The National Strategies
Secondary

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Teaching mental mathematics from Level 5: Statistics

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department for
children, schools and families

Teaching mental mathematics from Level 5: Statistics

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Contents

Introduction

What is mental mathematics?

Almost all of mathematics could be described as 'mental' in the sense that engaging in a mathematical task involves thinking. Thus every mathematical problem a pupil tackles must involve several stages of mental mathematics. Pupils actively involved in mental mathematics might be engaged in any combination of:

interpreting	visualising	analysing	synthesising	explaining	hypothesising
inferring	deducing	judging	justifying	making decisions	

These ideas are prevalent throughout mathematics and underpin mathematical processes and applications.

If the definition is so wide-ranging, how have we produced a few brief booklets with this title? The answer is that we have been very selective! The 'mental mathematics' supported through the teaching approaches described in these booklets is aimed at a subset of mental mathematics in its broadest sense. We have chosen a few key areas likely to influence pupils' progress beyond level 5. These selections have been informed by annual standards reports from QCA and the experience of teachers and consultants. The initial ideas have also been supported by classroom trials.

How do I help pupils to improve the way they process mathematics mentally?

Individual pupils will be at different stages but all pupils develop some strategies for processing mathematical ideas in their heads. Many of the activities suggested in these booklets increase the opportunities for pupils to learn from one another by setting them to work collaboratively on tasks that require them to talk. Often pupils develop and enhance their understanding after they have tried to express their thoughts aloud. It is as if they hear and recognise inconsistencies when they have to verbalise their ideas.

Equally, new connections can be made in a pupil's 'mental map' when, at a crucial thinking point, they hear a different slant on an idea. A more discursive way of working often allows pupils to express a deeper and richer level of understanding of underlying concepts that may otherwise not be available to them. In this way pupils may:

- reach a greater facility level with pre-learned skills, for example, becoming able to solve simple linear equations mentally;
- achieve a leap in understanding that helps to complete 'the big picture', for example, seeing how the elements of a function describing the position-to-term relationship in a sequence are generated from elements in the context of the sequence itself.

The activities are designed to engage pupils in group work and mathematical talk.

Is mental mathematics just about the starter to the lesson?

Developing mental processes is not simply about keeping some skills sharp and automating processes through practice. The activities described in this booklet support the main part of the lesson. Developing a mental map of a mathematical concept helps pupils to begin to see connections and use them to help solve problems. Developing the ability to think clearly in this way takes time. Once in place, some aspects of mental mathematics can be incorporated into the beginning of lessons as a stimulating precursor to developing that topic further.

The activities are intended to support the main part of the lesson.

Is mental mathematics just about performance in tests?

Using these materials will help pupils to perform more successfully in tests, but the aim is more ambitious than that. Developing more effective mental strategies for processing mathematical ideas will impact on pupils' progress in mathematics and their confidence to apply their skills to solve problems.

Secondary teachers recognise the importance of pupils' mathematical thinking and application, but few have a range of strategies to support its development. The expectations described, and the activities suggested in the accompanying mental mathematics resources aim to create a level of challenge that will take pupils further in their thinking and understanding. These materials should provide the chance for pupils to interact in such a way that they learn from each other's thinking, successes and misconceptions and thereby become increasingly confident and independent learners.

Pupils need to transfer mathematics confidently and apply it whenever they need to use it. This needs to be taught. Most commonly, pupils will use mental mathematics in solving problems as they occur in their lives, in other areas of their studies and as they prepare for the world of work. To support pupils in doing this, teachers will frequently need to set both large and small mathematical problems in real, purposeful and relevant contexts. Pupils will need to solve increasingly complex and unfamiliar problems using mathematics, apply more demanding mathematical procedures during their analysis and do so with increasing independence. These materials support teachers in planning a structured and progressive approach to do this. If learning is planned with mental mathematics as a significant element, pupils will develop increasing confidence in applying mathematics.

Improving mental mathematics will improve pupils' confidence to apply what they know.

Can mental mathematics involve paper and pencil?

Mathematical thinking involves drawing on our understanding of a particular concept, making connections with related concepts and previous problems and selecting a strategy accordingly. Some of these decisions and the subsequent steps in achieving solutions are committed to paper and some are not. When solving problems, some of the recording becomes part of the final solution and some will be disposable jottings.

Many of the activities involve some recording to stimulate thinking and talking. Where possible, such recording should be made on large sheets of paper or whiteboards. This enables pupils, whether working as a whole class or in pairs or small groups, to share ideas. Such sharing allows them to see how other pupils are interpreting and understanding some of the big mathematical ideas. Other resources such as diagrams, graphs, cards, graphing calculators and ICT software are used in the activities. Many of these are reusable and, once developed in the main part of a lesson, can be used more briefly as a starter on other occasions.

Progress may not appear as written output. Gather evidence during group work by taking notes as you listen in on group discussions. Feed these notes into the plenary and use them in future planning.

The materials

Each attainment target in mathematics is addressed through its own booklet, divided into separate topic areas. For each topic, there is a progression chart that illustrates expectations for mental processes, broadly from level 5 to level 8. Mathematical ideas and pupils' learning are not simple to describe, nor do they develop in a linear fashion. These are not rigid hierarchies and the degree of demand will be influenced by the context in which they occur and, particularly for the number topics, by the specific numbers involved. For this reason ideas from one chart have to interconnect with those in another. The aim is that the charts will help teachers to adjust the pitch of the activities that are described on subsequent pages.

The main guidance is in the form of suggested activities for developing mental processes in each topic. These activities are designed to be used with pupils who are achieving at level 5 and beyond. There are many National Strategy materials which reinforce and extend these ideas but, to ensure that these booklets are straightforward and easy for teachers to use, cross-referencing has been kept to a minimum. The most frequent referencing throughout the booklet is to the *Supplement of examples*, which is now connected to the Framework for Secondary Mathematics, www.standards.dcsf.gov.uk/nationalstrategies. The page numbers of the original supplement have been retained and the examples can be downloaded as a complete document or in smaller sets from the related learning objectives.

Teaching mental mathematics from level 5: statistics

'You can prove anything with statistics' is a commonly-held public belief. It reflects both the perceived power and weakness of this area of mathematics that is built on the study of uncertainty. Statistics play a key role in the development of our society, for example, knowing how many doctors we will need in the future is very important for planning within the NHS, and we need to ensure pupils do not mistrust statistics but are able to interpret data confidently and accurately to check and support hypotheses and substantiate opinions together with an ability to communicate conclusions and opinions. These are key elements of Citizenship.

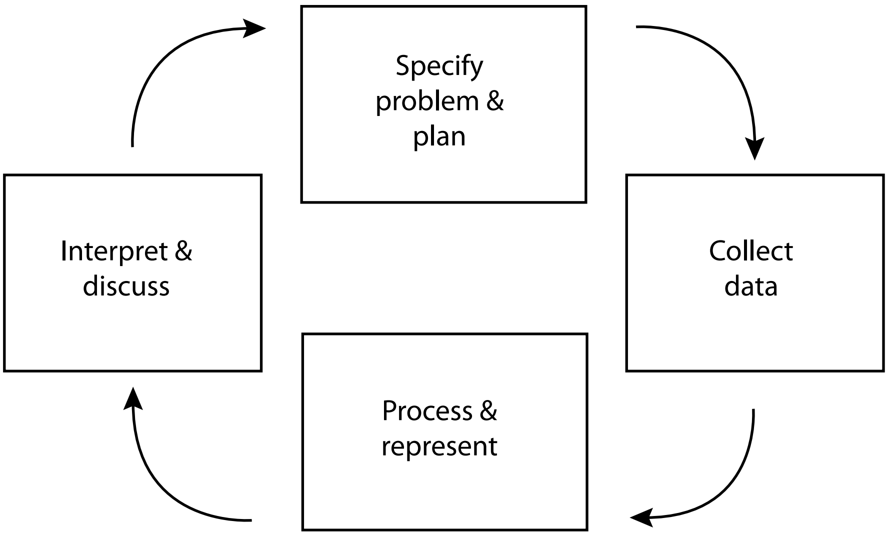
Dealing with uncertainty holds the potential for a wide variety of conclusions, which in turn demands discussion to reveal the different perspectives and reasoning. Pupils need to be encouraged to articulate and share their thinking in order to develop a fuller understanding. The activities presented in this booklet have been chosen to stimulate mental activity by challenging pupils' understanding. Teachers need to be aware that the language of 'uncertainty and inference' is different to the language of 'precision and proof' that is used more generally in mathematics.

> *'(In statistics) decisions tend to be inferred (rather than deduced) on the basis of the most likely explanation or outcome.'*

Alan Graham, *Developing thinking in statistics*, ISBN 1412911672

Explanation based on evidence has to be modelled by the teacher and must be expected of the pupils. The handling data cycle offers a structure for testing and verifying hypotheses and highlights the contribution of each stage to robust statistical enquiry as well as their interrelationship. For example, in considering how to identify the number of doctors required in the future, the questions: 'What data would you need? ', 'How would you gather it? ', 'How would you organise it? 'and 'How would you analyse it?' provides a framework within which to work. The handling data cycle also helps to provide the 'big picture' and is referred to throughout this booklet.

Handling Data Cycle



The handling data cycle is closely linked to the key processes:

- specifying the problem and planning (representing)
- collecting data (representing and analysing)
- processing and representing the data (representing and analysing)
- interpreting and discussing the results (interpreting and communicating).

Key Processes in statistics

Representing

In a statistical enquiry, representing is part of almost all elements of the handling data cycle. It involves:

- suggesting a problem to consider using statistical methods, framing questions and raising conjectures
- deciding what data are relevant and identifying primary or secondary sources
- designing ways of capturing the required data, including minimising sources of bias
- creating representations of the data, including the use of ICT, for example, tabulation, grouping, arrays, diagrams and graphs.

If pupils can represent data as part of a statistical enquiry then they are better positioned to become responsible citizens who can select and sift information thoughtfully and use mathematics with confidence to inform decision-making. Representation is a major focus of statistics, important in tying together the decisions pupils make at the different stages of the handling data cycle.

Analysing – use mathematical reasoning

Mathematical reasoning is required at all stages of the handling data cycle:

- when specifying and planning by working logically, identifying constraints and considering available techniques; also by exploring conjectures and using knowledge of related problems
- when collecting data by working systematically, exploring the effects of varying values in situations where there is random or systematic variation
- when processing and representing data, making connections within mathematics and identifying patterns and relationships, and making use of feedback from different audiences
- when interpreting and discussing results, explaining and justifying inferences drawn from the data, recognising the limitations of any constraints or assumptions made; using feedback to reassess initial conjectures and adjust aspects of the handling data cycle.

Analysing – use appropriate mathematical procedures

Using appropriate procedures involves manipulating data into suitable forms for accurate representation, calculation and communication. This will involve monitoring the accuracy of methods and solutions.

Appropriate procedures in a statistical enquiry are:

- using systematic methods for collecting data from primary and secondary sources
- constructing tables, graphs and diagrams to present data in an organised form
- calculating summary statistics, for example, measures of average and spread
- calculating experimental and theoretical probabilities.

Interpreting and evaluating

Interpreting and evaluating results is fundamental to any statistical enquiry. It includes:

- interpreting tables, graphs and diagrams, and drawing inferences to support or cast doubt on initial conjectures
- interpreting probabilities when assessing the likelihood of a particular outcome
- comparing distributions and making inferences
- looking at data to find patterns and exceptions
- considering the effects of changes to the data (e.g. removing outliers, adding items, making proportional changes)
- appreciating why the interpretations placed on data have a degree of uncertainty and can be misleading
- appreciating convincing arguments, but knowing that these do not constitute proof.

Communicating and reflecting

Effective communication and reflection is of particular relevance in statistics. It includes:

- preparing a brief report of a statistical enquiry, using tables, graphs and diagrams to summarise data and support interpretations and inferences drawn from the data
- using precise language to summarise key features pertinent to the conjectures raised
- presenting support for conclusions in a range of convincing forms
- presenting a balanced conclusion where results are not convincing
- considering alternative approaches if results do not provide sufficient evidence.

Functional skills and statistics

Statistics provides a wealth of opportunities for pupils to learn and problem solve using real, purposeful and relevant contexts. Pupils should be given opportunities to apply the handling data cycle whenever they are required to, whether it is in their lives, their further studies or when preparing for future employment.

Teachers will need to plan so that pupils gain confidence and competence as they are presented with more complex and unfamiliar situations that require the application of the handling data cycle. In addition pupils will need to increasingly apply more demanding statistical procedures in solving problems and do so with increasing independence. Mental activities that involve the handling data cycle not only provide pupils with the opportunity to build confidence, but as importantly to develop decision-making skills.

In 2001, a US government agency reported that, as cycling declined and helmet use increased over the previous decade, head injuries rose by 10%. This is both surprising and interesting. What can the explanations be and what are the ramifications for the public?*

Outcomes are often not obvious; they do not necessarily match our intuition. Pupils need to be made aware of this and need to know what can be done to gain a more informed view.

As well as being a stimulus for discussion the context of the statistical enquiry can also provide a real purpose to the individual stages of the handling data cycle. Often pupils are required to practise calculating a particular statistic for a set of numbers without discussing its significance. It is useful to ask pupils to tell each other a story around the problem which could describe the stage of the handling data cycle before and after the processing stage. This puts the calculation into a possible context. Thus, when dealing with an element of the cycle, it is useful to speculate; 'What came before and what comes after?'

* Evidence suggests cyclists are safer in numbers and statutory use of helmets discourages cycling.

Contents of the booklet

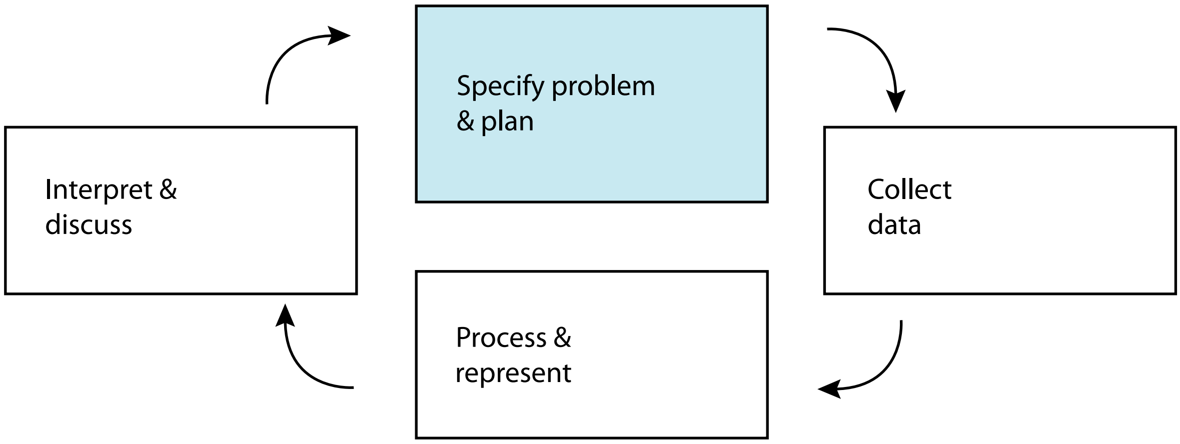
The topics covered in this booklet are selected from the statistics strand learning objectives in *The Framework for secondary mathematics:*

- Specifying a problem and planning
- Collecting data
- Processing and representing data
- Interpreting and discussing results
- Probability.

Each section contains:

- A rationale and some key messages which help to clarify the position of this aspect as part of the cycle and how it helps to build understanding.
- Progression charts which focus on the fact that developments in learning in mental mathematics are reliant on gaining fuller understanding of the concept and processes, rather than acquiring more complicated calculation skills. (The objectives in these charts are taken from the Statistics strand of the Framework for mathematics. By accessing these through the Framework more exemplification is offered via the links to the supplement of examples.)
- References to further resources.
- Activities which have been developed to support this topic.

Specifying a problem and planning



This stage is perhaps the most neglected of those described in the handling data cycle. Here the focus is on planning and decision-making processes, not common to many mathematics lessons, and more generally reserved for investigations. However, a recent HMCI report states that the best schemes of work

> *'...integrate reasoning, investigative and problem-solving skills'* (*Mathematics in secondary schools,* HMCI, 2004–05).

The handling data cycle provides many opportunities to integrate tasks which develop these skills, not least in the preliminary specification and planning.

One way of maintaining the focus on the stages of the cycle is to develop the habit of asking:

What came before this stage and what could come after?

Thus pupils get the opportunity to develop thinking and understanding of all elements of the cycle regularly and at the same time reference how the structure of the full cycle supports effective statistical enquiry. This approach will also help to identify the 'big picture' and promote thinking around next steps.

As an example, consider a lesson such as Unit 30 'How do I handle data?' from *Thinking Maths: Cognitive Acceleration in Mathematics Education*, ISBN 978 0 435307 80 6. In this activity pupils are given three different contexts and come to recognise, through class discussion, that different modes of data representation are needed to reveal the hidden meanings. This is a rich activity using preselected data sets and focusing primarily on comparing and contrasting representations of the data. This could be opened out by asking pupils to consider 'What came before?' For example, pupils could be challenged to think about this statement for the first data set:

Girls are doing better than boys at GCSE in English.

Groups could discuss whether the statement could be true in their school and what issues would need to be considered in order to test the hypothesis. They could then decide the nature of the evidence which would produce a convincing test of the hypothesis, what data they would need and how they would collect it. Decisions on planning could then be shared and the merit of each discussed before pupils considered the value of the data which would be provided. This level of discussion would prepare the pupils for the activity of determining the most effective forms of representation for given data and its subsequent analysis.

Pupils should have a sense of why they are undertaking a problem and the intended purpose of the solution. If it is possible to set these in a real context, with pupils increasingly able to formulate how they start to tackle the problem, then pupils' confidence to transfer problem-solving skills is developed.

The progress chart shown below for 'Specifying a problem and planning' has been separated into 'Framing the question' and 'Choosing the data'. The separation seeks to clarify the development in each area but does not imply that the stages are discrete. By definition they must overlap and equally must be considered alongside 'Collecting data' which is discussed in the next section.

Specifying a problem and planning

Specifying a problem and planning	Notes
Suggest possible answers, given a question that can be addressed by statistical methods.	These are all about framing the questions. This is a challenging step in thinking through the problem before detailed planning can begin.
Discuss a problem that can be addressed by statistical methods and identify related questions to explore.	
Suggest a problem to explore using statistical methods, frame questions and raise conjectures.	
Break a task down into an appropriate series of key statements (hypotheses), and decide upon the best methods for testing these.	
Decide which data would be relevant to an enquiry and possible sources.	These statements are about choosing the data. They describe decisions which need to be made before the data is collected. This stage of the cycle is often rushed but is crucial to the success of the other stages.
Decide which data to collect to answer a question, and the degree of accuracy needed; identify possible sources; consider appropriate sample size.	
Discuss how different sets of data relate to the problem; identify possible primary or secondary sources; determine the sample size and most appropriate degree of accuracy.	
Identify possible sources of bias and plan how to minimise it.	
Understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn.	
Identify what extra information may be required to pursue a further line of enquiry.	

Further ideas can be found through the learning objectives and linked examples in substrand 5.1 'Specifying a problem, planning and collecting' in the *Framework for secondary mathematics*, which is one of the sections found at www.standards.dcsf.gov.uk/nationalstrategies.

ACTIVITIES

Seven hypotheses and questions

1. More goals in football matches are scored in the last ten minutes than at any other time.
2. You can't rely on a bus turning up on time.
3. Young people watch more television than adults.
4. Out-of-town shopping centres encourage people to make more journeys by car.
5. In freshwater habitats, most invertebrates live near the surface.
6. Penguins keep warm by huddling together.
7. More people go abroad the year after we have a bad summer in Britain.

For chosen hypothesis:

- What data is needed to test the hypothesis?
- How would you collect it?
- How would you represent it?
- What other hypotheses could you suggest?

Hypotheses 1 to 6 are based on questions from the *Supplement of examples*, page 249, where related questions can be found.

Sufficient to test?

- Are the sets of collected data sufficient to test the stated hypothesis?
- If not what other data should be collected?

Hypothesis: Young people watch more TV than adults.

100 males were asked how many hours they spent watching television in one week

Number of hours watched	Number of males
0–9	18
10–19	19
20–29	33
30–39	17
Over 39	13

100 females were asked how many hours they spent watching television in one week.

Number of hours watched	Number of females
0–9	13
10–19	15
20–29	37
30–39	21
Over 39	14

- Are the sets of collected data sufficient to test the stated hypothesis?
- If not, what other data should be collected?

Hypothesis: Males who cycle to school are taller than those who do not.

Height in cm	Total number of males
Less than 140	4
$140 \le h < 150$	26
$150 \le h < 160$	68
$160 \le h < 170$	109
$170 \le h < 180$	89
$180 \le h < 190$	40
$190 \le h < 200$	9
200 or more	0

Height in cm	Males who cycle
Less than 140	2
140≤ h <150	3
150≤ h <160	20
160≤ h <170	34
170≤ h <180	23
180≤ h <190	11
190≤ h <200	1
200 or more	0

Discuss a problem that can be solved by statistical methods; identify related questions to explore, in mathematics or other subjects. For example:

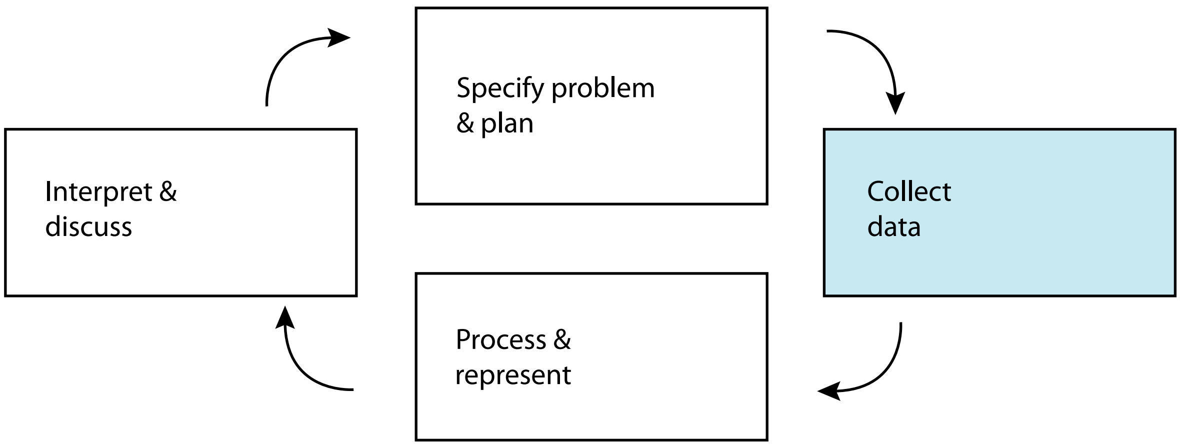
At what time during a football match is there most likely to be a goal?

