

# **Report of Professor Sir Adrian Smith's review of post-16 mathematics**

**July 2017**

## Foreword

In March 2016, I was asked to undertake, on behalf of HMT and DfE, a review of 16-18 mathematics education, prompted by two related issues: first, the increasing importance of mathematical and quantitative skills to the future workforce; and secondly, by comparison with competitor economies, the low percentage of students in England continuing mathematics post-16.

In particular, I was asked to consider the case for and feasibility of all students continuing some form of mathematics until 18 – mathematics being interpreted in its broadest sense, including quantitative skills, statistics and data analysis.

I have structured my approach under four broad headings: the appropriate range of 16-18 mathematics pathways needed; the factors that encourage or discourage individual students to participate; levels of attainment and progression; and capacity to deliver, both in terms of provision of courses and teaching capability.

The review has involved wide-ranging consultation with stakeholders and I am grateful to everyone who has contributed. I am also indebted to the team from DfE who undertook much of the analysis and the interactions with stakeholders. The report is written in the third person, but I take full responsibility for the judgments made and recommendations.

The 16-18 period of education is just part of the wider educational journey and cultural context in which attitudes to and achievements in mathematics take place. I hope that the evidence provided in this report will also prove useful to wider policy debates.

Two such wider policy issues have emerged in the course of this review. First, the need to pay close attention to local variations in provision and attainment, and to develop appropriate interventions; secondly, the need to recognise more explicitly, including through a re-examination of funding levels and mechanisms, the fundamental importance of Further Education in the post-16 landscape, an importance underlined by the publication during the course of this review of Lord Sainsbury's proposals for technical education.

In relation to the issue of most or all students continuing mathematics until 18, my clear conclusion is that we do not yet have the appropriate range of pathways available or the capacity to deliver the required volume and range of teaching. However, I would hope that if we were able to move forward over the next few years with many of the recommendations in this report, we might realistically aspire to such a vision within a decade.

Adrian Smith, London, July 2017.

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## **Executive Summary and recommendations**

1. This review considers ways of improving mathematics education for 16-18 year olds in England with the aim of ensuring that the future workforce is appropriately skilled and competitive. This includes considering the case and feasibility for more or all students continuing to study mathematics to age 18 in the longer-term.
2. Throughout this report, the term 'mathematics' is used in its broadest sense – referring to mathematical and quantitative skills. It includes numeracy, statistics and data analysis.

### **The case for mathematics**

3. There is strong demand for mathematical and quantitative skills in the labour market at all levels and consistent under-supply, reflecting the low take-up of and achievement in 16-18 mathematics in England relative to other developed countries. The quantitative demands of university courses in both STEM (science, technology, engineering and mathematics) and non-STEM subjects are increasing and are set to increase further. The problem of insufficient STEM skills is exacerbated by a shortage of women studying key STEM subjects post-16.
4. Adults with basic numeracy skills earn higher wages and are more likely to be employed than those who fail to master basic quantitative skills. Higher levels of achievement in mathematics are associated with higher earnings for individuals and higher productivity. Increased productivity is a key determinant of economic growth.
5. The increasing sophistication of technology is driving change to the economy and the nature of work. This is increasing the demand for mathematical and quantitative skills.

### **The case for improving 16-18 mathematics**

6. England remains unusual among advanced countries in that the study of mathematics is not universal for all students beyond age 16. Almost three quarters of students with an A\*-C in GCSE mathematics at age 16 choose not to study mathematics beyond this level. England was the only country in a 2013 sample of developed economies where young adults performed no better than older adults in numeracy proficiency.
7. With the exception of mathematics degrees, more than 40 per cent of English 19 year olds studying STEM subjects in UK universities do not have a mathematics qualification beyond GCSE. This increases to over 80 per cent for students on non-

STEM degree courses, many of which have a significant quantitative element. A lack of confidence and anxiety about mathematics/statistics are problems for many university students; and many have done little or no mathematics pre-university for at least two years.

## **Mathematics pathways and options**

8. The majority of students who progress from GCSE to further study of mathematics, study an AS or A level. Significant changes to A levels in mathematics (and further mathematics) are due in 2017. The changes to mathematics and further mathematics A levels are marked and significant support for teachers of A level mathematics and further mathematics is needed in the form of professional development and resources.

9. Core maths qualifications were introduced in 2014, designed for students with an A\*-C at GCSE but who do not study AS/A level mathematics. Core maths is an umbrella term covering a number of qualifications offered by awarding organisations; all offer the opportunity for students to build their knowledge to a higher level, with a focus on the use and application of mathematics and statistics.

10. Core maths plugs a critical gap for students who are progressing to higher education and into higher technical study with a quantitative element.

**Recommendation 1: The Department for Education should seek to ensure that schools and colleges are able to offer all students on academic routes and potentially students on other level<sup>1</sup> 3 programmes access to a core maths qualification.**

**Recommendation 2: The Department for Education and Ofqual should consider how the core maths brand could be strengthened with the aim of improving awareness and take-up of the qualification.**

11. Functional skills are the most common mathematics qualifications after GCSEs at level 2 and below. They have a high take-up with apprentices, adults and 16-18 year olds yet to achieve a standard GCSE pass (typically those with an E or below at 15). Given the important role that functional skills qualifications play for 16-18 students not taking GCSE or level 3 mathematics, it is essential that the new qualifications, currently being developed, have a clear purpose and fit appropriately alongside others in the 16-18 mathematics landscape – particularly in relation to GCSE mathematics.

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<sup>1</sup> Qualifications are conventionally referred to as 'levels'. The numbers reflect their level of difficulty. AS/A levels are level 3 qualifications.

12. Significant reforms to the technical education landscape were outlined in the government's Post-16 Skills Plan in response to the recommendations of the Sainsbury panel. Panels of industry professionals will determine the mathematical content of the 15 technical education routes, presented by Sainsbury, where these directly relate to occupational requirements. Panels will need to ensure that this reflects the needs of the profession, society and the emerging economy.

**Recommendation 3: The Institute for Apprenticeships should work with the Royal Society Advisory Committee on Mathematics Education to ensure appropriate expert advice is available to the panels of professionals developing technical routes.**

## **Participation and achievement**

13. There is a significant gender gap in progression to AS/A level, despite good GCSE achievement by girls. In 2015/16, only 50 per cent of girls with GCSE A grades continued to AS/A level compared to 70 per cent of boys.

14. Despite increases in recent years, AS/A level further mathematics numbers in state schools are still low. This is of concern, given the role of further mathematics as a 'passport' to mathematical study at some research-intensive universities.

15. There are concerning differences between local areas in level 3 mathematics participation, many of which cannot be explained by prior attainment. London students who achieved A\*-C are the most likely to carry on to level 3 mathematics (34 per cent) and students in the North East (20 per cent) and Yorkshire and the Humber (23 per cent) least likely.

16. For students who achieved A/A\* at age 16, Luton, Bournemouth, and Leicester are the only authorities outside London and the South East where more than 80 per cent progress to level 3 mathematics. The figure is 88 per cent in Redbridge. Local authority areas with poor levels of A/A\* grade progression to level 3 mathematics include Salford, Barnsley and York.

17. Among pupils with an A grade in GCSE mathematics at age 16, white pupils are the least likely of all ethnic groups to progress to AS/A level mathematics (55 per cent), with Asian students the most likely (80 per cent).

18. Encouraging students to make informed decisions to choose level 3 mathematics is critical. Employers have stressed to the review the need to get co-ordinated and clear messages about the importance of mathematics to young people.



19. Schools and colleges are heavily influenced by requirements set by universities in deciding which courses to offer to students. This is illustrated by the rise over recent years in the number of students studying core academic subjects following publication of the Russell Group's Informed Choices report.

**Recommendation 4: The Department for Education should work with UK learned societies to encourage universities to better signal and recognise the value of level 3 mathematics qualifications for entry to undergraduate courses with a significant quantitative element.**

20. Since the introduction of the condition of funding in 2014, 16 to 18 year old students without A\*-C in GCSE mathematics are required to continue studying the subject. This has resulted in many more young people successfully retaking their GCSEs and a significant increase in GCSE maths entries. However, this has also resulted in a decline in the post-16 GCSE pass rates with many more students resitting GCSE examinations without success.

21. The proportion of resit students passing at grade C or above has declined as entries have risen. Students taking GCSEs again have already experienced failure and may be less motivated or confident to achieve in the subject as a result. These challenges are most likely to be felt in FE colleges, which take students with lower average grades than school sixth forms or sixth form colleges and where there has been the largest increase in numbers studying qualifications in maths at level 2.

22. Many who teach GCSE mathematics in FE colleges lack appropriate experience and training. This and the low A\*-C pass rates for GCSE resits point to a need to do more to support teachers' professional development, especially in colleges.

**Recommendation 5: In view of the low GCSE success rates and new GCSE requirements, the Department for Education should review its 16-18 resit policy with the aim that a greater proportion of students without a grade C or equivalent attain appropriate mathematical understanding by age 18. Specifically, there should be fresh consideration of appropriate curricula and qualifications for these students and the extent to which current policy incentivises these to be offered.**

## **Capacity to deliver**

23. There have been consistent increases in the number of students taking further mathematics AS and A level qualifications over recent years and the Further Maths Support Programme has had a positive impact on the quality of provision and student outcomes.

24. A combination of changes to A levels ('decoupling' of AS) and changes to funding (per student rather than per qualification) are combining to present, what many consulted during the course of the review see as, serious risks to the provision of AS/A level further mathematics. If state school entries in further mathematics were reduced this could impact on the admission of state school pupils to mathematical courses at research intensive universities.

25. Core maths qualifications are always studied as an addition to a student's main programme. Providers delivering core maths at scale are in the minority, and those with smaller cohorts find it difficult to resource, especially smaller school sixth forms. The current funding model does not incentivise core maths provision.

**Recommendation 6: The Department for Education should reconsider the institutional incentives and disincentives arising from the 16-19 funding model for schools and colleges, with a view to removing disincentives for mathematics provision;**

**As an urgent and immediate measure, it should consider increasing the financial incentive for both AS and A level further mathematics within four/five A level programmes and consider providing a funding incentive for student programmes which include core maths.**

26. Schools report significant challenges recruiting skilled mathematics teachers. Meeting the current demand for mathematics specialists is challenging and demand for mathematics in the labour market will continue to create pressure on supply from a relatively small graduate pool. The Department for Education is putting in place a range of additional measures to improve teacher supply, including mathematics subject training for non-mathematics specialists and for those returning to the profession.

27. Little is known about the workforce teaching mathematics and quantitative skills in FE colleges, but colleges highlight challenges in recruiting the right staff. The new and significant demand for teaching as a result of GCSE resits creates pressures on FE colleges in particular. The government has invested £30 million to train thousands of mathematics teachers in the FE sector, but the low pass rates for GCSE resits suggests significant ongoing need for investment to improve teaching.

**Recommendation 7: The Department for Education should improve the evidence base on the FE workforce teaching mathematics and quantitative skills in order to assess supply, teaching quality and the effectiveness of current recruitment measures.**

**Recommendation 8: The Department for Education should expand its support to develop excellence in GCSE mathematics teaching across the FE sector. This should be informed by evidence of effective pedagogy for students who have not succeeded in the subject within secondary education and emerging evidence about the needs of the workforce.**

28. Continued government funding for a support programme for core maths is essential. Significant numbers of teachers, including specialists in other quantitative subjects, should be trained to teach core maths, and the qualification needs to gain wide reach, covering all local areas and types of school and college.

**Recommendation 9: The Department for Education should continue to fund a central core maths programme until the qualification becomes embedded and enhance this to upskill large numbers of teachers of other quantitative subjects to teach core maths.**

29. Substantial reforms to AS/A levels in mathematics and further mathematics will mean that schools will need significant further support to teach these new qualifications effectively.

**Recommendation 10: The Department for Education should continue to fund centrally-delivered professional development programmes for teachers of AS/A level mathematics and further mathematics at least at their current level.**

30. Universities also have the potential to make a positive impact on mathematics attainment in other ways, for example by developing curriculum and teaching materials and enhancing teachers' subject knowledge. There is a good case for universities to consider providing support for disciplines of strategic importance or vulnerable subjects, not only mathematics but also computing and modern foreign languages.

**Recommendation 11: The Department for Education, in issuing any new guidance to the independent Director of Fair Access to Higher Education, should continue to encourage universities to support 16-18 mathematics education in the context of access for and success of students from disadvantaged backgrounds.**

**Recommendation 12: The Department for Education, in supporting the Prime Minister's desire for higher education to engage more with schools, should seek ways to encourage universities to consider specialism in 16-18 mathematics if establishing new schools, sponsoring existing schools or providing other support to schools, particularly in local areas where level 3 mathematics participation and achievement is poor.**

31. The stark regional and sub-regional differences in attainment and participation in 16-18 mathematics require more targeted and intensive responses in some areas. One potential solution may be to link work to improve attainment and progression in 16-18 mathematics to wider, high profile action to improve local outcomes and increase social mobility.

**Recommendation 13: The Department for Education should commission and fund interventions in local areas with low level 3 mathematics participation, co-ordinating local work to provide the best training and support for schools and colleges to build capacity at GCSE and level 3. This should include developing partnerships between schools and local and national sources of support.**

32. Technology is already adding value to 16-18 teaching but there appear to be no widely adopted technological solutions to specialist teaching capacity issues. Schools and colleges are free to use the teaching approaches that meet the needs of their students. This has led to variation in approach, scale and effectiveness of the use of technology across the system. Nonetheless, there is evidence of potential for technology to address some of the challenges.

**Recommendation 14: The Department for Education should seek to improve the evidence base on the role and effectiveness of technology in the teaching of 16-18 mathematics.**

**Recommendation 15: The Department for Education, in conjunction with partners such as the Institute for Apprenticeships, should fund online professional development resources and materials aimed at increasing the numbers of teachers of mathematics and quantitative skills within new technical education routes and core maths.**

## **Policy and System Issues**

33. There is a strong case for higher uptake of 16-18 mathematics. Increased participation would be likely to deliver significant payback in terms of labour market skills, returns to individuals, increased productivity and longer-term economic benefits.

34. The government should set an ambition for 16-18 mathematics to become universal in 10 years. There is not a case at this stage, however, for making it compulsory.

35. Participation should be guided by the principle that all students should study the mathematics they need for the future. If delivered in practice, this would result in a significant boost to the number of students taking mathematics at level 3.

36. In the medium term (5 years), strong provision should become established across all mathematics pathways, with core maths becoming widely available for students who can benefit and appropriate mathematics courses in place for technical routes. Other provision should be strengthened with the aim that pass rates improve – notably for GCSE resits.

37. In the shorter term, high priority should be placed on the challenges of continuing to expand teaching capacity through recruitment and upskilling of teachers, including non-mathematics specialists, and on encouraging schools and colleges to offer mathematics options and encouraging students to take these.

38. There are risks to achieving improvement to 16-18 mathematics in the context of changes to GCSE and AS/A level mathematics and other recent reforms, including decoupling of A levels and 16-19 funding constraints. The government should keep progress in strengthening provision and improving uptake under close review.

39. Funding levels across 16-18 education are of particular concern as this limits the range of courses that can be provided for students, particularly in smaller schools.

40. This review was limited in scope to 16-18 mathematics, but the poor performance of GCSE resit students and the lack of confidence in mathematics among many students with good GCSE grades reflect problems at earlier stages. Due consideration should be given to how secondary mathematics teaching could be improved so that more students gain numeracy by age 16 and secure understanding of concepts taught at GCSE.

41. Negative attitudes towards mathematics generally are a cause for concern. Gender has a heavy influence on mathematics participation, reflecting entrenched cultural attitudes towards mathematics.

**Recommendation 16: The Department for Education should commission a study, from pre-school onwards, into the cultural and other root causes of negative attitudes to mathematics, including gender and other sub-group effects.**

42. Despite the commendable strengthening in recent years of requirements on schools to provide high quality, independent careers advice and guidance to young people, there remain significant challenges in getting good information to young people to inform their subject and life choices.

43. Given the demand for mathematical skills in the economy, it is essential that young people are provided with far clearer information about the mathematical needs of degree courses, technical fields and, importantly, their future careers.

