concerned that the session would go too slowly and learners would lose interest, and also learners would sometimes put the teacher under pressure to tell them

Table 15: Types of question found in classroom transcripts

'Lower order' closed questions

Type of question	Example
Asking directly	What is 6 x 4?
Reassuring	Are you OK with that?
Checking (prior knowledge)	Do you know what the numerator means?
Clarifying	Is it one AND a half or one half?
Reminding	Is it always, or sometimes?
Prompting and guiding	Have you thought of using half a square?

'Higher order' open questions

Type of question	Example
Creating examples	Can you show me an example of a square number?
Evaluating and correcting	What is wrong with this statement, 'When you multiply by 10 you add a nought', and how can you correct it?
Comparing and organising	What is the same and what is different about these objects? Can you explain why 1/3 and 2/6 are the same?
Modifying and changing	How can you change this shape to give it a line of symmetry?
Generalising and conjecturing	Is this statement, 'When you multiply by 10 you add a nought', always, sometimes or never true?
Explaining and justifying	Can you give me a reason why a square is also a rectangle?
Describing methods and reasons	How did you work that out? Can you explain why you think that?

the answer. The temptation therefore is to allow few pauses and to answer one's own question:

➜ So who does know what is perimeter and area? [pause for fraction of a second] I think we're going to have to explain it. The perimeter is round the outside, look at the word 'rim' in the word. And the area is the space within the shape. (AD)

Teachers reported that they had to make a conscious effort to change this behaviour:

➜ I'm very, wanting to dive in when I see them struggling, so I have really concentrated on trying not to tell them the answers too quickly. (YY, oral)

LL was one teacher who challenged his own behaviour and began to allow learners much more time to think. He also constantly demanded explanations through his repeated use of the question 'why?':

➜ LL: What is the largest number that is 700 when rounded to the nearest 100?

LL waits and as learners slowly respond with 750, 699 [2 people], 754, 749, LL lists these on the board without comment.

LL: Which should be discarded?

Learner: 699

LL: Why?

Learner: 700s are bigger. Has to be in 700s

LL: Discount another.

Learner: 754

LL: Why?

Learner: It's higher. It goes to 800

LL: Are those two OK?

Learners disagree

LL: Some say 'yes' – why?

Learner: 750 is more than 749. Therefore, it's 750 that goes to the nearest 100.

LL: What are the rules of rounding?

Learner: So 750 would go up to 800.

LL: So what is the largest number that rounds to 700?

Learners: 749, 749

All agree.

There was evidence that, when teachers persisted in this practice, learners became more independent:

→ Learners initially expected me to tell them; now they are prepared to work it out themselves. (CC)

Organises cooperative small group work
Twelve of the teachers were evaluated to be using this principle effectively and consistently.

One half of the teachers were observed using group work effectively, and only one teacher appeared to find it genuinely difficult to organise her class in this way. For some, group work was a significant change in their existing practice; previously they had attempted to meet individual learning needs by asking learners to work on separate activities – they *differentiated by task*. Almost every teacher was pleased with the response of learners when group work was introduced:

→ It has been a bit of a success story for me oddly enough because I've suddenly discovered, they [the learners] love group work! They just love working together and they actually bounce off one another, like you were saying, if one of them finds a method that they can all relate to then it takes the pressure off me in a way because I can stand there till I'm blue in the face trying to explain it and I'll often follow the session plans, as they were laid out, try to stick to the way it is and they've not really understood it, but then they've come up with an idea themselves, sometimes it starts off as one idea but I let them develop it a bit just to see how far it goes and then I chip in and sort of redirect them. I think that's been the best part of it, just seeing them gel together as a group and I think if it wasn't for this project I don't think it would have happened. (DD, oral)

However, it should be noted that observers reported that some learners continued to work as individuals, even when they were asked to sit in groups. Teachers did not always explain to learners why they were adopting such practices.

Teachers adopted different methods for organising groups. Some allowed learners to organise themselves into friendship groups, others structured groups by ability/competence, sometimes matching learners in equal partnership, and sometimes deliberately asking learners of different abilities to work together.

One learner, Sally, (in RR's class) told the observer how much she valued working in 'mixed level' groups and found that it forced her to look at things from a different perspective:

→ Everybody knows something that somebody else doesn't know.

I've got to look at it from your point of view, because I know what you're saying but I don't understand it. It was really complicated that time. So I had to learn. (Sally, a learner)

There was evidence from only four classes of some learners declining to work with other members of the group, either for reasons of gender or because they preferred to work individually and, even here, this was not a regular occurrence. Occasionally, however, learners told observers that they felt they were being 'held back' when they were asked to work either with learners working at lower levels, or with learners with language difficulties:

> *Chris:* I found it hard working with Rani because she doesn't talk a lot of English ... so I know what I'm doing and I'm explaining and like Lisa would probably understand but because English is her (Rani's) second language she can't say anything back. ... I felt a bit miffed because I felt it was holding *me* back ... it's double your work isn't it?... Last week I didn't learn anything because I was working with Rani.

Emphasises methods rather than answers where learners are encouraged to explain and articulate their reasoning
Eight of the teachers were seen using this principle effectively and consistently.

Eight of the teachers were observed regularly encouraging learners to justify their decisions by offering reasons, rather than giving answers, suggesting that teachers found this a difficult aspect to incorporate into their practice. The observation reports contain many examples of teachers asking learners to explain their thinking. For example:

➜ I've not said you are right or wrong; it's about talking – If you can justify your feelings. (RR)

For most teachers, this was a considerable change in their practice, particularly remembering that many learners come to numeracy classes with the expectation of working through an exercise to get a correct answer, rather than working on a process or idea. Sometimes, the temptation to provide learners with short cuts proved irresistible, even when this may have actually reinforced a misconception. The quotation below comes from an observer's case study.

➜ The explanations given by the tutor were always designed to help understandings but there were occasions when he would emphasise quick ways of getting answers. The clearest example of this was the 'add a nought' model for multiplying by ten. He was aware of when this did not work but in some cases he felt that the learners needed a quick way of doing something. (about FF)

Emphasising methods is linked to the skilful use of questioning. This next passage of data comes from another observer's case study, with an extract from their observation notes.

➜ [The teacher] says that he has enjoyed teaching the sessions from the Resource box and finds that the activities generate lots of questions from the learners which he is enjoying fielding – a big difference from the way he used to work. In the past [the teacher] would have sat down with the learner and given an explanation with an example when asked 'What does it mean?' but in the extract below he handles it differently:

> *Learner 1:* What does it mean?
>
> *CC:* You said before that there is a 17% increase....
>
> *Learner 2:* But I worked it out and now I don't think so. ...
>
> *CC:* Why didn't it work?
>
> *Learner 2:* It won't bring it down by the same amount because the start number is bigger... *[Both learners start work on reassessing the problem. Teacher walks away.]*

Uses rich, collaborative tasks
Fifteen of the teachers were observed using this principle effectively and consistently.

By 'rich' tasks, we mean tasks that are accessible, yet admit further challenges; tasks which invite learners to make decisions; which involve learners in speculating, hypothesising, explaining, proving, reflecting and interpreting; which promote discussion and questioning; which encourage originality and invention; and which have an element of surprise and are enjoyable (Ahmed, 1987). In designing the activities, for this project, we sought to develop activities which would naturally encourage such behaviours and it is therefore hardly surprising that we were able to observe nearly two-thirds of the teachers using this principle consistently and effectively.

At the beginning of the project, we became aware that not all teachers were able to discriminate between activities that were intended to develop skills for fluency and those which were intended to develop conceptual discussion. This was particularly true when the superficial appearance of the two forms of activity were similar (e.g., when both involved matching cards). For example, we would not consider an activity that involves matching cards showing multiplication questions (6 x 3 =; 5 x 4 = ...) to cards showing answers (18, 20, ...) to be 'rich' in the above sense, as these would not encourage the discussion of concepts. An example of a rich activity is where cards showing mixed multiplications and divisions (2 x 6 =; 6 x 2 =; 6 ÷ 2 =; 2 ÷ 6 =) are matched to cards showing diagrams of these operations and corresponding problems for them, followed by a discussion of the alternative meanings of the concepts. We therefore explicitly addressed this issue in one of the meetings to encourage teachers to look more critically at what would comprise a 'rich' collaborative task.

When the task chosen was appropriate for the group, the learners clearly enjoyed working collaboratively:

> *Catherine:* I'd say we all worked.
>
> *Brent:* You all do, don't you? There's nobody carried by anybody else. We're all contributing and learning from each other really.
>
> *Catherine:* Yeah, if one of us don't know, then one of us will explain it to them. And if we don't get it that way we explain it another way.
>
> *Brent:* I think it worked quite well, because we took it in turns really.
>
> *Catherine:* Both had to agree. (RR)

However, there were times in the project when some learners were part of a group, but were content to remain 'passengers' and not become too involved in making decisions. Others, in contrast, enjoyed working together, but were concerned about the time taken up by discussion.