Related questions:

- Where could you find the necessary data?
- Are there differences between football divisions?
- When is the best time to buy a snack if you don't want to miss a goal or to queue at half-time?
- What is the likelihood of missing a goal if you leave 10 minutes early?

The above is taken from page 249 of the *Supplement of examples* where further problems can be found for pupils to discuss.

Collecting data



This stage of the cycle is closely related to the planning stage in that the main element of work is focused on decision making. The operational element of collecting and organising the data can only be considered when the right choices have been made relating to:

- what data is relevant to the enquiry
- what are the best sources
- how best to collect it.

Pupils need to appreciate that the 'collecting stage' of the cycle must provide suitable data for solving the problem or testing the hypothesis. To that extent it is essential to be explicit in describing this purpose and referencing the given or imagined context. Pupils should realise that their choice of the most appropriate collection method will be influenced by whether the data is discrete or continuous and whether the source is primary or secondary. They need to understand that the method of collection and type of organisation will eventually affect the quality of the analysis and resulting outcome of the enquiry.

Decision making of this kind is best supported through dialogue, and, by creating tasks and allowing time which force reflection on the quality of choice made. Some choices are not simple, and we need to expose pupils to tricky discussion about contentious issues, for example:

If we read times on a digital clock should we treat the data as discrete?

This section deals with pupils making and justifying decisions around methods for the collection of data and then reviewing and amending procedures. It also supports pupils in deciding on how to organise collected data by considering 'What is gained and what is lost?' using different structures.

Collecting data

Collecting Data	Notes
Plan how to collect and organise small sets of data from surveys and experiments. Design data collection sheets or questionnaires to use in a simple survey.	Start with small sets of data from simple surveys. Move on to experiments and include consideration of sample size.
Use frequency tables.	Begin with discrete data, grouped (equally) where appropriate. Move on to continuous data with given equal class intervals.
Use two-way tables.	For discrete data.
Select from a range of data.	Choose a suitable method (e.g. observation, controlled experiment, data logging using ICT, or questionnaire).
Design a survey or experiment to capture the necessary data from one or more sources.	Data from one or more sources; design, trial and if necessary refine data collection sheets.
Construct tables for large sets of raw data.	Discrete and continuous sets of raw data, choosing suitable class intervals.
Gather data from specified secondary sources.	Including printed tables and lists, and ICT-based sources, including the internet.
Gather data from primary and secondary sources.	Using ICT and other methods, including data from observation, controlled experiment, data logging, printed tables and lists.

Further ideas can be found through the learning objectives and linked examples in substrand 5.1 ‘Specifying a problem, planning and collecting’ in the *Framework for secondary mathematics*, which is one of the sections found at www.standards.dcsf.gov.uk/nationalstrategies.

ACTIVITIES

Classifying hypotheses and samples

Pupils use prepared statements describing situations where sampling methods are suggested. For example:

Heather wanted to know how much people were prepared to spend on Christmas presents for their families. She asked the people on the street where she lives.

(For further statements see Resource sheet: Collecting 1, page 47.)

Pupils work in pairs to rank the statements according to how suitable the collection method is to the hypothesis or question posed. Justification for answers should be written on the back of each card in pencil. Statements on card could be placed along a continuum such as this

Less suitable .... → .... More suitable

Allow scope for a pile of cards for which the pupils are unsure. Move pupils from pairs to fours and ask them to explain:

- the position of the highest and lowest ranked card
- why they are unsure about some cards.

Finally the group should consider one or two statements on the continuum and discuss how to improve upon the method of sampling so that it would move up their ranking.

Specific and sharply-focused hypotheses are tricky to compose. On the cards provided for this task it is important that the pupils recognise the kind of questions or statements as being close to the choices they might have made. In this way they are supported in strategies to improve steps in their own planning for the collection of data.

For further ideas see *The Supplement of examples* pages 252 to 255, available in the *Framework for secondary mathematics*, which is a section at www.standards.dcsf.gov.uk/nationalstrategies.

Comparing samples

Pupils discuss a hypothesis along with a pair of suggested samples.

For example:

Ali, a Year 8 pupil believes: 'The 50-minute lunch break at school is not long enough'.

Sample 1: Ask all the people at the local residential home for the elderly for their views.

Sample 2: Ask the opinion of a random sample of pupils from every year group at the school.

Pupils work in pairs and make entries on large tables which are then displayed in the classroom.

Hypothesis
Less suitable sample:
More suitable sample:
Reason:

Finally ask pairs to compose some of their own hypotheses with two suggested sampling methods for other pairs to critique.

This display could be interactive:

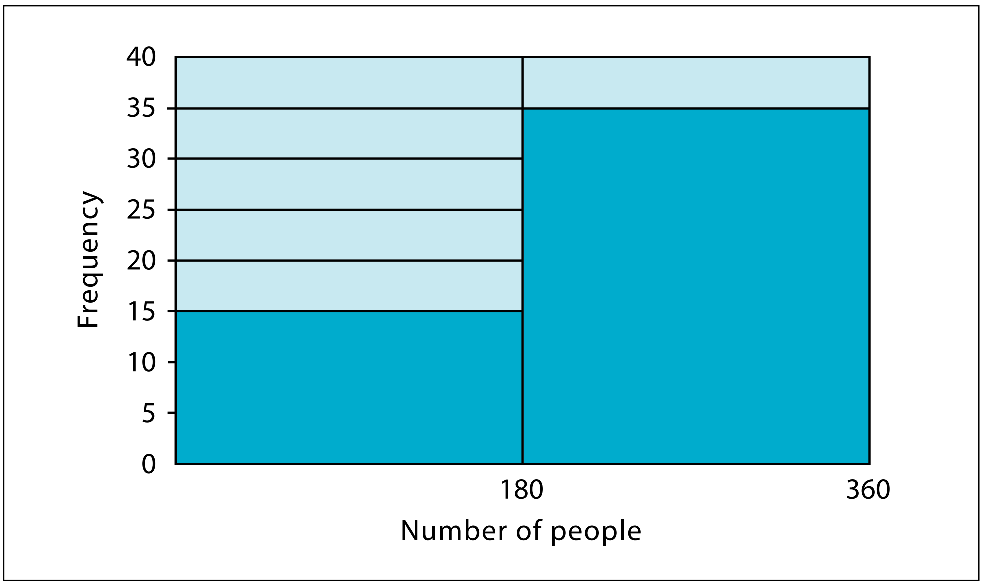
Leave some entries in the table blank, e.g., for the reasons to be added.

Provide hypotheses as cards to be placed in the 'more suitable' or 'less suitable' sections with a reason already in place.

'From broad to narrow'
Pupils engage with the consequences of decisions about grouping at the data collection stage.

Use the bar charts on Resource sheet: Collecting 2 page 48. Show the first chart with two bars:

Chart 1



and ask :

- If this chart is a picture of exactly how the data was collected what could the frequency table look like?
- What could the data collection chart look like?

Now show the second and third charts.

Chart 2

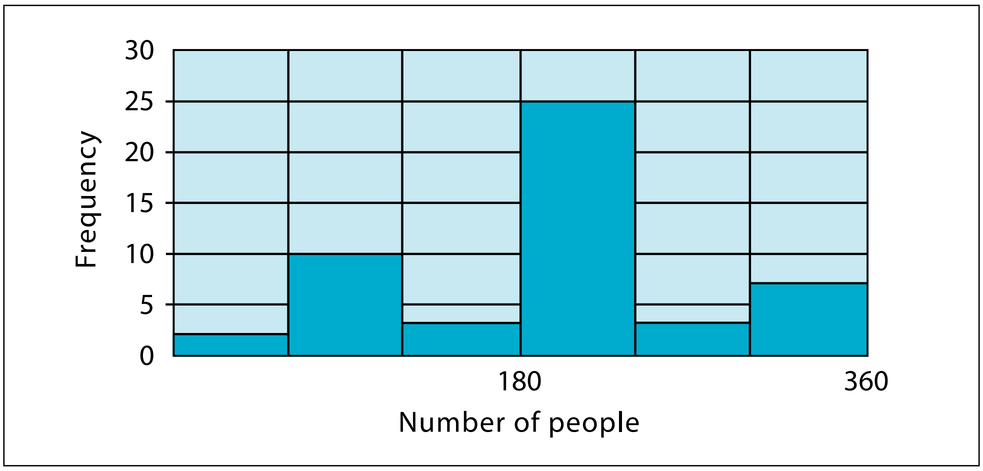
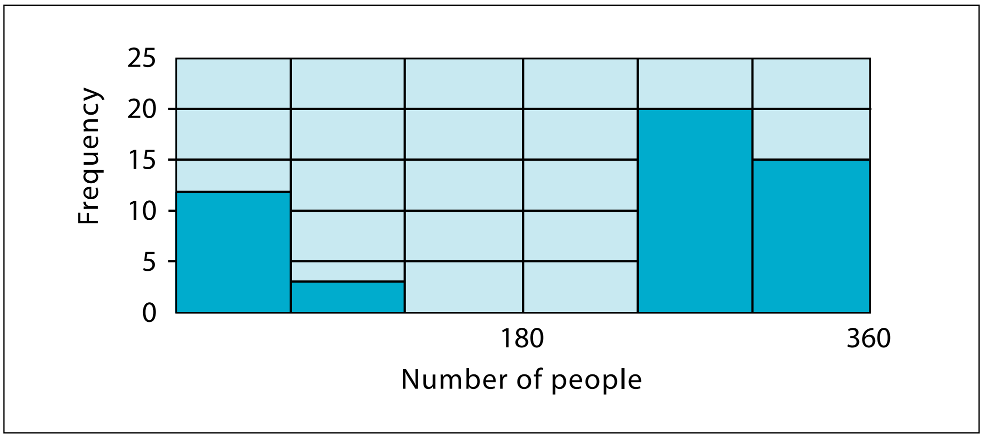


Chart 3



Ask which diagram represents the same data as that illustrated in chart 1. Allow pupils to work in pairs to present a reasoned argument. They should give reasons and examples to show why we cannot know for certain that chart 2 or 3 represent the detail beneath chart 1. Press them to be precise in their conclusions, perhaps choosing from the following stems:

The charts may have come from the same data set because...

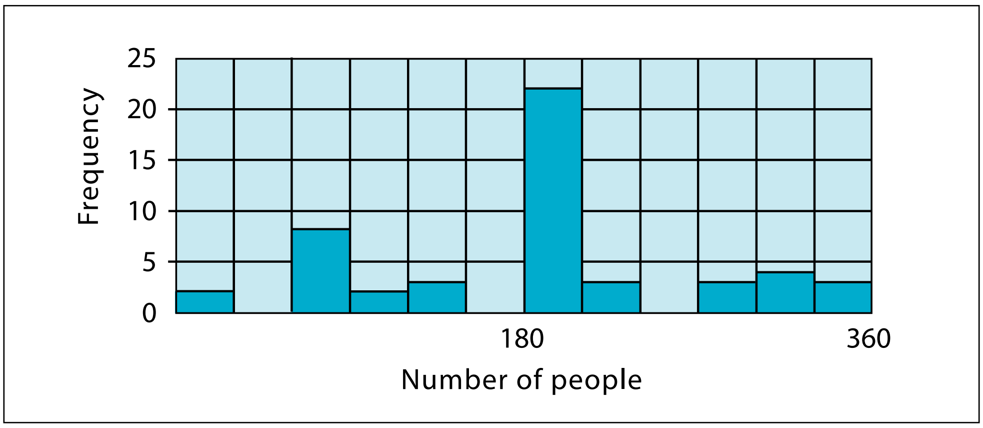
The charts may have come from different data sets because...

If we collect data in very large group intervals then...

'From narrow to broad'

Use the bar charts on Resource sheet: Collecting 2 page 48. Show chart 4.

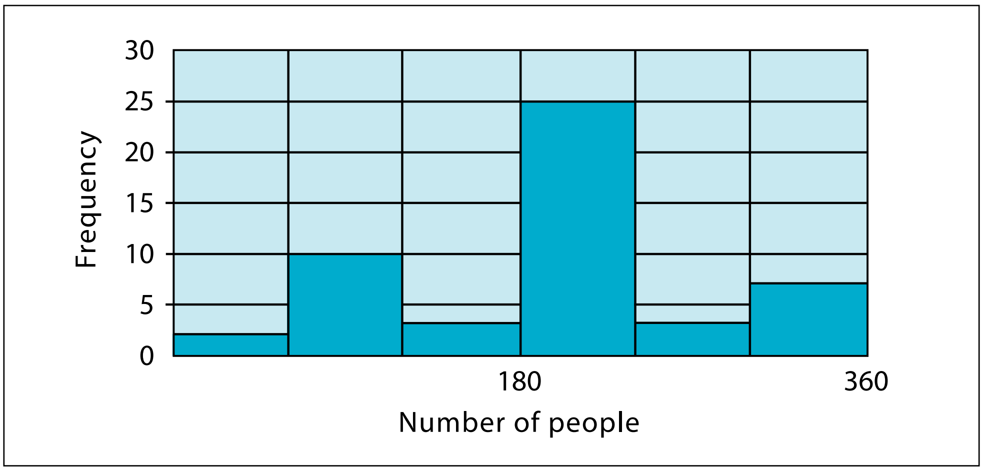
Chart 4



Ask :

- If this chart is a picture of exactly how the data was collected what could the frequency table look like?
- What could the data collection chart look like?

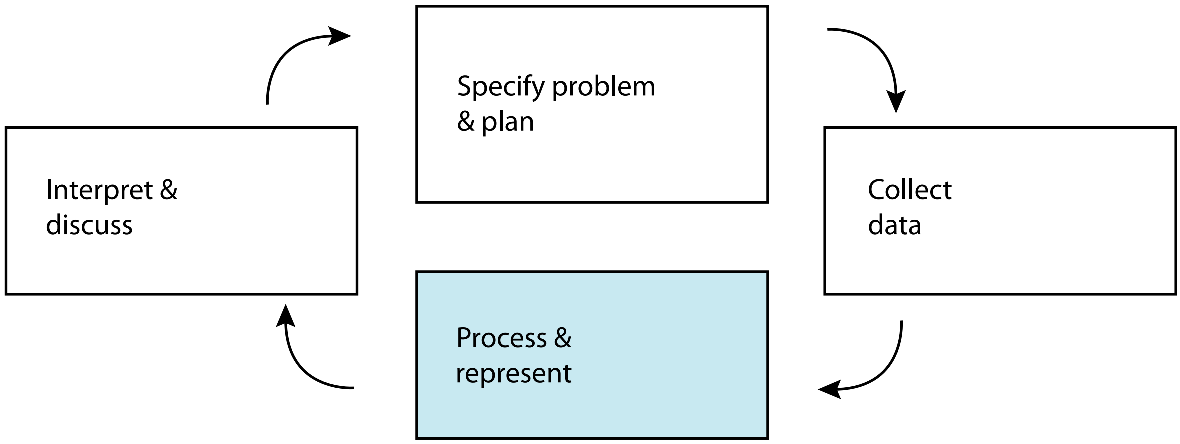
Chart 2



Show chart 2 and ask if this represents the same data. Allow pupils to work in pairs to present a reasoned argument for why, in this case, we know that this is certain. Press them to draw a conclusion about what they need to be cautious about when designing a data collection chart.

Consider what is lost and what is gained through grouping at the data collection stage. The simplicity and efficiency of collecting data in large group intervals is balanced against the fact that the groups cannot be 'undone', so some detail is lost and cannot be used at the interpreting stage.

Processing and representing data



Over the years analysis of assessment at the end of Key Stage 3 has indicated that teachers need to emphasise the difference between measures of average. To address this issue with pupils it is important to do more than ask pupils to calculate the mean of a number of different data sets and repeat for the median and other statistics. A context can help this understanding and this also means more than simply adding units or a heading to the tables of data which pupils are processing.

Using a context can be very effective in exemplifying the different statistics and so enhance the understanding. For example, changing headings and units on the original data after a calculation has been made can provoke pupils to reconsider and lead them to make a different choice of appropriate measure of location (average). When considering processing as part of the handling data cycle the contribution of each statistic to the testing of the hypothesis can be evaluated and compared. Again a key question to ask as processing and representing decisions are being made is:

What came before this stage and what could come after?

This forces a consideration of the meaning of the numbers in the context of the problem and also prompts pupils to consider what the calculated statistic or diagram offers in summary to the analysis and what detailed information about the initial raw data/data set is lost.

Visualising the numerical data on a number line can develop understanding of the value of different statistics and help pupils to estimate and justify the value of the measures prior to calculation. This thinking helps to check the outcome of processing the data and can stimulate thinking about the effects of translating the data set along the line or magnifying the values perhaps by changing the units from cm to m. These kinds of discussions are precursors to the notion of assumed means and to considering the effects of errors in the data.

The progression chart has been divided into separate charts for processing and representing to more clearly identify progression in these two aspects. The activities do not focus on the complexities of calculation or construction but rather on identifying and comparing different statistics and representations for the same data sets and different data sets for the same statistics and representations. These comparisons demonstrate the individual properties and relationships. Matching representation to context encourages sharing and developing understanding by justifying and challenging choice.

Processing Data	**Notes**
Calculate statistics for small sets of discrete data: • find the mode, median and range • calculate the mean, including from a simple frequency table.	Use a calculator or spreadsheet for a larger number of items.
Calculate statistics such as the range, mean, median and mode and, for grouped data, the modal class.	Use a calculator and spreadsheet where appropriate. Recognise and select the statistic(s) most appropriate to the context of the problem and the type of data.
Explore and summarise data.	
Estimate and find the mean, median, quartiles and interquartile range for large data sets.	
Explore data; including calculating an appropriate moving average for a time series.	

Representing Data	**Notes**
Construct, on paper and using ICT, graphs and diagrams to represent data.	• bar-line graphs • frequency diagrams for grouped discrete data • simple pie charts.
Construct graphical representations, on paper and using ICT, and identify which are most useful in the context of the problem.	• pie charts for categorical data • bar charts and frequency diagrams for discrete and continuous data • simple line graphs for time series • simple scatter graphs • stem-and-leaf diagrams.
Select, construct and modify, on paper and using ICT, suitable graphical representations to progress an enquiry and identify key features present in the data.	• line graphs for time series • scatter graphs to develop further understanding of correlations • cumulative frequency tables and diagrams box plots, graphs and lines of best fit (by eye) • histograms for grouped continuous data with equal class intervals and progressing to those requiring unequal class interval.

Further ideas can be found through the learning objectives and linked examples in substrand 5.2 'Processing and representing data' in the *Framework for secondary mathematics*, which is one of the sections found at www.standards.dcsf.gov.uk/nationalstrategies.

Further support is available through the Royal Statistical Society, Centre for Statistical Education in its publication *Data4Learning* by D. Connor and C. Turner, (Claire publications, ISBN 1 904572 17 0). The ATM disk *Integrating ICT into the mathematics classroom* (ISBN 1 898611 40 8) includes an application in the 'MMR file' which supports the style of activities described in this section.

ACTIVITIES

Classifying cards

Pupils build understanding about the nature of sets of data which have one feature in common. For example, a set of cards could be composed as below, such that some have the same mean but many other aspects are different such as the number of data items or the range of the data items. Allow pupils to classify the cards according to their own criteria and take feedback. Possible categories will include those cards which have:

- the same mean
- the same median
- the same mode
- the same range
- the same maximum value.

Encourage pupils to cross-reference or overlap their categories if that helps. One pair should interrogate another pair to check groups for accuracy and consistency.

6, 1, 4, 10, 20	8, 8, 8, 8, 8	10, 0, 10, 0
15, 0, 6, 18, 6, 2	8, 5, 13, 8	7, 3, 6, 4, 1, 9
2, 1.9, 10, 7.8	8, 10, 8, 10, 8	5, 5, 5
3¼ , 6½ , 8¾, 5½	0, 2, 8, 8, 8, 4	3 ¾ , 6 ¼
6.4, 6.1, 5.9, 3.2	8.3, 8.0, 8.1, 9, 8.0	2.3, 12.4, 2.7, 2.6

The *Framework for teaching mathematics: Years 7, 8 and 9* supplement of examples pages 260 and 261 extends this activity by asking pupils to consider incomplete sets of data and to complete them to match a given statistic.

Matching attributes of visual representations of data

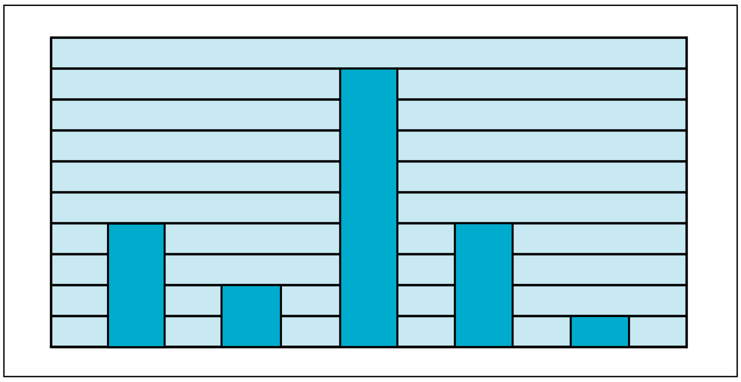
Pupils build mental pictures of the types of representation available to them when writing a report. This also helps in the process of diagnosing misconceptions and therefore assists teacher assessment.

From a set of separate cards of diagrams, names and situations, pupils could:

- match a diagram with a name and a situation (these might match more than one diagram)
- write down another situation to match the diagram
- discuss in groups gains and losses that happen when raw data is translated into diagrammatic form
- record in a table decisions made during the activity.

A set of cards and a table for recording is given on pages 50-52. (Resource sheets: matching 1 and matching 2.)

A matched set might look like this:



Bar chart

This represents the favourite terrestrial TV channels of Year 9 pupils

A bar chart could represent the frequency of how pupils travel to school

Clouding the picture

This is described and illustrated in *Teaching mental mathematics from level 5: algebra* (DfES 1287-2005PDF-EN-01). It is also shown as an example on page 53 of this document (Resource sheet: Clouding the Picture). In the context of handling data pupils are asked to express data sets in as many ways as they can while maintaining equivalence in some sense. For example, the title of the diagram could be 'Mean of 5' and all the data sets on the diagram would have that mean. In this case, ways of clouding the picture along a branch could be to:

- increase the number of data items in the set
- keep the number of items the same but change the range
- keep the median the same but vary the number of data items.

Pupils should be encouraged to be precise about the ways that they 'cloud the picture' along the branches and to keep the branches simple. See the example given on page 53.

Other diagrams might be titled 'Range of 3' or 'Median of 7' and the central starting point could be a small frequency table such as:

Ht (cm)	Frequency
5	4
6	4
7	10
8	2

In this case, the entries on the branches would involve adjusting frequencies for existing cells or introducing new cells while maintaining the equivalence stated in the title.