**Recommendation 17: The Department for Education should, in any future work to improve careers provision and related advice, prioritise and make clear the importance of mathematics to a wide range of future careers.**

44. The increasing sophistication of technology is driving change to the economy and the nature of work. This is not only increasing the demand for mathematics and quantitative skills but is also changing the nature of required skillsets, in particular those relating to the analysis and use of 'big data'.

**Recommendation 18: The Department for Education and the Department for Business, Energy & Industrial Strategy should commission a study into the long-term implications of the rise of data science as an academic and professional field, looking at skills required for the future and the specific implications for education and training in mathematics and quantitative skills.**

# Chapter 1: Introduction

## Background and terms of reference

45. This review considers ways of improving mathematics education for 16-18 year olds with the aim of ensuring that the future workforce is appropriately skilled and competitive. This includes considering the case and feasibility for more or all students continuing to study mathematics to age 18 in the longer-term.

46. Throughout this review, the term ‘mathematics’ is used in its broadest sense – referring to mathematics and quantitative skills. It includes numeracy, statistics and data analysis.

47. In examining these issues, this report sets out:

- the rationale for change to 16-18 mathematics education: the economic and labour market context and employment trends, and what this means for current and future demand for mathematical and quantitative skills in light of these;
- issues in 16-18 mathematics provision: the range and suitability of current and proposed mathematics routes and qualifications, and patterns of participation and progression, including progression by different groups of students; and
- delivery challenges: potential barriers to increasing the provision of 16-18 mathematics, including specialist teacher supply and funding, and the potential for the use of technology to help address related issues.

48. Finally, this review considers the case and feasibility for all students continuing to study mathematics to 18 and the issue of whether mathematics should become a compulsory subject for all 16-18 year olds.

49. Professor Adrian Smith reviewed post-14 mathematics education in 2004,<sup>2</sup> providing a root and branch overview of the challenges facing mathematics at that time. The inquiry led to several changes, including:

- revisions to A level mathematics to improve take-up;
- a redesign of GCSE mathematics to improve rigour and offer everyone the chance of achieving a grade C;

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<sup>2</sup> Smith A (2004) *Making mathematics count. The report of Professor Adrian Smith's inquiry into post-14 mathematics education* (Department for Education and Skills), <http://dera.ioe.ac.uk/4873/>

- the establishment of a National Centre for Excellence in Mathematics Teaching (NCETM);
- an enhanced role for the Advisory Committee on Mathematics Education (ACME), an independent committee providing advice on mathematics education policy;<sup>3</sup>
- higher targets for the recruitment of mathematics teachers; and
- new professional development programmes for teachers.

50. The 2004 review also explored potential mathematics pathways to age 19, setting out principles for these with the aim that every student might at some future time study mathematics beyond age 16. The 14-19 education landscape has changed considerably since 2004, but the principles are still relevant.

51. Many of the issues identified in the review have been addressed. Since 2004, for example, entries to A level mathematics have risen by 77 per cent,<sup>4</sup> with mathematics now the most popular A level<sup>5</sup>. Far more students continue the study of mathematics in 16-18 education and training – mainly through functional skills and GCSE qualifications. New ‘core maths’ qualifications have been launched as an option for students with a GCSE grade A\*-C or equivalent in mathematics at age 16 who want to use and apply mathematics and statistics at a more advanced level but who do not take A or AS level mathematics in their main study programme.

52. Despite this significant progress, however, there are continuing challenges for mathematics education in England. There continues to be a shortage of specialist mathematics teachers and there are still relatively low numbers of students studying mathematics after age 16. When students do study mathematics, achievement can often be weak, particularly in the case of GCSE resits. Changes in the labour market are also presenting the need for new skills – in particular in the use and analysis of data.

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<sup>3</sup> Established in 2002 by the Royal Society and the Joint Mathematical Council of the UK, ACME is now a Royal Society Committee.

<sup>4</sup> Department for Education (2017) *A level and other 16 to 18 results: 2015 to 2016 (revised)*, Subject time series: SFR05/2017, Table 11, <https://www.gov.uk/government/statistics/a-level-and-other-16-to-18-results-2015-to-2016-revised>

<sup>5</sup> Department for Education (2017) *Revised A level and other 16-18 results in England, 2015/2016*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/584124/SFR05\\_2017\\_A\\_level\\_and\\_other16-18\\_results\\_in\\_England\\_SFR\\_revised.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/584124/SFR05_2017_A_level_and_other16-18_results_in_England_SFR_revised.pdf)



## Scope, challenge and limitations of the review

53. The review looks at mathematics for the range of students in 16 to 18 education and training, which includes:

- Students with an A\*-C grade or equivalent grade in GCSE mathematics at age 16 – this group progresses to both academic study (in school or college settings) and to technical programmes (mainly in further education). Most, but particularly students with a grade B or C, drop mathematics at age 16. Others progress to AS/A level mathematics or other level 3 mathematics qualifications.
- The GCSE 'resit' group – students aged 16-18 with a grade D or equivalent who are required to study GCSE mathematics as a condition of funding.
- Other students needing to secure skills below GCSE level, who generally achieved grade E or below at age 16. The large majority of these students study in FE colleges; and
- Students looking to gain additional mathematics for the workplace – mathematics that is specific to a technical area, such as financial mathematics or technical measurement.

54. 16-18 education is a pivotal period in young people's education. It is the final full-time education experience that many students have and an important staging post towards higher education or training.

55. 16-18 students study a range of programmes: academic (A levels, International Baccalaureate), applied (applied general qualifications) and technical (soon to be defined through technical routes). They do this in different settings: school sixth forms, sixth form colleges, further education colleges, and through apprenticeships, other providers and workplace settings. They have differing aims – for example, to progress to mathematical study in higher education, or to support social science study with a quantitative element; to work in technical fields requiring mathematics and quantitative skills; or to acquire the basic qualitative skills required for work and everyday life.

56. The guiding principle for this review is that every student should access the mathematics needed for their future and be offered appropriate support to achieve in this. By age 18, every young person should have secure numeracy – the ability to use and apply basic mathematical knowledge to make decisions and engage in society. Most should have gained, and retained securely, the fundamental mathematics needed to thrive in the modern workplace – for example, the ability to

analyse, interpret and present quantitative and statistical information and reason with data.

57. Beyond these basic requirements, students progressing to university study or higher technical fields with a mathematical or statistical element require more advanced (level 3) mathematics and quantitative skills. These students need the ability to apply mathematics and quantitative skills in a variety of contexts, and certain technical careers demand further specific knowledge and skills for students to succeed within that technical area.

58. This review has taken place at a time of some change. Qualification reforms are at early stages of embedding, and other reforms are in progress, including:

- The introduction of a new mathematics GCSE, which will be examined for the first time in 2017.<sup>6</sup> Content was reformed to raise standards, provide greater assurance of fundamental mathematical skills and knowledge, and provide stretch at the top end. The grading structure has also changed.<sup>7</sup>
- The 'D-Grade requirement' for students without A\*-C in GCSE mathematics to study the GCSE post-16. Since 2014, it has been a condition of school and college funding for students aged 16-19 without GCSE A\*-C grade to continue to study towards this. In September 2015, this requirement was strengthened so that full-time students with a grade D retake mathematics GCSE.<sup>8</sup>
- First teaching of the new mathematics and further mathematics A levels in 2017. The new A levels are set at a similar level of demand, but there is a common compulsory A level mathematics curriculum.<sup>9</sup>
- A review of functional skills qualifications. Mathematics (and English) functional skills qualifications are being reformed to improve their relevance and rigour, and improve recognition and credibility in the labour market, with new qualifications due to be in place by 2019.<sup>10</sup>

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<sup>6</sup> Department for Education (2017) *Get the facts: GCSE reform*, <https://www.gov.uk/government/publications/get-the-facts-gcse-and-a-level-reform/get-the-facts-gcse-reform>

<sup>7</sup> Department for Education (2014) *Setting standards for new GCSEs in 2017*, <https://www.gov.uk/government/news/setting-standards-for-new-gcse-in-2017>

<sup>8</sup> Department for Education (2016) *16 to 19 funding: maths and English condition of funding*, <https://www.gov.uk/guidance/16-to-19-funding-maths-and-english-condition-of-funding>

<sup>9</sup> A Level Content Advisory Board (2014) *Report of the ALCAB panel on mathematics and further mathematics*, <https://alevelcontent.files.wordpress.com/2014/07/alcab-report-on-mathematics-and-further-mathematics-july-2014.pdf>

<sup>10</sup> Education & Training Foundation (2016) *ETF launches Functional Skills Reform consultation*, <http://www.et-foundation.co.uk/news/etf-launches-functional-skills-reform-consultation/>

Lord Sainsbury's review of technical education in England and the government's response in its Post-16 Skills Plan, published in 2016. The planned reforms will encourage a strong focus on mathematics (and English) post-16.<sup>11</sup>

59. Further details about these changes are set out in Chapter 4, and a timeline can be found in Annex A. Many of these developments present leadership and subject challenges for schools and colleges, and come with professional development implications for teaching staff. Some will demand greater specialist mathematics teaching capacity – something which is in relatively short supply – or will have implications for up-skilling of non-specialists.

## **Approach taken**

60. This review considered a wide range of evidence and research of relevance to 16-18 mathematics education. Much of this evidence is referenced within this report and its annexes. The Department for Education published a related ad-hoc statistical notice to accompany this report.<sup>12</sup>

61. The review team also talked to a large number of individuals and organisations with an interest in 16-18 mathematics, including employer groups, university representatives and mathematics organisations. The list of consulted stakeholders is set out at Annex B.

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<sup>11</sup> Department for Education (2016) *Post-16 skills plan and independent report on technical education*, <https://www.gov.uk/government/publications/post-16-skills-plan-and-independent-report-on-technical-education>

<sup>12</sup> Department for Education (2017) *Ad-hoc notice: Post 16 maths participation in 2015/16*, <https://www.gov.uk/government/publications/post-16-maths-participation>

## Chapter 2: The case for mathematics

62. This chapter sets out the case for mathematics as a special discipline – fundamentally important to many aspects of our lives. Mathematics is a formal discipline that underpins and is fundamental to all scientific disciplines and technical fields. The reach and importance of mathematics extends well beyond the physical sciences, with data and algorithms increasingly pervading all aspects of modern life and an increasingly large range of fields of study.

### Mathematics and quantitative skills are essential in modern life

63. Everyone needs good numeracy – the ability to use and apply basic quantitative skills to make decisions and engage in society. Such skills are necessary elements of citizenship, for example to participate in or follow public debate individuals need the ability to understand and potentially challenge arguments based on quantitative evidence.<sup>13</sup>

64. At work, we are often required to use analytical reasoning, make calculations and create, use and interpret data. To be effective in these areas requires mathematics achievement beyond basic numeracy.<sup>14</sup>

65. Quantitative skills are required in a wide range of occupations and activities, embracing not only the mathematical and physical sciences but also the social sciences, the humanities and the creative arts. There has long been a divide between disciplines, with students often making early decisions to follow either a sciences or a humanities path, but mathematics is important for a wide range of disciplines across both arenas. Mathematics is now intrinsic to some aspects of the creative arts, for example<sup>15</sup>, and learned societies argue that students across the sciences, social sciences and humanities need significant quantitative skills, and these should be a central component of their education.<sup>16</sup>

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<sup>13</sup> British Academy (2015a) *State of the Nation: a review of evidence on the supply and demand of quantitative skills*, <http://www.britac.ac.uk/sites/default/files/BA-NIESR%20State%20of%20the%20Nation%20-%20A%20review%20of%20evidence%20on%20the%20supply%20and%20demand%20of%20QS.pdf>

<sup>14</sup> Advisory Committee on Mathematics Education (2011) *Mathematical needs: mathematics in the workplace and in higher education*, <https://royalsociety.org/~media/policy/topics/education-skills/Maths/Mathematical-needs-2011.pdf?la=en-GB>

<sup>15</sup> Plus Magazine (2010) *Teacher package: Maths and art*, Millennium Mathematics Project, <https://plus.maths.org/content/teacher-package-maths-and-art>

<sup>16</sup> British Academy (2015b) *Count Us In: Quantitative Skills for a New Generation*, [http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report\\_0.pdf](http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report_0.pdf)

### **Mathematics, technology and change – forecasting and simulation**

Complex mathematical models are used to forecast weather systems, for example to predict the path of destructive storms to provide warnings and help plan emergency responses.<sup>17</sup>

Mathematics underpins the 3D modelling software written to create cutting-edge special effects in the latest movies.<sup>18</sup> Improved sensors and the creation of 3D road maps are now opening up possibilities for automated vehicle navigation.<sup>19</sup>

Complex flight simulators allow pilots to practise for the unexpected. In the future, virtual reality will open the way to new ways of training people in other spheres such as fire-fighting, first aid, nuclear decommissioning, deep-sea welding and high-tech manufacturing.<sup>20</sup>

66. Accuracy in measurement and data analysis are central to the performance of many organisations and are sometimes critical, for example for managing public health or enforcing safety – nurses calculate precise dosages of medications; railway technical staff measure and monitor track quality.<sup>21</sup>

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<sup>17</sup> Deloitte (2012) *Measuring the Economic Benefits of Mathematical Science Research in the UK*, <https://www.epsrc.ac.uk/newsevents/pubs/deloitte-measuring-the-economic-benefits-of-mathematical-science-research-in-the-uk/>

<sup>18</sup> Deloitte (2012) *ibid.*

<sup>19</sup> Deloitte (2014) *London Futures Agiletown: the relentless march of technology and London's response*, <https://www2.deloitte.com/content/dam/Deloitte/uk/Documents/uk-futures/london-futures-agiletown.pdf>

<sup>20</sup> Baker K (2016) *The digital revolution*,

[http://www.edge.co.uk/sites/default/files/documents/digital\\_revolution\\_web\\_version.pdf](http://www.edge.co.uk/sites/default/files/documents/digital_revolution_web_version.pdf)

<sup>21</sup> British Academy (2015a) *ibid.*

## Mathematics and technology underpin machine learning and artificial intelligence

Machine learning underpins how the internet works and helps tailor users' web experiences.<sup>22</sup>

'Smart motorways' in the UK use feedback on road conditions from embedded sensors and artificial intelligence systems to anticipate and manage traffic flow.<sup>23</sup>

Cars can now park themselves, perform adaptive cruise control on highways, steer themselves during stop-and-go traffic, and alert drivers about objects in blind spots during lane changes. Vision and radar technology are used to develop pre-collision systems that let cars autonomously brake when risk of a collision is detected. Deep learning has also been applied to improve cars' capacity to detect objects in the environment and recognise sound.<sup>24</sup>

In financial markets, 'high-frequency trading' algorithms use mathematical code to respond to changes in markets many times faster than human traders. Financial advisers use similar algorithms to spot investment opportunities for clients.<sup>25</sup> 'Robo advisers' provide automated investment advice through web-based platforms at a lower cost than human financial advisers.<sup>26 27</sup>

67. Many technical occupations, often accessed via apprenticeships, require strong quantitative skills. Chemistry technicians prepare solutions to specific concentrations and analyse them. Production line operators use statistical process control methods of data collection and analysis.<sup>28</sup> Mechanical engineering technicians assist with the design, development, operation, installation and maintenance of engineering systems, for example, making parts, and installing and

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<sup>22</sup> Government Office for Science (2015) *Artificial intelligence: opportunities and implications for the future of decision making*,

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/566075/gs-16-19-artificial-intelligence-ai-report.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/566075/gs-16-19-artificial-intelligence-ai-report.pdf)

<sup>23</sup> Government Office for Science (2015) *ibid.*

<sup>24</sup> Stanford University (2016) *Artificial intelligence and life in 2030*,  
<https://ai100.stanford.edu/2016-report>

<sup>25</sup> Government Office for Science (2015) *ibid.*

<sup>26</sup> Reuters (2016) *Wells Fargo to roll out pilot robo adviser in first half of 2017*,  
<http://www.reuters.com/article/us-wells-fargo-wealth-robo-adviser-idUSKCN1002E9>; *Exclusive: Wells Fargo to unveil robo-adviser partnership with SigFig*, <http://www.reuters.com/article/us-wells-fargo-technology-exclusive-idUSKBN13A1YB>