> *Observer:* How did this lesson compare with what you usually do in maths?
>
> *Rosemary:* We worked more as a group now. Before we worked more as an individual.
>
> *Jane:* It's better working as a group
>
> *Sharon:* One person should not do it all.
>
> *Observer:* So how did you organise yourselves?
>
> *Sharon:* We took it in turns to work out the cards.
>
> *Rosemary:* The cards make you focus.
>
> *Sharon:* We weren't sure about how to work out areas but we discussed it. ...
>
> *Jane:* We did try mostly ...
>
> *Observer:* What do you think about how you were working?
>
> *Sharon:* I don't want to do this too often *[Other two nod].*
>
> *Observer:* What working as a group or...?
>
> *Jane:* It's too much ...
>
> *Rosemary:* ... Want to feel you will have moved onto another topic ...

> *Sharon:* ... Takes a long time.

(referring to a session in TT's class)

It is our view that learners need just as much induction, guidance and/or training about how to collaborate together as teachers do. Learning to work together, to listen and share ideas and responsibilities, is a practice that most learners are unfamiliar with, at least in mathematics sessions.

Creates connections between mathematics topics
Eight of the teachers were seen using this principle effectively and consistently.

The ANCC curriculum specification compartmentalises mathematics into discrete topics, and some teachers plan the order of their teaching according to this conceptual layout. Only eight of 24 teachers were, in our judgement, making connections between topics effectively and consistently.

In *TTM,* we found that some teachers were unwilling to tackle topics that had been placed at a higher level in the curriculum than they were currently working at; this resulted in an inflexible approach to teaching. Some teachers, however, had used this project to free them from the core curriculum specification:

- I used to teach, until fairly recently, from topic to topic, I've got the core curriculum here, I know I've got to cover these topics and I'll go through them. I don't do that any more. [I now] combine all these elements together and I think it's wonderful. (CC)

Some of the teaching sessions were written with the deliberate intention of causing learners to make connections between different mathematical areas and/or representations (see section 3.2.2). Some teachers not only made connections to other mathematical topics but also to work covered in previous sessions, for example, 'You remember what we did on Thursday?' They also made reference and connections both to other areas of the learners' course (e.g., a cookery or financial component), and also to the world outside the classroom, to make the mathematics more relevant. The data below comes from an observer's case study, and shows how the teacher uses figures from a learner's life instead of from the activity, which enables them to access their knowledge about percentages. (Teacher is RR)

RR is working with Brian, Catherine and Alison.

> *RR:* If it goes from £30 to £60 how much is it?
>
> *Alison:* 50%. [...]

RR stops and thinks.

> *RR:* Don't answer if you don't want to, but how much do *you* earn an hour?'
>
> *Catherine:* £4.34.
>
> *Helen:* If you got a raise of 100% what would that be?
>
> *Brian:* £8.68.

The learner's initial suggestion of 50% is a common error. The personal relevance of the teacher's questions seems to be a turning point for this small group.

However, while teachers recognised making connections as one of the key features of the resources provided, we have little evidence of them making connections beyond those prompted by the use of these materials.

Uses technology in appropriate ways
Only four of the teachers were judged to be using this principle effectively and consistently.

We have little evidence of teachers using technology during the project, whether these were interactive whiteboards, computers or calculators. In total we only observed four teachers using technology for exposition (using PowerPoint and downloaded software) or exploration (using spreadsheets and the internet). In some cases, teachers had no access to IT at the centre where they worked. For other teachers, technology was either given a low priority or, where learners were encouraged to access information using computers, this was done on an individual basis and the information did not form part of the group activities. Although calculators were seen, they were usually used to check work rather than to facilitate or formulate concepts.

7 The impact on teachers and learners

This section considers how individual teachers' beliefs and practices have evolved over the course of the project. The final section presents the impact of the project on the learners, in terms of how they responded to the approaches and the materials.

7.1 Teachers' evolving practices

The teachers' point of view; from the questionnaires

Teachers were asked to rate the relative frequency of 28 teaching behaviours both before and at the end of the project, using a five-point scale. The results are shown in Table 16. Fourteen of these behaviours were categorised as learner-centred and 14 as teacher-centred. (Only 17 of the 24 teachers were present at both days and completed both questionnaires. Responses to items when they were present, however, suggest that they are representative of the whole sample.)

On the pre-questionnaire, teachers rated themselves as generally learner-centred in orientation. This was not only more learner-centred than their own learners rated them (as we have already discussed in section 5.6) but also much more so than earlier research into FE teachers who were teaching GCSE retake courses (Swan, 2006). The teachers in our project saw themselves as enabling learners to work collaboratively, discussing ideas and mistakes, and addressing individual needs. They did not tend to restrict themselves to single methods, hurry learners or closely follow textbooks or worksheets. Interestingly, however, they did claim to often simplify work by starting with easy questions and working up to harder ones. These responses may be to some extent a reflection of the fact that these teachers had smaller classes than the GCSE teachers and were also used to dealing with more severe learning difficulties (including reading difficulties, so textbooks would be less appropriate). Their response (which may be perceived as caring) was to simplify the demands made on learners.

On the post-questionnaire teachers reported substantial changes to their practices that made them considerably more learner-centred in their approaches. In fact every practice occurring more than half the time may be considered to be learner-centred, with the exception of one; they still had a tendency to start with the easy questions, but not nearly so much as before.

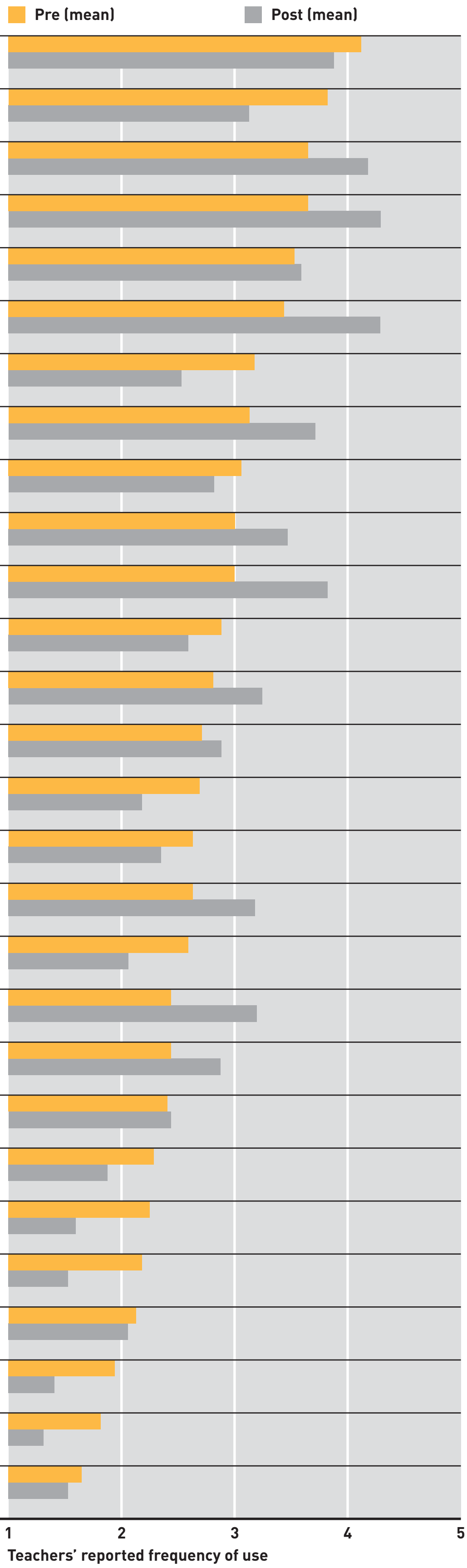
From pre to post, the greatest increases in emphasis may all be described as *learner-centred* behaviours:

- L16 Learners work collaboratively in pairs or small groups (+0.86)
- L28 I jump between topics as the need arises (+0.82)
- L6 I encourage learners to work more slowly (+0.76)
- L15 Learners learn through discussing their ideas (+0.65)
- L7 Learners compare different methods for doing questions (+0.58)
- L17 Learners invent their own methods (+0.55)
- L27 I encourage learners to make and discuss mistakes (+0.53)

Conversely, the greatest decreases reported are all in teacher-centred behaviours:

Table 16: **Teachers' self-reported changes in behaviour (n = 17)**

L11	I draw links between topics and move back and forth between topics
T4	Learners start with easy questions and work up to harder questions
L27	I encourage learners to make and discuss mistakes
L15	Learners learn through discussing their ideas
L23	I teach each learner differently according to individual needs
L16	Learners work collaboratively in pairs or small groups
T26	I know exactly what maths the lesson will contain
L7	Learners compare different methods for doing questions
T9	I teach the whole class at once
L12	I am surprised by the ideas that come up in a lesson
L28	I jump between topics as the need arises
T1	Learners learn through doing exercises
L18	Learners work on substantial tasks that can be worked on at different levels
T2	Learners work on their own, consulting a neighbour from time to time
T25	I tend to teach each topic separately
T19	I tell learners which questions to tackle
L17	Learners invent their own methods
T13	I avoid learners making mistakes by explaining things carefully first
L6	I encourage learners to work more slowly
L5	Learners choose which questions they tackle
L22	I find out which parts learners already understand and don't teach those parts
T3	Learners use only the methods I teach them
T10	I try to cover everything in a topic
T8	I teach each topic from the beginnning, assuming they know nothing
L24	I only cover important ideas in a topic
T14	I tend to follow the textbook or worksheets closely
T20	I find myself encouraging learners to work more quickly
T21	I only go through one method for doing each question



Teachers were asked to rate 28 practices on a 5-point scale: 1 = almost never, 2 = occasionally, 3 = half the time, 4 = most of the time, 5 = almost always.
L = Learner centred; T = Teacher centred. Mean ratings for each statement are shown for the 17 teachers that provided pre-post questionnaire data.