This task may cause pupils to reflect on what they actually know when they are given a mean value and what they do not know. They may also come to realise how extreme data items influence the mean value.

Sequencing

Putting cards of data sets in order of the size of their mean value is a starting point for this activity. Pupils could then be asked:

'How does the order change if the median value is used as the criterion for sorting?'

1, 12, 18	0, 0.8, 1.2
9, 10, 11, 12	5, 5, 5, 5, 5
¼, ½, ¾, 1½, 2½	

The activity could be used as a precursor to working with data sets which include extremes that distort an average or the range, such as wages in a company, or waiting times for buses.

A visual sense of this can be gained from the presentation slides to support the 'Photos' activities in *Securing progression in Handling Data* – school pack DfES 0656–2003; presentation slides DfES 0662–2003CD. Pupils can get a feel for how the values in the data sets may have a bearing on the most suitable type of average to use.

What happens if a constant is added to each data item?

Pupils could consider applying a constant to each number in the data set and studying the effect this has on an average. Pupils could explore this in a diagram and explain or generalise about why the effect occurs.

Mean = 4	Mean =	Mean =
1, 3, 5, 7	11, 13, 15, 17	

- What happens to the mean if 10 is added to each data value?
- Explain why this happens
- Use this effect to find the mean of 62, 65, 68, 69
- Try starting a chain with the same values and doubling them each time
- Do the same effects happen when finding the median or the mode?

Discrete grouping: what does it show?
This is a task which looks at different ways of grouping the same set of discrete data and asks what effect this has on the interpretation of the data as shown in a chart. Use frequency charts such as those on Resource sheet: Collecting 2, page 48.

Chart 1

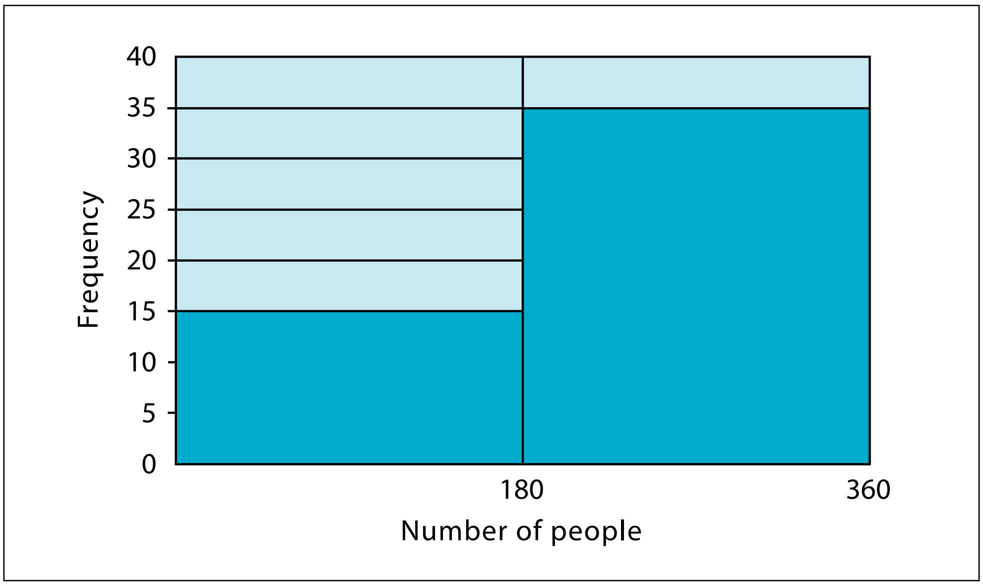
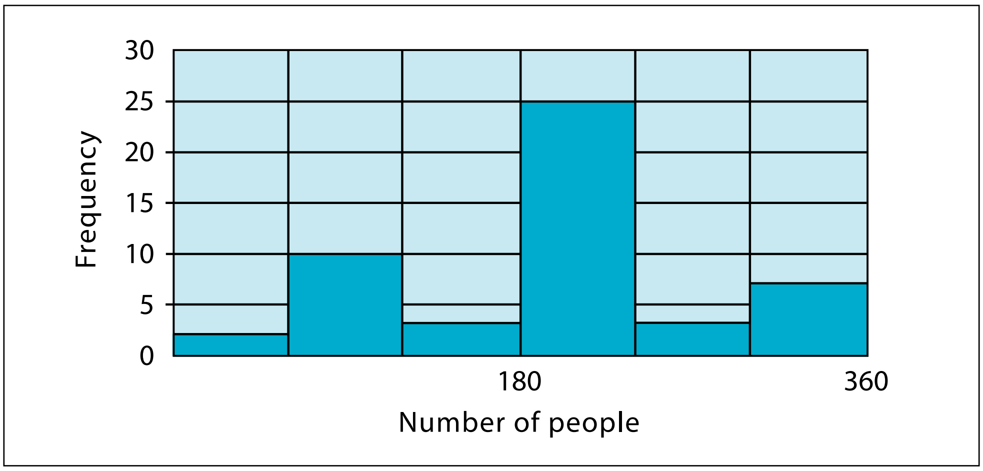


Chart 2

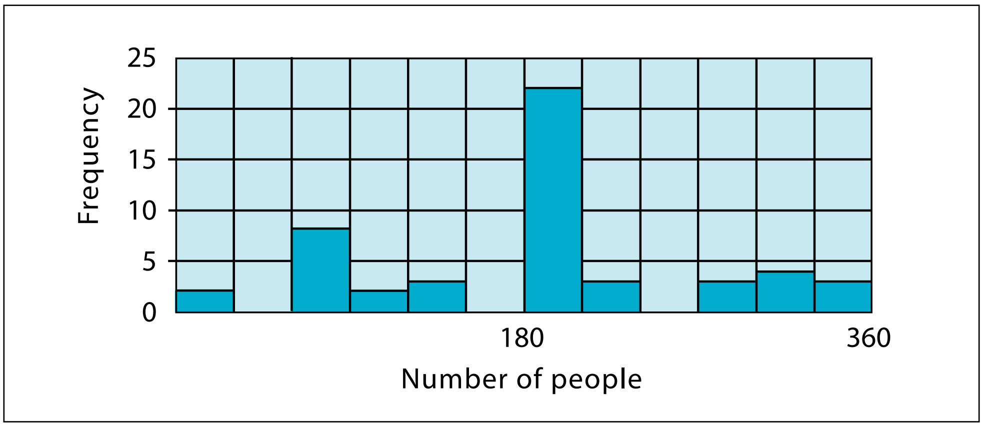


Tell pupils that the data is the same in both charts and ask pupils to discuss in pairs:

- What does the first chart show that the second one does not?
- What question could the first chart help to answer?
- What does the second chart show that the first one does not?
- What question could the second chart help to answer?

Now show the third chart and ask when this chart might be a better choice than the other two.

Chart 4



Pupils could come to see that the representation in a chart emphasises key features and that grouping hides or reveals detail in the data. The decision about grouping depends on the question that is being asked.

Consider whether it is possible to calculate an accurate value for the mean, median and modal class if the data is only represented in the groups as shown and the calculation cannot be done from any greater detail. What is the effect on the estimate of these values by the different grouping?

Ask pupils to decide a context and compose statements involving values for the mean, the median and the mode which would result from calculating from each of the different charts. Stems could be offered as a start point.

The most common attendance was between... and...

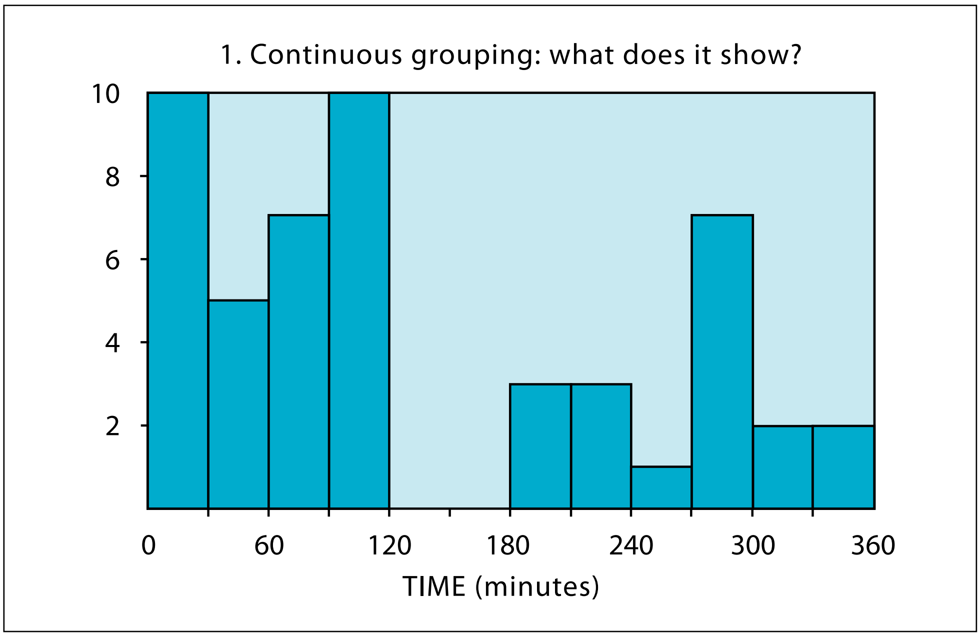
Half of the attendances were greater than...

The mean attendance was...

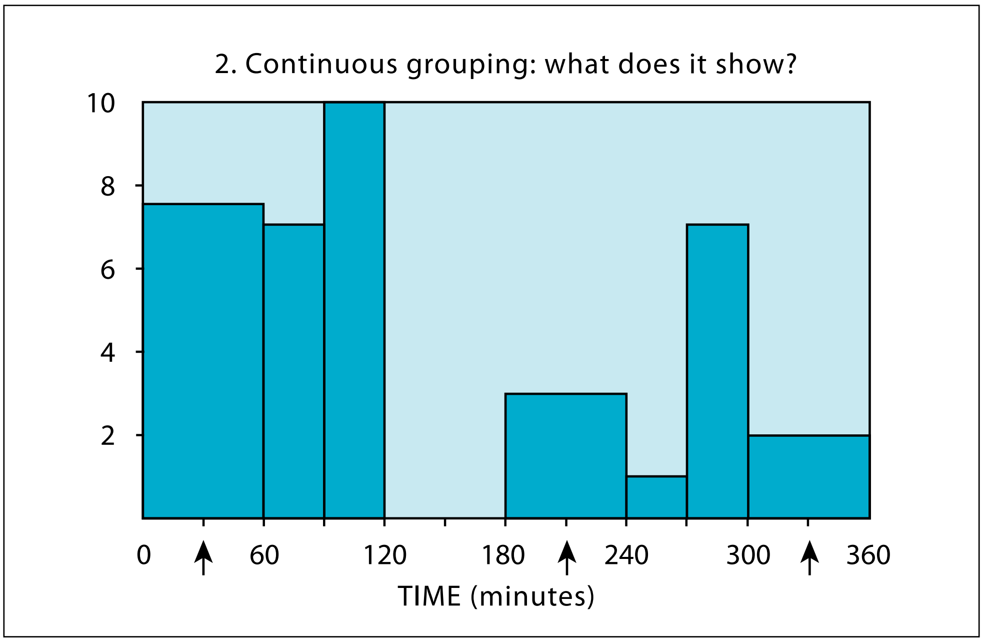
Pupils' statements could be pulled together into a summary discussion about the effect of grouping on measures of average. A discussion of this kind can help pupils to see that grouping reduces the accuracy with which they are able to estimate or calculate averages.

Continuous grouping: what does it show?
This is a task which looks at different ways of grouping the same set of continuous data and challenges pupils to establish the conventions of representing the data on a histogram.

Show chart 1 and ask pupils to speculate about the source of the data, the overall title and the scale and label on the vertical axis.



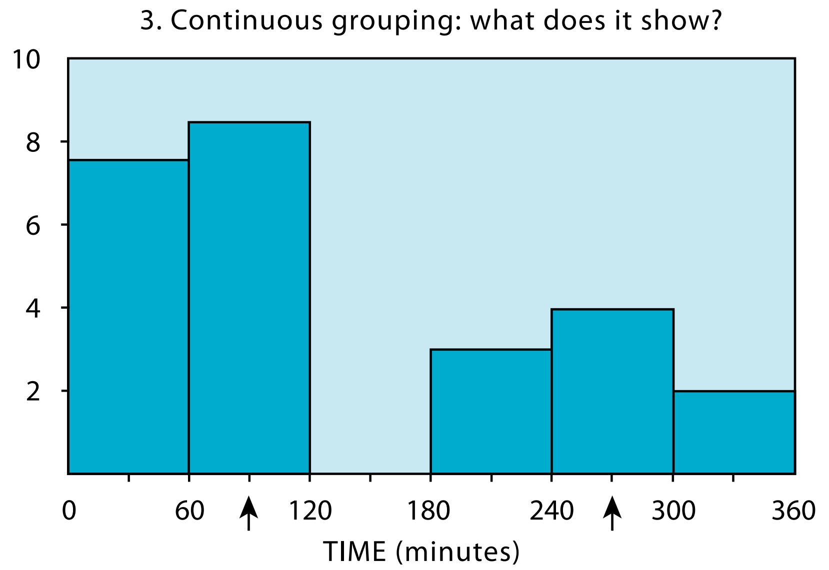
Now show chart 2. Say that it represents the same data and ask what changes the arrows are indicating.



Ask pupils to discuss in pairs:

- Does this seem like a fair representation of the data?
- How does this change the scale and label on the vertical axis?

Show chart 3. Say that it represents the same data and ask what changes the arrows are indicating.



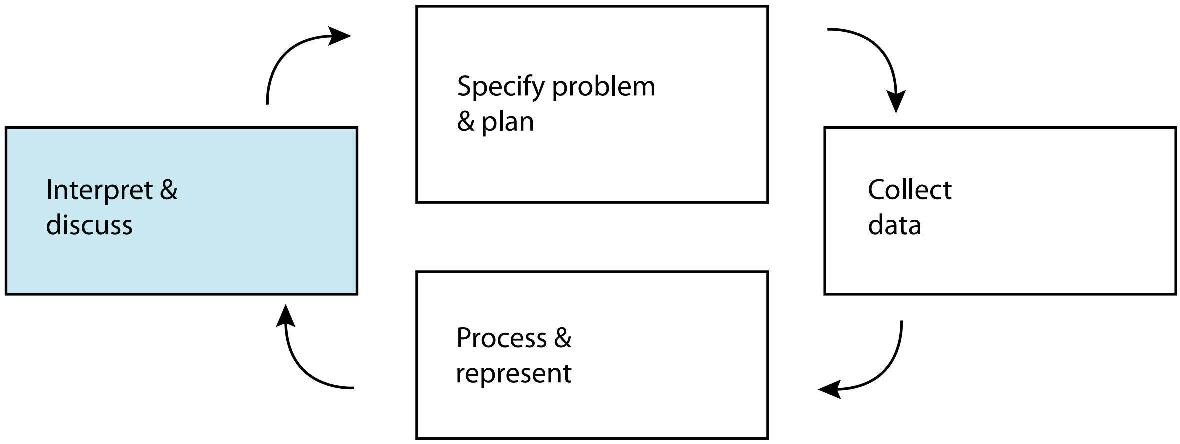
Ask pupils to discuss in pairs:

- Does this seem like a fair representation of the data?
- How does this change the scale and label on the vertical axis?

Finally, ask when and why it might be useful to group the data in different ways.

Constructing and interpreting histograms with unequal intervals is exceptional performance as described in the National Curriculum. Approached in this way, however, the idea can seem a natural convention through which to represent continuous data. Sowing early seeds of big ideas can accelerate understanding when these topics appear for more formal treatment later in the scheme of work.

Interpreting and discussing results



The purpose of any enquiry is to draw conclusions based on robust evidence. It is at the fourth stage of the handling data cycle that results are interpreted and evaluated in order to confirm or challenge the hypothesis. This may identify the need for further refinement or a repeat of other elements of the cycle. It is important to emphasise to pupils that this final stage is critical to the success of the enquiry and that it is dependent on the quality of the outcomes of all the previous stages. Effective interpretation is based on two aspects:

- The identification of key features within the representation and processing of the data.
- The interpretation of these features within the context being considered.

Both of these, especially the second, require the pupils to work mentally in linking such features to previous experiences/knowledge and use them to 'summarise' the outcomes.

Presenting a new form of representation can in itself be very stimulating and pupils should have this opportunity. However, the challenge is to choose representations and perform calculations which help communicate the key features to as many people as possible and pupils must be open in their thinking. This is helped by sharing ideas, discussing their representations and hearing other interpretations.

If pupils know the purpose of the problem they are tackling then they are more likely to make effective decisions about how they should interpret and communicate a solution; for example, producing a report to the school's leadership team.

The skills of interpretation can be honed through dialogue and by drawing out both the general and the extreme. This is a strong argument for interpreting pre-drawn graphs and charts before the skills of construction have been developed. A diagram that can be used to develop skills of interpreting through charts which pupils are unlikely to have seen before is illustrated on Resource sheet: Interpreting on page 66. It is an extract from:

- Representing Mortality levels (Florence Nightingale).
  Developing Thinking in Statistics, page 73, Alan Graham, Open University ISBN 1 4129 1167 2.

The progress chart for interpreting and discussing results has been separated into:

- Interpreting results
- Comparing results
- Communicating results.

The activities in this section focus primarily on interpretation and to some extent communicating conclusions. Matching diagrams, graphs and charts to particular interpretations and to one another forces attention on key features of the data, how we see them and the different ways they can be revealed.

Interpreting results	Notes
Interpret diagrams and graphs, and draw simple conclusions based on the shape of graphs and simple statistics for a single distribution.	Include pie charts.
Interpret tables, graphs and diagrams for discrete and continuous data, relating summary statistics and findings to the questions being explored.	
Interpret graphs and diagrams and make inferences to support or cast doubt on initial conjectures; have a basic understanding of correlation.	
Analyse data to find patterns and exceptions, and try to explain anomalies.	Include social statistics such as index numbers, time series and survey data.
Interpret and use cumulative frequency diagrams to solve problems.	
Use, interpret and compare histograms.	Include those with unequal class intervals.

Comparing results	Notes
Compare two simple distributions using the range and one of the mode, median or mean.	Increase the challenge by moving to more complex distributions or by increasing the numbers of distributions to be compared.
Compare two or more distributions and make inferences, using the shape of the distributions and appropriate statistics.	Increase the challenge by including various (suitable) measures of average and spread, including median and quartiles.

Communicating results	Notes
Write a short report of a statistical enquiry, including appropriate diagrams, graphs and charts, using ICT as appropriate; justify the choice of presentation.	Start with a short written report and move to oral presentations and more detailed written submissions with justifications.
Review interpretations and results of a statistical enquiry on the basis of discussions; communicate these interpretations and results using selected tables, graphs and diagrams.	Justify the choices included in presentations, with reference to the purpose of the task, the audience and to the data itself.
Examine critically the results of a statistical enquiry; justify choice of statistical representations and relate summarised data to the questions being explored.	Make decisions about which type of tables, graphs or diagrams is best suited to the task in the light of the intended audience.
Recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis.	

Further ideas can be found through the learning objectives and linked examples in substrand 5.3 'Interpreting and discussing results' in the *Framework for secondary mathematics*, which is one of the sections found at www.standards.dcsf.gov.uk/nationalstrategies.

ACTIVITIES

Say what you see
This activity helps pupils to understand the process of drawing graphs. Use a set of cards which could include for discrete data:

- bar charts
- pie charts.

Or for continuous data:

- histograms
- cumulative frequency graphs
- percentage charts
- box plots.

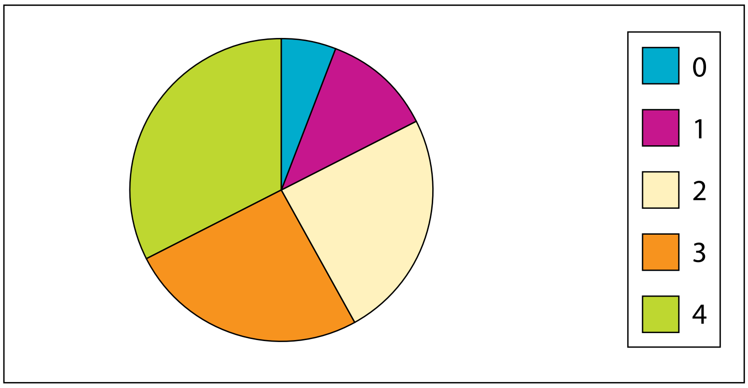
Annotating blank charts, to add the missing title, labels and graduation to the axes is suggested so that the pupils can 'tell the story of the graph'. Subsequent discussions can provide the chance to revisit and explain target vocabulary such as mode, median, mean and range in the context of the data as suggested by the pupils.

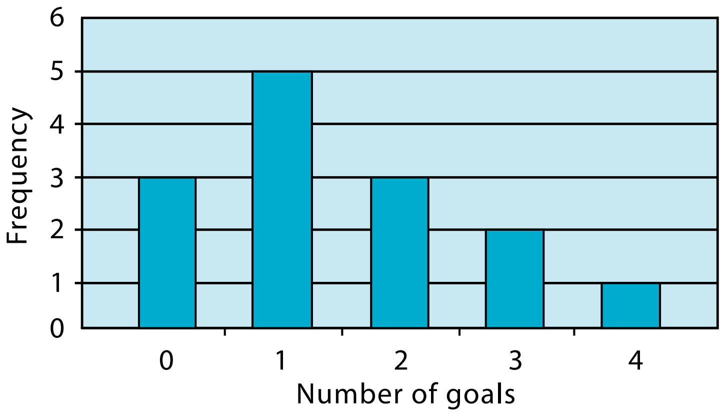
This task is open to many levels of response and can be easily adjusted to suit the existing knowledge of the pupils.

Diagrams annotated from the discussion of 'gains and losses' provide excellent displays. The display can be made interactive by providing additional graphs and charts and posing questions alongside them.

Matching pairs

This task takes different forms of graphical representation and challenges pupils to interpret each chart. As pupils try to match the different forms in a collection of cards they must ask themselves whether, for example, the bar chart and pie chart shown below represent the same data.





See Resource sheet: Matching pie charts and bar charts pages 55-58. (Note that the first pair of charts do not match.) The task is made more challenging by reducing the labelling on the charts.