<sup>27</sup> BBC (2015) *How artificial intelligence is transforming the financial industry*,  
<http://www.bbc.co.uk/news/business-34264380>

<sup>28</sup> British Academy (2015a) *ibid.*

testing instruments or machinery to make sure they run smoothly, safely and meet performance targets.<sup>29</sup>

### **Mathematics, big data, medicine and digital health**

The collection and analysis of complex biological data, such as genetic codes, provides extraordinary opportunities to understand the causes of a range of different health conditions. Mathematical modelling is increasingly used to aid diagnosis and the development of new treatments.<sup>30</sup>

Algorithms are used to minimise blur in 3D X-ray imaging of patients to enable the location of a tumour to be better pinpointed, helping to target radiation treatment and protect healthy tissue.<sup>31</sup>

Digital techniques are used to accelerate drug discovery. Mobile health uses mobile apps to give real-time access to sensors monitoring vital signs. Wearable biosensors are able to monitor chemicals and biomarkers to let doctors determine how well a drug is metabolised and adjust dosage and frequency accordingly.<sup>32</sup>

Pharmaceutical companies are able to exploit large volumes of data on social media from patient communities to understand drug side effects and to improve patient care.<sup>33</sup>

For patients with heart disease, diabetes, depression and other chronic conditions, telehealth technologies such as home tele-monitoring have the potential to reduce hospital readmissions and help people to live independently and adhere to their drug schedule.<sup>34</sup>

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<sup>29</sup> UK Commission for Employment and Skills (2015a) *Reviewing the requirement for high level STEM skills*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/444052/stem\\_review\\_evidence\\_report\\_final.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/444052/stem_review_evidence_report_final.pdf)

<sup>30</sup> BBC (2016) *Maths becomes biology's magic number*, <http://www.bbc.co.uk/news/science-environment-37630414>

<sup>31</sup> Biguri A (2016) *Engineering and design student insights: algorithms to improve medical imaging*, <http://blogs.bath.ac.uk/engdes-student-insights/2016/10/06/algorithms-to-improve-medical-imaging/>

<sup>32</sup> Deloitte (2015) *2016 global life sciences outlook*, <https://www2.deloitte.com/content/dam/Deloitte/global/Documents/Life-Sciences-Health-Care/gx-lshc-2016-life-sciences-outlook.pdf>

<sup>33</sup> Deloitte (2015) *ibid.*

<sup>34</sup> Deloitte (2015) *ibid.*

68. Access to a workforce with appropriate qualifications in science, technology, engineering and mathematics (STEM), is vital for many growing parts of the economy<sup>35</sup> – and there is no ‘STEM’ without the ‘M’.

69. While mathematics underpins the STEM disciplines, its impact is often less obvious than in other areas where impacts are realised in the form of products or technologies. Knowledge exchange in UK mathematical sciences is ‘patchy’ and harder to access by industry. As a result, Innovate UK and the Engineering and Physical Sciences Research Council are reviewing how to overcome those barriers and make it easier for industry to benefit from mathematical science expertise.<sup>36</sup>

70. In the era of online transactions, software has already automated many routine transaction processes. As technology becomes more sophisticated, it will be able to undertake increasingly complex tasks. Digitisation and data will be central to many future skills requirements, and understanding these processes will be valuable to employability and citizenship. Citizens’ interactions with the public sphere and the commercial world are increasingly digital by default. Mathematical and quantitative skills form part of the foundation for understanding how this world works, and how citizens interact with it and with each other. Without this understanding, individuals risk being excluded or reduced to passive consumers of services, finding it difficult to think critically about their interactions with the digital world.

## **Mathematics can enhance individual life opportunities**

71. Well-established evidence shows that adults with basic numeracy skills earn higher wages and are more likely to be in employment than those who fail to master these skills. Acquiring basic numeracy skills supports achievement of qualifications which in turn support employment and further upskilling and development.<sup>37,38</sup> In England, the link between better basic skills, higher wages and lower risk of unemployment is particularly strong.<sup>39</sup>

72. Amongst individuals whose highest qualification was an A level, pupils performing well on mathematics tests (British Cohort Study BCS70) at age 10 earn

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<sup>35</sup> HM Treasury (2011) *The Plan for Growth*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/31584/2011budget\\_growth.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/31584/2011budget_growth.pdf)

<sup>36</sup> Innovate UK (2016) *Are we making the most of our mathematical masterminds?* <https://ktn-uk.co.uk/articles/are-we-making-the-most-of-our-mathematical-masterminds>

<sup>37</sup> OECD (2016a) *Skills matter: further results from the survey of adult skills*, [https://www.oecd.org/skills/piaac/Skills\\_Matter\\_Further\\_Results\\_from\\_the\\_Survey\\_of\\_Adult\\_Skills.pdf](https://www.oecd.org/skills/piaac/Skills_Matter_Further_Results_from_the_Survey_of_Adult_Skills.pdf)

<sup>38</sup> Heckman J J (2008) *Schools, Skills, and Synapses*, *Economic Inquiry* 46(3) 289-324, [http://jenni.uchicago.edu/papers/Heckman\\_2008\\_EI\\_v46\\_n3.pdf](http://jenni.uchicago.edu/papers/Heckman_2008_EI_v46_n3.pdf)

<sup>39</sup> OECD (2016b) *Building skills for all: a review of England*, <https://www.oecd.org/unitedkingdom/building-skills-for-all-review-of-england.pdf>



more in later life, even after accounting for holding A level qualifications. Compared to scoring in the lowest mathematics test quartile at age 10, a female in the top quartile earns a 23.9% premium, while a male achieves a 12.5% premium.<sup>40</sup> Comparing individuals who have achieved A\*-C in at least 5 GCSEs and who hold at least an A level or higher qualification, top quartile individuals are also more likely to be in work between ages 29 and 42 (based on a subset of individuals).<sup>41</sup>

73. Individuals who achieve five or more good GCSEs (including English and mathematics) as their highest qualification have a lifetime productivity gain worth around £100,000 compared to those with below level 2 or no qualifications.<sup>42</sup> Attainment at 16 (GCSE) also plays a crucial role in access to higher education in England.<sup>43</sup>

74. A 2012 survey by the Confederation of British Industry reported that, for numeracy, 43 per cent of responding employers considered GCSE A\*-C grades (i.e. 'good' passes for these level 2 qualifications) to be best for equipping young people for work, followed by A levels (level 3 qualifications) in STEM subjects (25 per cent of respondents).<sup>44</sup> (See boxes following paragraph 107 for further information on level 2 and level 3 qualifications.)

75. It is generally accepted that attaining A level mathematics bestows a significant wage premium. The often-cited work by Dolton and Vignoles<sup>45</sup> demonstrated that graduates from 1980 with a mathematics A level were earning 7-10 per cent higher wages in their early thirties compared to those who took A levels

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<sup>40</sup> London Economics (2015) *The earnings and employment returns to A-levels*, <http://londoneconomics.co.uk/wp-content/uploads/2015/03/London-Economics-Report>Returns-to-GCE-A-Levels-Final-12-02-2015.pdf>

<sup>41</sup> London Economics (2015) *The earnings and employment returns to A-levels* *ibid*.

<sup>42</sup> Department for Education (2014) *The economic value of key intermediate qualifications: estimating the returns and lifetime productivity gains to GCSEs, A levels and apprenticeships*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/387160/RR398A\\_-\\_Economic\\_Value\\_of\\_Key\\_Qualifications.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/387160/RR398A_-_Economic_Value_of_Key_Qualifications.pdf)

<sup>43</sup> Vignoles A, Dearden L, Crawford C and Micklewright J (2016) *Family Background and University Success*, <https://www.ifs.org.uk/uploads/Presentations/Family%20Background%20and%20University%20Success.pdf>

<sup>44</sup> Confederation of British Industry (2012) *Learning to grow: what employers need from education and skills: Education and skills survey 2012*, [http://www.ucml.ac.uk/sites/default/files/shapingthefuture/101/cbi\\_education\\_and\\_skills\\_survey\\_2012.pdf](http://www.ucml.ac.uk/sites/default/files/shapingthefuture/101/cbi_education_and_skills_survey_2012.pdf)

<sup>45</sup> Dolton P J and Vignoles A (2002) *Is a Broader Curriculum Better?*, *Economics of Education Review* 21(5) 415–29, [http://ac.els-cdn.com/S0272775701000462/1-s2.0-S0272775701000462-main.pdf?\\_tid=9241a358-a5c0-11e6-9017-00000aab0f01&acdnat=1478615898\\_73c2f5ae43f2ab0affb7925dd121cc38](http://ac.els-cdn.com/S0272775701000462/1-s2.0-S0272775701000462-main.pdf?_tid=9241a358-a5c0-11e6-9017-00000aab0f01&acdnat=1478615898_73c2f5ae43f2ab0affb7925dd121cc38)

in other subjects, even after controlling for undergraduate and postgraduate degrees. A repeat of this work using a later cohort found a return of around 11 per cent.<sup>46</sup>

76. Mathematical concepts and techniques are required in higher education for a wide range of disciplines, not just the sciences.<sup>47</sup> By continuing to study mathematics post-16, students are better prepared for studies in higher education by ensuring their skills are current when they arrive and giving them confidence in tackling the mathematical and statistical content within their university course.

77. A recent study<sup>48</sup> showed that, 10 years into the labour market, graduates were a small proportion of those with very low earnings, and around 15 per cent of graduates had no earnings compared to 20-30 per cent of non-graduates. Average earnings for mathematics and computer science graduates and for business studies graduates were roughly the same, with both doing slightly better than graduates as a whole.

78. Where students continue on to STEM disciplines in higher education, labour market statistics show that they have lower unemployment rates and higher median salaries when compared to graduates who study social sciences, arts and humanities subjects.<sup>49</sup> STEM skills are in widespread demand, and achieving 2 or more A levels in STEM subjects can boost earnings.<sup>50</sup> Around half of individuals in jobs where mathematical sciences qualifications are essential were found to have salaries of £29,000 or more, compared with only 19 per cent of the UK workforce overall.<sup>51</sup>

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<sup>46</sup> Adkins M and Noyes A (2016) *Reassessing the economic value of advanced level mathematics*, [http://eprints.nottingham.ac.uk/31453/1/WP1\\_Post\\_Print.pdf](http://eprints.nottingham.ac.uk/31453/1/WP1_Post_Print.pdf)

<sup>47</sup> Hodgen J, McAlinden M and Tomei A (2014) *Mathematical transitions: a report on the mathematical and statistical needs of students undertaking undergraduate studies in various disciplines* (York: Higher Education Academy), [https://www.heacademy.ac.uk/system/files/resources/hea\\_mathematical-transitions\\_webv2.pdf](https://www.heacademy.ac.uk/system/files/resources/hea_mathematical-transitions_webv2.pdf)

<sup>48</sup> Institute for Fiscal Studies (2016) *How English domiciled graduate earnings vary with gender, institution attended, subject and socio-economic background*, <https://www.ifs.org.uk/uploads/publications/wps/wp201606.pdf>

<sup>49</sup> Department for Business, Innovation & Skills (BIS) (2017) *Graduate labour market statistics: 2016*, <https://www.gov.uk/government/statistics/graduate-labour-market-statistics-2016>  
STEM is defined in the Labour Force Survey.

<sup>50</sup> Greenwood C, Harrison M and Vignoles A (2011) *The labour market value of STEM qualifications and occupations*, an analysis for the Royal Academy of Engineering, [http://www.raeng.org.uk/publications/reports/the\\_labour\\_market\\_value\\_of\\_stem](http://www.raeng.org.uk/publications/reports/the_labour_market_value_of_stem)

<sup>51</sup> Council for Mathematical Sciences (CMS) (2015) *The mathematical sciences people pipeline*, [http://www.cms.ac.uk/files/News/article\\_5630c69e789971.96989222.pdf](http://www.cms.ac.uk/files/News/article_5630c69e789971.96989222.pdf)

## Higher-level skills are central to economic growth and productivity

79. Increased productivity is a key determinant of economic growth and, together with higher employment rates and hours worked, is the primary route to higher living standards.<sup>52</sup> The UK has relatively poor productivity performance<sup>53</sup> and has long lagged behind other major economies in this respect. The Office for National Statistics estimates that on an output per hour basis, UK productivity was 18 percentage points below the average for the rest of the G7 advanced economies (Canada, France, Germany, Italy, Japan, the UK and the USA) in 2014.<sup>54</sup>

80. Higher levels of education and skills raise productivity *directly* by enabling individuals to accomplish difficult tasks and address complex problems. They also raise productivity *indirectly* by facilitating technological diffusion and innovation.<sup>55</sup> Mathematics and quantitative skills are a vital component of this. Furthermore, analysis of student performance in international mathematics and science tests from 50 countries over 40 years shows that higher performance is significantly and positively related to growth in real output per capita.<sup>56</sup>

81. In the UK, around seven in ten employees report that quantitative skills are essential or important to carry out their work.<sup>57</sup> Recognising the need for skills is different to having these skills, and the annual cost to the economy due to low numeracy skills in the working age population is estimated to be between £7 billion

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<sup>52</sup> OECD (2015a) *The Future of Productivity*, <https://www.oecd.org/eco/growth/OECD-2015-The-future-of-productivity-book.pdf>

<sup>53</sup> HMT (2015) *Fixing the Foundations: Creating a more prosperous nation*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/443898/Productivity\\_Plan\\_web.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/443898/Productivity_Plan_web.pdf)

<sup>54</sup> Office for National Statistics (2016) *Statistical bulletin: international comparisons of UK productivity – final estimates: 2015*, <https://www.ons.gov.uk/economy/economicoutputandproductivity/productivitymeasures/bulletins/internationalcomparisonsofproductivityfirstestimates/2015>

<sup>55</sup> Department for Business, Innovation & Skills and the National Institute of Economic and Social Research (2015) *UK Skills and Productivity in an International Context*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/486500/BIS-15-704-UK-skills-and-productivity-in-an-international\\_context.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/486500/BIS-15-704-UK-skills-and-productivity-in-an-international_context.pdf)

<sup>56</sup> Hanushek E A and Woessmann L (2012) *Do Better Schools Lead to More Growth? Cognitive skills, economic outcomes, and causation*, J. Econ. Growth 17 267-321, [http://hanushek.stanford.edu/sites/default/files/publications/Hanushek%2BWoessmann%202012%20J%20EconGrowth%2017\(4\).pdf](http://hanushek.stanford.edu/sites/default/files/publications/Hanushek%2BWoessmann%202012%20J%20EconGrowth%2017(4).pdf)

<sup>57</sup> British Academy (2015a) *ibid.*

and £33 billion.<sup>58</sup> In 2012, around 20 per cent of young people in the UK did not have basic skills.<sup>59</sup>

82. The economic benefits of increasing the skills of the UK workforce are potentially huge. The Organisation for Economic Co-operation and Development (OECD) estimates that if *all* young people in the UK acquired basic skills by 2030, by 2095 the economy would be 13 per cent larger than would be expected with the current labour force (143 per cent of current UK GDP).<sup>60</sup>

83. Advanced quantitative skills are also important to the economy, especially in the growing need to exploit big data, where there is an urgent need for graduates with a combination of quantitative, computing, analytical and business skills.<sup>61</sup>

84. Jobs where mathematical science is a significant element comprise around 10 per cent of all jobs in the UK and contribute over £200 billion in terms of Gross Value Added (around 16 per cent of total UK GVA). Mathematical science occupations include mathematicians and statisticians, engineers and scientists, IT professionals, social scientists, finance professionals, medical practitioners, and many administrators and senior managers. Productivity is significantly higher in sectors that make use of such occupations compared to other sectors.<sup>62</sup>

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<sup>58</sup> Pro Bono Economics (2014) *Cost of outcomes associated with low levels of adult numeracy in the UK*, <http://www.probonoeconomics.com/sites/default/files/files/PBE%20National%20Numeracy%20costs%20report%2011Mar.pdf>

<sup>59</sup> OECD (2015b) *Universal Basic Skills: What Countries Stand to Gain*, <http://www.oecd-ilibrary.org/docserver/download/9115061e.pdf?expires=1492077300&id=id&accname=guest&checksum=95C25E956AC14BE2DCB4ED519D7AF88B>. The OECD defined basic skills as “the acquisition of at least level 1 skills (420 points) on the OECD Programme for International Student Assessment (PISA).”