T4 Learners start with easy questions and work up to harder questions (-0.70)

T10 I try to cover everything in a topic (-0.65)

T26 I know exactly what maths the lesson will contain (-0.65)

T8 I teach each topic from the beginning, assuming they know nothing (-0.65)

T13 I avoid learners making mistakes by explaining things carefully first (-0.53)

T14 I tend to follow the textbook or worksheets closely (-0.53)

T25 I tend to teach each topic separately (-0.51)

T20 I find myself encouraging learners to work more quickly (-0.51)

Changes by teacher

Table 17 below shows the changes from teacher-centred behaviours towards learner-centred behaviours, using a scale from 0 (extreme learner-centred) to 100 (extreme teacher-centred), for each teacher who completed pre- and post-questionnaires at the final project meeting in June[21]. The table shows the teachers ranked in order according to the degree to which they claimed to have changed in their practices from pre-to post questionnaire. Teachers at the top of the list claim to have made significant changes from teacher-centred to learner-centred behaviours. Teachers at the bottom claim to have made no significant change in their behaviour (but it is noticeable that these teachers already claimed to be working in learner-centred ways).

In order to check the reliability of these self-reported statements, the reports were checked for consistency by an independent researcher using the qualitative descriptive reports obtained from oral interviews and other written accounts, completed by some of the teachers. These are still, therefore, analyses of self-reports by teachers. The outcomes of this are shown in column 5 of the table. The figures show whether the researcher feels that these accounts show that the teacher has become more learner-centred: not at all (0); to a limited extent (1); to some extent (2); to a considerable extent (3); to a great extent (4).

A final independent analysis was carried out using the classroom observation reports. These do not reflect the views of the teachers and are thus a check on the validity of these self-reports. They are shown in column 6 of the table. Change was judged by researchers according to how well teachers had integrated the principles of the project into their general practice on a consistent and effective basis, with researchers using the same ratings as for column 5. When the researcher found it difficult to make an accurate assessment about a particular principle, the session observation sheets were used to find the data that corroborated or contradicted the assertion.

It may be seen that, for ten of the teachers, these reports are broadly in agreement. For four, the teachers claim to have changed in approach more than the observers described[22]; for the remaining three teachers, the observers would claim that they changed more than the teachers themselves claimed. In both these cases, it should be noted that the researchers' observation differs from the teachers' written and/or oral accounts by only one point (on the scale of 0–4).

We would like to point out that each source of data is based on general impressions and none may be considered completely reliable. There are several other difficulties that cloud interpretation:

- Teachers may have reported general approaches to all their teaching while the observers saw their practices while they were using project activities. Thus, although we cannot be sure of this, there may have been a distinction between *teachers' general practice* and *teachers' project practice.*

21 Briefly, only the 25 statements that gave the most reliable scale were used. Ratings for learner-centred statements were reverse scored and added to the ratings for teacher-centred statements, giving a total score from 25 to 125. Finally, 25 was subtracted to arrive at the range 0 to 100.

22 Shown by T>R in Table 17

Table 17: Reported and observed changes in practice, by teacher and observer

Teacher	Pre	Post	Change	Other written and oral reports	Observers' observations	Reasonable agreement?
CC	51	21	-30	2	3	Yes
DD	49	24	-25	2	1	T>R
LL	53	31	-22	2	4	Yes
BB	54	34	-20	1	1	T>R
TT	46	28	-18	n/a	2	Yes
GG	66	49	-17	2	1	T>R
AB	28	13	-15	0	2	Yes
XX	53	41	-12	1	0	T>R
JJ	39	31	-8	1	1	Yes
AD	50	43	-7	1	1	Yes
FF	61	54	-7	0	0	Yes
MM	41	35	-6	1	1	Yes
YY	35	34	-1	1	2	R>T
ZZ	14	15	1	0	0	Yes
AA	33	34	1	1	2	R>T
PP	28	31	3	1	0	Yes
HH	29	33	4	2	2	R>T

Pre, post and change scores are on a scale from 0 to 100. 100 means giving a rating of 5 (almost always) to every teacher-centred statement, and 1 (almost never) to every learner-centred statement. Other written and oral report data and researcher observation data are recorded using a 5-point scale (0 = no change; 4 = great change).

- Teachers may have been prone to exaggerate some of the changes in the post questionnaire, because they had a fuller understanding of what the project was about, and were 'second-guessing' what they thought researchers wanted to hear.
- Teachers may have conflated beliefs and practices in their oral reports. Thus, when interviewed, AB was adamant that she had not changed in her teaching, while other data (including reports from her learners) confirmed that she had modified her approaches considerably. She may have been claiming that her *beliefs* about teaching had not changed.
- In some cases, observers may have confused states with changes. Thus, when observers saw teachers adopting the principles well, they may have considered this as a change, when the teacher was already using learner-centred approaches to begin with. (This may account for some of the R>T rows in the table.)

Researchers reported that seven teachers intended to implement more of the approaches than they eventually did, and were generally using these ineffectively. This may have been partly due to a lack of subject-specific pedagogical knowledge. In some cases there were additional external factors, e.g., lack of support from senior management, or learners unwilling to discuss. These teachers were supportive towards the project in general and appeared open-minded but appeared to have only a limited understanding of the underlying principles[23]. They clearly needed more support and professional development.

23 This may have been for a number of reasons, including, possibly, an insufficient amount of time reflecting on the principles during project workshops/training days.

The design of the activities themselves clearly 'forced' teachers to stop using worksheets, to organise their classes into groups and encourage collaborative learning. This was a major change for teachers:

➜ I now place more value on mathematical discussion rather than the main purpose of the lesson being: teacher explains method, learner learns and practises the method, learner hopefully gets the answers right, end of lesson. (JJ)

Before, I thought, the best way of them to learn what to do loads of, was worksheets 'cause that's the impression they gave me when I first started teaching numeracy, which hasn't been very long, it's only been for three years. So this totally turned that belief around because in fact they learn more, they understand more by doing this and discussing it amongst one another, than I could ever do spending years and years of doing worksheets with them. (DD)

But many teachers did more than this: they began to stand back, observe and listen to learners working together, gave the learners more choices, and began to encourage learners to ask each other questions, argue together and reach their own solutions without giving them the answers.

➜ We all whenever we're teaching we always encourage learners to ask questions, but never to the extent, or I've never done it to the extent that we do with this. (CC, oral)

I now feel more strongly that the most important thing in maths is not just to get the answer right but to look for ways of getting to the answer. (JJ, oral)

One of the most difficult things teachers found was knowing what to do with the information they uncovered from learners. It was all very well asking learners what they knew about a particular mathematical area or concept; it was knowing what to do with this information, how to build on it and be flexible enough to incorporate the findings into the coming session, which may have already been pre-planned. Similarly, although many teachers found themselves being able to stand back more and not interrupt learners' conversations, it was much harder to know when to step in and how to guide them by using further, reflective discussion, to 'move them on'.

Teachers and observers were in broad agreement that 18 of the 24 teachers had changed their practice towards becoming more learner-centred and seven of these had introduced changes of a substantive and wide-ranging nature. (It should be noted that some teachers were already working in learner-centred ways and one would not expect their practices to have been so noticeably affected.)

The main way that teachers' practice moved was in a change from passive to more active learning: there was greater learner involvement, discussion and decision making. This was, after all, what the materials had been designed for. Teachers also spoke of the approaches resulting in increased learner interest and motivation, and an improvement in relations between learner-learner and learner-teacher (see section 7.3).