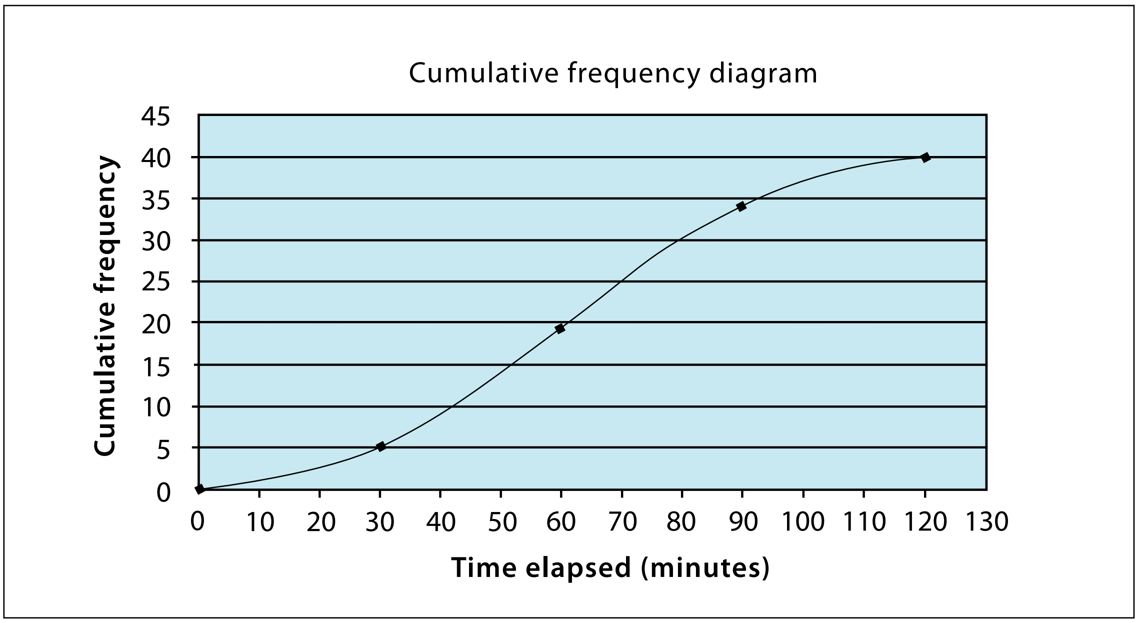
As an extension to the task pupils could be asked to sketch a bar chart from a given pie chart or a pie chart from a given bar chart.

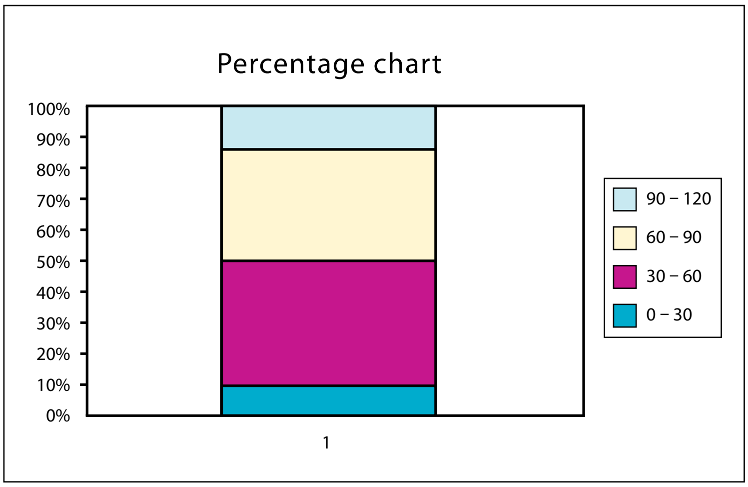
Matching fours

This is principally the same as the preceding task. This time the data is continuous and the charts more complex including, for example the same data represented by:

- a histogram
- a percentage chart
- a cumulative frequency graph
- a box plot.

Again this is made more challenging by reducing the labelling on the charts. As pupils try to match the different forms they must ask themselves, for example, whether the cumulative frequency graph and percentage chart shown below could represent the same data. Feedback on this task should include discussion around the importance of labelling the grouped continuous data correctly on the histogram and the relationship between the group frequencies and the steepness of sections of cumulative frequency graph.





Resource sheets: Matching more complex graphs, on pages 59-65, illustrate representations of four data sets. Each set is illustrated through a histogram, a cumulative frequency graph, a percentage chart and a box plot.

An extension question could be, 'Given the estimated nature of the medians and quartiles, what other box plots could represent the same data as the cumulative frequency diagrams?'

As an extension to the task pupils could be asked to sketch an alternative graph given one particular form. For example:

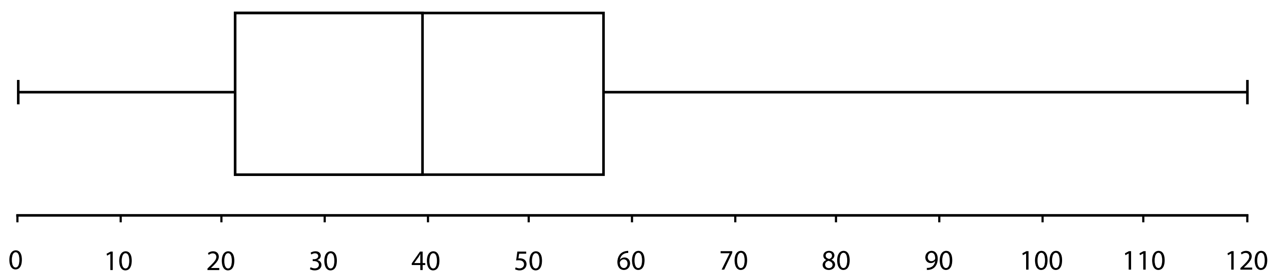
- sketch a cumulative frequency graph given a histogram
- sketch a box plot given a percentage chart.

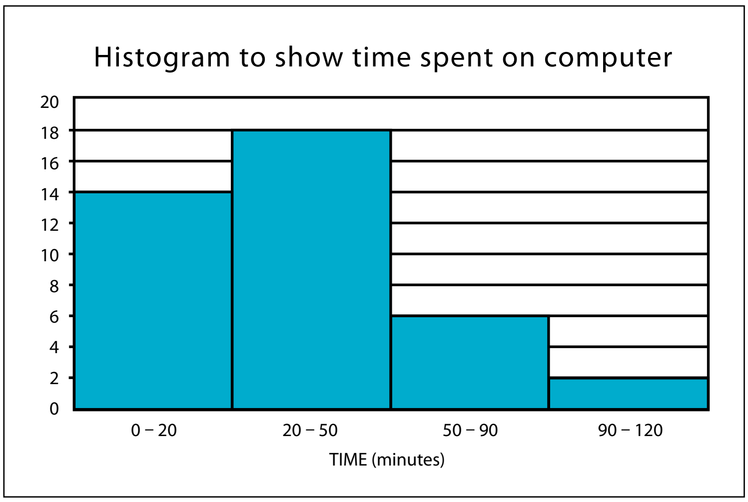
In this way pupils have to visualise a connection between the various graphs and begin to predict the shape of one from the other.

What do you lose, what do you gain?
This is an activity based on the same set of resources as the matching tasks. This time, ask pupils to discuss what are the gains and losses of choosing one graph or chart rather than another. For example:

- Which graph is most appropriate, say for estimating the median?
- What detail is lost by representing data in a box plot?
- What can be easily seen by representing data in a box plot?

Wise Words
This is a versatile task using the same set of resources as the matching tasks. It is suitable for developing understanding of most visual forms, including graphs and charts. Pupils work in pairs with a set of cards. Pairs compose one or two statements to describe a card, which another pair must try to identify. Each statement may use only one key word and may not say what form the graph takes (for example, whether it is a box plot or a percentage chart). Each pair passes their statements to another pair, for them to work out which card is being described. The second pair can write one 'yes/no' question and, after getting a response to this, they must identify the card.

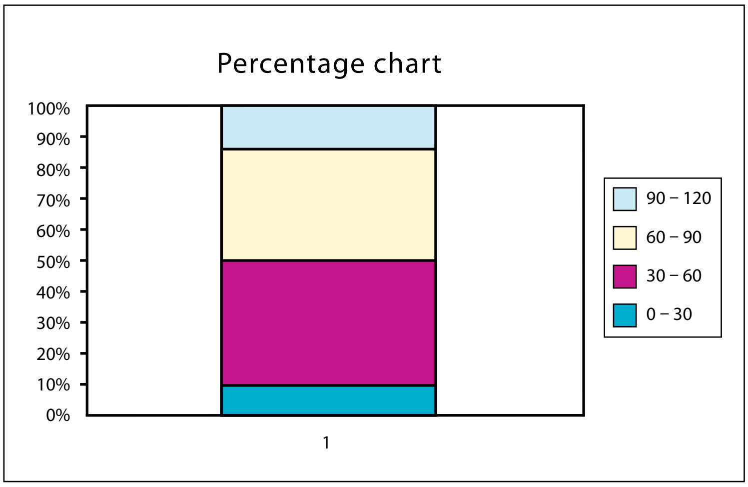




'What lies beneath'

This is a task which asks how much detail is needed in order to convince or inform through a statistical diagram. In essence it is about audience and purpose. It engages pupils in the decision-making stages of drawing a statistical chart. Present each stage as a layer of detail underneath a final presentation. Start with the final layer and ask 'What lies beneath?'

Use any of the charts on resource sheets, 'Matching more complex graphs', for example, a percentage chart.



Say that this is the final layer; it is stripped of some of the detail in order to focus on key features, but to be mathematically correct the detail must lie beneath the surface. Encourage pairs of pupils to find more than one answer to the following:

- What could have been the title on the layer beneath and what would the key have said?
- What could have been the grouped frequency table on the layer beneath that and what could the groupings have been?
- What could the collection chart and the raw data on the layer beneath that have looked like?
- In a sense this task is similar to 'Say what you see' and 'What do you lose, what do you gain?', on page 33.

In this context however, the emphasis is on the layers of processing and representing that are hidden beneath a final form. In particular pupils should think about the decisions which are made according to the audience for the data. Clippings from newspapers, magazines and journals could further inform this discussion. Emphasise that mathematical rigour and consistency underpin the statistical process even though this may not be evident in the final 'layer'.

Discuss the difference between the detail we need to present in an examination to show that we can make the correct mathematical decisions and the detail we might present in order to convince or inform another person.

Probability

'What is the probability that it will rain in this area tomorrow?'

The challenge of capturing hearts and minds when considering the future or when engaging in statistical analysis is best tackled through dialogue so that pupils can consider and articulate their thinking and teachers can tap in to their reasoning. With this in mind, perhaps a better start point might be:

'In this area it may rain tomorrow or it may not, and yet the probability that it will rain tomorrow is generally not 50% – discuss and explain your thinking with evidence.'

The expectation that pupils support the value they choose for a probability with evidence gives them the opportunity to reveal their understanding. This kind of reasoning with evidence needs to be modelled through critical dialogue between the teacher and the pupils.

One way to help pupils develop the skills of reasoning and explanation is to work more frequently on the mental aspects of handling data including probability. Explicit links can be made between the handling data cycle and the way we work with probability. There are useful parallels with the cycle both in examples where we use an experiment to find the estimate for a probability or where we solve a problem using theoretical probabilities. In addition teachers need to plan for discussions which compare theoretical and experimental methods: their appropriateness, drawbacks and advantages in particular circumstances.

It is clear that skills in using fractions, decimals and percentages as part of a probability calculation need to be considered as a precursor to tackling probability problems. In the main, however, progress in probability depends largely on understanding ideas, rather than acquiring further skills, as the following charts show. Finally, as probability is a measure of what might happen, it is important to carefully choose language so that the event described is placed in the future. For example, 'What is the probability that I will score 7 on two dice?' makes more sense to pupils than, 'What is the probability that I scored 7 when I rolled two dice?'

Theoretical Probability	Notes
Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally-likely outcomes in simple contexts; identify all the possible mutually-exclusive outcomes of a single event.	Develop language and communication by using the language of probability, drawing on pupils' experience; appreciate that random processes are unpredictable.
Know that if the probability of an event occurring is p then the probability of it not occurring is $1 - p$; use diagrams and tables to record in a systematic way all possible mutually-exclusive outcomes for single events and for two successive events.	
Identify all the mutually exclusive outcomes of an experiment; know that the sum of probabilities of all mutually-exclusive outcomes is 1 and use this when solving problems.	Use tree diagrams to represent outcomes of two or more events and to calculate probabilities of combinations of independent events.
Recognise when and how to work with probabilities associated with independent and mutually-exclusive events when interpreting data.	Recognise when events are independent and distinguish between contexts involving selection with and without replacement.

Making connections	Notes
Compare experimental and theoretical probabilities beginning with simple contexts. Increase the range of contexts and, from this range appreciate the difference between mathematical explanation and experimental evidence.	Solutions to include the language of probability as part of more complex reasoned solutions. Move from simple problems to more complex contexts.

Experimental probability	Notes
Estimate probabilities by collecting data from a simple experiment and recording it in a frequency table;	Develop language and communication by using the language of probability, drawing on pupils' experience; appreciate that random processes are unpredictable.
Compare estimated experimental probabilities with theoretical probabilities, recognising that: • if an experiment is repeated the outcome may, and usually will, be different • increasing the number of times an experiment is repeated generally leads to better estimates of probability.	
Understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.	

Further ideas can be found through the learning objectives and linked examples in substrand 5.4 'Probability' in the *Framework for secondary mathematics*, which is one of the sections found at www.standards.dcsf.gov.uk/nationalstrategies.

ACTIVITIES

Matching

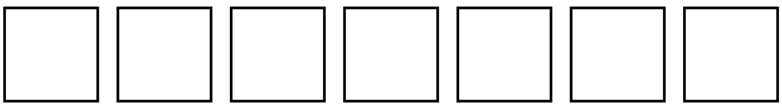
Linking different circumstances to a given probability is an activity based around the number and colour of otherwise identical counters in a bag. This engages pupils in working out the possible number and range of colours of counters in a bag given a certain probability such as those shown below. Initially the work is in pairs moving to larger groups to share thinking.

P(Red) = $^1/_2$	P(Red) = $^1/_2$ and P(Blue) = $^1/_2$
P(Red) = $^1/_2$ and P(Blue) = $^1/_4$	P(Blue) = P(Green)
P(Blue) = P(Red) and P(Green) = $^1/_2$	P(Red or Green) = $^2/_5$
P(Yellow) = $^1/_2$ and there are 6 red counters	P(Red) = $^3/_7$ and P(Green) = $^1/_3$
P(Green) = $^1/_4$ and there are at least 8 yellow counters	P(Pink) = $^1/_5$ and there are 4 different colours

Together pupils should seek to find as many ways as they can of responding to the task, discussing results as a whole class with pupils taking on a critical role to discern similarities and differences between the solutions and to deduce the important features of the counters in the bag in order to satisfy the given probability. In other words, the joint thinking gives them the opportunity to generalise the solutions.

To simplify the task, the number of possible colours could be limited. To extend it, consider giving the probability of an event not occurring, for example P(not Red) = ½

Which chair: trees to grouping branches.



This is a simple scenario which produces some unexpected results and so promotes further thinking about calculating combinations of outcomes.

One pupil sits on the middle chair of a row of seven:

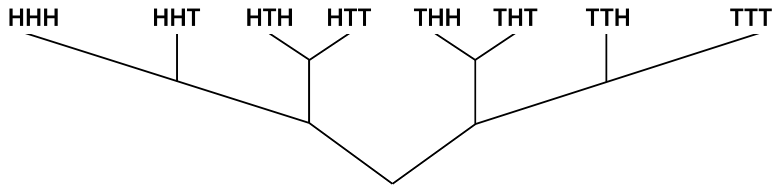
- an unbiased coin is flipped
- a head means move one chair to the left
- a tail means move one chair to the right.

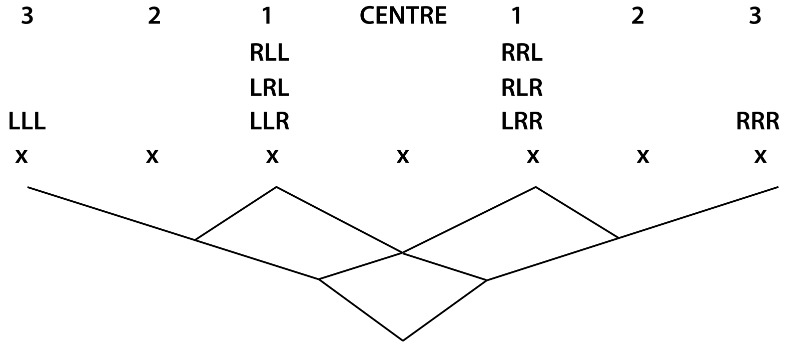
Repeat the process twice more.

Pupils work in pairs to answer the question:

- How many of the chairs is it possible to finish on after the three flips of the coin?

A 'tree diagram' could be used to build on the movement and visualisation to identify all possible sets of movement. It is interesting to discuss with pupils how the two forms of diagram both illustrate different aspects of the problem; see Resource sheet: Which chair? on page 67.





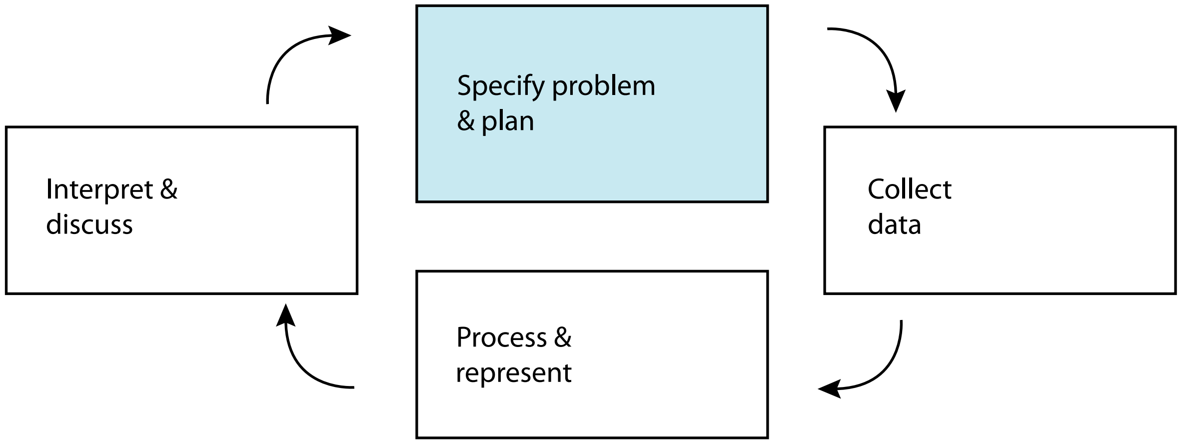
The ability to find and record all possible outcomes for successive events or a combination of two or more experiments is essential if pupils are to understand, find and use probabilities or estimates for probabilities in more complex situations.

ACTIVITIES

HD cycle and experimental probability

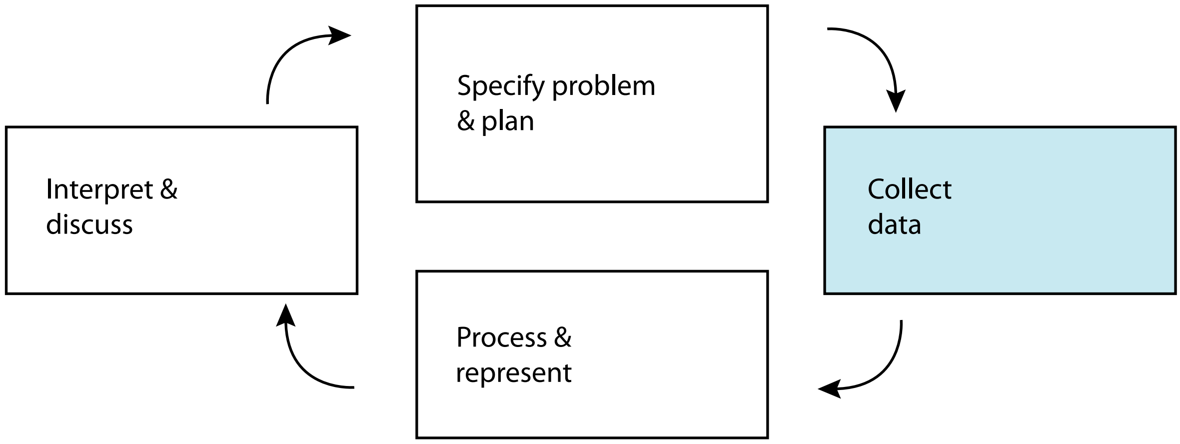
This is a longer activity but an important one because it reinforces the links between the handling data cycle and probability.

Dropping the drawing pin



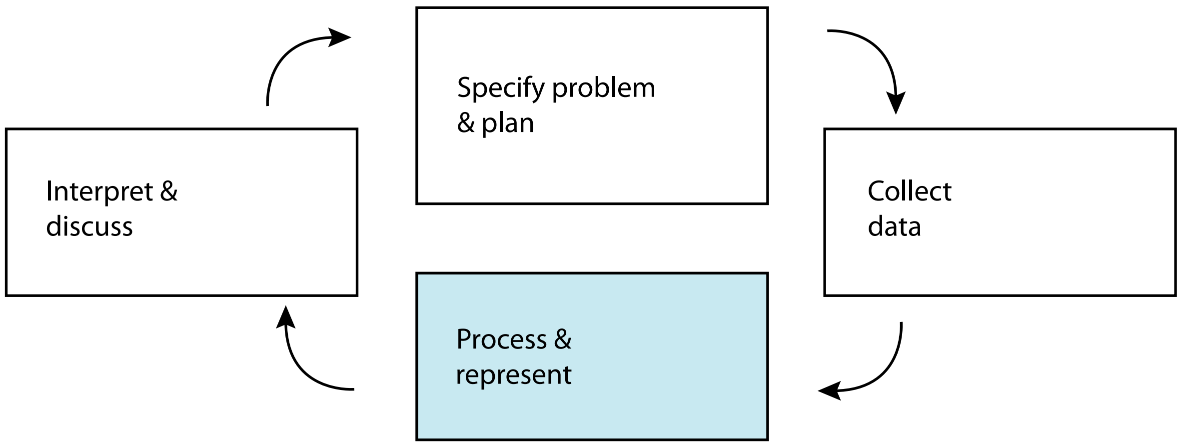
Address this aspect of the cycle by estimating the probability of landing 'point up' and discussing and explaining initial ideas. Ask:

- Will it land 'point up' or 'point down'?
- How often will it land 'point up'?
- How can we find out?