<sup>60</sup> OECD (2015b) *ibid.*

<sup>61</sup> British Academy (2015a) *ibid.*

<sup>62</sup> Deloitte (2012) *ibid.*; and discussion of Deloitte’s results by British Academy (2015a) *ibid.*

## Chapter 3: The case for improving post-16 mathematics

### England lags behind internationally in mathematics

85. In late November 2016, results were published from the 2015 Trends in International Mathematics and Science Study (TIMSS) assessments of year 5 and year 9 students.<sup>63</sup> In both years 5 and 9, pupils in England performed, on average, above the international average in mathematics, remaining in the second highest-performing group of countries. Although performances had improved from the 2011 report, the increase was not significant.

86. On 6 December 2016, the OECD's Programme for International Student Assessment (PISA) published its results of the (comparative) mathematical attainment of 15 year olds.<sup>64</sup> England's results in mathematics assessments remained level with the OECD average, broadly the same as in 2012. There are 18 countries where the mean score is at least a third of a year of schooling ahead of England, and 36 countries where the mean score is at least a third of a year of schooling below.

87. Despite considerable government focus on improving the performance of schools, educational reforms have not yet translated into significant improvement in performance in either TIMSS or PISA mathematics tests.

88. The mixed results in international tests show that continuing work and investment are needed pre-16 to bring England up to the standards of the world's best. Improved 16-18 mathematics education has the potential to address problems arising from this relatively poor performance, but England remains unusual among advanced countries in that the study of mathematics is not continued by most students beyond age 16. In 2010, the Nuffield Foundation<sup>65</sup> published a report showing that the three nations – England, Wales and Northern Ireland – were alone among 24 economies (mostly OECD members) in having fewer than 20 per cent of upper secondary students participating in mathematics (see Table 1). Note, however,

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<sup>63</sup> Department for Education (2016) *TIMSS 2015: national report for England*, <https://www.gov.uk/government/publications/timss-2015-national-report-for-england>

<sup>64</sup> Department for Education (2016) *PISA 2015: national report for England*, <https://www.gov.uk/government/publications/pisa-2015-national-report-for-england>

<sup>65</sup> Nuffield Foundation (2010) *Is the UK an outlier? An international comparison of upper secondary mathematics*, [http://www.nuffieldfoundation.org/sites/default/files/files/Is%20the%20UK%20an%20Outlier\\_Nuffield%20Foundation\\_v\\_FINAL.pdf](http://www.nuffieldfoundation.org/sites/default/files/files/Is%20the%20UK%20an%20Outlier_Nuffield%20Foundation_v_FINAL.pdf)

that the Nuffield study omitted post-16 students who were retaking GCSE mathematics.<sup>66</sup>

**Proportion of students in post-16 (or ‘upper secondary’) education or training studying any mathematics**

<b>All (95-100 per cent)</b>	Czech Republic, Estonia, Finland, Japan, Korea, Russia, Sweden, Taiwan
<b>Most (81-94 per cent)</b>	Canada (BC), France, Germany, Hungary, Ireland, USA (Mass.)
<b>Many (51-80 per cent)</b>	Australia (NSW), Netherlands, New Zealand, Singapore
<b>Some (21-50 per cent)</b>	Hong Kong, Scotland, Spain
<b>Few (6-20 per cent)</b>	England, Wales, Northern Ireland

Table 1. Proportion of students in post-16 (or ‘upper secondary’) education or training studying any mathematics, based on the 2010 Nuffield Foundation report.<sup>67</sup>

89. The government has taken action to support lower attaining pupils post-16. Since 2014, funding conditions on schools and colleges in England have meant that nearly all students aged 16-19 without a GCSE A\*-C in mathematics at 16 need to continue studying the subject. Since 2015, a further condition means that students with a GCSE D grade in mathematics at 16 now study GCSE mathematics post-16.

90. There has been an increase in the proportion of students in England at academic age 16 studying mathematics, reaching 47 per cent in 2015/16 (including students resitting GCSE mathematics).<sup>68</sup> Despite this increase, 72 per cent of students with a standard pass of A\*-C in GCSE mathematics still choose not to study mathematics beyond this level.<sup>69</sup> Furthermore, although in international terms England has a relatively well *qualified* cohort of young adults, the basic skills of this cohort remain weak.<sup>70</sup> In 2013, the OECD<sup>71</sup> reported that English 16-24 year olds

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<sup>66</sup> Nuffield Foundation (2010b) *24 Country Profiles*, [http://www.nuffieldfoundation.org/sites/default/files/files/Country\\_profiles\\_outlier\\_NuffieldFoundation18\\_04\\_11.pdf](http://www.nuffieldfoundation.org/sites/default/files/files/Country_profiles_outlier_NuffieldFoundation18_04_11.pdf)

<sup>67</sup> Nuffield Foundation (2010a) *ibid.*

<sup>68</sup> Department for Education (2017) *Ad-hoc notice: Post 16 maths participation in 2015/16* (accompanying statistical release to this report), <https://www.gov.uk/government/publications/post-16-maths-participation>

<sup>69</sup> Department for Education (2017) *Ad-hoc notice*, *ibid.*

<sup>70</sup> OECD (2016b) *Building skills for all: a review of England*, <https://www.oecd.org/unitedkingdom/building-skills-for-all-review-of-england.pdf>

ranked 20<sup>th</sup> for numeracy out of 22 OECD countries, and England was the only country where young adults performed no better than older adults in numeracy proficiency. A 2016 update<sup>72</sup> placed England 22<sup>nd</sup> of 29 OECD countries and economies for numeracy of 16-24 year olds.

91. The number of students entering apprenticeships has grown rapidly in recent years,<sup>73</sup> and the government sets minimum requirements for mathematics. However, apprenticeships in England and other parts of the UK have long been notable for requiring much less general education in mathematics and other core subjects for trainees in comparison to apprenticeship training in continental European countries.<sup>74</sup>

92. There are around nine million working age adults in England (more than a quarter of adults aged 16-65) with low literacy or numeracy skills or both. Weak basic skills reduce an individual's productivity and employability. They also damage citizenship and are implicated in challenges of equity and social exclusion.<sup>75</sup> For example, poor numeracy reduces the ability of individuals to make sound financial decisions.<sup>76</sup>

## There are mathematics and quantitative skills shortages

93. Weaknesses in basic numeracy in the workforce are reported in various employer surveys. For example, a quarter of skill shortage vacancies in England in 2015 were found to be due to a lack of numeracy.<sup>77</sup> Poor basic skills hinder employees' ability to perform everyday tasks, including being able to work through

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<sup>71</sup> OECD (2013) *OECD Skills Outlook 2013: First Results from the Survey of Adult Skills*, [https://www.oecd.org/skills/piaac/Skills%20volume%201%20\(eng\)--full%20v12--eBook%20\(04%2011%202013\).pdf](https://www.oecd.org/skills/piaac/Skills%20volume%201%20(eng)--full%20v12--eBook%20(04%2011%202013).pdf); Table A2.7, <http://dx.doi.org/10.1787/888932897230>

<sup>72</sup> OECD (2016a) *Skills matter: further results from the survey of adult skills*, [https://www.oecd.org/skills/piaac/Skills\\_Matter\\_Further\\_Results\\_from\\_the\\_Survey\\_of\\_Adult\\_Skills.pdf](https://www.oecd.org/skills/piaac/Skills_Matter_Further_Results_from_the_Survey_of_Adult_Skills.pdf); Table A3.5(N), <http://dx.doi.org/10.1787/888933366463>

<sup>73</sup> Department for Education (2016) *Further Education and Skills in England 2016*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/577119/SFA\\_SFR36\\_2\\_016\\_Ofqual\\_Update.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/577119/SFA_SFR36_2_016_Ofqual_Update.pdf)

<sup>74</sup> British Academy (2015a) *State of the Nation: a review of evidence on the supply and demand of quantitative skills*, <http://www.britac.ac.uk/sites/default/files/BA-NIESR%20State%20of%20the%20Nation%20-%20A%20review%20of%20evidence%20on%20the%20supply%20and%20demand%20of%20QS.pdf>

<sup>75</sup> OECD (2016b) *ibid.*

<sup>76</sup> OECD and the International Gateway for Financial Education (2016) *International Survey of Adult Financial Literacy Competencies*, <http://www.oecd.org/daf/fin/financial-education/OECD-INFE-International-Survey-of-Adult-Financial-Literacy-Competencies.pdf>

<sup>77</sup> UK Commission for Employment and Skills (2016) *UK Commission's Employer Skills Survey 2015: UK Results*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/525444/UKCESS\\_2015\\_Report\\_for\\_web\\_May\\_.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/525444/UKCESS_2015_Report_for_web_May_.pdf)

calculations and make sense of numerical data.<sup>78</sup> Tasks requiring basic mathematics skills challenge many employees, even those with a standard GCSE pass. As a result, many employers look for the study of mathematics at a higher level to guarantee that employees have the confidence and versatility to use mathematics in a variety of unfamiliar situations.<sup>79</sup>

94. Employers value achievement in STEM subjects but struggle to find recruits with these qualifications.<sup>80</sup> Analysis of graduate destinations shows that up to 60 per cent of STEM graduates enter non-STEM roles.<sup>81</sup> The proportion of young people selecting STEM A levels has increased over the past decade.<sup>82</sup>

95. Insufficient STEM skills are exacerbated by a shortage of women studying such subjects post-16. The gender profile of the UK workforce as a whole is slightly skewed towards men (53 vs 47 per cent), but strongly skewed where mathematical science qualifications are considered essential (65 vs 35 per cent).<sup>83</sup>

96. Almost 30 per cent of skill shortage vacancies in 2015 were linked to a lack of 'complex' numerical/statistical skills.<sup>84</sup> When UK businesses were surveyed about the level of skills held by their technology specialists, half (equivalent to 182,000 firms)

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<sup>78</sup> Confederation of British Industry and Pearson (2015) *Inspiring Growth: CBI/Pearson Education and Skills Survey 2015*, <http://www.cbi.org.uk/cbi-prod/assets/File/Education-and-skills-survey-2015.pdf>

<sup>79</sup> Advisory Committee on Mathematics Education (2011) *Mathematical needs: mathematics in the workplace and in higher education*, <https://royalsociety.org/~media/policy/topics/education-skills/Maths/Mathematical-needs-2011.pdf?la=en-GB>

<sup>80</sup> Confederation of British Industry and Pearson (2016) *The Right Combination: CBI/Pearson Education and Skills Survey 2016*, <http://www.cbi.org.uk/cbi-prod/assets/File/pdf/cbi-education-and-skills-survey2016.pdf>

<sup>81</sup> UK Commission for Employment and Skills / Bosworth D L, Lyonnette C, Wilson R A, Bayliss M and Fathers S (2013) *Supply of and demand for High-Level STEM skills*, UKCES Evidence Report 77 (UK Commission for Employment and Skills),

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/302973/evidence-report-77-high-level-stem-skills\\_1\\_.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/302973/evidence-report-77-high-level-stem-skills_1_.pdf)

Referenced in UKCES (2014) *The Labour Market Story: The State of UK Skills*, Briefing Paper, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/344440/The\\_Labour\\_Market\\_Story- The\\_State\\_of\\_UK\\_Skills.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/344440/The_Labour_Market_Story- The_State_of_UK_Skills.pdf)

<sup>82</sup> UK Commission for Employment and Skills / Hogarth T, Bosworth D, Davis C, Price S and Garrett R (2010) *Strategic Skills Needs in the Bio-medical Sector: A report for the National Strategic Skills Audit for England 2010*, UKCES Evidence Report 14 (UK Commission for Employment and Skills),

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/336842/UKCES\\_Bio-medical\\_Main\\_Report\\_v5a\\_2.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/336842/UKCES_Bio-medical_Main_Report_v5a_2.pdf)

Referenced in UKCES (2014) *The Labour Market Story: The State of UK Skills*, Briefing Paper, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/344440/The\\_Labour\\_Market\\_Story- The\\_State\\_of\\_UK\\_Skills.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/344440/The_Labour_Market_Story- The_State_of_UK_Skills.pdf)

<sup>83</sup> Council for Mathematical Sciences (2015) *ibid.*

<sup>84</sup> UK Commission for Employment and Skills (2015b) *ibid.*



identified a shortfall among their staff. Businesses operating within financial services were most likely to be experiencing such gaps.<sup>85</sup>

### **The demand for high level quantitative skills continues to grow**

There is a computer science crisis in the UK, but it is a “crisis of opportunity”. The UK will need 10,000 computer science scholars to improve its global position in the knowledge economy.<sup>86</sup>

The proportion of employees saying advanced mathematics or statistics is important in their jobs rose from 29 per cent in 1997 to 38 per cent in 2012.<sup>87</sup> The skill most in demand in the US in 2014 was statistical analysis and data mining, falling to 2nd place in 2016, but remaining the only skill consistently ranked in the top 4 in all countries analysed by LinkedIn.<sup>88</sup>

Services requiring the highest levels of quantitative skills, such as finance, real estate, insurance and business services, are growing fastest.<sup>89</sup> The majority of private sector organisations believe the use of data analytics will be the most important factor in increasing growth in UK businesses.<sup>90</sup>

Shortages of people with STEM skills have been a longstanding concern for businesses across the UK. CBI/Pearson<sup>91</sup> argue that these skills shortages must be tackled effectively if the pace of future economic growth is not to be held back.

## **Many students are inappropriately prepared for the mathematics in their university courses**

97. A wide range of disciplines, including business and management, chemistry, economics, geography and sociology, require mathematics – from applied statistics through to advanced mathematical modelling.

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<sup>85</sup> Tech Partnership (2015) *Employer insights: skills survey 2015*, [https://www.thetechpartnership.com/globalassets/pdfs/research-2015/tec\\_employer\\_skill\\_survey\\_web.pdf](https://www.thetechpartnership.com/globalassets/pdfs/research-2015/tec_employer_skill_survey_web.pdf)

<sup>86</sup> Times Higher Education News, 24 Oct 2016, <https://www.timeshighereducation.com/news/eric-schmidt-uk-needs-10000-computer-science-academics>

<sup>87</sup> British Academy (2015b) *Count Us In: Quantitative Skills for a New Generation*, [http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report\\_0.pdf](http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report_0.pdf)

<sup>88</sup> Utts J (2016) American Statistical Association President’s Address, Joint Statistical Meetings 2016, Chicago, 30 July – 4 August, <https://www2.amstat.org/meetings/jsm/2016/webcasts/index.cfm>

<sup>89</sup> OECD (2013) *ibid.*

<sup>90</sup> Deloitte (2012) *Measuring the Economic Benefits of Mathematical Science Research in the UK*, <https://www.epsrc.ac.uk/newsevents/pubs/deloitte-measuring-the-economic-benefits-of-mathematical-science-research-in-the-uk/>

<sup>91</sup> Confederation of British Industry and Pearson (2016) *ibid.*

98. With the exception of mathematical sciences, more than 40 per cent of English 19 year olds studying STEM subjects (engineering and technology; physical sciences) in UK universities do not have a mathematics qualification beyond GCSE. This increases to over 80 per cent for students on non-STEM degree courses, many of which have a significant quantitative element.<sup>92</sup>

99. Students are often surprised at the extent of the mathematical demands of their university programmes and some struggle to cope with those demands. For example, over 80 per cent of economics students surveyed stated there was much more mathematics involved in their economics degree programme than they expected.<sup>93</sup> A lack of confidence and anxiety about mathematics/statistics are problems for many students; and many have done little or no mathematics pre-university for at least two years.<sup>94</sup> In a STEM survey of sociology degree students, over a third said that they struggled with quantitative methods. More than half felt “anxious” when using mathematics/statistics and a slightly smaller proportion found working with numbers “challenging”.<sup>95</sup>

100. In 2011, the Advisory Committee on Mathematics Education (ACME), estimated that of students in England entering higher education in any year, some 330,000 would benefit from recent experience of studying mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 did so.<sup>96</sup> Many degree programmes do not set out information about mathematical content but only specify a minimum requirement, most often a grade C in GCSE mathematics or equivalent.<sup>97</sup>

101. Q-Step centres (see the box on the following page) bolster social science programmes in universities by integrating quantitative skills and knowledge into their courses. In this way, students acquire the skills needed for statistical analysis of data

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<sup>92</sup> Hodgen J, Adkins M and Tomei A (2017) *The mathematical backgrounds of undergraduates: Interim report* (Nottingham: University of Nottingham),

<http://www.nottingham.ac.uk/research/groups/crme/projects/mathematical-backgrounds.aspx>

The study examined English-domiciled students who were age 16 in 2007/8, took a level 3 qualification between 2008/9 and 2010/11, and then entered university between 2010/11 and 2012/13. The final report, due in 2017, will form the basis of a resource which will enable universities to compare the mathematical backgrounds of entrants across institutions in all disciplines.