➜ One of the reasons I think this project is absolutely great cos it's taught me a huge amount on how to generate much more interest, much more motivation amongst the learners than I was able to do myself. (CC)

In Table 18 we summarise the changes that teachers and researchers reported were occurring with a greater frequency as the project progressed.

7.2 Teachers' changing beliefs

Teachers were asked at the beginning and end of the project: What are your current views on mathematics, learning and teaching? They were asked to give each of nine statements a percentage weighting, so that the sum of the three percentages in each section totalled 100% (Table 19). They were also invited to add their own personal statements. The first statement in each

Table 18: Observable changes in teachers' practices

Type of practice	Increasing emphasis on:
Organisation	• Learners working in groups • Teachers integrating topics and making connections between mathematical areas such as fractions, decimals and percentages • Teachers building on what learners know in group or whole class discussions • Teachers structuring learner-learner discussions • A reduction in the number of worksheets
Atmosphere, ethos	• Learners feeling that it was acceptable to make, and admit to making, mistakes
Approaches	• Teachers standing back without intervening, observing and listening • Asking more questions • Using open questions • Emphasising greater understanding rather than getting the right answer • Making greater use of peer support
Learners' practices (behaviours)	• Being given more choices, more independence • Talking together and asking questions to each other • Justifying decisions to each other and to the teacher • Willingness to 'have a go'.

group of three corresponds to a *transmission* orientation, the second corresponds to a *discovery* orientation and the third corresponds to a *connectionist* orientation. The mean of the three transmission statements was calculated and this was deemed an overall transmission weighting for that teacher. The other two weightings were treated similarly. The results on the pre and post questionnaires is given in Table 20. The results show that teachers reported a significant movement away from a transmission orientation and a significant increase in the connectionist orientation ($p<0.05$; $p<0.01$ respectively).

Table 20 shows that, at the beginning of the project, five of the 17 teachers held a predominantly transmission orientation, six held a predominantly discovery orientation, three held a connectionist orientation and three held an equal mixture of more than one orientation. By the end of the project, no teachers held a transmission orientation, 11 teachers had a predominantly connectionist orientation, and a further two had an equal measure of connectionist and discovery. Four held a predominantly discovery orientation.

The data broadly supports the findings of Swan (2006) who found that teachers' beliefs tend to evolve from transmission to discovery, transmission to connectionist and discovery to connectionist. What seems to happen is that some teachers recognise the limitations of transmission methods and move from a 'telling' role into a 'not telling' role. They recognise that in the past they have not allowed learners to think for themselves and they react against this. They therefore stand back and begin to adopt a passive 'facilitating' role rather than a pro-active 'challenging' role. The teachers who move beyond this discovery orientation towards a connectionist one, begin to

Table 19: Beliefs about mathematics, teaching and learning

Mathematics is:	
Transmission:	a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to learners.
Discovery:	a creative subject in which the teacher should take a facilitating role, allowing learners to create their own concepts and methods.
Connectionist:	an interconnected body of ideas which the teacher and the learner create together through discussion.
Learning is:	
Transmission:	an individual activity based on watching, listening and imitating until fluency is attained.
Discovery:	an individual activity based on practical exploration and reflection.
Connectionist:	an interpersonal activity in which learners are challenged and arrive at understanding through discussion.
Teaching is:	
Transmission:	structuring a linear curriculum for the learners; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners fail to 'grasp' what is taught.
Discovery:	assessing when a learner is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.
Connectionist:	a non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.

interact collaboratively with learners, stimulating thinking and reasoning, without 'taking over'; and begin to make appropriate challenges and interventions after learners have had time to think for themselves.

It might be assumed that, in order to change a teacher's practice in any profound way, one has to first change his or her beliefs. Indeed, this forms the model of many pre-service and in-service professional development courses, where ideas and theories are introduced and practical implementation follows. However, as Swan (2006) and this project suggest, the relationship between practices and beliefs is more complex than this. In this project, almost all teachers stated at the outset that they felt constrained to teach in ways that were far from ideal. Reasons given varied: their own lack of subject knowledge; the individual learning needs of students; a lack of suitable resources; the need for syllabus coverage; the lack of time for preparation and 'delivery'; and the pedagogical expectations of learners.

➜ My beliefs are frequently let down by my own skills and understanding. The other factor that makes it difficult is the wide range of ability/previous knowledge in my classes – it's a challenge! (AC)

I need to cover specification/syllabus, get through a course, get students a qualification, cheat the system. (FF)

Lack of time to develop and tailor lessons is an inhibiting factor. (GG)

I feel my own beliefs are not important. The students in the classes each need to be taught in a way suitable for them which incorporates their needs and learning styles. (YY)

Table 20: Overall transmission, discovery and connectionist orientations, by teacher

	Before			After		
	Tran	Disc	Conn	Tran	Disc	Conn
JJ	57	23	20	17	23	60
AA	50	31	19	15	53	32
YY	38	32	30	22	43	35
LL	37	33	30	17	43	40
FF	37	30	33	5	43	52
CC	20	53	27	13	33	53
PP	4	52	44	7	30	63
MM	13	50	37	27	37	37
GG	30	45	25	33	27	40
AD	27	44	29	32	34	34
TT	15	43	42	10	43	47
ZZ	0	28	72	0	47	53
AB	15	42	43	17	47	37
AC	18	40	42	10	37	53
DD	27	37	37	27	20	53
XX	37	27	37	10	43	47
BB	30	35	35	32	33	35
Mean	**26.75**	**37.98**	**35.28**	**17.18**	**37.49**	**45.33**
SD	**15.16**	**9.14**	**12.05**	**10.06**	**9.08**	**9.92**

Professional development courses must take account of such factors. Teachers are unlikely to believe in a new approach until they have tried it out under the constraints that exist in their own context, with their own learners. Changes in beliefs, we suggest, are more likely to *follow* the successful implementation of well-engineered, innovative methods, as processes and outcomes are discussed and reflected upon.

7.3 The impact on learners

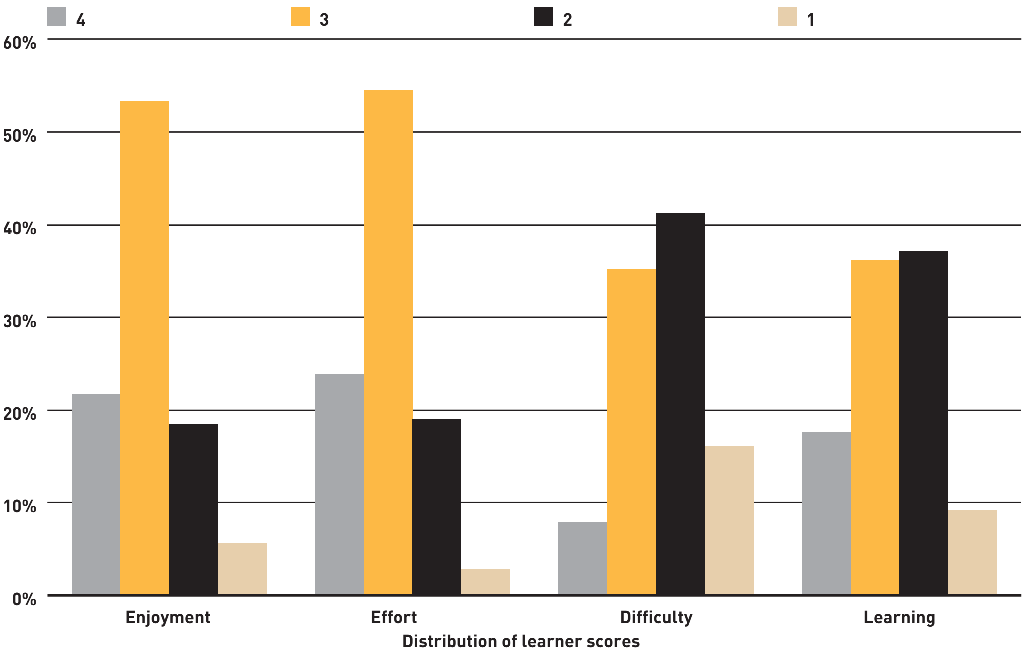
7.3.1 Learners' general impressions of the activities

The data in this section is drawn from researchers' classroom observations, interviews with learners, and from a short questionnaire completed after each session. The questionnaire asked learners their views on: how difficult the session had been; how enjoyable it was; how hard they had worked and how much they had learned. Learners were asked simply to rate each aspect on a 4-point scale. A score of 4 represented 'very enjoyable', 'I worked very hard', 'very difficult', 'I learned a great deal', while a score of 1 represented 'very boring', 'I was very lazy', 'very easy', 'I did not learn much at all'. There were 302 responses; these are summarised in Figure 8.