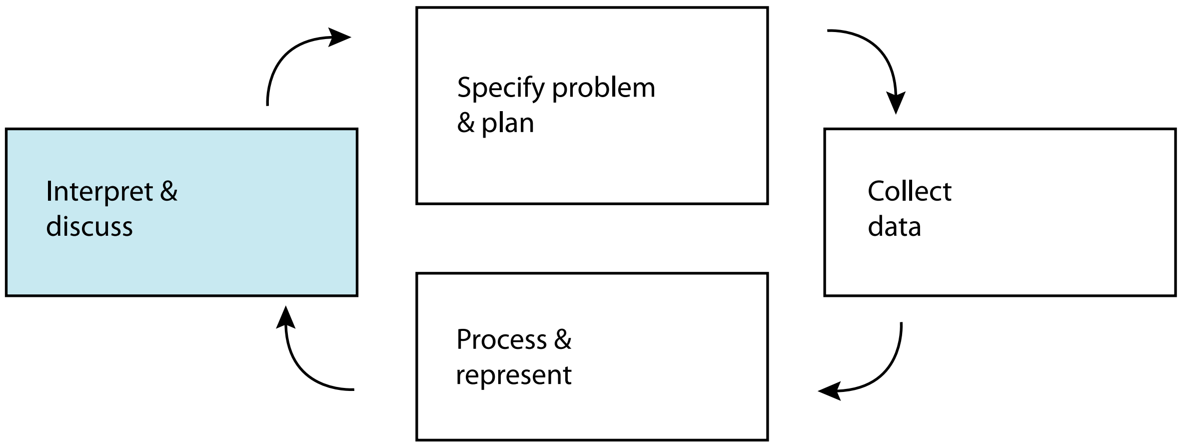
Pupils work in groups to perform a batch of trials, perhaps ten, and note the frequency of 'point up' and 'point down'.

- Have all the groups got the same frequencies?
- Is this what we would expect? Why?



Use further trials to produce estimates of the probability based on relative frequency. Plot these on a graph showing probability estimate against number of trials.

- Have all the groups got the same estimate?
- Is this what we would expect? Why?



Match the result against the hypothesis and note any trend and limiting value of the estimate. Decide whether to repeat with more trials, if so go to the collect data stage again.

Finally pupils decide further trials are not needed, results are comparable and limiting values are reached.

- How do we explain what has happened?
- Can we generalise this idea?
- If we dropped a thousand pins or one pin a thousand times what could happen? What would we expect?

The handling data cycle can be used to find other estimates of probability experimentally. There is minimal calculation or representation. Most of this task is about thinking through the probability as it emerges from a large number of trials. Generating the combined graph using ICT provides a very powerful interactive image.

ACTIVITIES

Using a probability fact

Two bags A and B contain identical coloured cubes. Each bag has the same number of cubes in it. An experiment consists of taking one cube from the bag.

- The probability of taking a red cube from bag A is 0.5.
- The probability of taking a red cube from bag B is 0.2.

All the cubes are put in an empty new bag.

- What is the probability of taking a red cube out of the new bag?

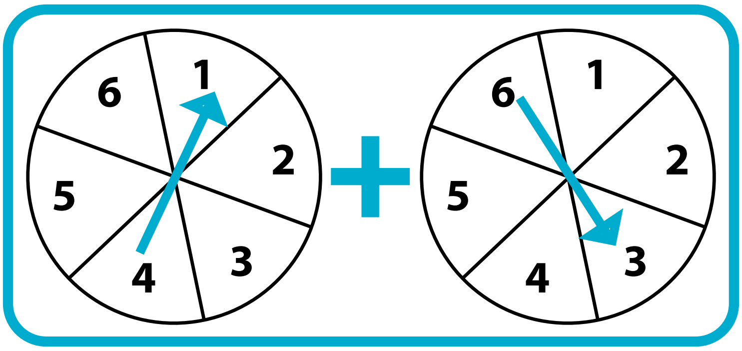
Pupils should individually write down a 'gut' response and then compare their answers in small groups. The use of specific examples to answer the above will be useful but pupils need to share these and be encouraged to generalise.

- What happens if the probability of picking a red cube is the same for both bags?
- What happens if you change the probability of picking a red cube from each bag?
- What happens if you change the number of red cubes in one bag? In both bags?

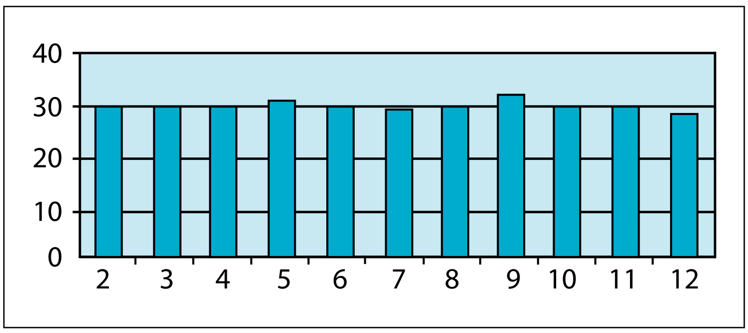
All stages of this problem demand that pupils identify the facts surrounding a situation. It has the potential to reveal misconceptions around probabilities of related events and offers the opportunity to generalise an outcome where the intuitive response is often incorrect.

Likely results?

This activity takes a context and a set of results and asks pupils to assess the likelihood of the given context producing the suggested results. Show the two spinners (larger version on Resource sheet: Spinners on page 68).



The results of spinning these spinners many times and adding their scores together are represented on a bar chart. Show one or more of the bar charts from Resource sheet: spinner outcomes on page 69.



Ask pupils to work in pairs or threes to decide whether the bar chart could represent these results. Ask them to give reasons for their response and to rate the match as more or less likely by placing the chart on a probability continuum such as:

Impossible **Unlikely** **Likely** **Certain**

Sequencing

Sequencing events according to their probability can reinforce the usefulness of the probability line as well as stimulating discussion about the relative chance of different events.

The probability of getting a tail and two heads when three coins are flipped

The probability of getting a multiple of 3 when one dice is thrown

The probability of getting at least one six when two dice are thrown

Give pupils a selection of statements on cards and ask them to sequence on a probability continuum such as this

Impossible **Unlikely** **Likely** **Certain**

0 **1**

The task gives practice in assessing an awareness of the outcomes which are possible in each context. Pupils may choose to calculate or may wish to illustrate some of the outcomes. Either will help to justify their ranking of the events relative to one another. We are sometimes expected to appreciate the chance of one event relative to the chance of another, quite different event, for example, 'You are more likely to die crossing the road than…'

Always, sometimes, never

This task challenges pupils' understanding of probability statements. Prepare some cards, for example :

It is harder to throw a 6 on a dice than a 2

If there are more blue counters in bag A than in bag B you are more likely to draw a blue from bag A than bag B

If a dice is thrown 6 times I am just as likely to get 1,1,1,1,1,1 as 1,2,3,4,5,6

The probability that a hockey team wins is one-third as they can win, lose or draw

Ask pupils to make decisions and give reasons about which statements are always true, which are sometimes true and which are never true. For those which are sometimes true could they be adjusted to make them always true?

A conundrum?

Three spinners A, B and C are designed so that each has three equal sectors which are marked up by placing one of the digits 1 to 9 on each sector using each digit once only. How should the digits be placed so that A has a better chance of getting a higher score than B; B has a better chance of a higher score than C; and C has a better chance of getting a higher score than A?

The puzzle may appear impossible on first impression but it is perfectly accessible to logical trial and improvement. It forces us to consider the language very carefully and what the question means.

Pupils will find that having a systematic way of representing the different outcomes is very helpful in keeping track of trials. It is important that pupils make these choices and come to realise the relative merits of lists, two-way tables and tree diagrams.

Engaging with this problem challenges our understanding of probability and the fact that it does not operate as a simple unit of measure. The key is to consider all possible outcomes.

Resource sheet: Collecting 1

To investigate the statement. 'Children no longer do sport', all the children at one school in Leeds were surveyed.
Heather wanted to know how much people were prepared to spend on Christmas presents for their families. She asked the people on the street where she lives.
To investigate the usage of mobile phones, people were stopped in the shopping centre on a Saturday afternoon.
A survey to find out about the popularity of a sports centre was conducted by telephoning local people during the day.
In order to study obesity in teenagers, a random sample of teenagers was asked to fill in a questionnaire.
To find out attitudes on the price of bus fares, an interviewer stopped people in a local shopping centre one weekday morning and asked their views.
Clare believes teachers spend less money on cars than golfers. She asks all the people arriving at the local golf club on a Saturday morning what car they drive.
People who eat breakfast do better in their examinations. Raj asks a selection of staff at his local comprehensive.
To investigate the popularity of goth music, a group of friends interviewed people dressed in black.
Young people are not interested in politics. A survey was conducted at a city shopping mall.
'Vegetarians don't like fox hunters!' John investigated this statement while dining at a vegetarian restaurant.
Football fans are fed up with all the live coverage of the premiership. Sarah, an avid Blackburn Rovers fan, asked people's opinions as they left the match on Saturday.

Resource sheet: Collecting 2
Attendances at hockey matches

Chart 1

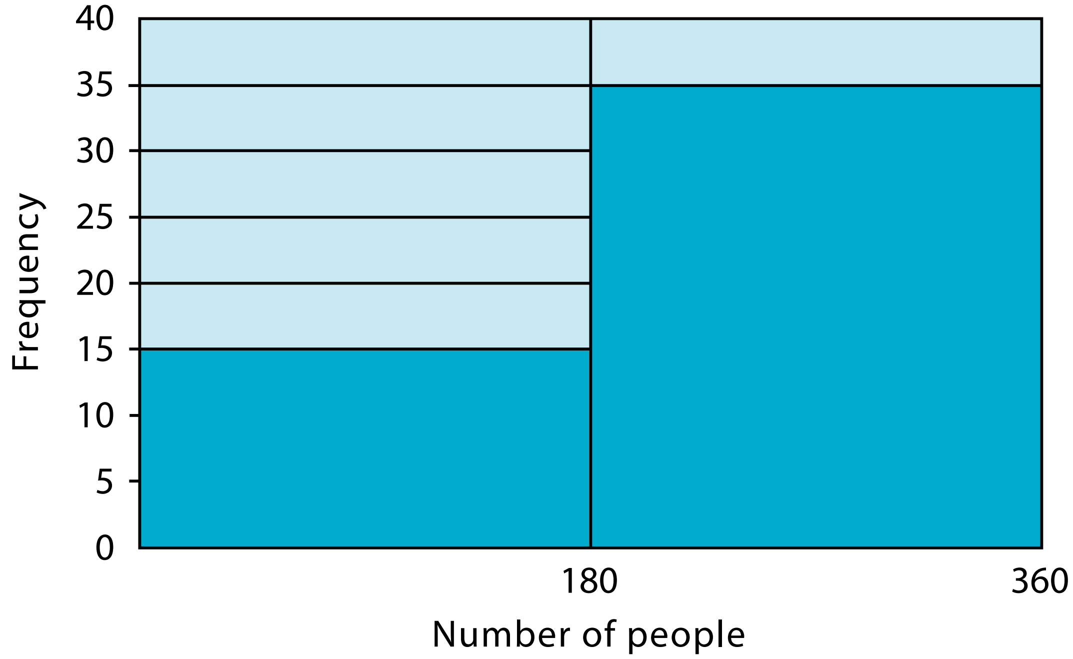


Chart 2

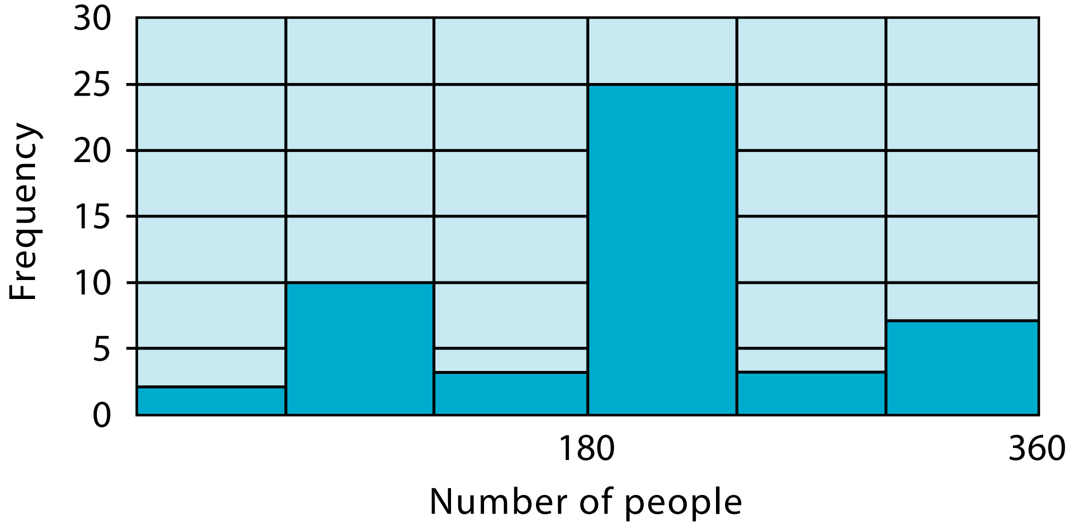


Chart 3

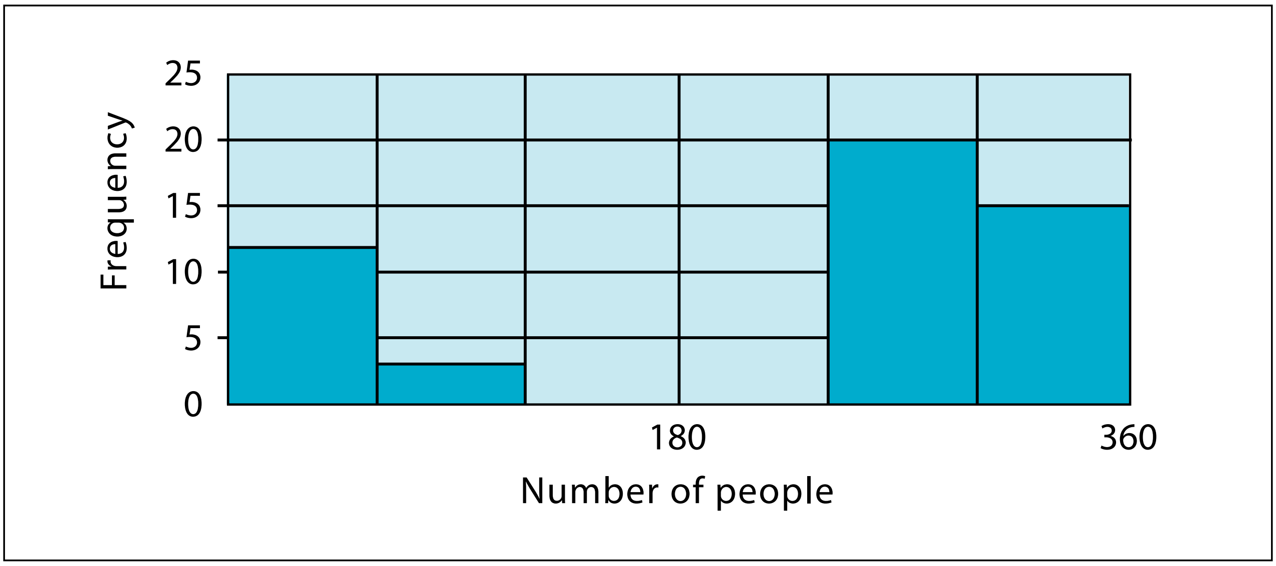
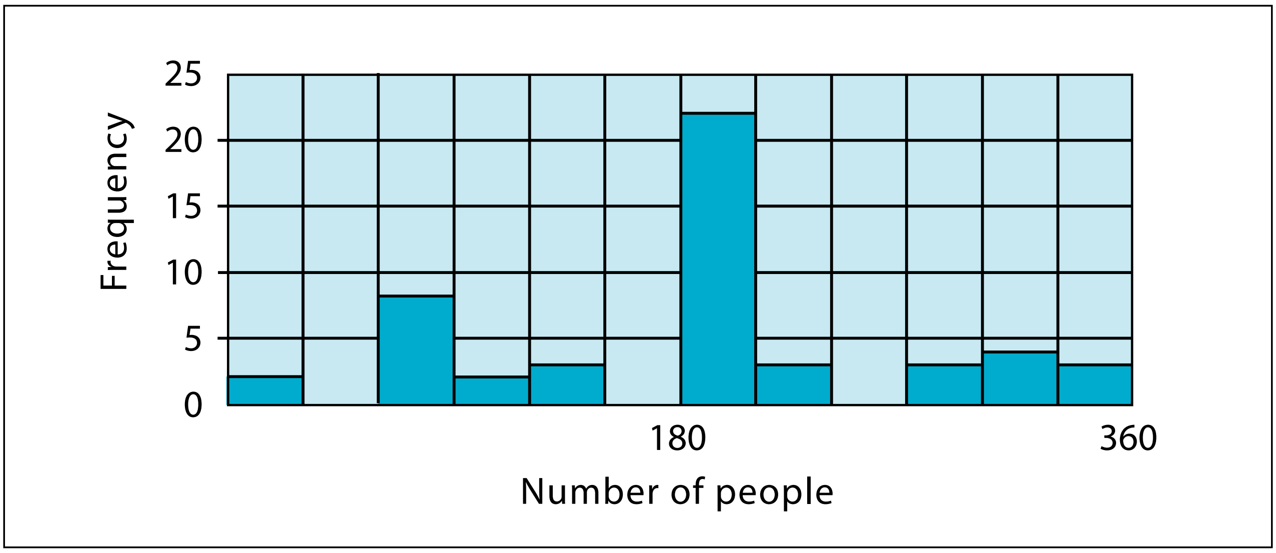
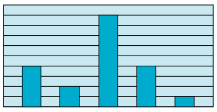


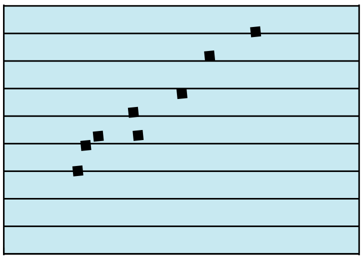
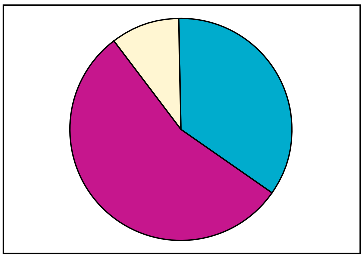
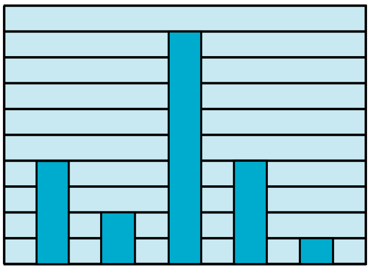
Chart 4



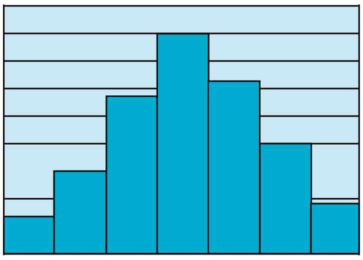
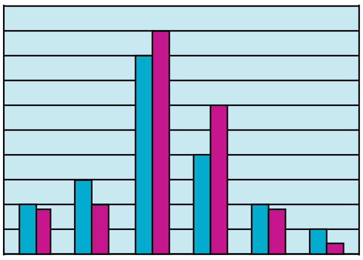
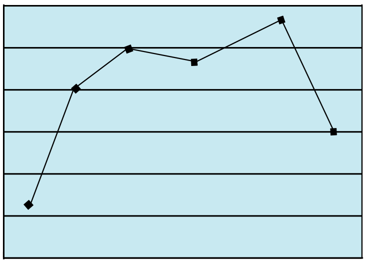
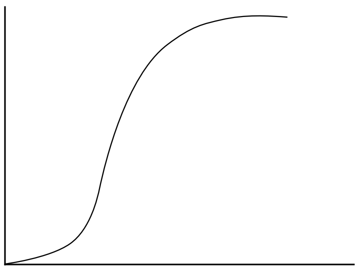
Resource sheet: Matching 1 (recording table)

The diagram looks like this….	The diagram is called a…	It can be used for this type of data …	It can be used to find …	Gains	Losses
	Bar chart	Categorical	Mode	Can read frequency for each category. Can see the mode.	Hard to see proportions.

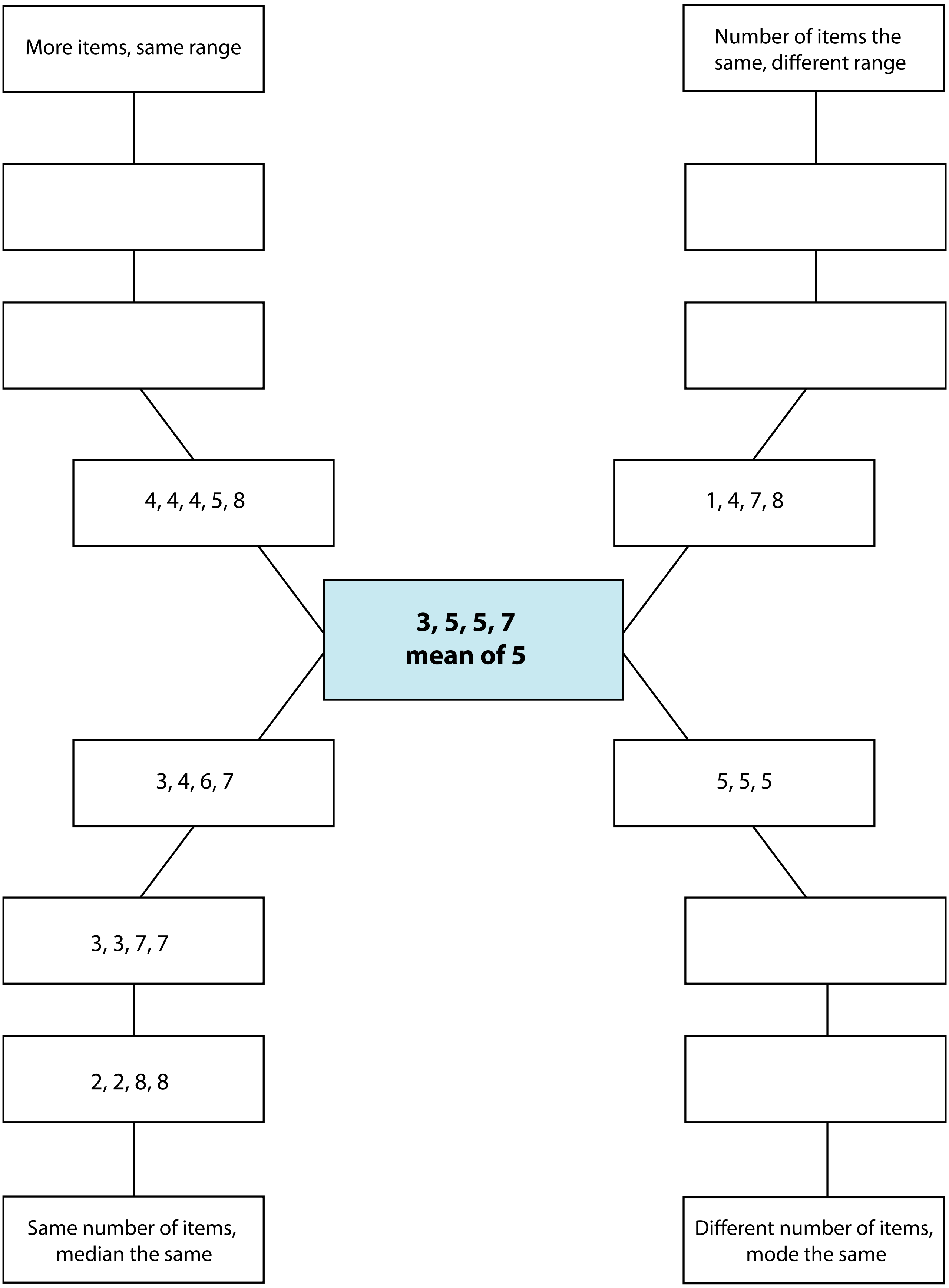
Resource sheet: Matching 2 (cards)

	**Scatter graph**	This compares the prices of digital cameras with the number of pixels.
	**Pie chart**	This represents the proportion of voters voting for the main political parties in Britain.
	**Bar chart**	This represents the favourite terrestrial TV channels of Year 9 pupils.
1	1, 3, 7, 9 2	2, 4, 6, 8, 8, 9 3	3, 3, 5, 7, 7, 7, 8 4	2, 4, 4, 9 5	6	**Stem and leaf diagram**	This represents the ages of people eating in a restaurant.