<sup>93</sup> Dawson P (2014) *Skills in mathematics and statistics in economics and tackling transition* (York: Higher Education Academy),

[https://www.heacademy.ac.uk/system/files/resources/tt\\_maths\\_economics.pdf](https://www.heacademy.ac.uk/system/files/resources/tt_maths_economics.pdf)

<sup>94</sup> Hodgen J, McAlinden M and Tomei A (2014) *Mathematical transitions: a report on the mathematical and statistical needs of students undertaking undergraduate studies in various disciplines* (York: Higher Education Academy), [https://www.heacademy.ac.uk/system/files/resources/hea\\_mathematical-transitions\\_webv2.pdf](https://www.heacademy.ac.uk/system/files/resources/hea_mathematical-transitions_webv2.pdf)

<sup>95</sup> Jones J S and Goldring J (2014) *Skills in mathematics and statistics in sociology and tackling transition* (York: Higher Education Academy),

[https://www.heacademy.ac.uk/system/files/resources/tt\\_maths\\_sociology.pdf](https://www.heacademy.ac.uk/system/files/resources/tt_maths_sociology.pdf)

<sup>96</sup> Advisory Committee on Mathematics Education (2011) *ibid.*

<sup>97</sup> Hodgen J et al. (2014) *ibid.*

and assessment of evidence. However, it is not only social science students who may benefit from the type of training provided by Q-Step and learned societies argue that science, social science and humanities students all need to have developed significant quantitative skills in school.<sup>98</sup>

### **Q-Step and the need for mathematical skills in the social sciences**

Q-Step is a £19.5 million programme that was developed by the Nuffield Foundation, the Economic and Social Research Council and the Higher Education Funding Council for England as a strategic response to the shortage of quantitatively-skilled social science graduates in the UK.

The programme has led to the launch of a range of new quantitative social science degrees in 15 partner universities, and a programme of support for social science students and faculty staff across a wide range of disciplines, including education, socio-legal studies, psychology, environmental planning, geography, management and business planning, international relations and social policy.

102. Many academics expressed to the review their feeling that humanities students will be more successful after university if they complement their humanities skills (for example, critical reasoning, weighing of historical evidence and analysis of text, language and meaning), with quantitative reasoning and data analysis. They argued that universities should learn from the impact of Q-Step in the social sciences and should work together to pioneer an initiative that incorporates a relevant understanding of data analysis into humanities curricula. The aim would be to build humanities students' confidence and their ability to compete in the future employment market.

103. International studies show that around 10 per cent of all university students in England have numeracy or literacy levels below GCSE level, indicating a major numeracy challenge, and suggest that this is often not resolved at the point of graduation.<sup>99</sup>

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<sup>98</sup> British Academy (2015b) *Count Us In: Quantitative Skills for a New Generation*, [http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report\\_0.pdf](http://www.britac.ac.uk/sites/default/files/Count-Us-In-Full-Report_0.pdf)

<sup>99</sup> OECD (2016b) *ibid.*

## The need for mathematics and quantitative skills will increase

104. Developments in technology will alter the nature of work and jobs. A study<sup>100</sup> of the susceptibility of US jobs to computerisation concluded that the jobs available in the future will increasingly require mathematical and quantitative skills. Those least at risk are in skilled management, financial services, computing, engineering, science, education, legal services, community services, the arts and media, and healthcare.

### The rise of data science

Between 2013 and 2020, employment in the UK is forecast to increase by 6 per cent, whilst demand for experts in software, data storage, retrieval and analysis is expected to rise by 160 per cent, reflecting the growing UK digital economy and the increasing amounts of data garnered from the internet of things.<sup>101</sup>

In the UK, the national institute for data science, The Alan Turing Institute<sup>102</sup>, brings together work in pure and applied mathematics, statistics, engineering and computing, the key disciplines underpinning the emerging field of data science. One of the Institute's key research priorities is to understand the societal implications of big data, including the ethics of data science, and privacy and security.

Most recently, The Alan Turing Institute announced that it is to research new methods to exploit financial data by analysing it with greater detail and accuracy, in order to aid economists, researchers, policymakers and businesses to better understand the UK economy and its interconnection with global markets.<sup>103</sup>

105. Many commentators have remarked on labour market 'polarisation' that has accompanied increasing technological innovation in the workplace in recent decades and is continuing.<sup>104,105,106</sup> Technology also enables offshoring of some jobs in global

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<sup>100</sup> Frey C B and Osborne M A (2013) *The future of employment: how susceptible are jobs to computerisation?*,

[http://www.oxfordmartin.ox.ac.uk/downloads/academic/The\\_Future\\_of\\_Employment.pdf](http://www.oxfordmartin.ox.ac.uk/downloads/academic/The_Future_of_Employment.pdf)

<sup>101</sup> Tech Partnership (2014) *Big Data Analytics: Assessment of Demand for Labour and Skills 2013–2020*, [https://www.thetechpartnership.com/globalassets/pdfs/research-2014/bigdata\\_report\\_nov14.pdf](https://www.thetechpartnership.com/globalassets/pdfs/research-2014/bigdata_report_nov14.pdf)

<sup>102</sup> The Alan Turing Institute, <https://www.turing.ac.uk/>

<sup>103</sup> The Alan Turing Institute (2016) *HSBC and The Alan Turing Institute to work together to advance research into economic data*, <https://www.turing.ac.uk/news/hsbc-alan-turing-institute-work-together-advance-research-economic-data/>

<sup>104</sup> Frey C B and Osborne M A (2013) *ibid.* and references therein

<sup>105</sup> McIntosh S (2013) and Holmes C (2013) *Hollowing out and the future of the labour market*, two presentations, <http://www.niesr.ac.uk/publications/hollowing-out-and-future-labour-market>

competition for labour. This polarisation is manifested as an increase in the share of high-income cognitive jobs and low-income manual occupations, at the expense of middle-income routine jobs, which are said to be ‘hollowed out’. In the UK, between 1981 and 2008, there were large increases in employment share of the top 20 per cent paid jobs (highest paid occupations associated with higher-level skills) and the lowest 10 per cent, with sharp decreases in medium-paid jobs.<sup>107</sup>

106. Over the next decade it is expected that the demand for newly created jobs will be focused even further on highly skilled occupations and lower skilled occupations that cannot be automated.<sup>108</sup> The long-term implications of technological change and the rise of big data will be profound. There are significant economic opportunities for the UK from data science and a need to examine the skills required in the future and what this means for future education and training.

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<sup>106</sup> Baker K (2016) *The digital revolution*,

[http://www.edge.co.uk/sites/default/files/documents/digital\\_revolution\\_web\\_version.pdf](http://www.edge.co.uk/sites/default/files/documents/digital_revolution_web_version.pdf)

<sup>107</sup> Holmes C and Mayhew K (2012), *The changing shape of the UK job market and its implications for the bottom half of earners*, report for the Resolution Foundation, [http://socialwelfare.bl.uk/subject-areas/services-](http://socialwelfare.bl.uk/subject-areas/services-activity/employment/resolutionfoundation/131865The_Changing_Shape_of_the_UK_Job_Market_1.pdf)

[activity/employment/resolutionfoundation/131865The\\_Changing\\_Shape\\_of\\_the\\_UK\\_Job\\_Market\\_1.pdf](http://socialwelfare.bl.uk/subject-areas/services-activity/employment/resolutionfoundation/131865The_Changing_Shape_of_the_UK_Job_Market_1.pdf)

<sup>108</sup> UK Commission for Employment and Skills (2014b) *The Future of Work: Jobs and Skills in 2030*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/303334/er84-the-future-of-work-evidence-report.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/303334/er84-the-future-of-work-evidence-report.pdf)

## Chapter 4: Mathematics pathways and options

107. The previous two chapters set out the case for improving participation and achievement in mathematics at all levels. This chapter looks at the range of mathematics qualifications that are in place, or planned, and considers the suitability of these for the needs of 16-18 year old students. These qualifications are conventionally referred to as 'levels', level 3 being more advanced than level 2, for example.

### Many of the necessary qualifications are in place or are planned

#### Mathematics pathways in the 16-18 landscape

- Level 3 qualifications
  - **AS/A level mathematics:** This is currently the most commonly chosen pathway in the level 3 mathematics landscape. It is designed to build on mathematics skills developed at level 2 and provide students with a thorough grounding in mathematical tools and techniques.
  - **AS/A level further mathematics:** This course is designed to broaden and deepen the mathematical knowledge and skills developed when studying A level mathematics.
  - **AS/A level statistics:** This course builds on the statistics and probability components of GCSE mathematics and employs the statistical enquiry cycle to help make sense of data trends in a variety of contexts. AS statistics is aimed specifically at facilitating the development of the statistical elements employed across the A level curriculum. It prepares students for further study and employment in a wide range of disciplines which use statistical analysis and reasoning with data.
  - **Core maths:** These qualifications are designed for students who achieve an A\*-C at GCSE but do not study AS/A level mathematics. Core maths is an umbrella term covering a number of qualifications offered by awarding organisations; all offer the opportunity for students to build their knowledge to a higher level, with a focus on the use and application of mathematics and statistics. They enable students to think mathematically and to apply mathematical techniques with confidence to a variety of unfamiliar situations.
  - **International Baccalaureate Diploma Programme (IBDP) mathematics certificates:** IBDP students complete assessments in six subjects and three core requirements, the latter of which include mathematics at level 3.

## Mathematics pathways in the 16-18 landscape

- Level 2 qualifications and below
  - **GCSE mathematics** (These courses are taken as resits by those who do not achieve an A\*-C at age 16): This course is designed to enable students to:
    1. develop fluent knowledge, skills and understanding of mathematical methods and concepts
    2. acquire, select and apply mathematical techniques to solve problems
    3. reason mathematically, make deductions and inferences and draw conclusions
    4. comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context. It provides a strong basis for successful transition to A level study.
  - **Functional skills**: These qualifications, mostly taken at level 1 and 2, develop practical skills in English, mathematics and ICT. They are often taken alongside apprenticeships or as an alternative to GCSE.

## Reformed AS/A level mathematics qualifications will support transition to HE study, but the challenge is to put this into practice

108. The majority of students who progress from GCSE mathematics to level 3 study an AS or A level in the subject.<sup>109</sup> A subset of students study further mathematics, generally as a fourth subject alongside or after taking A level mathematics.<sup>110</sup> A small proportion of students study statistics A level, although this is not offered by all providers.<sup>111</sup>

109. Significant changes have been made to A levels in mathematics and further mathematics, which are due to be taught for the first time in 2017. The new courses

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<sup>109</sup> Department for Education (2017) *Ad-hoc notice: Post 16 maths participation in 2015/16* (accompanying statistical release to this report), <https://www.gov.uk/government/publications/post-16-maths-participation>

<sup>110</sup> Lee S, Stevens A, Boylan M and Demack S (2016) *The fragility of A level Further Mathematics in schools/colleges in England*, <http://www.bsrlm.org.uk/wp-content/uploads/2016/11/BSRLM-CP-36-2-08.pdf>

<sup>111</sup> Department for Education (2017) *A level and other 16-18 results Statistical First Release*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/584125/SFR05\\_2017\\_National\\_tables\\_1\\_to\\_8.xlsx](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/584125/SFR05_2017_National_tables_1_to_8.xlsx)

will retain the same number of teaching hours and will be set at a similar level of challenge to the previous qualifications. The changes made reflect calls from universities for a common compulsory curriculum.<sup>112</sup>

110. All students will study pure mathematics, mechanics and statistics within mathematics A level. Additional content has been included on statistical analysis of large datasets.

111. As for all A levels, the AS level assessment has been decoupled from the A level, meaning that the latter will assess the full curriculum rather than just the final year. The AS level will still be a standalone qualification representing the first year of A level study. While some mathematics organisations have voiced concern about this decoupling (an issue explored further in chapter 6), many recognise the value of this change to enabling 'synoptic assessment' (testing understanding of connections between different elements of a subject). University departments we spoke to welcomed the intent of the new A level to deepen mathematical understanding and assess substantial mathematics problems presented in an unstructured form.

112. While this intent has been welcomed, A level mathematics reforms need to be put into practice by teachers. Given the extent of the changes, it is likely that a large proportion of teachers of A level mathematics will require professional development and other support, including high quality resources to introduce these changes in ways which realise the intent. Chapter 6 gives an overview of the support currently provided.

## **Mathematics and quantitative skills are assessed in other A levels and require skilled teaching**

113. As part of the A level reform process, other subjects now have a greater emphasis on the application of mathematics and statistics (drawn from the skills required for level 2). Assessment of mathematical and quantitative skills is now included in a third of all non-mathematics A levels: business, computer science, economics, geography, physical education, accounting, design and technology, electronics, environmental science, geology, psychology, biology, physics, and chemistry.<sup>113</sup> For each subject, the assessment of quantitative skills is worth a percentage of the final mark awarded.

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<sup>112</sup> The A level content advisory board (2014) *Report of the ALCAB panel on mathematics and further mathematics*, <https://alevelcontent.files.wordpress.com/2014/07/alcab-report-on-mathematics-and-further-mathematics-july-2014.pdf>

<sup>113</sup> Department for Education (2017) *GCE AS and A level subject content*: <https://www.gov.uk/government/collections/gce-as-and-a-level-subject-content>



**Subjects containing mathematical and quantitative skills with the percentage of the final mark awarded for their application**

<b>Subject</b>	<b>Marks awarded for mathematical and quantitative skills (%)</b>	<b>Number of entries 2015/16</b>
Physics	40	31,000
Electronics	30	900
Geography	20-30	32,200
Accounting	20	2,300
Economics	20	27,600
Chemistry	20	45,400
Design and technology	15	10,600
Computer Science	10	5,600
Business	10	25,100
Environmental science	10	800
Geology	10	1,900
Biology	10	54,300
Psychology	10	55,900
Physical education	5	10,200

**Table 2. A level subjects that include an assessment of mathematics and quantitative skills.**<sup>114,115</sup>

114. Although in many subjects these changes do not represent a large element of assessment, the recognition of mathematical and quantitative content within each subject is a significant development and has been led, in many cases, by learned societies and welcomed by universities. How well they are delivered will be down to the knowledge and skills of the teachers; the learned societies and others will have roles to play in providing guidance and resources for schools.

**Teaching mathematics within new AS/A levels in other subjects**

The increased mathematics content in new A levels means that some teachers may require extra support to teach mathematics effectively. The Royal Geographical Society and Institute of British Geographers’ programme ‘Data Skills in Geography’ is supporting this transition.

Running from 2015-2017, its programme to support teachers and students to improve their understanding of and use of data focuses on the value of these skills to further study and employment. The programme does this through online resources, face-to-face workshops and teacher CPD.

<sup>114</sup> Department for Education (2017) *GCE AS and A level subject content* ibid.

<sup>115</sup> Department for Education (2017) *Ad-hoc notice* ibid.

## Core maths plugs a critical gap for students progressing to HE courses and higher technical study with a quantitative element

115. In 2014, core maths qualifications were introduced, designed for students with an A\*-C at GCSE who do not study AS/A level mathematics. They focus on using and applying mathematics and statistics to realistic scenarios and problems. The qualifications follow the same number of guided learning hours as an AS and are graded A-E. They attract the same UCAS points as an AS qualification.