Most learners appear to have enjoyed the sessions and feel as though they have worked hard. Their responses for 'difficulty' and 'learning' were more evenly spread. We are unable to compare these responses with learners' responses to 'normal' sessions where they are not using project approaches and it may be the case that these responses are no different from those they would normally give. As one learner said in interview:

Learners were asked simply to rate each aspect on a 4-point scale. A score of 4 represented 'very enjoyable', 'I worked very hard', 'very difficult', 'I learned a great deal', while a score of 1 represented 'very boring', 'I was very lazy', 'very easy', 'I did not learn much at all'.

Figure 8: Learner ratings of the classroom activities



➜ I always work 100% whether it [the work] is easy or hard, whatever, I always put 100% effort in all the time. (From JJ's class)

Learners were given an opportunity to write open responses to sessions but they rarely did this. The tone of most comments was encouraging:

➜ Learnt that I know a lot more about % than I thought I did, and it was very useful.

Hard but enjoyable. It was a real eye-opener. Found it difficult to grasp but I think I will get it in time. (RR's class)

During the Pilot and Trial phases, the materials were under a constant state of review and different versions were developed. Some teachers found it difficult to judge the level of difficulty of these resources and this, coupled with the reluctance of some to challenge learners, meant that inappropriate activities were sometimes chosen by teachers. There is little evidence of teachers customising the materials to the needs and levels of their own learners, and some learners told us during interview that some activities were too easy for them, particularly when they had seen the ideas many times before:

➜ I'd already done it many times before. [...] I want to face a challenge but I've done this for, like, five years before so I can't seem to enjoy it. (From JJ's class)

This again underlines the importance of formative assessment, where teachers first discover learners' prior knowledge and build on this rather than start again from the beginning.

7.3.2 Learners' responses

Most of the data in this section is from classroom observations and from the 26 interviews that were conducted with learners in 14 of the classes: these involved 41 learners, or about one-fifth of the total number of learners that took part in the project. As already noted, we were unable to obtain much data from written questionnaires given to learners.

By the end of the project the vast majority of learners appeared very supportive towards the project and embraced the approaches, even when they were asked to organise themselves and work in a different way. A typical example comes from one researcher's case study:

→ There is a noticeable difference in their attitude and motivation over the observed sessions. For example, at the end of the session Kevin and Matthew were jubilant – they asked for more of the same to do in the afternoon and continued through their lunch-break. (Observation of DD)

Learners often became very engaged with the materials. In one session, two learners joined the class from another class, and the researcher observed the two newcomers as they used the materials:

→ It was interesting to watch these learners as one wrote down all her working when she was using the worksheet. However, as she became engrossed in the jigsaw she started doing all the calculations in her head. At the end of the lesson the two 'extra' adult learners refused to leave until they had completed the jigsaw. (Observation of XX)

In interviews, many learners contrasted the new collaborative approaches with negative experiences of lessons at secondary school, and with their teachers' normal transmission approaches. Although not every researcher was able to interview learners, we believe that the following quotes are representative of the views of the majority:

Many learners noticed a major change in their teacher's practice:

> We don't normally do activities [from YY's class]

> We are used to being nannied [from WW's class]

> We had to think for ourselves more [from WW's class]

> Normally, she would tell us methods more and then walk round helping people [from WW's class]

> Normally we do separate things. We don't normally do work as a whole group. It's usually different work for different students [from AA's class]

... but not everyone!

> *Researcher:* Did you think the class today was different from what you usually do?

> *Lee:* Much the same.

> *Mark:* [It was] just a basic JJ lesson.

> *Researcher:* So what is a normal JJ lesson like?

> *Mark:* Well, like it was this morning. All normal.

The reason why some learners did not detect a change may reflect the fact that some teachers were already using similar approaches before the project began.

As we have seen, the great majority of learners enjoyed working on the activities:

> It was interesting enough to keep me wanting to do it [from XX's class]

> We kept going; we were busy so the time passed quickly [from AA's class]

> There wasn't anything I disliked. In fact the more I did, the more I enjoyed it [from XX's class]

> It makes learning maths a lot easier and more fun [from GG's class]

Many learners enjoyed working collaboratively; they felt less vulnerable and more relaxed when they worked towards making a group decision. They also felt that they learned from each other, particularly when they needed to explain their thinking.

> It's great because it's like double the brain work [from GG's class]

> I take it in better when we discuss [from WW's class]

> I liked not working by myself; having different options [from YY's class]

> It helped to talk about it – we each had contributions [from YY's class]

> If I'd been on my own I would have been scared of it [from DD's class]

> It's good in a group; no one laughs at you, we're all very relaxed [from DD's class]

> It makes you think from somebody's point of view [from RR's class]

> I like to notice what they [the other learners] can do and what I can do, and put them together and maybe there is something I cannot do but they can do, or I can do and they cannot do, so we put them together and see what we get [from RR's class]

Learners liked activities that challenged them to think:

> You can't just do it, you have to work it out, which is good. Better it being that way or you won't learn nowt. [from RR's class]

> I don't like going below what I can do. I love challenges. I want to challenge my strengths and weaknesses, I want to just challenge everything. [from JJ's class]

> This [the approaches] challenges what you know and makes you think about what you are doing. [from LL's class]

> I think in terms of what you have made us do [with] Always, Sometimes, Never. So I was thinking as that as Sometimes, Always, Never rather then just seeing it as a sum. Got me thinking, how would it be Never? How would it be Always? What is the rule?

One teacher (AC) reported that learners complained if an activity was too easy:

> … the ones they were doing, actually, were really quite difficult, but I found a lot of the learners really like it. It is challenging. If it is too easy they quickly say – this is too easy, I can do it. [from AA's class]

Learners began to recognise the value of explanation in helping them to organise their own thinking and also to recognise what they had learned. (The extract below comes from a learner in EE's class.)

> By explaining the work to someone else, it helps you to absorb it.

> *Researcher:* How do you actually know if you have learned something? […] You know sometimes you fill in a form and it says – do you think you have learned something? And it says nothing, a bit, a lot, how do you know?

> *Marilyn:* If you have just learned it and someone asks you for help and you can explain it then you know you have learned it.

> *Researcher:* So when you actually have to explain it to someone.

> *Marilyn:* Well, if you explain it, and you have explained it correctly, then obviously you know it.

To summarise, most learners were generally very positive about the collaborative activities. They recognised that this was a big change from normal mathematics teaching, they enjoyed the mutual support and learning that took place, and some had begun to recognise the importance, for their own learning, of actively explaining ideas to others. Whereas some teachers were concerned about giving learners activities that were too difficult, learners appeared to enjoy the challenge and a few even complained when the work given was too easy.

8 Issues for professional development

In this section, we discuss the issues that arose during the project which have implications for initial teacher training and continuing professional development. These issues are: teachers' mathematical knowledge and subject-specific pedagogy; the need for flexibility, linked to formative assessment and questioning; the need to differentiate work; and the skills required to set up collaborative work and manage learners' discussions. The final issue of teachers' general pedagogical knowledge runs through many of these.

8.1 Teachers' mathematical knowledge and subject-specific pedagogy

Although the majority of teachers in our project were well qualified to teach *Skills for Life* (see section 5.4) a few had not been trained in teaching basic mathematical concepts. These teachers did not have a 'profound understanding of fundamental mathematics' (e.g., appreciating that division may be seen as partition (sharing) or quotition (grouping)) that Liping Ma, in her research with US and Chinese secondary mathematics teachers, argues are essential (Ma, 1999). There is no requirement in the current Subject Specifications for Adult Numeracy for teachers to have a deep understanding of basic concepts, such as place value and division, comparable with the understanding required of primary school teachers working with learners at similar levels. The activities designed for this project certainly challenged the teachers to think more deeply themselves about mathematical content and this sometimes had the effect of making them realise that they did have gaps in their knowledge:

> *Researcher:* What do you think has changed?
>
> *Teacher:* I'm more certain about my lack of confidence and knowledge in maths, definitely. (BB)

In section 5.7.2, we distinguished between *mathematical knowledge* (knowledge of the subject itself), and *mathematics-specific pedagogical knowledge* (knowledge about how to teach the subject). Teachers were also challenged to think about this latter form of knowledge. For example, some teachers were unsure what to do when they had uncovered a learner's misconception, and they lacked the 'know-how' to select appropriate activities that would create cognitive conflict and help to foster reflection and discussion. Again, we feel, at present, that this is a missing element of CPD or initial teacher training and, although we are pleased to note that 'Specialist learning and teaching in the teacher's own specialist area' is a recommendation in *Application to Professional Standards, for teachers of mathematics (numeracy): new overarching professional standards for teachers, tutors and trainers in the lifelong learning sector* from LLUK[24], the precise meaning of this phrase remains unclear[25].