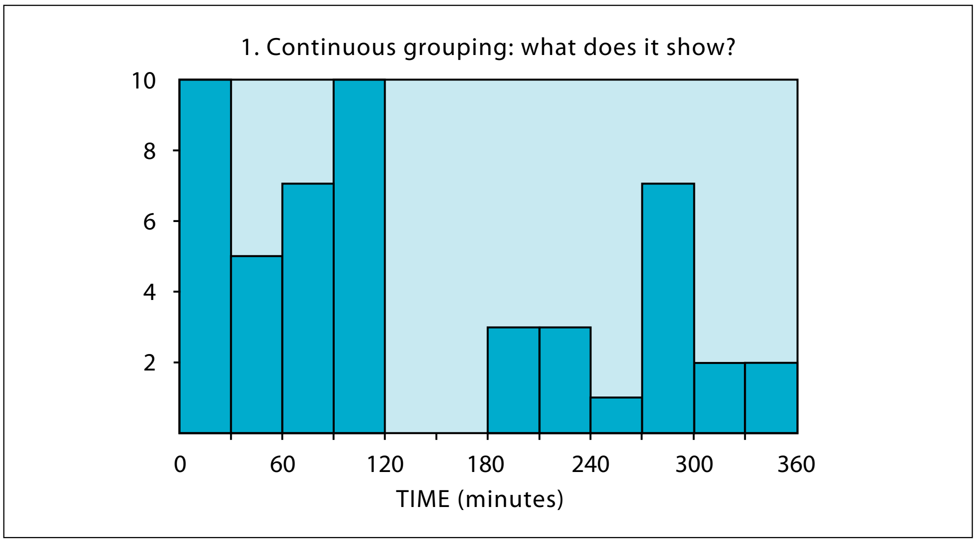
Resource sheet: Matching 2 (cards)

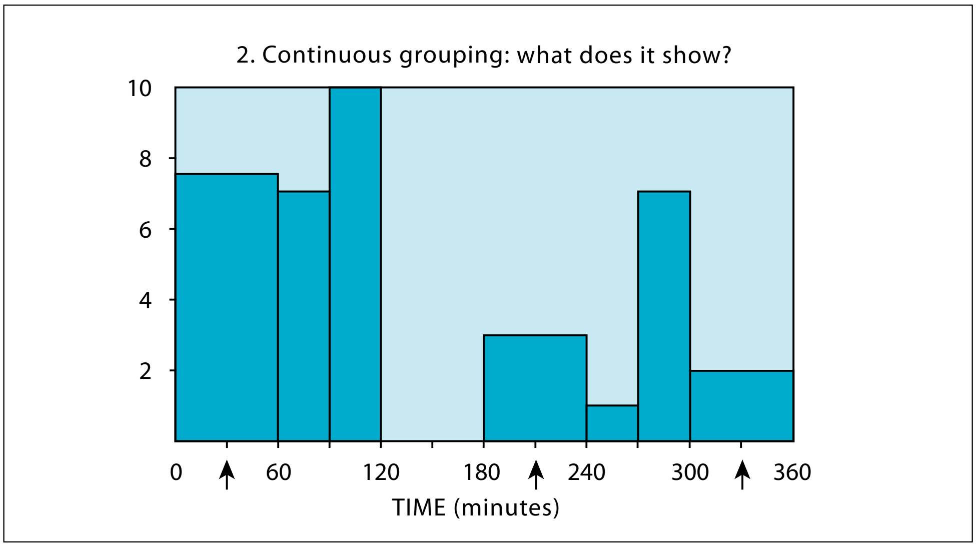
	**Frequency diagram**	This shows the heights of the pupils in a school.
	**Dual bar chart**	This compares the boys' and girls' Key Stage 3 results in mathematics.
	**Time series graph**	This shows how the earnings of a rock band vary over time.
	**Cumulative frequency graph**	This shows the distribution of the time Year 9 pupils spend on their homework each week.

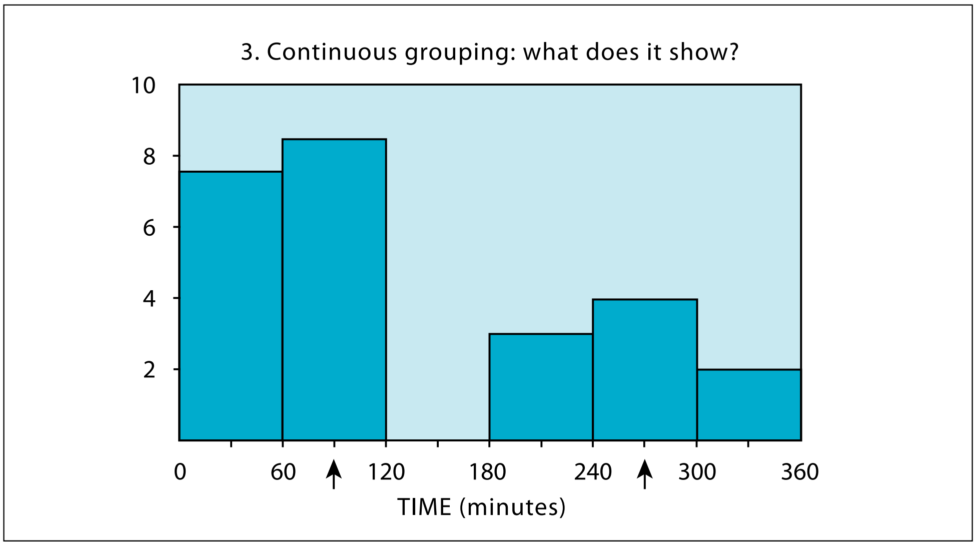
Resource sheet: Clouding the Picture – Mean of 5



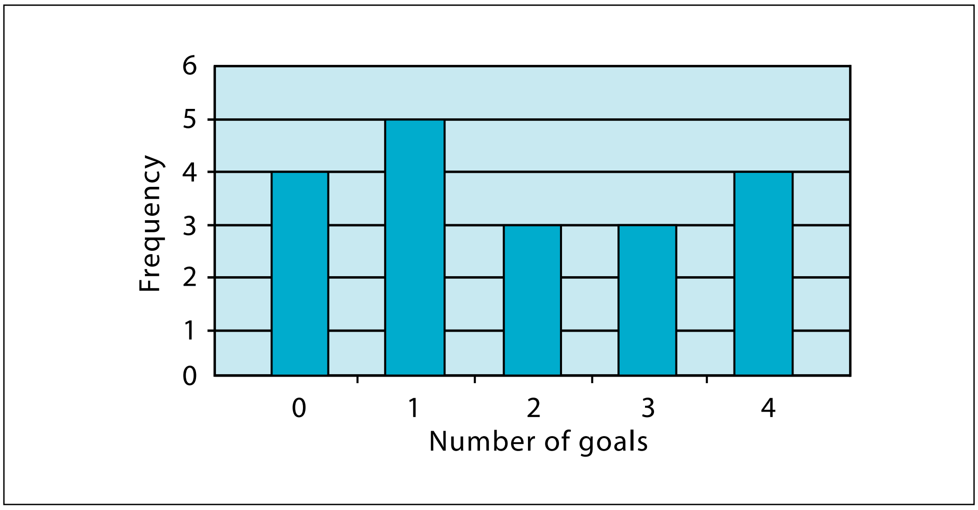
Resource sheet: Continuous grouping: what does it show?

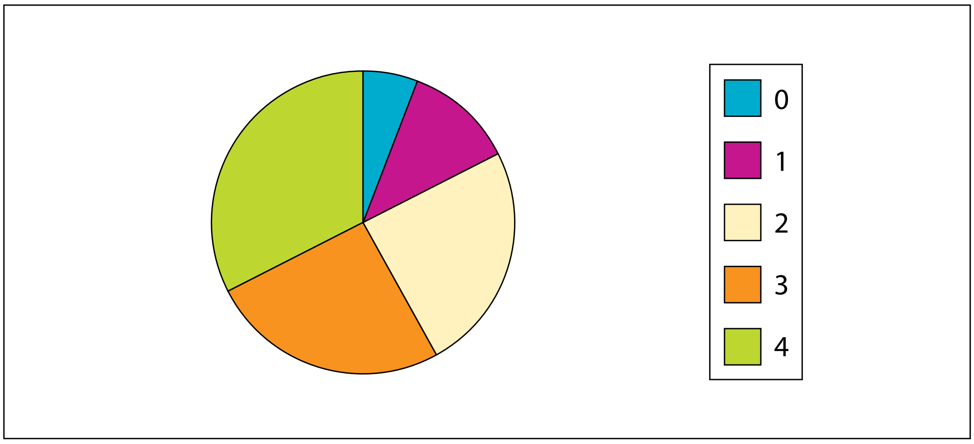


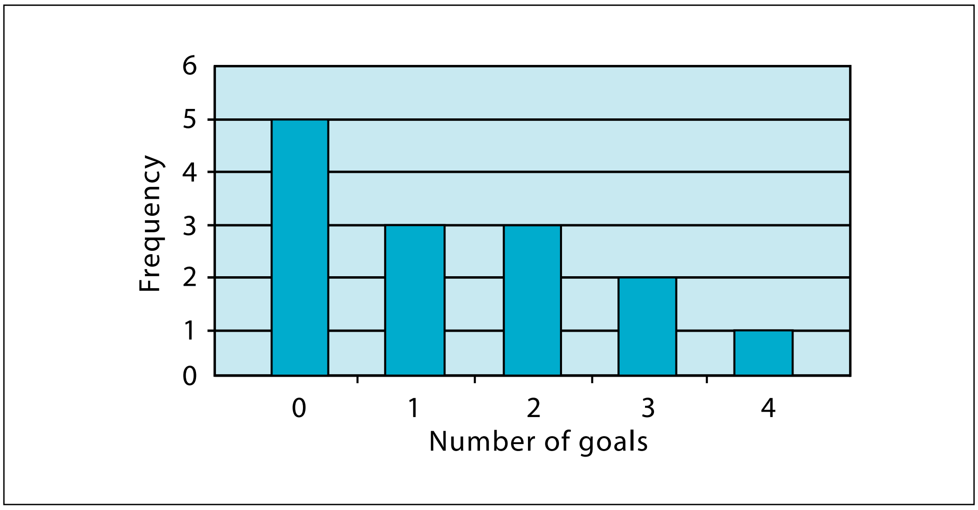


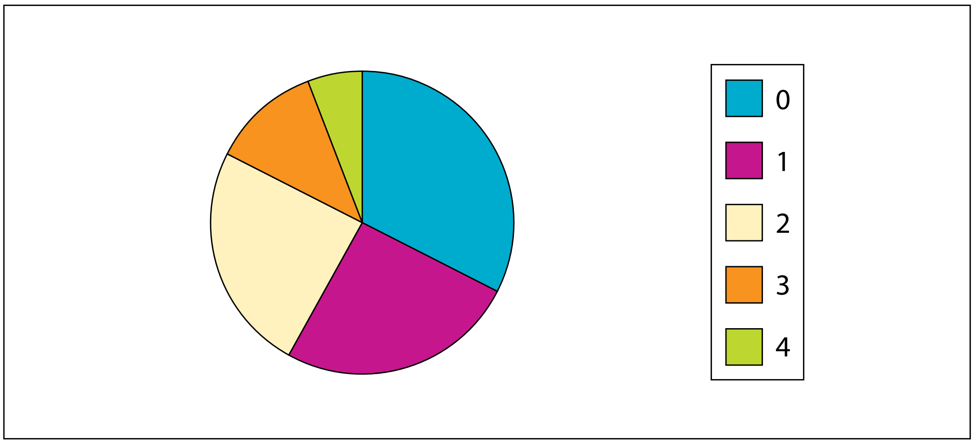


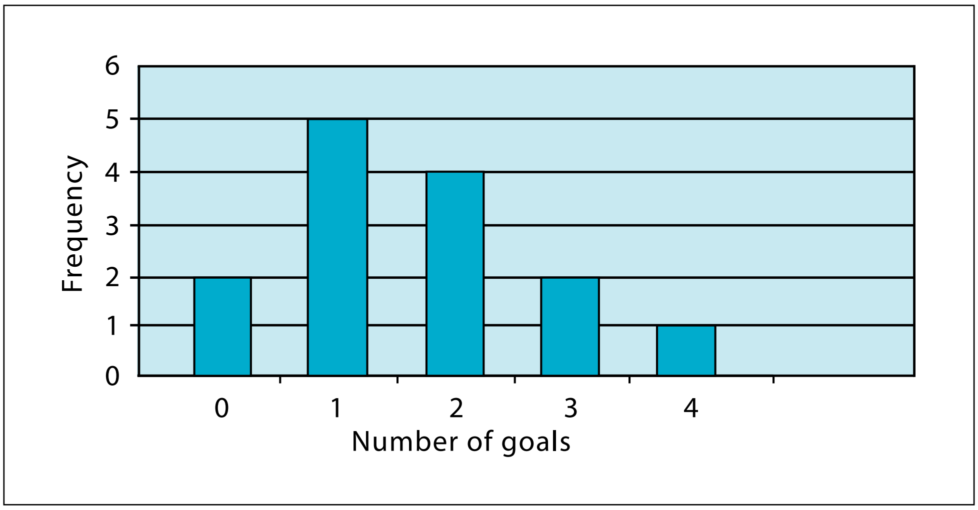
Resource sheet: Matching pie charts and bar charts

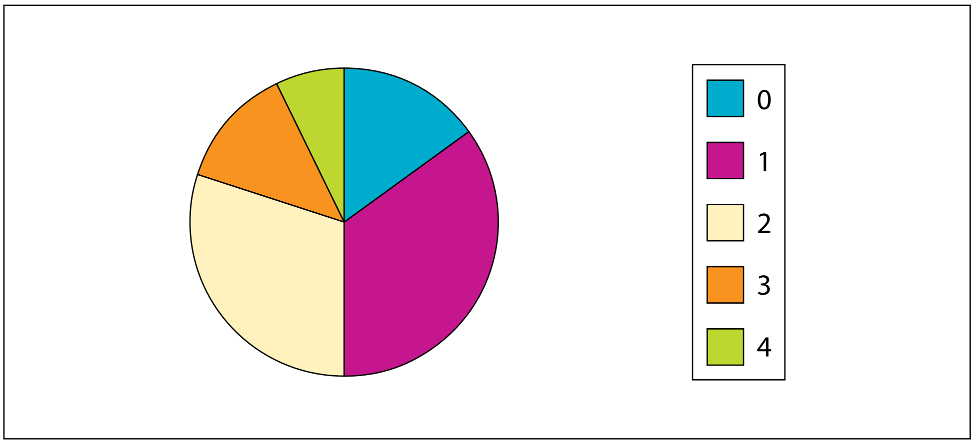







0
1
2
3
4


Frequency
0
1
2
3
4
5
6
0
1
2
3
4
Number of goals


0
1
2
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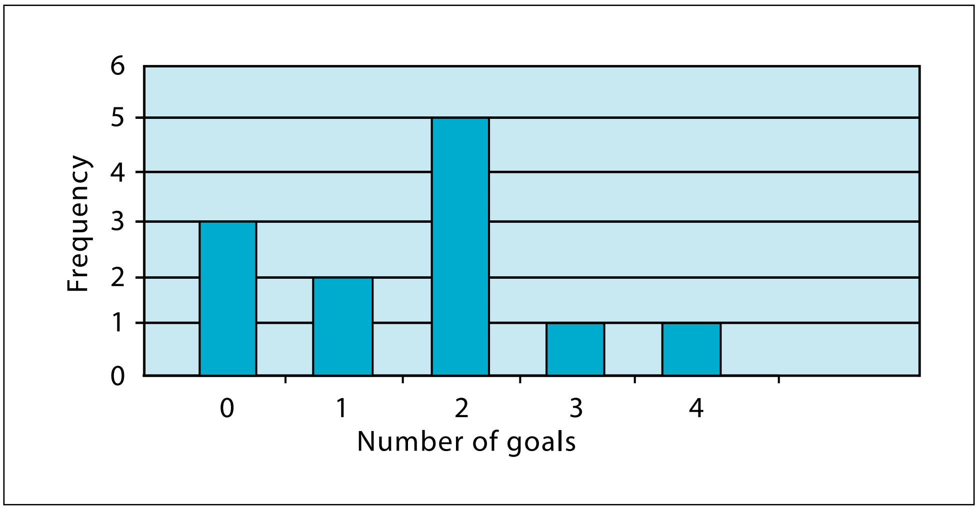
Frequency
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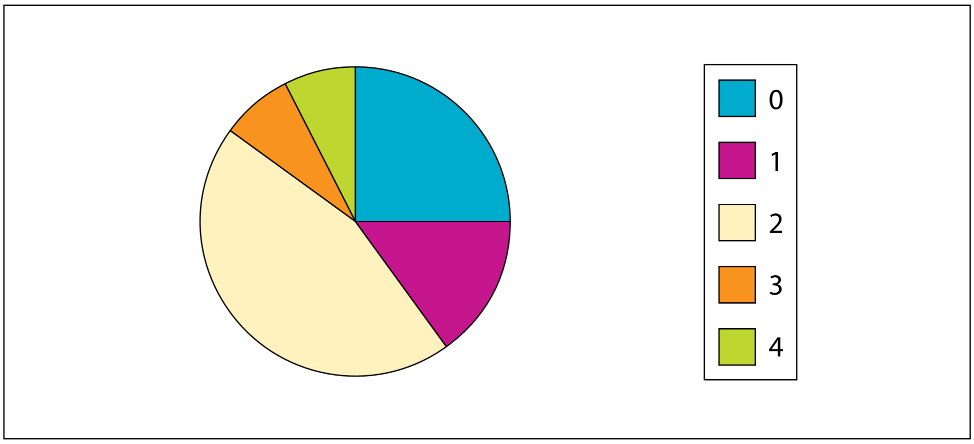
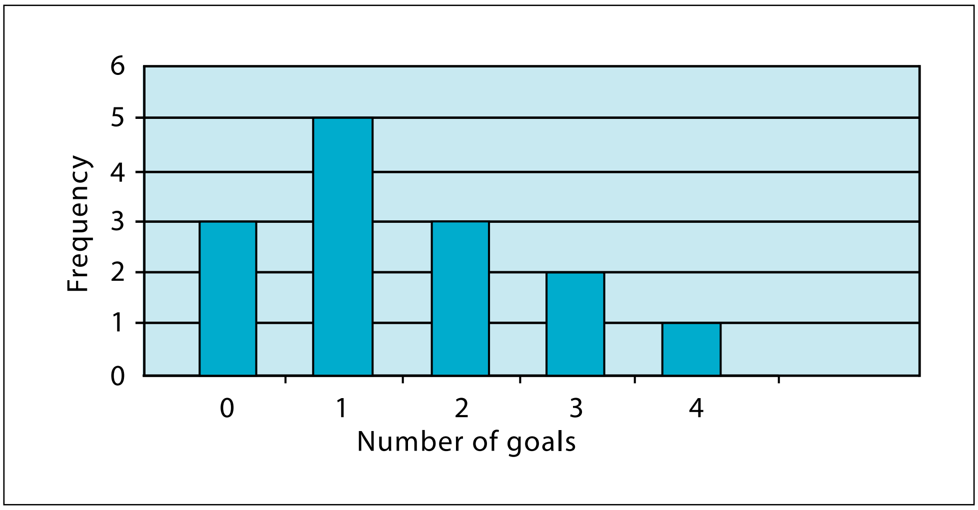
1
2
3
4
5
6
0
1
2
3
4
Number of goals