### What is core maths?

- Core maths refers to a set of qualifications, designed to provide opportunities for students who achieved an A\*-C at GCSE but who are not taking AS/A level mathematics to continue with the subject.
- Given the value placed on mathematical and quantitative skills by universities and employers, schools and colleges are being encouraged to offer these new qualifications, intended for the approximately 271,000 eligible students each year who would not otherwise continue to study mathematics.<sup>116</sup> These students could be those who intend to go on to do further study of quantitative subjects or those with interests mainly in non-mathematical areas.
- Core maths is intended to complement a range of academic and technical programmes and is designed to strengthen and build on students' existing skills, with a focus on using and applying mathematics and statistics.
- From September 2014, the Department for Education funded around 150 early adopter schools and colleges to begin teaching the new qualifications, with almost 3,000 exam entries in 2016.<sup>117</sup>

116. The content of core maths qualifications have been well received by stakeholders, and feedback has been widely positive. Core maths offers the opportunity to apply mathematics and statistics to examples from economics, sociology, psychology, chemistry, geography, computing, and business and

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<sup>116</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>117</sup> Data collected from individual exam boards:

Pearson <http://qualifications.pearson.com/content/demo/en/support/support-topics/results-certification/grade-statistics.html?Qualification-Family=mathematics-in-context>,

AQA <http://www.aqa.org.uk/exams-administration/results-days/results-statistics>,

OCR <http://www.ocr.org.uk/ocr-for/exams-officers/stage-4-results/results-statistics/results-statistics-archive/>

management. The example below shows how core maths qualifications create mathematical problems out of real life scenarios.

**Example core maths exam question**<sup>118</sup>

The male to female sex ratio at birth is the number of males that are born for every female born.

The table below shows the countries with the two highest male to female sex ratios at birth in 2013.

Country	Male to female sex ratio at birth	Population (thousands)	Births per 1000 of population
Liechtenstein	1.26	37	10.67
Azerbaijan	1.13	9590	17.17

Data: CIA World Factbook

- (i) Use the information in the table to show that the total number of births in 2013 in Liechtenstein can be estimated as 395. Show that approximately 175 are girls and approximately 220 are boys.

A researcher is investigating whether the number of boys born in some countries is distinctly different from the number of girls.

Her initial model is that the long-term probability of a new baby being a boy is 0.5.

- (ii)
- (A) For Liechtenstein investigate whether the figures of 175 girls and 220 boys provide strong evidence that the initial model is incorrect.  
Explain your reasoning and show your working clearly.
- (B) In 2013 in Azerbaijan 77 305 girls and 87 355 boys were born. Do these figures provide strong evidence that the initial model is incorrect?

117. Core maths qualifications were examined for the first time in June 2016, with approximately 3,000 students entering the qualifications. The Russell Group's 'Informed Choices'<sup>119</sup> – the Russell Group of 24 leading UK universities' guide to making decisions about post-16 education – has publically endorsed the development; additionally, at the time of drafting this report, a total of 43 universities had shown their individual support for core maths, including 20 Russell Group universities and 23 others.<sup>120</sup>

<sup>118</sup> OCR (2014) *Quantitative reasoning; Sample paper question* <http://www.ocr.org.uk/Images/174107-unit-h866-02-critical-maths-sample-assessment-material.pdf>

<sup>119</sup> Russell Group (2016) *Informed choices*, <http://russellgroup.ac.uk/media/5457/informed-choices-2016.pdf>

<sup>120</sup> Core Maths Support Programme (2017) <http://www.core-maths.org/about-core-maths/universities/>

**Recommendation 1: The Department for Education should seek to ensure that schools and colleges are able to offer all students on academic routes and potentially students on other level 3 programmes access to a core maths qualification.**

**Recommendation 2: The Department for Education and Ofqual should consider how the core maths brand could be strengthened with the aim of improving awareness and take-up of the qualification.**

118. A small percentage of students in England complete the International Baccalaureate (IB), which includes a compulsory mathematics element at level 3. The IB is equivalent in size to completing AS or A level courses, depending on the certificate studied. Given the small numbers, this route is not considered further within the review.

119. Students in England who do not study exclusively A level programmes often follow academic options such as Applied General Qualifications (AGQs). AGQs are designed for 16-19 year old students to develop transferable knowledge and skills through applied learning, allowing entry to a range of higher education courses. AGQs include brands such as BTECs, Cambridge Technicals, UAL and others. The number of students studying AGQs is increasing, with take-up of BTECs<sup>121</sup> at key stage 5 increasing from 45,000 to 150,000 since 2006<sup>122</sup>.

## **GCSE mathematics has undergone significant change**

120. Since 2014, it has been a condition of funding for schools and colleges that students aged 16-19 yet to achieve a GCSE A\*-C grade continue to study towards this end. In September 2015, the condition was strengthened to require full-time students with a grade D to retake the GCSE. Students with grade E and below can study alternative approved qualifications. Most study functional skills qualifications, which are currently being reformed (see the section on the following page).

121. The large majority of students resitting GCSE mathematics take foundation tier papers. In September 2015, GCSE mathematics content was redesigned to raise expected standards, provide stretch, and ensure that fundamental mathematics and numeracy skills are in place to reassure employers who raised concerns.

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<sup>121</sup> BTECs are the largest brand of AGQs.

<sup>122</sup> Richards, B (2016): *Passports to Progress: How do vocational qualifications help young people in building their careers. Part 1* (Social Market Foundation), <http://www.smf.co.uk/wp-content/uploads/2016/07/Social-Market-Foundation-Passports-to-Progress-Vocational-Qualifications-Embargoed-0001-070716.pdf>

122. The consequence of these changes needs to be considered fully, in particular the grade at which students are required to resit the GCSE and the role of alternative qualifications for weaker students. For 2017/18 and 2018/19 academic years the resit threshold has been set at grade 3 and below – roughly equivalent to the current grade D. These issues are further discussed in the next chapter.

## **Functional skills reforms aim to improve qualification rigour and credibility with employers**

123. Functional skills are the most prevalent non-GCSE maths qualifications at level 2 and below for students who have not achieved a standard pass in the subject.<sup>123</sup> Take-up is particularly high with apprentices, adults and 16-18 year olds yet to achieve a standard GCSE pass.

### **What are functional skills qualifications?**

- Functional skills qualifications are designed to enable students to develop the skills they need in work and daily life, and to apply these skills to solve problems.
- Functional skills are available in English, mathematics and ICT.
- They are available in five levels: entry 1, entry 2, entry 3, level 1, level 2.
- Functional skills have become the most widely used non-GCSE qualifications in English and mathematics.

124. Functional skills qualifications are often studied within apprenticeship programmes. As a minimum, apprentices are required to achieve a mathematics qualification at one level below their apprenticeship level. For example, students completing a level 2 apprenticeship are required to achieve level 1 mathematics in order to gain the apprenticeship and study towards a level 2 qualification after that. Recently reformed, apprenticeships are increasing in popularity. In 2015/16 there were 509,400 apprenticeship starts, of which 131,400 were under 19 years of age.<sup>124</sup>

125. In 2015, Ofqual raised concerns about the quality of assessment and standard setting within functional skills qualifications, setting out necessary improvements that

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<sup>123</sup> Department for Education (2017) *A level and other 16-18 results Statistical First Release* [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/584125/SFR05\\_2017\\_National\\_tables\\_1\\_to\\_8.xlsm](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/584125/SFR05_2017_National_tables_1_to_8.xlsm)

<sup>124</sup> Department for Education and the Skills Funding Agency (2014) *FE data library, apprenticeships*, <https://www.gov.uk/government/statistical-data-sets/fe-data-library-apprenticeships>

awarding organisations needed to make to improve rigour.<sup>125</sup> Subsequently, the government commissioned the Education and Training Foundation (ETF) to lead a programme to update and further improve the relevance and rigour, and recognition of the qualifications, and their credibility in the labour market. ETF consulted extensively with employers to understand the skills needed of employees.

126. Reformed functional skills qualifications will assess mathematical knowledge, as set out in the subject content documents, and the ability to apply that knowledge. The knowledge will demonstrate employer-informed skills at various levels from entry level 1 to level 2. The Department for Education and Ofqual will publish specific content and revised Conditions of Assessment that all functional skills qualifications will need to observe. Teaching of the new qualifications is due to commence in 2019.

127. Given the important role that functional skills qualifications play for 16-18 year old students not taking GCSE qualifications again, or moving on to level 3 mathematics, it is essential that the new qualifications have a clear purpose and fit appropriately alongside others in the 16-18 mathematics landscape – particularly in relation to GCSE mathematics.

## **Panels of professionals will need expert mathematical advice to determine the quantitative skills within technical education routes**

128. Significant reforms to the technical education landscape are outlined in the government's Post-16 Skills Plan (published in July 2016).<sup>126</sup> The government accepted all the recommendations of the Sainsbury panel – 34 in total – which, collectively, are far reaching, and represent the most ambitious changes to post-16 education since A levels were introduced 70 years ago.

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<sup>125</sup> Ofqual (2015): *Improving functional skills qualifications*, <https://www.gov.uk/government/publications/improving-functional-skills-qualifications>

<sup>126</sup> Department for Education (2016) *Post-16 skills plan and independent report on technical education*, <https://www.gov.uk/government/publications/post-16-skills-plan-and-independent-report-on-technical-education>

### **Technical education routes**

An expert panel, chaired by Lord Sainsbury, advised the government on reforms needed to improve the technical education system. The Sainsbury panel recommended streamlining the system and that a new set of 15 technical education routes – for example “Construction”, “Engineering and Manufacturing”, “Childcare and Education” and “Catering and Hospitality” – should be introduced. The routes will be based upon a common framework of standards and will group occupations together to reflect shared training requirements. Each route will have two modes of learning which will be equally valid ways of progressing: employment-based – typically an apprenticeship; and college-based – typically starting with a two-year, full-time study programme. Routes will extend to the highest skills levels.

129. For technical routes, employers will be able to determine the mathematics (and English) content where these directly relate to occupational requirements. This will serve the dual purpose of ensuring students have both transferable and occupation-specific mathematics and numeracy.

130. Some occupations, particularly those relating to STEM, may require higher standards of mathematics than others, including achievement at level 3. The panels of professionals developing technical routes will need a good understanding not only of mathematical demands across the field in question, but also how this may develop in future.

131. Technical routes at level 3 will provide a critical pathway to training at levels 4 and 5 in subjects where quantitative skills are central. To help meet the needs of employers in local areas for STEM skills at levels 3, 4 and 5, the government is supporting the introduction of Institutes of Technology (IoTs) from 2017. IOTs will be aligned to new technical routes to provide students with a clear pathway to employment or higher education where STEM skills are in demand.

132. During the course of the review, many stakeholders stressed the importance of rigorous qualifications at all levels and pointed to the recommendation in the Sainsbury review to have a single awarding body or consortium for each technical qualification at levels 2 and 3. While it is outside of the remit of this review, several stakeholders said they would welcome a similar singular organisation approach to developing qualifications for mathematics in future.

133. This review commissioned the Gatsby Foundation, working with Nottingham University, to look at mathematics within technical routes in a number of countries with successful technical education systems. These include Norway, the Netherlands, Singapore and Germany. Mathematics is central to technical education for all young people up to the age of 18 in these countries. In the Netherlands,

students are obliged to work towards a national applied numeracy examination and towards gaining further specific knowledge and skills in mathematics to achieve occupational competencies. In Norway, students are required to study mathematics as a subject alongside their technical or applied course.

### **Mathematics in technical routes: the Netherlands**

In the Netherlands, all technical routes incorporate the study of mathematics. Technically-specific mathematics is integrated into each course where appropriate. Since 2015/16, regardless of the particular technical course, all students have been required to study towards a national applied numeracy examination as described by the Dutch Literacy and Numeracy framework (LaNF) and assessment syllabus. Additional mathematics teaching is built into technical courses to prepare students for the test.

Students are often taught mathematics by the teachers of their technical courses, and teacher training programmes are responding to this change in policy by increasing the mathematics and numeracy content in their courses.

134. Defining the appropriate mathematics for each of the technical routes is likely to be complex. The mathematics should be designed to reflect the requirements of the relevant occupations, wider society and the emerging economy. It needs to be coherently structured, taught and assessed.

**Recommendation 3: The Institute for Apprenticeships should work with the Royal Society Advisory Committee on Mathematics Education to ensure appropriate expert advice is available to the panels of professionals developing technical routes.**



## Chapter 5: Participation and achievement

### Overall picture of 16-18 mathematics participation

135. Participation in mathematics study among academic age 16-17 year-olds is driven largely by prior attainment.<sup>127</sup>

136. Of 16 year olds who have achieved an A\*-C grade in GCSE mathematics, nearly three quarters – around 271,000 in 2015/16 – do not continue with mathematics post-16. The majority of these students who do not continue (57 per cent), go on to study academic level 3 qualifications, while a third move on to study vocational qualifications.<sup>128</sup>

137. However, those with a higher grade at GCSE were also more likely to progress with their participation at age 17. In 2015/16, over three quarters (79 per cent) of A\* students studied the subject at a higher level at 17 than they did at 16. This falls to 52 per cent of students with an A and to 22 per cent of B students. Those with a C grade at 15 were the most likely (92 per cent) not to study the subject at 16 or 17.<sup>129</sup>

138. Figure 1 on the following page shows the highest level of participation in mathematics at academic age 16 or 17<sup>130</sup> for the 2013/14 GCSE cohort. This shows that the higher grade a student achieves in their mathematics GCSE, the higher level of mathematics they participate in.

139. As a result of the funding requirement for students with a grade D to retake the GCSE, entries to GCSE mathematics by 16-18 students in 2016 increased by 53 per cent when compared to 2015.<sup>131</sup> Larger numbers of students with an E grade or below are also being entered for GCSE mathematics, compared to previous years, despite this not being a funding requirement.

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<sup>127</sup> Department for Education (2017) *Ad-hoc notice: Post 16 maths participation in 2015/16* (accompanying statistical release to this report), <https://www.gov.uk/government/publications/post-16-maths-participation>

<sup>128</sup> Department for Education (2017) *Ad-hoc notice*, *ibid.*

<sup>129</sup> Department for Education (2017) *Ad-hoc notice*, *ibid.*

<sup>130</sup> Age at the start of the academic year.

<sup>131</sup> Department for Education (2017) *A level and other 16 to 18 results: 2015 to 2016 (revised)*, <https://www.gov.uk/government/statistics/a-level-and-other-16-to-18-results-2015-to-2016-revised>

## Highest level of participation in mathematics pathways at academic age 16 or 17

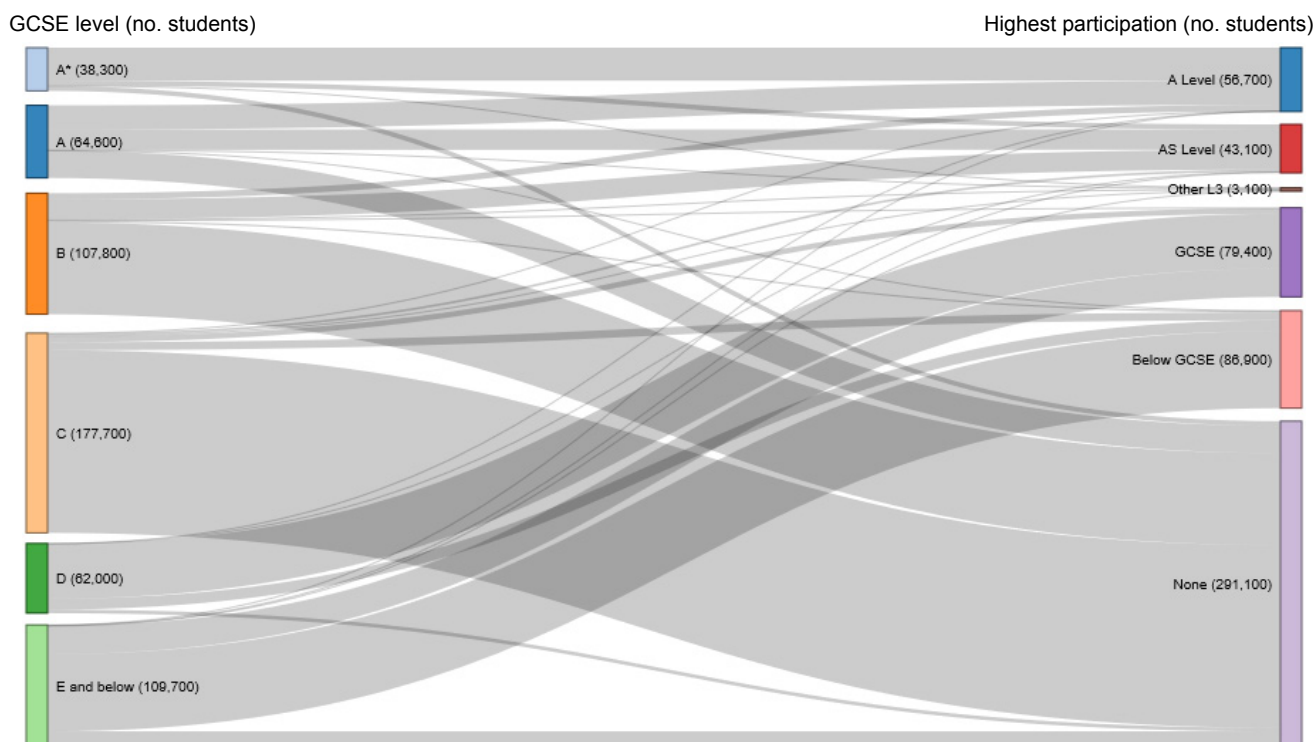


Figure 1. The highest level of participation in mathematics at academic age 16 or 17 for the 2013/14 GCSE cohort. GCSE mathematics grades are shown on the left; highest level of participation on the right for the same cohort.<sup>132</sup>

### **Mathematics is the most popular A level and there is scope to improve uptake further by reducing barriers to entry**

140. A level results data show that 81,533 students entered A level mathematics in 2016 – the highest number of A level entries out of all subjects. 731 pupils entered A level statistics.<sup>133</sup>

141. Students who achieve the highest grades in GCSE mathematics are *disproportionately* likely to study the subject at A level. 75 per cent of all students who achieved an A\* at mathematics GCSE in 2013/14, in the state sector, went on to enter A level mathematics in 2015/16, whereas only 42 per cent of students who achieved an A\* in History GCSE, in the state sector, went on to enter an A level in history. Similarly, 92 per cent of students who entered A level mathematics had

<sup>132</sup> Department for Education (2017) *Ad-hoc notice*, *ibid*.