8.2 Formative assessment and questioning

There is a great deal of research on formative assessment (see, for example, Black et al., 2003; Ecclestone, 2002a, 2002b) which is defined as assessment designed to inform subsequent teaching. This can take many forms, from paper-based diagnostic tests to oral classroom questioning.

Formative assessment is an integral part of the *TTM* approaches and involves teachers developing a broader repertoire of questioning techniques (see section 3.2). For example, teachers need to find out

24 The Professional Standards are due to be introduced in September 2007.

25 There are also plans for new entry requirements to teaching, including mathematics / numeracy skills. See http://www.lifelonglearninguk.org/currentactivity/itt/ptlls.html

about their learners' use of mathematics in contexts outside the classroom, and are required to ask learners what they know about a topic/area of mathematics before it is taught, so that they can build on this and integrate this knowledge into the forthcoming session. Sometimes, this means that session plans have to be radically changed.

➜ The lesson started with WW asking the learners what they knew about measurement and measuring. She was amazed by how much they knew and said that she would have to scrap a couple of lessons, which she had planned. (Observer report)

During the session teachers have to assess learners' skills and reasoning processes. They need to ask higher-order questions that challenge learners' knowledge and beliefs and accepted ways of thinking and working; they also need to ask probing, diagnostic, questions such as 'can you tell me how you got that?' and 'can you show me another way of doing this?'

Formative assessment requires the teacher to 'think on her feet', respond flexibly and appropriately, and even change direction within a session. This might happen at any point, such as when a learner reveals a greater or lesser understanding of a concept or procedure than the teacher anticipated.

Such flexibility requires both confidence and subject-specific pedagogical knowledge. Teachers who lack a deep knowledge of possible response strategies tend to miss many opportunities for discussion and learning. Again, this is an important area where professional development is needed.

8.3 Differentiating work to meet the needs of all learners

The project resources were designed to be used with learners working at Level 1 and below, with the majority of the learners working in the Entry Level spectrum. In the previous *Improving Learning in Mathematics* project, it was found that the 'rich' nature of the activities ensured that most learners could find a suitable challenge in them.

In this project, however, classes varied so greatly in attainment that teachers struggled to select and use activities that were appropriately challenging:

➜ The other big thing with my class is just the range of the students' previous knowledge and level of maths is enormous and I don't know how you do this, bring it all together at the end, I'm sorry how do I do that with people that are Entry 2 and people that are Level 1 or above, [...] how do you bring all that together? (AC)

We actually did the consecutive numbers and before that we did counting numbers up to 100. I had one student counting up to 10, right up to someone trying to do a Level 2 task. So it's a case of a student at this end is absolutely bored rigid, give it another 10 minutes and that student hasn't got a clue what's happening. What are you supposed to do in that situation? Because it was planned to teach as a whole group, in my particular situation I know at our college it's not practical. (AA)

We frequently found teachers using tasks that were too easy for particular learners (who consequently became bored) or (less often) too difficult for learners (who lost interest and commitment). Over one-half of the 24 classes contained a mixture of learners who were working between Entry Level 2 and Levels 1 or 2, and this meant that, for at least some parts of the session, teachers needed to reorganise their classes and customise the materials to provide different groups with appropriately differentiated activities. Some teachers found that planning for this was difficult and time-consuming. In our revisions of the resources, we have therefore decided to list activities in a notional order of difficulty to signify the level of challenge posed to the learner.

8.4 Setting up collaborative work and managing discussion

Terms such as 'mathematical discussion' and 'collaborative learning' are often taken for granted, or interpreted superficially.

Although some teachers told us they had changed their practice by setting up group work, there were many occasions when researchers noted that the learners were neither collaborating *as a group*, nor *discussing* in constructive ways. For example, in one class, TT asked nine learners to work in groups. The task was of the multiple representations type; learners were asked to work together to match different representations of the same dataset: tables, bar charts, descriptive statistics. Learners worked in four different ways:

- Three learners worked as a genuinely cooperative group discussing and comparing the representations on the cards.
- Two learners sat together but worked independently of each other except for occasional conferences to check accuracy.
- Three learners sat together, but shared out the cards, then worked silently on their individual subset.
- One learner worked completely on his own, asking help from only the teacher when he needed it.

Even when learners do talk together, they may not be having constructive discussions. As Mercer (2000) has noted, many learners engage in *disputational* and *cumulative* talk rather than constructive *exploratory* talk. Disputational talk consists of disagreement and individualised decision-making. In cumulative talk, speakers build positively but uncritically on what each other has said. Both can be unhelpful for learning. Instead, mathematical discussion should be characterised by *exploratory talk*, which consists of critical and constructive exchanges, where challenges are justified and alternative ideas are offered. In this, participants work on and elaborate each other's reasoning in a collaborative rather than competitive atmosphere. Exploratory talk enables reasoning to become audible. Professional development is needed so that teachers are more able to recognise and stimulate exploratory talk.

Teachers also needed some guidance on how to structure groups. In the early stages, many teachers spoke of organising learners into friendship groups, but in the later stages many had begun to group learners on ability or mathematical experience. As one teacher said:

➜ I found it [friendship grouping] didn't work very well because very often one of the students was very good and the other not so good. And I thought, well that will work because the good one will help the poor one and the good one will gain because she's giving explanations. But it didn't work at all, because first of all the explanations might not have been very good, and very often went over the top of the head of the other one. Secondly, on the social side, some of the students did not know the names of other students in the same group, they'd been together for months and they didn't know the names. Now I tell them who's going to sit next to each other, what pairs there'll be and it works far, far better I find. And of course it's brought the whole group together because it constantly changes around, and there's a lot more social interaction and a lot more conversations. (CC, oral)

Learners also need to be taught how to discuss in helpful ways. While some learners found it difficult to work collaboratively, possibly due to a lack of confidence and/or social skills, others lacked an understanding of what they were supposed to do in group situations. We found that teachers needed help in knowing how to introduce learners to the purpose of discussions.

8.5 Teachers' general pedagogical knowledge

A common theme underlying these last issues (sections 8.2 to 8.4) is the issue of teachers' general pedagogical knowledge. Alexander (2004) writes that the concept of pedagogy has remained relatively unexplored and untheorised in the English-speaking world, and Zukas and Malcolm claim that 'Lifelong learning pedagogies do not, as yet, exist in the UK' (2002, p. 203).

Here we are using the term to refer to the ways in which teachers organise and manage their classes.

In many ways, the task of the *Skills for Life* teacher is a particularly demanding one. In this project they have been observed teaching learners from Entry Level 1 up to Level 2; teaching learners aged between 16 and 65; teaching learners with physical and mental health problems, and social and behavioural difficulties; and teaching in formal classroom contexts like FE colleges and in informal contexts like Drug Re-integration Centres. We can see that responding to, and catering for, each learner's needs is a sensitive and skilful business, and that teachers need a great deal of training and experience in learning how to organise and manage their classes to cope with these demands.

9 Conclusions

In this final section we return to the original research questions from the beginning of the report (see section 2.2) to see how they have been addressed. The section ends with implications for the future design of resources, and the implications for the professional development of teachers.

9.1 The status of the research

This report describes a design-based research study carried out over nine months with *SfL* numeracy learners working at Level 1 and below. This involved 24 teachers from 12 organisations. While we have no reason to suspect that these situations are atypical, we recognise that such a small sample does not, by itself, permit us to generalise our findings to the whole sector.

It is important, however, to stress that this project builds on other research in the FE sector, and its findings do seem consistent with this earlier work. In particular, this study builds on the work commissioned by the DfES Standards Unit in 2005, *Improving Learning in Mathematics* (DfES, 2005). Subsequently, the Ofsted report, *Evaluating Mathematics Provision for 14–19-year-olds* (Ofsted, 2006), quotes in its 'key findings' that a specific factor that contributes to raising learners' achievement is the effective use of 'high quality learning resources, including new resources devised by the Standards Unit in the DfES' (p.2). Many principles that the report recommends should be used to raise achievement in mathematics are striking in their similarity to the principles we advocate in *TTM*:

> ➜ The best teaching gave a strong sense of the coherence of mathematical ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning. Careful questioning identified misconceptions and helped to resolve them, and positive use was made of incorrect answers to develop understanding and to encourage students to contribute. Students were challenged to think for themselves, encouraged to discuss problems and to work collaboratively. Effective use was made of information and communication technology (ICT).
>
> (Ofsted, 2006, executive summary, p.1)

In addition, the Advisory Committee on Mathematics Education (ACME) published a report *Mathematics in Further Education (FE) colleges* (ACME, 2006). This report made further recommendations to improve the overall quality of teaching and learning in the FE sector.