0
1
2
3
4


Frequency
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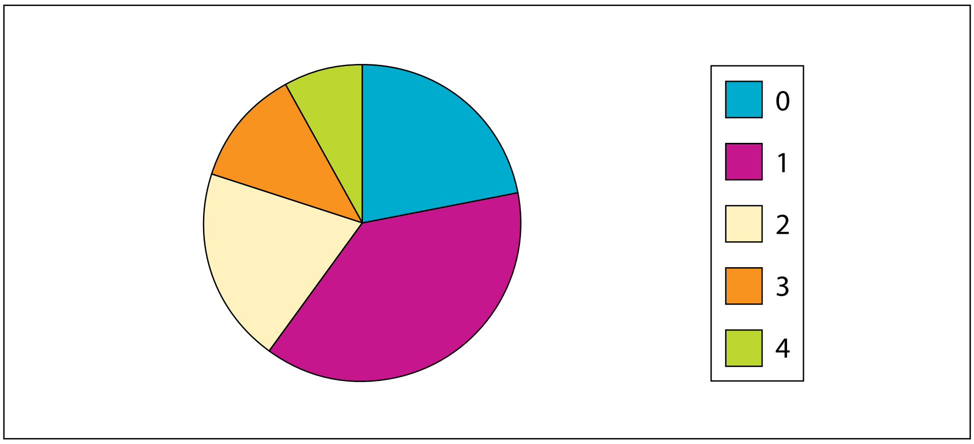
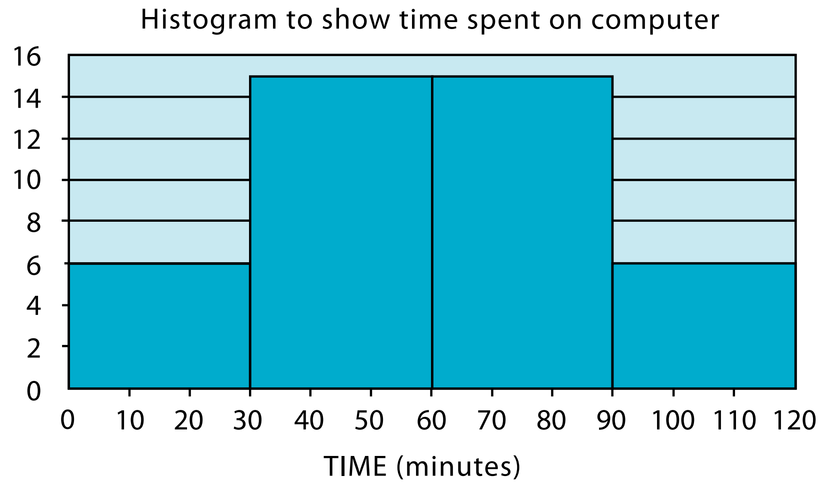
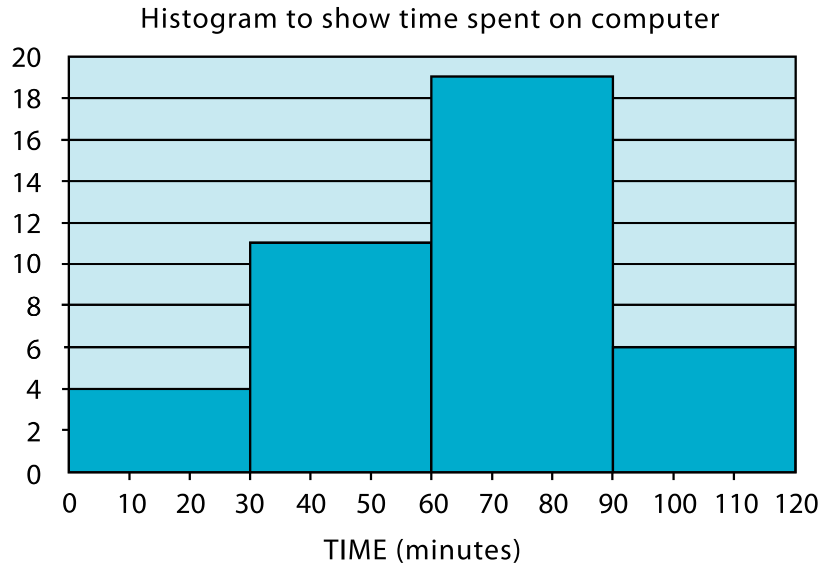
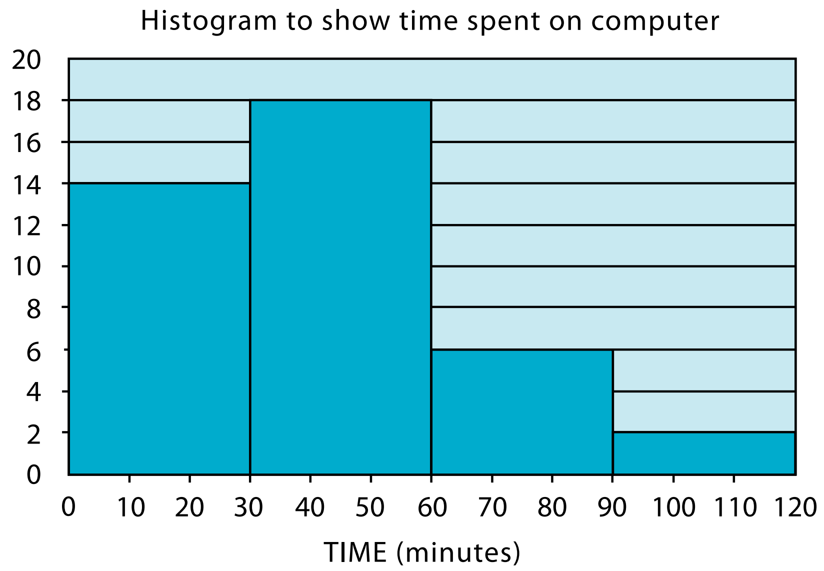
1
2
3
4
5
6
0
1
2
3
4
Number of goals

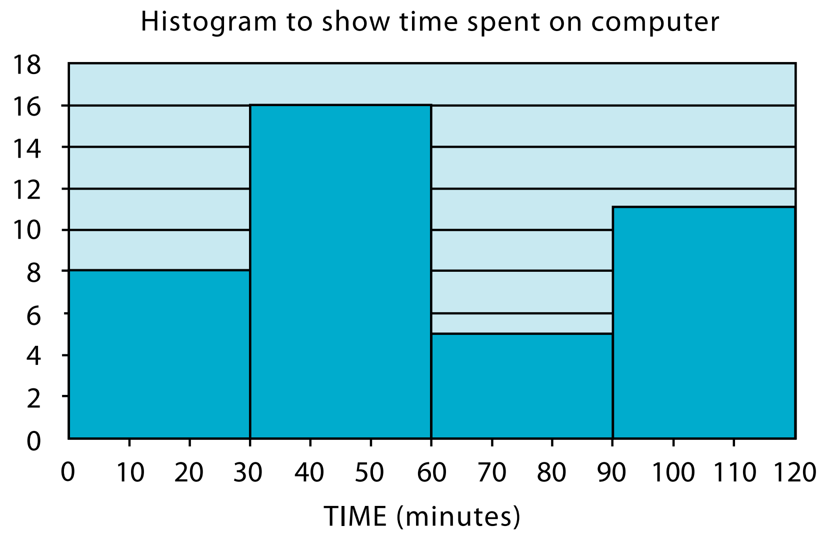

0
1
2
3
4

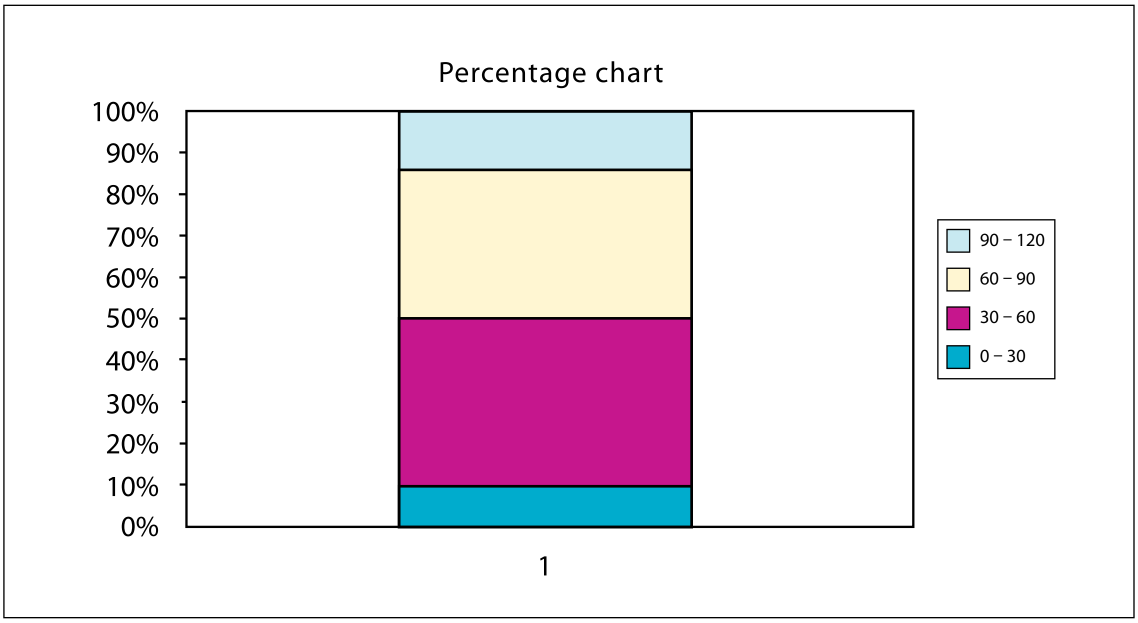
Resource sheet: Matching more complex graphs

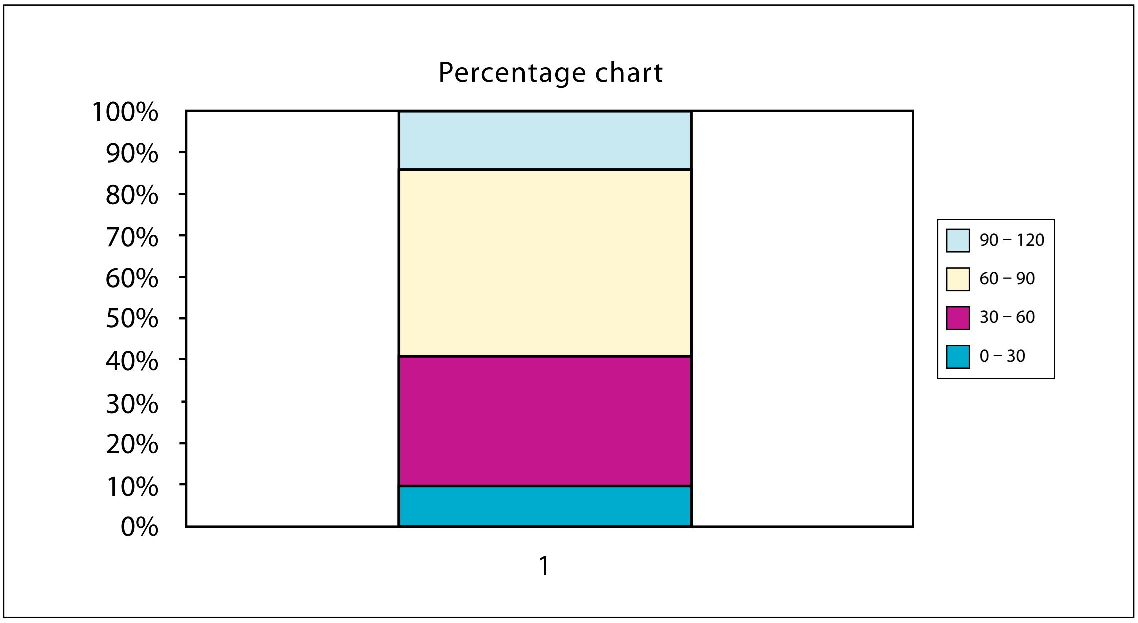


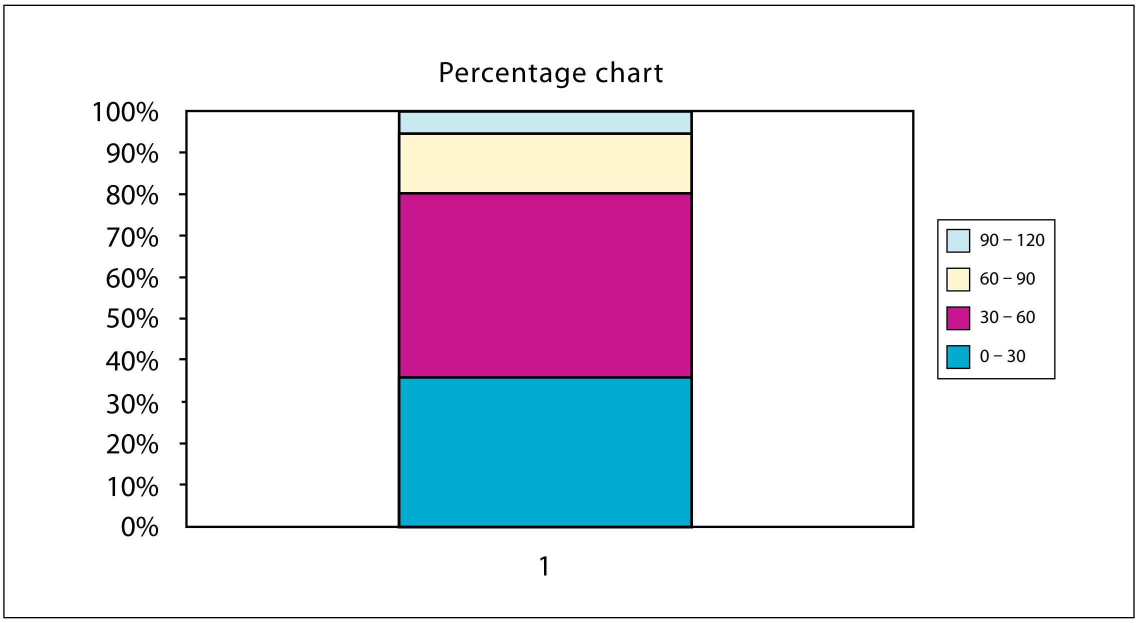



Histogram to show time spent on computer
20
18
16
14
12
10
8
6
4
2
0
0 10 20 30 40 50 60 70 80 90 100 110 120
TIME (minutes)


Histogram to show time spent on computer
18
16
14
12
10
8
6
4
2
0
0 10 20 30 40 50 60 70 80 90 100 110 120
TIME (minutes)


Percentage chart
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1
90 – 120
60 – 90
30 – 60
0 – 30


Percentage chart
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1
90 – 120
60 – 90
30 – 60
0 – 30


Percentage chart
100%
90%
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60%
50%
40%
30%
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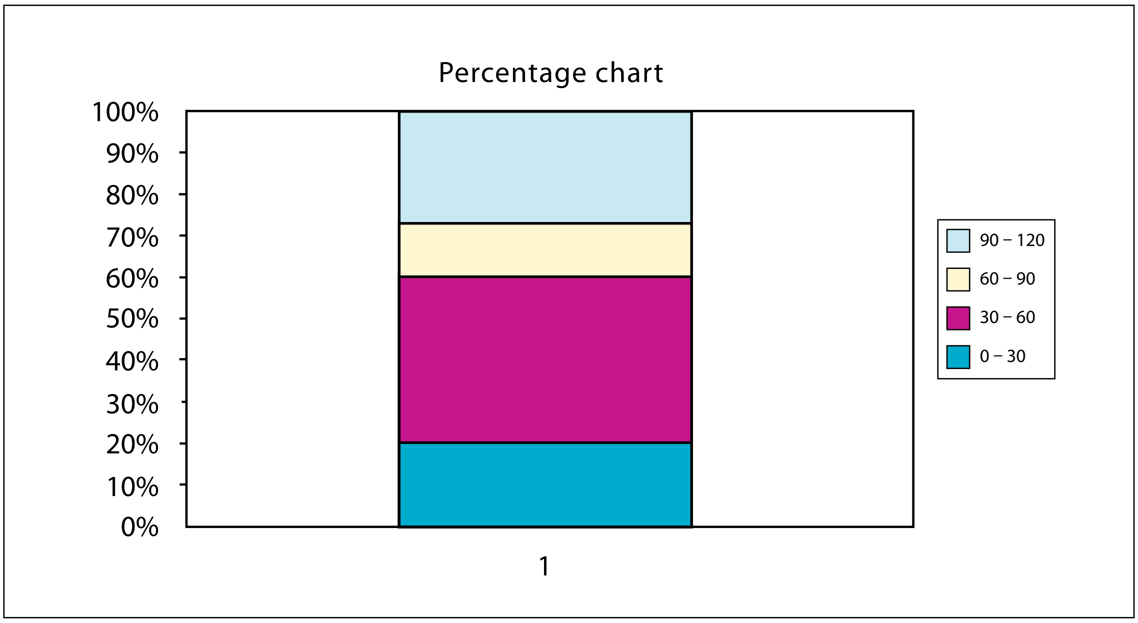
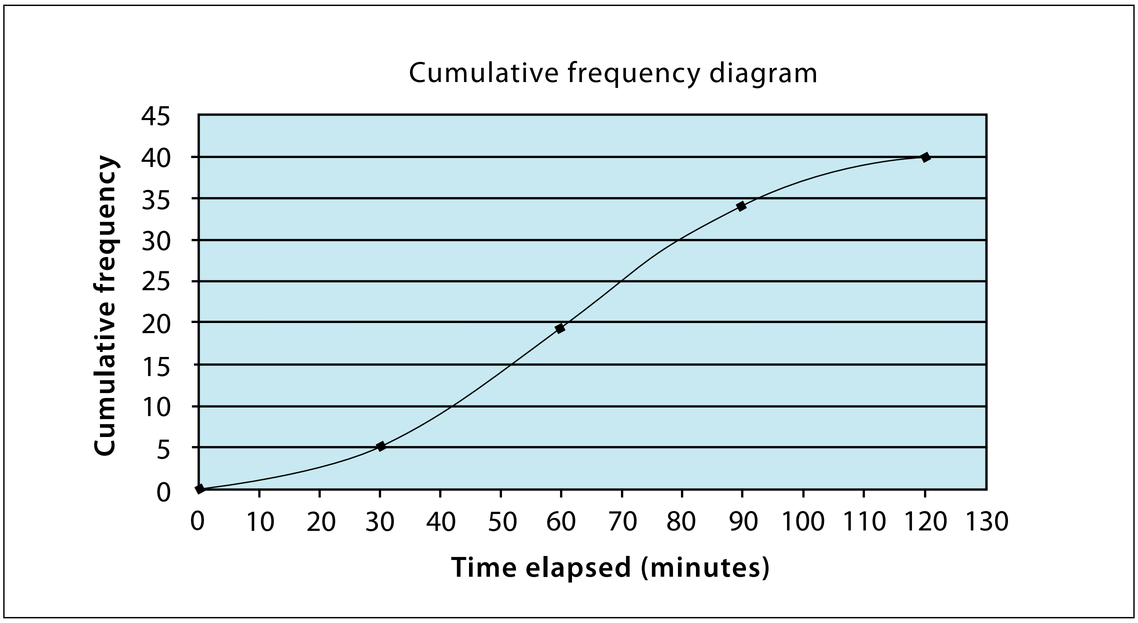
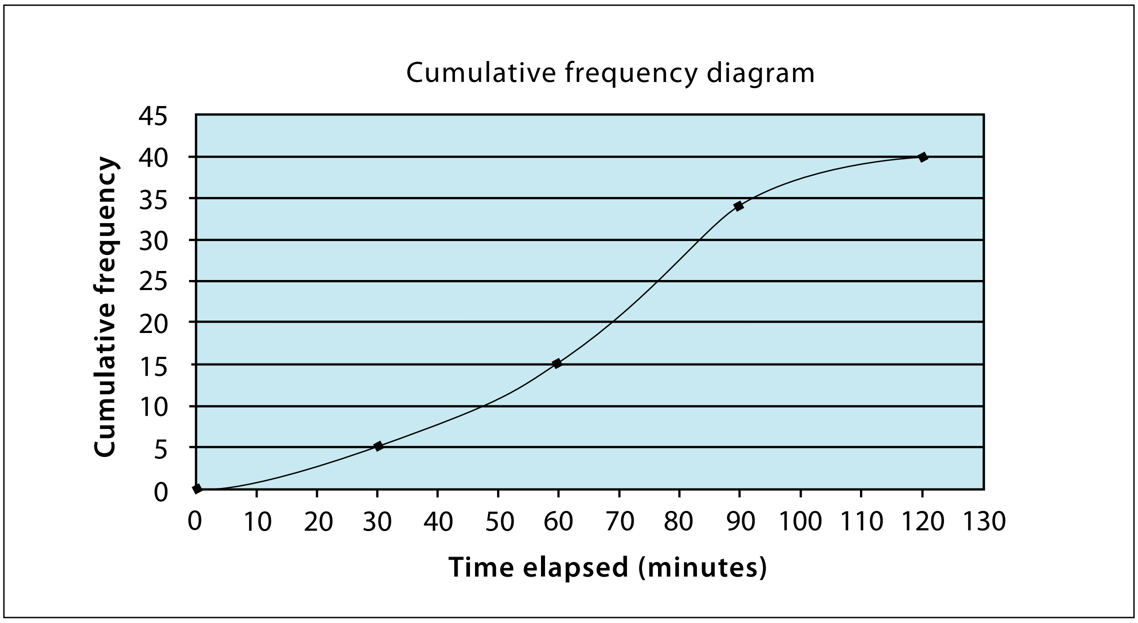
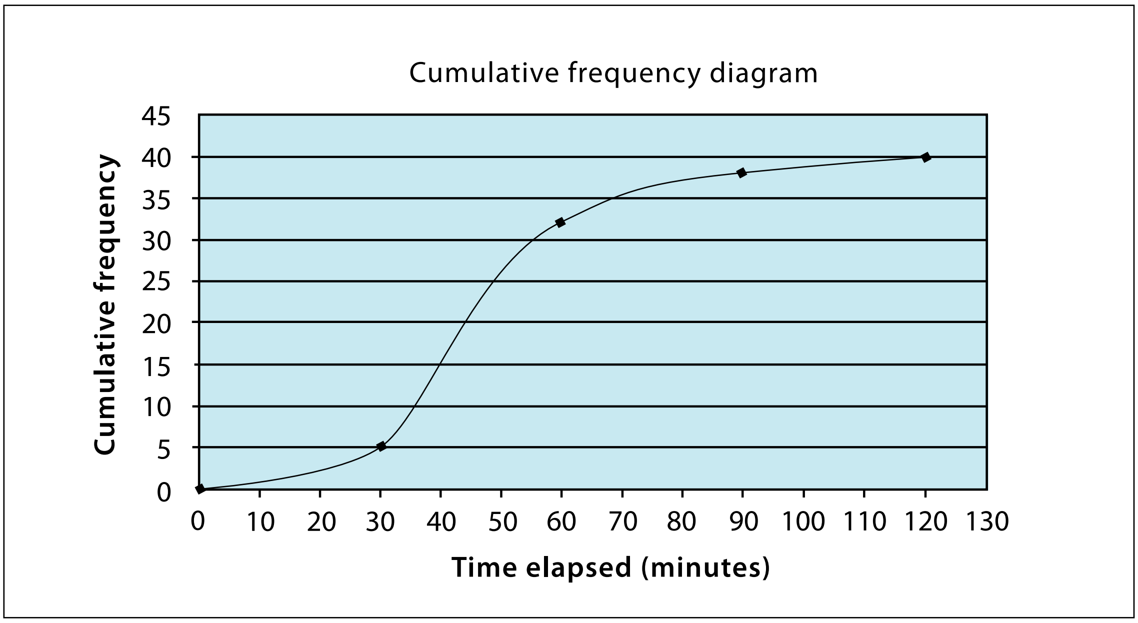
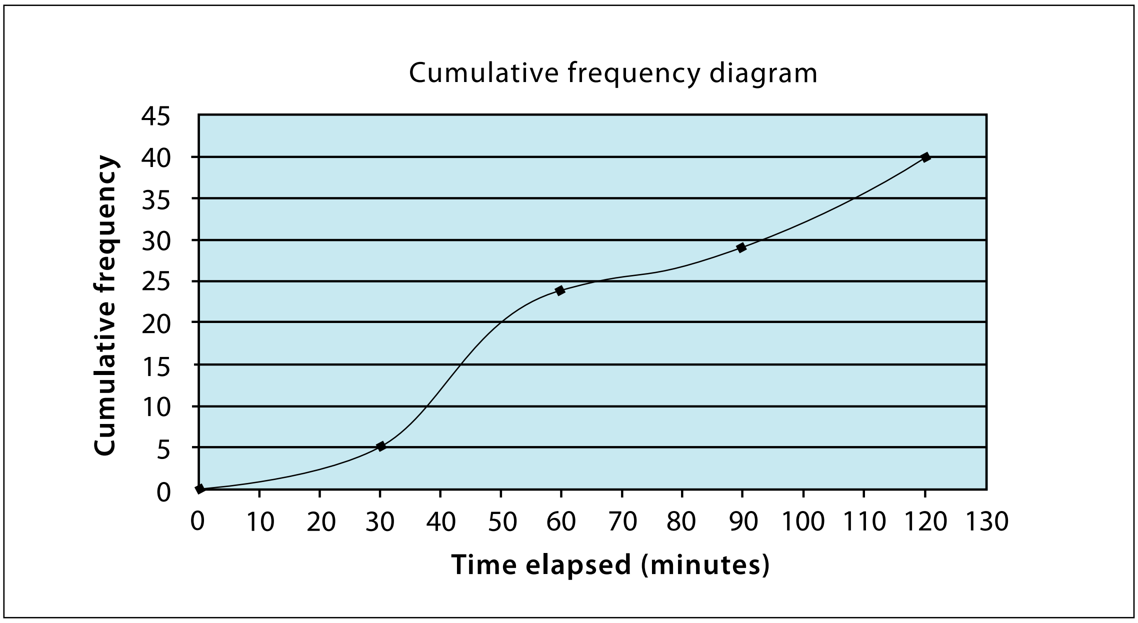
10%
0%
1
90 – 120
60 – 90
30 – 60
0 – 30