<sup>133</sup> Department for Education (2017) *Revised A level and other 16-18 results in England 2015/16* [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/584124/SFR05\\_2017\\_A\\_level\\_and\\_other16-18\\_results\\_in\\_England\\_SFR\\_revised.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/584124/SFR05_2017_A_level_and_other16-18_results_in_England_SFR_revised.pdf)

achieved an A or A\* in GCSE mathematics, whereas just 53 per cent of students entering English Literature A level achieved an A or A\* in English at GCSE.<sup>134</sup>

142. On the face of it, there is considerable scope for increasing the number of students taking AS/A level mathematics. A level mathematics is regarded by many schools and students as challenging and, as a result, entrance grades are often B-A\*, rather than C-A\*, with a large proportion of schools appearing to set the bar at A-A\*.

## **Despite growth, AS/A level further mathematics numbers in state schools are still low**

143. The proportion of students studying A level further mathematics is also increasing. Between 2004 and 2016, the number of students taking the A level more than doubled to over 14,000<sup>135</sup> and of state-funded institutions offering A level mathematics, 68 per cent had students taking A level further mathematics in 2014/15, compared to only 40 per cent in 2004/5.<sup>136</sup>

144. Despite this, however, overall numbers are small. This is of concern given the role of further mathematics as a 'passport' to mathematical study at research-intensive universities. Although A level further mathematics is not a requirement for students at most universities, statistics show that a majority of students studying at these universities have achieved the qualification. In 2015/16, 68 per cent of 19 year olds from England who progressed to mathematics degree courses at Russell Group universities had achieved further mathematics A level.<sup>137</sup>

145. For students studying mathematics in non-Russell Group universities, 36 per cent had the further mathematics A level.<sup>138</sup> Not all state sixth forms and colleges offer further mathematics A level. This is concerning, in that those students who do not have access to the subject at sixth form may be at a disadvantage when applying to enter certain research-intensive universities.

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<sup>134</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>135</sup> Joint Council for Qualifications, (2016) *Provision GCE A level results 2016*, <http://www.jcq.org.uk/Download/examination-results/a-levels/2016/a-as-and-aea-results>

<sup>136</sup> Lee S, Stevens A, Boylan M and Demack S (2016) *The fragility of A level Further Mathematics in schools/colleges in England*, <http://www.bsrlm.org.uk/wp-content/uploads/2016/11/BSRLM-CP-36-2-08.pdf>

<sup>137</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>138</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

## **A level statistics uptake is also low, but this qualification is an essential part of the mix and is likely to be important in the future**

146. In 2016, 731 pupils entered A level statistics.<sup>139</sup> The level of uptake is considerably smaller than A level mathematics but stakeholders are clear that A level statistics is an important element in the 'mixed economy' of numerically rich 16-18 qualifications in England. While participation is currently low, the rise of big data and associated data science developments make it important to retain this qualification.

## **There is a significant gender gap in progression to AS/A level, despite good GCSE achievement by girls**

147. There are significant differences in the likelihood of progressing to AS/A level mathematics between girls and boys, even where they achieve the same GCSE grade. Girls are more sensitive to the grade achieved in mathematics at GCSE than boys in informing their decision to continue: while a high proportion of girls achieving an A\* progress to A/AS level, of students achieving A grades in 2014/15, only 50 per cent of girls continued to AS/A level compared to 70 per cent of boys.<sup>140</sup>

148. Research has found several causes of this including a suggestion that girls hold a lower 'mathematics self-concept' than boys with the same prior achievement. They are also more likely to cite stereotypical images associated with mathematicians as reasons not to do it.<sup>141</sup>

149. But the issues are complex and it is likely that numerous factors contribute to differences in choices. It is likely, for example, that girls' high achievement in other subjects at GCSE plays a role in their decisions. Evidence also suggests that girls are more likely to continue the subject if they enjoyed it and felt they were succeeding. Boys were more likely to cite instead that mathematics is easier than other subjects. School culture and behaviours play a role. Girls are more likely to choose the subject if they attend a single sex school, for example, or if their school has a sixth form.<sup>142</sup>

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<sup>139</sup> Department for Education (2017) *Revised A level and other 16-18 results in England 2015/16* *ibid.*

<sup>140</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>141</sup> Smith C (2015) *Gender and participation in mathematics and further mathematics A-levels: a literature review for the Further Mathematics Support Programme*, <http://furthermaths.org.uk/docs/Research%20Report%20UCL%20IOE%20Case%20Studies%20FINALE.pdf>

<sup>142</sup> Smith C (2015) *ibid.*

## There are concerning differences in participation levels between local areas, many of which are not explained by prior achievement

150. There are significant regional differences in level 3 mathematics participation. Students who achieved A\*-C, including both boys and girls, are most likely to carry on to level 3 mathematics in the London region (34 per cent). This contrasts starkly with the North East (20 per cent) and Yorkshire and the Humber (23 per cent).<sup>143</sup>

151. Similar differences are found in level 3 mathematics participation in local authority areas. The regions with the highest level 3 mathematics participation for students with A\*-C at age 16 are in London and the South East and the lowest participation rates are in the North.<sup>144</sup>

152. The tables below show the ten local authorities in 2015/16 where 40 per cent or more of those with A\*-C progress to level 3 mathematics at 16 and the ten local authorities with the lowest participation rates.<sup>145</sup>

### **Proportion of those that achieved A\*-C at 15 studying level 3 mathematics at 16 by local authority**

<b>LAs with lowest L3 mathematics participation</b>	
<b>Local Authority</b>	<b>% study L3</b>
Derby	17
York	17
Halton	16
Sunderland	16
Newcastle Upon Tyne	15
Kingston Upon Hull, City of	15
Salford	15
Middlesbrough	12
Knowsley	10
Barnsley	10

<b>LAs with highest L3 mathematics participation</b>	
<b>Local Authority</b>	<b>% study L3</b>
Reading	57
City of London	50
Redbridge	47
Barnet	46
Harrow	45
Ealing	44
Newham	43
Brent	43
Sutton	43
Hounslow	41

Table 3a (left) and Table 3b (right). Local authorities with the lowest (left) and highest (right) proportions of those that achieved A\*-C at 15 studying level 3 mathematics at 16.<sup>146</sup>

<sup>143</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>144</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>145</sup> Department for Education (2017) *Ad-hoc notice* ibid.

153. The picture is similar for students who achieved A/A\* at age 16. Luton, Bournemouth and Leicester are the only authorities outside London and the South East where more than 80 per cent progress to level 3 mathematics. 56 per cent of pupils with an A/A\* in Salford in north west England go on to study level 3 mathematics, compared to 88 per cent in Redbridge. Other local authority areas with relatively poor levels of progression to level 3 mathematics include Barnsley, York and Derby.<sup>147</sup>

**Proportion of those that achieved A\*/A at 15 studying level 3 mathematics at 16 by local authority**

<b>LAs with lowest L3 mathematics participation</b>	
<b>Local Authority</b>	<b>% study L3</b>
Somerset	62
East Sussex	61
Knowsley	61
Halton	61
Southampton	61
Derby	60
York	60
Barnsley	57
Salford	56

<b>LAs with highest L3 mathematics participation</b>	
<b>Local Authority</b>	<b>% study L3</b>
Redbridge	88
Reading	87
Newham	85
Brent	85
Harrow	84
Hounslow	84
Waltham Forest	83
Ealing	83
Enfield	83

Table 4a (left) and Table 4b (right). Local authorities with the lowest (left) and highest (right) proportions of those that achieved A\*/A at 15 studying level 3 mathematics at 16.<sup>148</sup>

154. Though not all local authorities with the lowest level 3 participation are in the north, there is a clear north-south divide in attainment but equally no clear explanations for these differences. These figures show that local authorities with the lowest proportion of A\*-C students taking level 3 mathematics qualifications tend to have lower than average achievement at mathematics GCSE. However, the significant variation in level 3 participation between some local authorities with similar mathematics GCSE attainment shows that these differences cannot be explained by prior attainment. Amongst the bottom ten areas, there are also no clear patterns in terms of local industry, prosperity, socio-economic make-up and geographical location.<sup>149</sup>

<sup>146</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>147</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>148</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>149</sup> Department for Education (2017) *Ad-hoc notice* ibid.

## **There are differences in progression to AS/A level based on ethnic background and eligibility for free school meals**

155. White pupils with an A grade at GCSE at age 16 are the least likely to progress to AS/A level (55 per cent) with Asian pupils the most likely (80 per cent).<sup>150,151</sup>

156. Findings on participation and socio-economic background may appear counterintuitive: students eligible for free school meals with an A grade at GCSE are *more* likely to choose mathematics at level 3 than others. 67 per cent of free school meals students progressed to AS/A level, compared to 59 per cent of others. Overall 69 per cent of students with an A grade from the 25 per cent most deprived areas took an AS/A level compared to 57 per cent of those from the least deprived areas.<sup>152</sup>

157. There is significant scope to improve the GCSE achievement of pupils eligible for free school meals in particular. Given the evidence that this group may be more likely than others to choose to continue the subject, this could lead to a significant increase in students progressing into AS/A level mathematics.

## **Many students not studying AS/A level need to study mathematics beyond GCSE**

158. The need for quantitative and mathematical skills across academic subjects and technical fields is of growing importance. However, of all students who continued on to the A level route in 2014/15, 52 per cent who had achieved A\*/A GCSE in mathematics did not enter the subject at A level at age 17.<sup>153</sup>

159. Furthermore, of the 388,400 academic-age 16 year olds who achieved A\*-C in mathematics in 2013/14, over half of these (57 per cent or 223,300) had a level 3 academic course as their highest study aim at age 17 in 2014/15, but just over a quarter of these (27 per cent) studied mathematics at age 17.<sup>154</sup>

160. The table on the following page shows that 24 per cent entered a mathematics A level at 17 and 12 per cent entered another STEM A level. A further 27 per cent sat an A level with some numerate content. A quarter sat a different A level.<sup>155</sup>

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<sup>150</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>151</sup> Categories used in Department for Education school census data collection

<sup>152</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>153</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>154</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<sup>155</sup> Department for Education (2017) *Ad-hoc notice* *ibid.*

<b>Total</b>	<b>223,300</b>	<b>100%</b>
Mathematics A level entry	53,000	24%
Other STEM A level entry	27,800	12%
Other numerate <sup>156</sup> A level entry	60,300	27%
Other A level entry	55,600	25%
Other academic L3 course	26,700	12%

Table 5a. Proportions of students entering different types of courses at academic age 17.<sup>157</sup>

<b>Mathematics split of the other academic L3 courses:</b>	<b>26,700</b>	<b>12%</b>
AS Level mathematics	5,300	2%
Mathematics A level aim but not mathematics entry	1,000	0%
Other mathematics study	700	0%
No mathematics study	19,700	9%

Table 5b. Breakdown of other academic level 3 courses in Table 5a.<sup>158</sup>

161. The most popular A level subjects for those who achieved an A/A\* GCSE in mathematics, who took neither mathematics nor further mathematics at A level at age 17 in 2015/16, were English and biology, followed by history, psychology and chemistry. The table below shows the A level subjects entered by more than 15 per cent of students who did not opt to study A level mathematics.<sup>159</sup>

<b>Most popular A level subjects</b>	
<b>Subject</b>	<b>%</b>
Biology	32
English	32
History	28
Psychology	27
Chemistry	20
Geography	19

Table 6. The most popular A level subjects for pupils who had achieved A/A\* GCSE in mathematics at academic age 15, and who entered an A level at academic age 17 in 2015/16 but had not entered mathematics or further mathematics<sup>160</sup>

<sup>156</sup> Subjects containing mathematical or quantitative content.

<sup>157</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>158</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>159</sup> Department for Education (2017) *Ad-hoc notice* ibid.

<sup>160</sup> Department for Education (2017) *Ad-hoc notice* ibid.



162. As can be seen in the table above, the proportion of students studying an A level with some numerate or STEM content is significant. However, they are not continuing further with any mathematics to support their course. Core maths fills a gap where previously there were limited options for this group to study mathematics.

## **Employer and university ‘pull’ for mathematics needs to be articulated more strongly**

163. Improving uptake of mathematics at level 3 rests in the available qualifications being chosen by students. This is critical to the success of core maths and important to A level mathematics and further mathematics.

164. The employer and business representatives we spoke to as part of the review were clear of the need for better mathematics and numeracy at all levels. They were keen to promote the value of mathematics across all careers, not just those relating to STEM. Employers noted the challenge of getting co-ordinated and clear messages about the importance of mathematics to young people. More needs to be done to make this employer ‘pull’ for mathematics and quantitative skills effective.

165. Mathematics-specific messages are needed for students, schools and others about the increasing importance of mathematics and quantitative skills to a broad range of careers, future employability and resilience in a changing labour market.

166. The same is true of universities. Many students arrive at university with unrealistic expectations of the mathematical content of degree courses. A recent study<sup>161</sup> showed that, of all English university students who graduated in 2013, between 5 and 7.5 per cent of all university students did not have a grade C or above in GCSE mathematics. This is especially noteworthy given that all level 3 apprentices are required to complete level 2 mathematics.

167. In addition, large numbers of students studying STEM subjects at university do not have a mathematics qualification beyond GCSE. Between 2010/11 and 2012/13, 61 per cent of biology undergraduates, 75 per cent of computer science students, 47 per cent of physical sciences students and 40 per cent of engineering and technology students do not have mathematics beyond GCSE. For subjects allied to medicine,

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<sup>161</sup> Hodgen J, Adkins M and Tomei A (2017) *The mathematical backgrounds of undergraduates: Interim report* (Nottingham: University of Nottingham), <http://www.nottingham.ac.uk/research/groups/crme/projects/mathematical-backgrounds.aspx>  
The study examined English-domiciled students who were age 16 in 2007/8, took a level 3 qualification between 2008/9 and 2010/11, and then entered university between 2010/11 and 2012/13. The final report, due in 2017, will form the basis of a resource which will enable universities to compare the mathematical backgrounds of entrants across institutions in all disciplines.

the figure is 74 per cent. These figures are concerning as mathematical and quantitative analysis is central to these disciplines and related professional fields.