> Good materials are an essential prerequisite for good teaching. What is new about the Standards Unit materials for mathematics is that they quite deliberately foster an active mode of learning, while many materials, even of apparent good quality, may not.
>
> The design of mathematics textbooks and resources should learn from the DfES's Standards Unit approach exemplified in *Improving Learning in Mathematics*.
>
> (ACME, 2006, Recommendation 18, p. 26)

We therefore believe that the findings in this report are a valuable contribution to a growing body of evidence that has the potential to impact on policy and practice, and will be of interest to teachers, teacher educators, managers, policy-makers and learners.

Moreover, we advocate that the design of the professional development element in this project can be used at any level across all CPD and initial teacher training, not only in mathematics/numeracy, but also in other curriculum areas. We also believe that almost all of the issues that have arisen, and have been highlighted in the report, are directly relevant to teachers and learners at higher levels of mathematics teaching, and across the *SfL* sectors.

9.2 What do we know about the context?

The organisations and their courses, the teachers, the classes, and the learners that took part in the project were heterogeneous. Organisations were from various *SfL* sectors across different geographical regions in England. Seven of the 12 institutions were FE colleges, one was a private training company linked to Jobcentre Plus and one a drugs re-integration centre. The courses catered for learners working between Entry Level 1 and Level 1, with over half also containing mixed-ability learners designated to be working between Entry Level 2 and Level 1 or 2. The length of the courses ranged from three to nine months, and the great majority of these were separate numeracy courses, not embedded into other curriculum areas. Over one-third were presented on a 'roll-on, roll-off' basis.

Teachers were nominated by their organisations, and we did not seek to recruit either particularly high- or low-performing teachers. The 24 teachers came from a wide variety of backgrounds and had a range of backgrounds and experiences. The mean number of years of professional experience was 6.5 years, ranging from under a year to 29 years, and fewer than half held a Level 4 subject-specific teaching qualification in numeracy. Although we found some gaps in the subject knowledge of a few of the teachers in regard to basic mathematical concepts, it was teachers' experience of subject-specific pedagogy which had a greater effect on how well they managed to successfully integrate the approaches into their practice.

Over 200 learners participated in the project. The vast majority were white British and two-thirds were women. Their ages ranged from 16 to their mid-60s, although one-third were 16–19-year-olds. Many learners had a poor experience of learning mathematics at school, and a significant minority had low level literacy skills and/or physical and/or mental health problems. Class sizes ranged from two to 16 learners, with an average size of eight learners. The duration of the classes ranged from 45 minutes to three hours; most were in the daytime, and about one-third of the classes had an LSA to support learners with special needs.

9.3 How do we design appropriate activities for this context?

Teaching materials were developed by Malcolm Swan and Susan Wall (with additional materials from Teresa Kent) using the same eight principles developed for the *Improving Learning in Mathematics* resources (DfES, 2005). Each activity was accompanied by a detailed session plan offering suggestions on classroom pedagogy, including organisation, guidance on the aims of the session, and a range of questions designed to promote discussion, reflection and a review of learning that had taken place.

The materials that were designed during the first three months of the project were trialled by the teachers in Phase 1, and feedback was received from the teachers and observers/researchers at the workshops and professional development sessions. These comments were taken on board and a series of changes incorporated before the materials were re-piloted in Phase 2.

During the course of the project, 30 teaching sessions were designed, although not every one was trialled by at least one of the 24 teachers. During the Trial and Pilot phases of the project the session materials had a variety of names, and this sometimes made it more difficult for the teachers to match the materials to the level the learners were working at. The final resource is presented as a series of packs of materials in a rough order of challenge and difficulty, from easiest to hardest.

9.4 What is the impact on teachers?

Teachers' understanding of the aims of the project, and the principles of the approaches, developed slowly, and some teachers had different interpretations of what was involved. There were also difficulties over some of the terminology used, and some teachers felt that it was inappropriate to over-challenge some learners, particularly those they felt had had a poor experience of learning mathematics at school.

Teachers rated their own practice before the project had begun, but whereas they tended to rate it as being learner-centred, their own learners saw them as being more teacher-centred. Teachers' own ideas of how much they had changed were broadly similar to, and in agreement with, the classroom observations.

The main ways in which teachers' practice had changed was in terms of their organisation (with more group work), classroom ethos (where learners were relaxed, felt comfortable to interact, and were less worried about making mistakes), and learners' practices (where learners were given more choices and were encouraged to ask questions). Another major change, of course, was that using the approaches had meant that the teachers had stopped using worksheets.

In terms of the implementation of the eight principles, observers concluded that teachers had most difficulty in integrating 'exposing and discussing common misconceptions' and 'building on what learners already know'. In the case of the latter principle, some teachers did not know how to use learners' responses to shape the remainder of the session. The most straightforward principles to introduce appeared to be 'using rich and collaborative tasks' and 'asking probing questions to assess what learners already know and think' and 'organising cooperative small group work': over half the teachers were seen to be using these principles on a regular and effective basis. However, although the majority of teachers were asking a broader range of higher-order questions which both diagnosed and challenged learners' thinking, and teachers' 'wait' time increased, we still found that learners were not given enough time to reflect and to discuss their own interpretations and methods before the teacher intervened with the 'correct' viewpoint. Less than one-third of teachers were seen making connections, or encouraging learners to justify decisions on a regular basis. Few classes were evaluated to be using technology appropriately but, as many classes had neither smartboards nor computers, it was difficult to make a proper assessment.

At the end of the project, teachers were also asked to rank what they regarded as being the most important principles. Two of the three highest positions were 'exposing and discussing common misconceptions' and 'building on knowledge learners bring to sessions', which were the principles that observers assessed teachers having the most difficulty with. A major difficulty was that teachers did not know how to react to learners' answers and guide them in further learning.

Teachers' beliefs also changed and their self-report questionnaires showed that they reported a significant movement away from transmission orientations and a significant increase in connectionist orientations over the course of the project. The data suggest that some beliefs develop from transmission to connectionist via the discovery orientation.

9.5 What is the impact on learners?

Our data on this research question is limited, and comes mostly from researchers' observations and learner interviews. Obtaining pre and post written evidence from learners proved difficult for several reasons. In some cases, teachers felt that their learners would not be capable of completing questionnaires and were reluctant to ask them to do so. The learner population varied considerably in some classes (particularly in roll-on, roll-off courses), and many learners had left their courses before the post questionnaires. Others were in the middle of exam revision, and did not feel able to find the time to complete the questionnaires a second time.

Our evidence suggests that learners appeared very supportive towards the project and, as with the teachers, their practices and behaviours had changed. They enjoyed working with the resources and felt that they had worked hard. Learners particularly enjoyed working collaboratively in groups and some recognised the benefits of learning from each other, particularly when they were asked to explain their thinking and strategies of working.

Data from the beginning of the project showed that learners' attitudes to learning numeracy/mathematics as an *SfL* learner was generally very positive, and that their main reasons for attending numeracy classes were to improve their skills and obtain higher grades. However, we have insufficient data to analyse any changes that had occurred at the end of the project.

9.6 What are the implications for the future design of resources?

The resources designed for this project were based on eight principles. Using the resources, most teachers were able to apply the following principles consistently and effectively:

- Use rich, collaborative tasks
- Use cooperative small group work
- Use probing questioning to assess what learners know and how they think.

These principles are the ones that refer to teachers planning and using the plans that we provided. Teachers found the following principles more difficult to apply:

- Expose and discuss common misconceptions
- Create connections between topics
- Build on the knowledge learners already have
- Encourage reasoning rather than 'answer getting'
- Use technology[26].

These principles refer to teachers *responding* to learners during sessions in appropriate and flexible ways. This suggests that the future design of resources, and professional development should pay particular attention to supporting teachers while they try to adopt these latter ways of working.

The different types of mathematical activity provide conceptual tools which give teachers and curriculum developers the opportunity to adapt these resources for their own particular needs. These are:

- *Classifying* mathematical objects
- *Evaluating* mathematical statements
- *Interpreting* multiple representations
- *Creating and solving* problems
- *Analysing* reasoning and solutions.

All the materials were reviewed before the start of the Pilot phase, and many minor revisions were made. All sessions were revised again at the end of the project. Most underwent minor changes again (simplification of language, addition of easier/harder tasks), but a few sessions which had not worked consistently well were given major re-writes to take into account what had happened during the project.

We believe that the teaching and learning would have been more effective if certain types of activity had been used more frequently. In particular we noted the underuse of *Evaluating mathematical statements*, where learners are confronted with generalisations (often common misconceptions) and are asked to justify their reasoning by creating examples and counterexamples. (A typical statement might be: *If you divide a number by 10, the answer will be less than the number.)*

We feel that the iterative model of – design trial with first hand observation, reflection, and further modification – has been successful during the project and should be used as a model for all curriculum development.

It is important to emphasise that the resource packs produced by this project are not intended to be definitive or complete. Rather, they should be seen as an

26 This principle was often difficult to implement because there was a lack of appropriate resources.

introduction to the approaches, with some accompanying examples. Ideally, teachers need to become more involved in the design and creation of future materials for their own learners, though we think that at the present time most may not have the time or pedagogical knowledge to do this without the support of experienced designers[27].