Percentage chart
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1
90 – 120
60 – 90
30 – 60
0 – 30

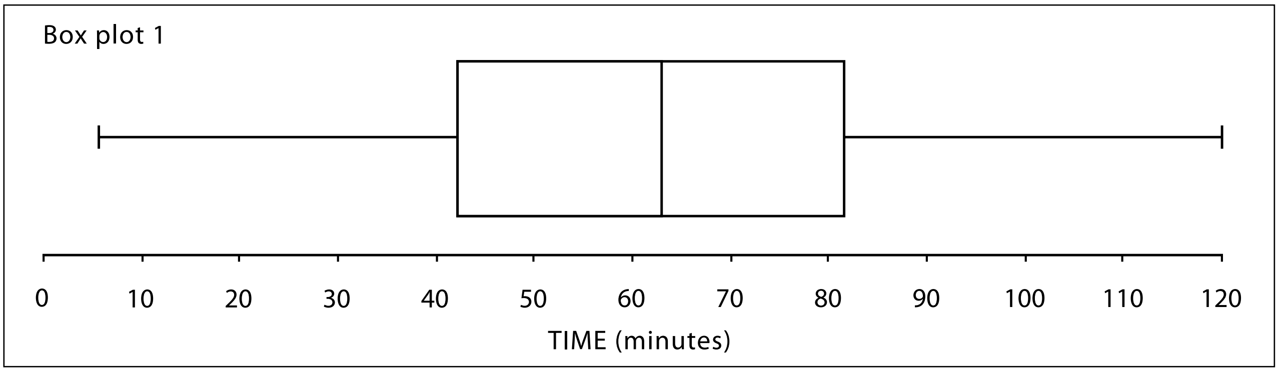

Cumulative frequency diagram
Cumulative frequency
0
5
10
15
20
25
30
35
40
45
Time elapsed (minutes)
0 10 20 30 40 50 60 70 80 90 100 110 120 130

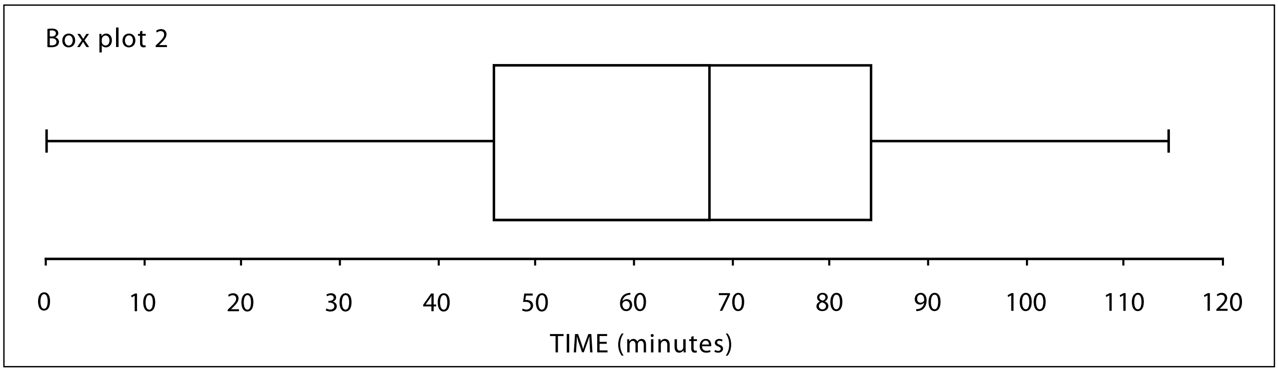

Cumulative frequency diagram
Cumulative frequency
0
5
10
15
20
25
30
35
40
45
Time elapsed (minutes)
0 10 20 30 40 50 60 70 80 90 100 110 120 130

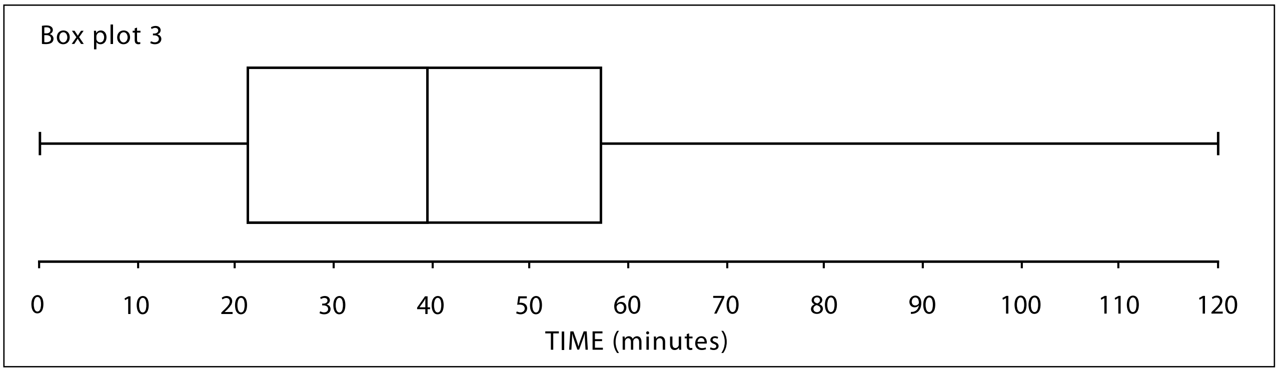

Cumulative frequency diagram
45
40
35
30
25
20
15
10
5
0
Cumulative frequency
0 10 20 30 40 50 60 70 80 90 100 110 120 130
Time elapsed (minutes)

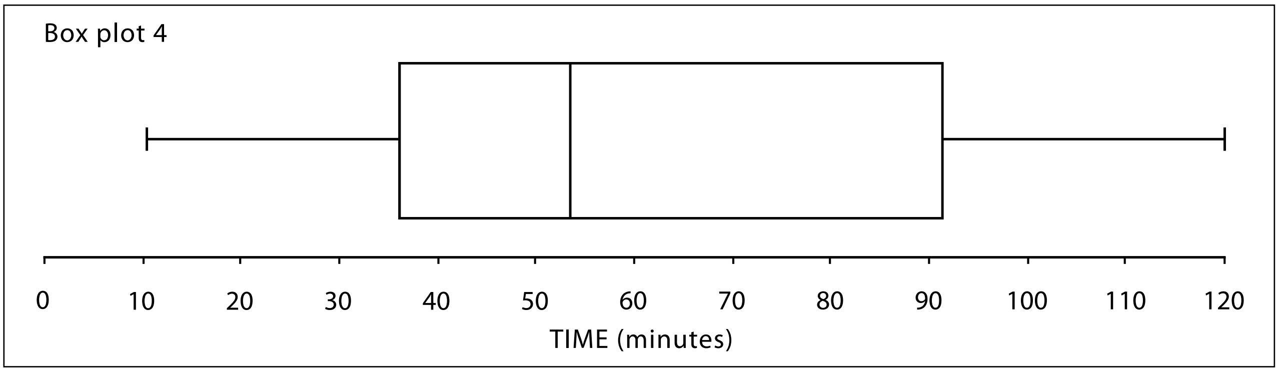

Cumulative frequency diagram
45
40
35
30
25
20
15
10
5
0
Cumulative frequency
0 10 20 30 40 50 60 70 80 90 100 110 120 130
Time elapsed (minutes)

Four box plots





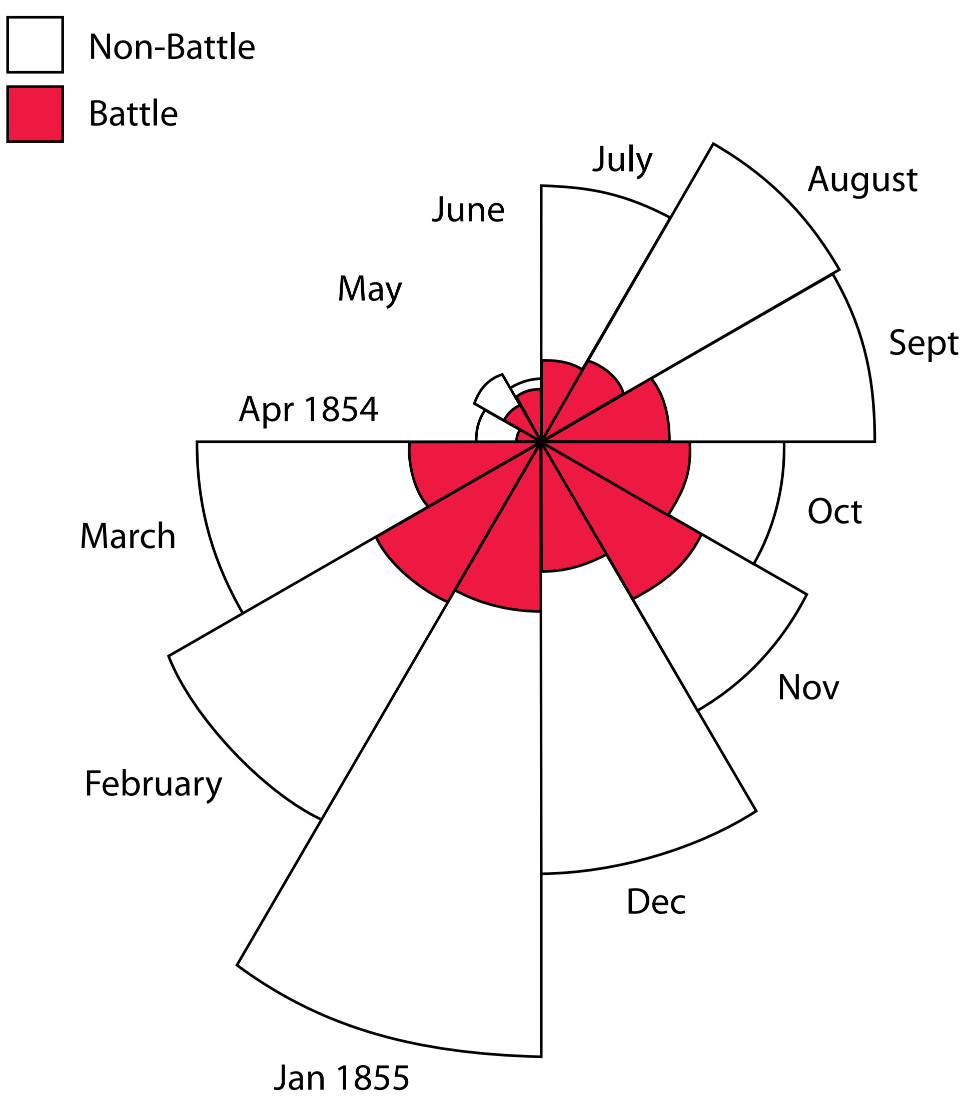




Resource sheet: Interpreting

Representing mortality levels (Florence Nightingale)

Cause of Mortality in the Army in the East
April 1854 to March 1855



From: F. Nightingale, 'Notes on Matters Affecting the Health, Efficiency and Hospital Administration of the British Army', 1855

Resource sheet: Which chair?

(Pupil starts at centre chair. The coin is flipped 3 times, if it is a head the pupil moves to the left, if it is a tail the pupil moves to the right)

Diagram to show all possible combinations of flipping the coin 3 times

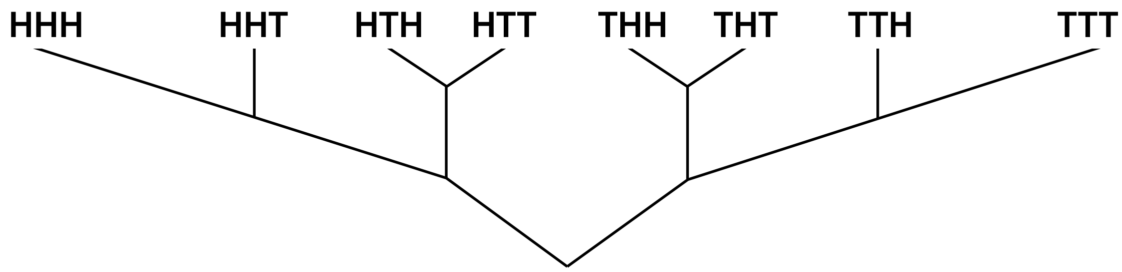
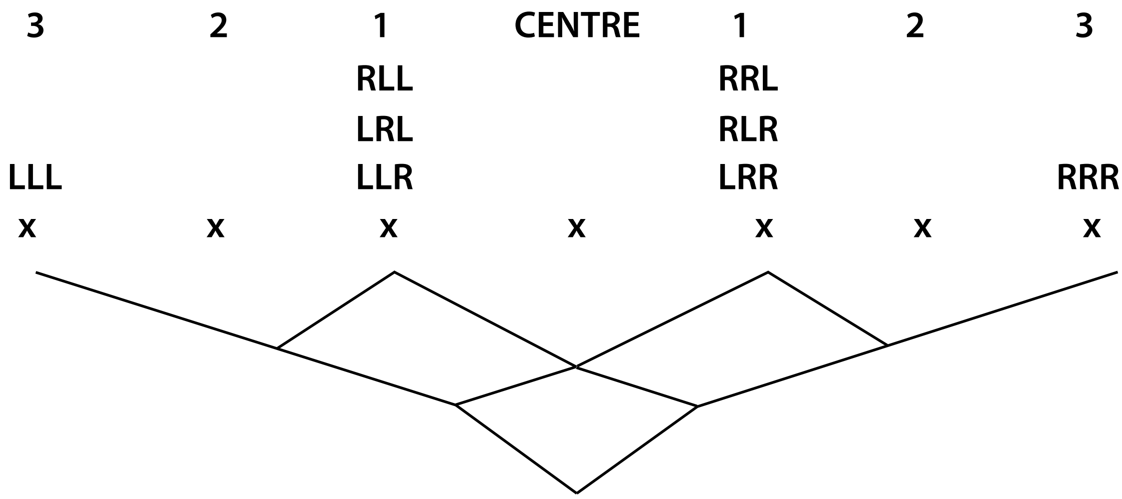
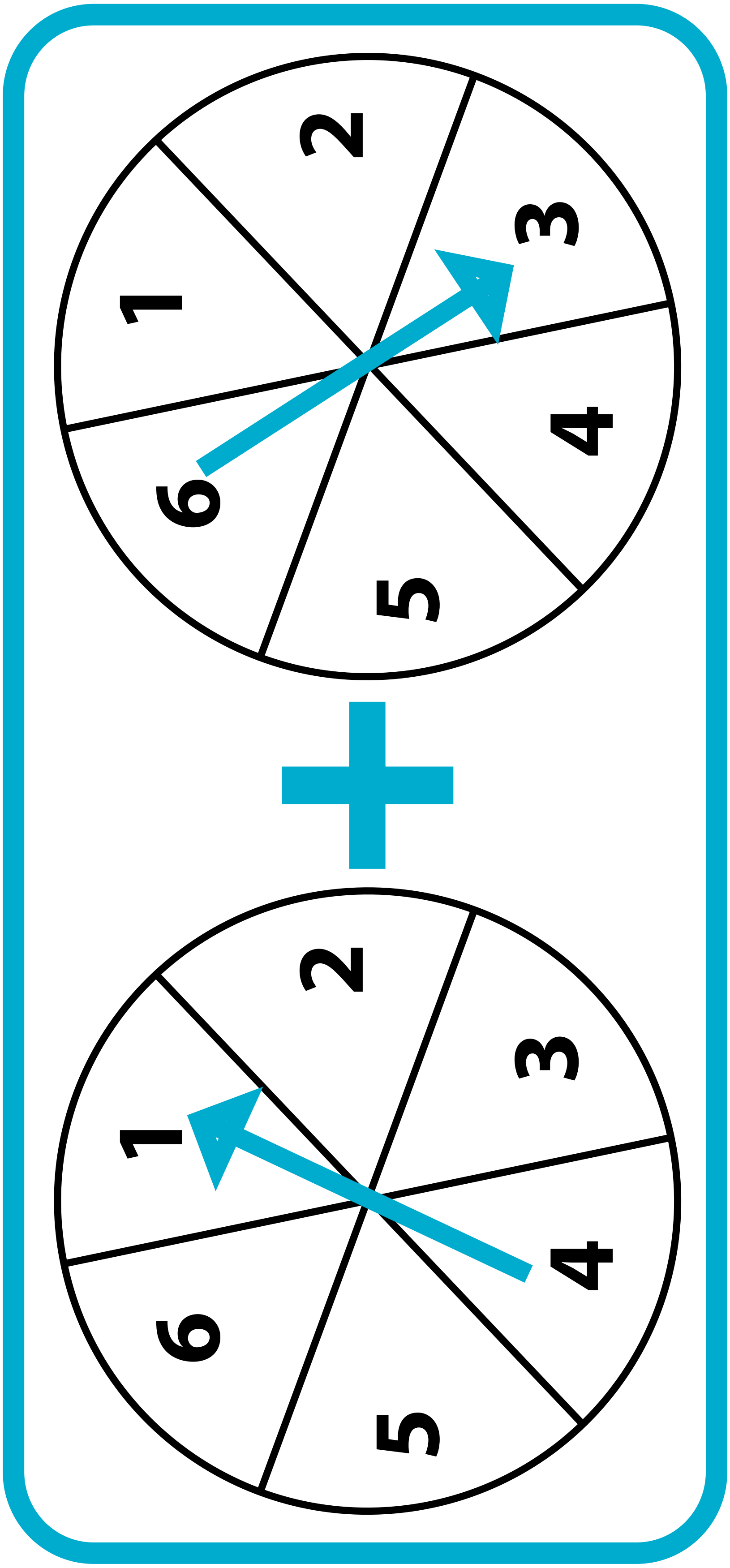


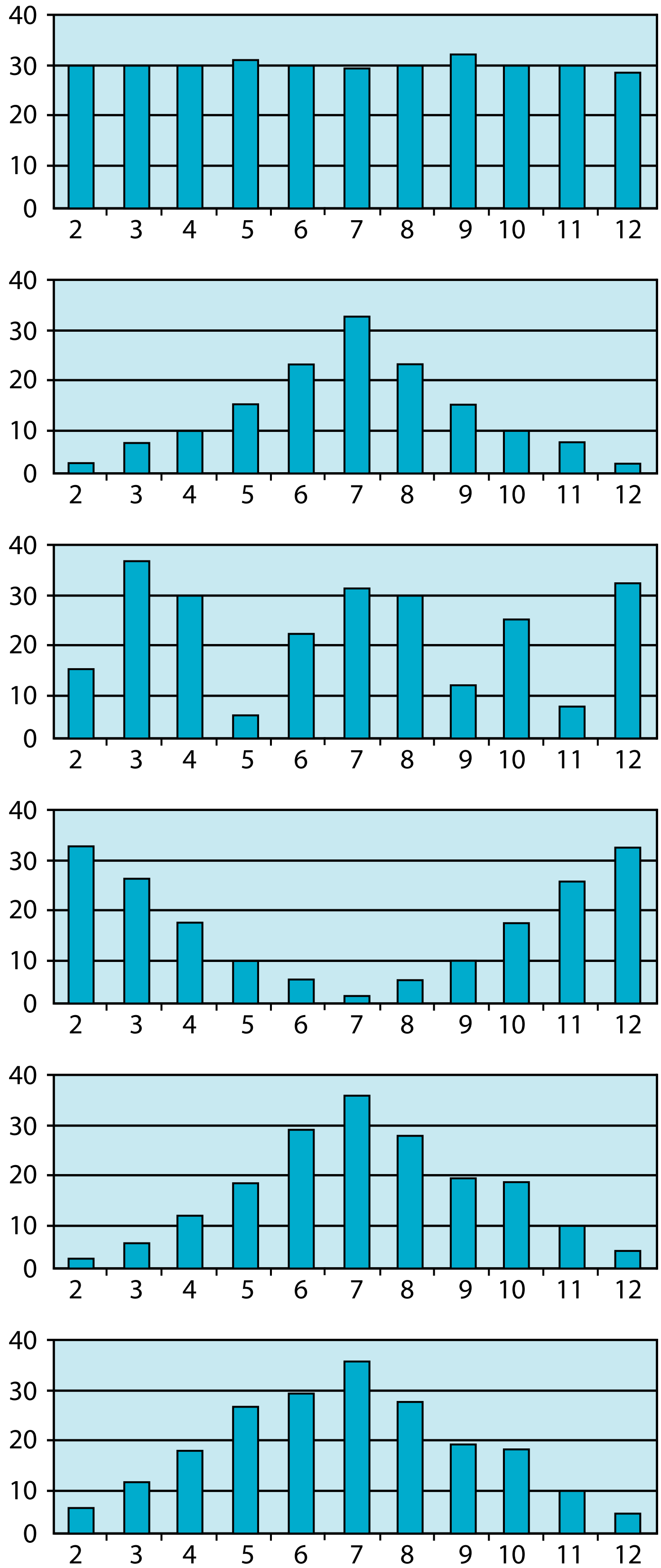
Diagram to show possible pupil's movement and final chair position



Resource sheet: Spinners



Resource sheet: Spinner outcomes



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