9.7 What are the implications for the professional development of teachers?

Most of the implications from the project are directed towards teacher educators in CPD and initial teacher training. We believe that professional development should include modules on: teaching basic mathematical concepts, particularly aimed at learners working at Entry Level; subject-specific pedagogy; and formative assessment, including asking higher-order questions. Training should also address the issues of teachers' flexibility so that they can listen and respond effectively to learners' needs; and the differentiation of learners' work with its organisational and managerial consequences. We also think that educators should ensure that terms such as 'discussion' and 'collaborative learning' in the numeracy classroom have shared definitions which are used with a greater precision and consistency. Teachers also need to be able to recognise and stimulate exploratory talk where learners elaborate on each other's reasoning. We also feel that teachers should be made aware that learners, like teachers, need induction and guidance in techniques of collaboration and working together. Finally, professional development should attempt to increase teachers' awareness of learners' most common misconceptions, and to give teachers a repertoire of actions for tackling them as opposed to the approach of 'diagnose and correct' that has been used in the past.

27 We would like to point out that we are talking about *SfL* numeracy teachers in general. We are aware that a few teachers are producing high-class materials based on these principles.

Glossary

ADHD	Attention Deficit Hyperactivity Disorder
ANCC	Adult Numeracy Core Curriculum, introduced in 2001
CPD	Continuing Professional Development (for teachers)
DfES	(Government) Department for Education and Skills
EMA	Educational Maintenance Allowance. This is a payment of up to £30 per week for 16–19-year-old learners from lower income families attending educational classes
ESOL	English for Speakers of Other Languages
GCSE	General Certificate of Secondary Education
ICT	Information and Communication Technology
ILPs	Individual Learning Plans
LSAs	Learning Support Assistants who work with individuals and groups of learners to support them in their learning
LLU+	London Language and Literacy unit, based at South Bank University
LLUK	Lifelong Learning UK is the Sector Skills Council responsible for the professional development of all those working in learning areas such as community learning and development; further education; higher education; and work-based learning
Maths4Life	A three-year project based at the NRDC, Institute of Education investigating the teaching and learning of mathematics. After March 2007 the NCETM ran it
MLD	Moderate Learning Difficulty
NCETM	National Centre for Excellence in the Teaching of Mathematics
NIACE	National Institute of Adult Continuing Education
NRDC	National Research and Development Centre for adult literacy and numeracy
SfL	*Skills for Life*. The national strategy for improving adult numeracy and literacy in England. Introduced in 2001
TTM	*Thinking Through Mathematics*

References

ACME. (2006). *Mathematics in Further Education Colleges* (No. ACME PR/08).

London: Advisory Committee on Mathematics Education, The Royal Society. http://reports.royalsoc.ac.uk/acme/FE_Report.htm

Ahmed, A. (1987). *Better Mathematics: A Curriculum Development Study*. London: HMSO.

Alexander, R. (2004). Still no pedagogy?: principle, pragmatism and compliance in primary education. *Cambridge Journal of Education, 34*(1), 7–33.

Askew, M., Brown, M., Rhodes, V., Johnson, D. and Wiliam, D. (1997). *Effective Teachers of Numeracy, Final Report*. London: King's College.

Askew, M. and Wiliam, D. (1995). *Recent Research in Mathematics Education 5–16*. London: HMSO.

Baxter, M., Leddy, E., Richards, L., Tomlin, A., Wresniwiro, T. and Coben, D. (2006) *Measurement wasn't taught when they built the pyramids – was it?: The teaching and learning of common measures in adult numeracy.* London: NRDC, Institute of Education.

Bereiter, C. (2002). Design research for sustained innovation. *Cognitive studies, Bulletin of the Japanese Cognitive Science Society, 9*(3), 321–327.

Bills, C., Bills, L., Mason, J. and Watson, A. (2004). *Thinkers: a collection of mathematical activities to provoke mathematical thinking*. Derby: Association of Teachers of Mathematics.

Black, P., Harrison, C., Lee, C., Marshall, B. and Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham: Open University Press.

Black, P. and Wiliam, D. (1998). *Inside the black box : raising standards through classroom assessment*. London: King's College London School of Education.

Brown, A. L. (1992). Design Experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences, 2*(2), 141–178.

Chanda, N., Griffiths, G. and Stone, R. (2005). *Integrating formative/diagnostic assessment techniques into teachers' routine practice in adult numeracy*. London: LLU and South Bank University.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R. and Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher, 32*(1).

Coben, D., Brown, M., Rhodes, V., Swain, J., Ananiadou, K. and Brown, P. (2007). *Effective Teaching and Learning: Numeracy.* London: NRDC, Institute of Education, University of London.

Coffield, F. (2000). Lifelong learning as a lever on structural change? Evaluation of White Paper: Learning to succeed: a new framework for post-16 learning. *Journal of Educational Policy, 15*(2), 237–346.

Collins, A. (1992). Towards a design science in education. In E. Scanlon and T. O'Shea (eds), *New directions in educational technology* (pp. 15–22). New York: Springer-Verlag.

DBRC. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational researcher, 32*(1), 5–8.

D*f*ES. (2005). *Improving Learning in Mathematics*. Standards Unit, Teaching and Learning Division.

DfES. (2007). *Thinking Through Mathematics: Strategies for teaching and learning*. Maths4Life.

Ecclestone, K. (2002a). *Formative Assessment in Lifelong Learning*. London: Routledge.

Ecclestone, K. (2002b). *Learning Autonomy in Post 16 Education*. London: Taylor and Francis.

Kelly, A. (2003). Theme issue: The role of design in educational research. *Educational Researcher, 32*(1), 3–4.

Learning and Skills Council (2005). *Learning and Skills – the agenda for change – the prospectus*. London: Learning and Skills Council.

Lifelong Learning UK (LLUK) (in press) *Application to Professional Standards, for teachers of mathematics (numeracy): new overarching professional standards for teachers, tutors and trainers in the lifelong learning sector*. London: LLUK.

LSC. (2005). *Learning and Skills – the agenda for change – the prospectus*. Coventry: Learning and Skills Council.

Lucas, N. (2004) *Teaching in Further Education: new perspectives for a changing context.* London: Institute of Education.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Mason, J. and Watson, A. (1998). *Questions and prompts for mathematical thinking*. Derby: Association of Teachers of Mathematics.

Mendick, H. (2005). Mathematical stories: why do more boys than girls choose to study mathematics at AS level in England? *British Journal of Sociology of Education, 26*(2), 235–251.

Mercer, N. (2000). *Words and Minds*. London: Routledge.

NCTM. (1988). NCTM Curriculum and Evaluation Standards for School Mathematics: Responses from the Research Community. *Journal for Research in Mathematics Education, 19*, 338–344.

Ofsted. (2006). *Evaluating mathematics provision for 14–19-year-olds*. London: HMSO. http://www.ofsted.gov.uk/publications

Shulman, L. (1986). Those Who Understand: knowledge and growth in teaching. *Educational Researcher*, *15*(2), 4–14.

Shulman, L. (1987). Knowledge and Teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–22.

Swain, J., Baker, E., Holder, D., Newmarch, B. and Coben, D. (2005). *Beyond the daily application: Making numeracy teaching meaningful to adult learners*. London: NRDC, Institute of Education.

Swain, J. (2005) Changes to adult learners' identities through learning numeracy, *Literacy and Numeracy Studies*, *14*(1) 5–16.

Swan, M. (2005a). *Improving Learning in Mathematics: Challenges and Strategies*. Sheffield: Teaching and Learning Division, Department for Education and Skills Standards Unit.

Swan, M. (2005b). *Learning Mathematics through Reflection and Discussion.* Unpublished PhD, University of Nottingham, Nottingham.

Swan, M. (2006a). *Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices*. London: National Institute for Adult and Continuing Education (NIACE); National Research and Development Centre for adult literacy and numeracy (NRDC).

Swan, M. (2006b). Describing mathematics teachers' beliefs and practices: resources for evaluating design interventions. *Research in Education*, 75(May).

van den Akker, J., Graveemeijer, K., McKenney, S. and Nieveen, N. (eds). (2006). *Educational Design Research*. London and New York: Routledge.

Zukas, M. and Malcolm, J. (2002). Pedagogies for lifelong learning: building bridges or building walls? In R. Harrison, F. Reeve, A. Hanson and J. Clarke (eds), *Supporting Lifelong Learning*. London: Routledge/Open University.

IOE LONDON
Leading education and social research
Institute of Education University of London

department for
education and skills
creating opportunity, releasing potential, achieving excellence

read·write·plus

LLU+
LONDON SOUTH BANK UNIVERSITY