168. Schools and colleges are heavily influenced by requirements set by universities in deciding which courses to offer to students as illustrated in the rise over recent years in the number of students studying core academic subjects following publication of the Russell Group's Informed Choices report.<sup>162</sup>

**Recommendation 4: The Department for Education should work with UK learned societies to encourage universities to better signal and recognise the value of level 3 mathematics qualifications for entry to undergraduate courses with a significant quantitative element.**

## **Students without at least a grade C or equivalent in GCSE mathematics at age 16**

169. Since 2014, the Government has required, through a condition of funding, that students aged 16 to 18 without A\*-C in GCSE mathematics and English should continue to study towards these qualifications. In 2015, the condition was tightened to require full time students with a grade D to retake the GCSE rather than study other mathematics and English qualifications such as functional skills.<sup>163</sup>

170. The policy has resulted in significant increases in the numbers resitting their GCSEs, with more young people successfully retaking their GCSEs. However, the proportion of resit students passing at a C or above has declined sharply, with many more students resitting GCSE examinations without success. Entries by 17 year olds and over for mathematics increased by 72.6 per cent between 2014, the year before the funding condition came into force, and 2016.<sup>164</sup> However, the number of students aged 17 and over achieving an A\*-C only increased by 30.9 per cent from 39,130 to 51,220.<sup>165</sup>

171. The proportion of resit students passing at a C or above has declined as entries have risen. Students taking GCSEs again have already experienced failure and may be less motivated or confident to achieve in the subject as a result. These

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<sup>162</sup> Russell Group (2016/17) *Informed choices*, <http://russellgroup.ac.uk/media/5457/informed-choices-2016.pdf>

<sup>163</sup> Funding for students not meeting the condition of funding is removed from future allocations at the national funding rate per student. <https://www.gov.uk/guidance/16-to-19-funding-maths-and-english-condition-of-funding>

<sup>164</sup> Joint Council for Qualifications (2016) *Provisional GCSE (full course) Results (UK candidates aged 17 and over)*, <http://www.jcq.org.uk/examination-results/gcses/2016/gcse-full-course-uk-by-age-2016>

NB. Derived from JCQ published entry data and cumulative percentages of subject results by grade

<sup>165</sup> Joint Council for Qualifications (2016) *ibid*.

challenges are most likely to be felt in FE colleges, which take students with lower average grades than do school sixth forms or sixth form colleges, and where there has been the largest increase in numbers studying qualifications in maths at level 2.<sup>166</sup>

172. Last summer, entries for mathematics increased by 43,000 compared to the year before, presumably as a result of the D grade rule coming into force; however, the number of students achieving A\*-C only increased by 4,330. The net result is an increase in the absolute numbers of students achieving a mathematics GCSE A\*-C by the age of 18, but at the expense of a much larger increase in the proportion of students entering GCSE examinations without success.<sup>167</sup>

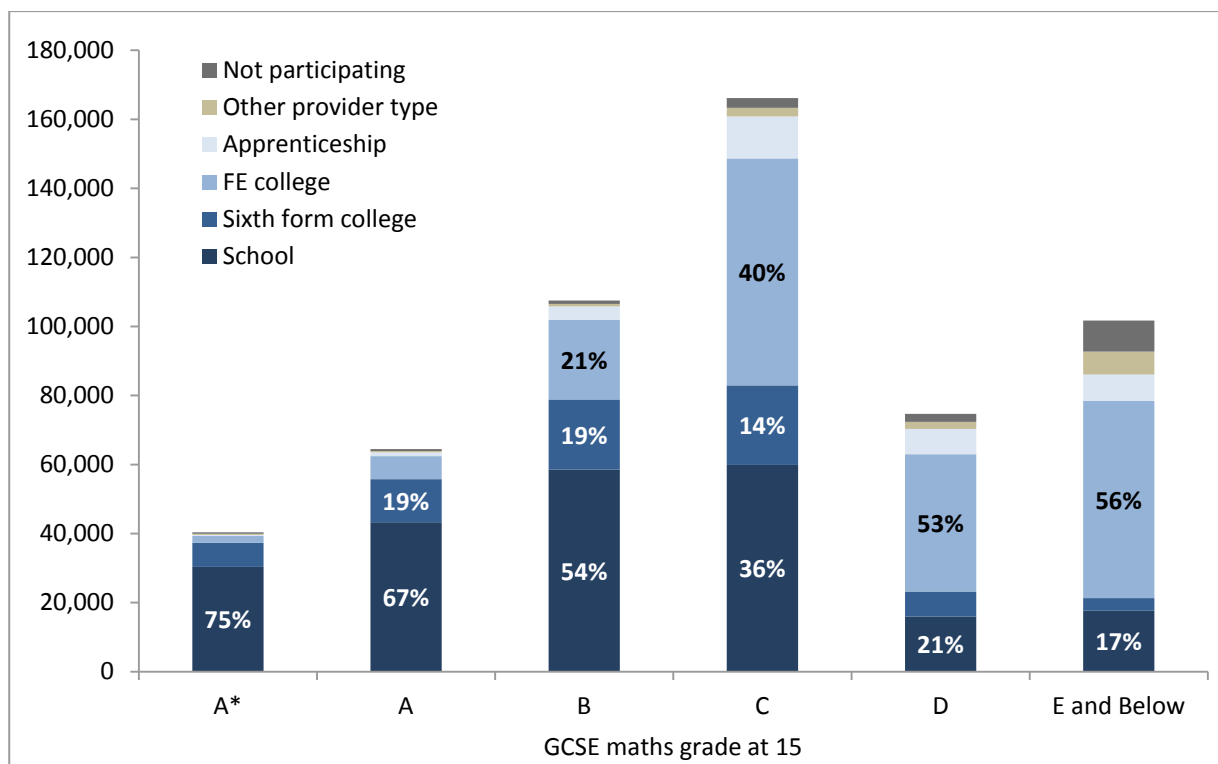
173. FE colleges receive by far the largest number of students with a D grade or below. In 2014, 54 per cent of D grade students went on to study at FE colleges compared to 22 per cent in schools (see figure 2 on the following page). At grade E and below, 57 per cent went on to study at FE colleges, in comparison to 18 per cent in schools. FE colleges consulted by the review have spoken positively about the principle of improving and securing a reasonable standard of numeracy by the end of compulsory education for those who do not achieve a GCSE A\*-C at 16. However, many also raised concerns about the focus on GCSE in the condition of funding rules and 16-18 accountability measures and made the case for an alternative pathway to securing an acceptable level of mathematics, alongside GCSE resits. Some suggested that a larger proportion of students could secure numeracy if they were able to opt for a more practical curriculum.

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<sup>166</sup> Ofsted, (2016) *The Annual Report of Her Majesty's Chief Inspector of Education, Children's Services and Skills 2015/16*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/574186/Ofsted\\_annual\\_report\\_education\\_and\\_skills\\_201516\\_web-ready.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/574186/Ofsted_annual_report_education_and_skills_201516_web-ready.pdf)

<sup>167</sup> Joint Council for Qualifications (2016) *ibid.*

**Numbers of students studying by type of institution at academic age 16 in 2015/16**



**Figure 2. The numbers of students studying by type of institution at academic age 16 in 2015/16, categorised by prior attainment at GCSE mathematics.<sup>168</sup> ‘Other’ includes, for example, private companies, charities, local authorities etc.**

174. There are a number of reasons that might explain low A\*-C pass rates amongst the additional students entered for GCSE mathematics as a result of the condition of funding. One of the most significant may be that some of the new students entered for the GCSE at age 17 or 18 are those who achieved a grade E or below at age 16. These students are likely to find it harder to achieve an A\*-C than those who attained a grade D.

175. A number of stakeholders suggested that some providers choose to enter almost all students for GCSE exams, regardless of their likelihood of achieving an A\*-C. Reasons cited for this include the challenge of switching from functional skills in year 12 to GCSE in year 13, the increased challenge of functional skills mathematics following intervention by Ofqual<sup>169</sup> to raise standards, and parental pressure to enter students for GCSE rather than alternative mathematics qualifications such as functional skills.

<sup>168</sup> Department for Education (2017) *Ad-hoc notice* *ibid*.

<sup>169</sup> Ofqual (2015) *Improving functional skills qualifications*,

<https://www.gov.uk/government/publications/improving-functional-skills-qualifications>

176. The significant increase in numbers of students entering GCSE mathematics exams aged 16-18 has created a new demand for teachers capable of teaching the subject, and colleges have needed to expand their teaching capacity in this area over the last four years. Ofsted has identified difficulties in recruiting and retaining skilled and experienced teachers, and the result of this is that many teachers of GCSE resits in colleges do not have significant prior experience and have had to develop the necessary knowledge and skills very quickly.

177. In their 2015/16 annual report, Ofsted highlighted poor mathematics teaching in both the lowest performing colleges and in many receiving the judgement 'good'.<sup>170</sup> The number of good or outstanding FE colleges declined from 77 per cent in 2015 to 71 per cent in 2016, in large part as a result of the new expectations in mathematics and English. Notably, independent training providers saw the opposite trend in their Ofsted profile, with an increase in good or outstanding from 81 per cent to 82 per cent; they mainly cater for part-time learners and apprentices, neither of whom are subject to the 16-18 mathematics and English conditions of funding.<sup>171</sup>

178. These circumstances and the poor GCSE A\*-C pass rates point to a significant need to address teachers' professional development needs and teacher shortages. Current steps to address these issues are detailed in the next chapter, but there is a clear need for significant intervention in this area.

179. Improving the number of skilled teachers able to teach GCSE mathematics to 16-18 students is likely to raise the rate of A\*-C attainment, but progress towards a scenario where much higher numbers of students succeed in attaining A\*-C is likely to be gradual. This means that for those who are not capable of achieving a grade C or above at GCSE in the time available, it is vital that there are alternative qualifications that will ensure they leave education with the mathematical skills they need. These alternative qualifications must not be easier; rather they must cover, at a sufficiently challenging level, the core curriculum content required to produce numerate citizens and make them readily employable. Ofsted has echoed this view.<sup>172</sup>

180. While GCSE A\*-C resit pass rates are so low, it is prudent for the government to consider how students without the capability to gain a C grade or equivalent by

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<sup>170</sup> Ofsted, (2016) *The Annual Report of Her Majesty's Chief Inspector of Education, Children's Services and Skills 2015/16*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/574186/Ofsted\\_annual\\_report\\_education\\_and\\_skills\\_201516\\_web-ready.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/574186/Ofsted_annual_report_education_and_skills_201516_web-ready.pdf)

<sup>171</sup> Ofsted 2016, *ibid.*

<sup>172</sup> Ofsted 2016, *ibid.*

age 18 can gain access to strong alternative qualifications, to help ensure they leave education with credit for the mathematics and quantitative skills they have.

**Recommendation 5: In view of the low GCSE resit success rates and new GCSE requirements, the Department for Education should review its 16-18 resit policy with the aim that a greater proportion of students without a grade C or equivalent attain appropriate mathematical understanding by age 18. Specifically, there should be fresh consideration of appropriate curricula and qualifications for these students and the extent to which current policy incentivises these to be offered.**

## Chapter 6: Capacity to deliver

181. This chapter sets out the delivery challenges that currently limit providers' ability to offer mathematics qualifications and teach them well. These challenges are national but there is considerable variation in the abilities of providers to respond to them.

### **The current funding model presents risks to AS/A level further mathematics provision and does not incentivise core maths**

182. There has been consistent progress towards increasing the number of students taking further mathematics AS and A level qualifications over recent years and the Further Maths Support Programme has had a positive impact on the quality of provision and student outcomes. This is significant, as further mathematics at AS/A level has a unique role as a 'passport' qualification to mathematical study at many research-intensive universities, as explained in chapter 5.

183. Stakeholders consulted during the review raised concerns that a combination of changes to A level (decoupling of AS) and changes to funding (per-student rather than per-qualification funding) are combining to present serious risks to further mathematics provision. They argue that the current funding model and funding levels do not reflect the cost of offering further mathematics, which is often taken as a fourth or fifth A level. It would clearly be undesirable for students from state schools to lose access to further mathematics, but this was perceived as a significant risk.

184. The government has recognised concerns that some programmes are necessarily larger than 600 hours. As a result, a 10 per cent uplift in funding for those taking 4 A levels and a 20 per cent uplift for those taking 5 A levels was introduced, as long as students achieved at least a grade 'B' in each.

185. However, this large programme funding has not allayed concerns raised by many stakeholders who contributed to this review. Further mathematics is studied by fewer students than many other A levels and is likely to have smaller class sizes. There are already many schools that do not enter students for further mathematics, while others are collaborating in order to deliver it. Stakeholders are concerned that it will no longer be affordable to schools and colleges to deliver it without thinking more creatively about solutions such as collaboration with other local schools.

### **Further mathematics provision delivered collaboratively across a multi academy trust**

ARK has 13 secondary schools with post 16 provision. All schools offer mathematics AS and A level but further mathematics AS and A level is not taught in every school. Where schools find it difficult to deliver further mathematics, regional partnerships allow students to study the subject in another local school.

The trust co-ordinates professional development across schools; a mathematics network has been established to provide opportunities for the sharing of expertise and efficient delivery. Moderation and professional development days are organised throughout the year creating opportunities for co planning and specific key stage 5 knowledge development. Teachers are drawn from across the network to deliver Saturday revision classes to students, enabling students to benefit from the strengths of a large group of teachers drawn from a wide variety of academies.

186. Core maths qualifications are studied by students in addition to their main programme. Providers delivering core maths at scale are in the minority, and those with smaller cohorts find it difficult to deliver economically. This is particularly true for smaller school sixth forms, which are disproportionately likely to say that they do not intend to offer core maths in future.<sup>173</sup>

**Recommendation 6: The Department for Education should consider the institutional incentives and disincentives arising from the 16-19 funding model for schools and colleges, with a view to removing any disincentives for mathematics provision;**

**As an urgent and immediate measure, it should consider increasing the financial incentive for both AS and A level further mathematics within four/five A level programmes and consider providing a funding incentive for student programmes which include core maths.**

### **Specialist mathematics teacher supply challenges will continue**

187. Teachers of mathematics comprise both subject specialists and non-specialists (or specialists in other areas) who teach mathematics.

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<sup>173</sup> IFF Research / Department for Education (2017) *Post-16 institutions omnibus: wave 3 survey*, <https://www.gov.uk/government/publications/post-16-institutions-omnibus-wave-3-survey>



188. In schools and sixth form colleges, most mathematics teaching is delivered by mathematics specialists – teachers with a relevant degree qualification or who have undertaken initial teacher training (ITT) in mathematics. This includes graduates with non-mathematical degrees who have done subject knowledge training, through which they acquire the necessary subject knowledge to teach mathematics. Specialist teachers of other subjects with some mathematics background (for example AS mathematics) can supplement this specialist capacity in secondary schools – generally at key stage 3.

189. Similarly, there are numeracy specialists in FE colleges, who are likely to spend much of their time teaching level 2 qualifications. But technical and other specialists also deliver mathematics and numeracy teaching in FE settings.

190. In 2016/17, the Department for Education fell short of its Teacher Supply Model (TSM) target (for an explanation of the teacher supply model, see the box on the following page), recruiting 2,605 teacher training places against a target of 3,102 (84 per cent).<sup>174</sup> Although the shortfall in recruiting skilled mathematics teachers is less than for other subjects, schools report significant challenges to their skilled mathematics teacher recruitment.<sup>175</sup> National figures may obscure regional challenges and in its response to the findings of the NAO review to the Public Accounts Committee in November 2016, the government committed to developing a better understanding of the local teacher supply challenges<sup>176</sup> and has recently published its first analysis of local supply issues.<sup>177</sup>

191. Department for Education analysis of the School Workforce Census shows that in 2015/16, 96.7 per cent of A level mathematics lessons in schools were taught by a teacher with a relevant post A level qualification.<sup>178</sup> This compares to 85.5 per cent for lessons in years 7-9 and 91.2 per cent in years 10-11. Information is only

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<sup>174</sup> Department for Education (2016) *Initial Teacher Training census for the academic year 2016 to 2017, England*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/572290/ITT\\_Census\\_16\\_17\\_SFR\\_Final.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/572290/ITT_Census_16_17_SFR_Final.pdf)

<sup>175</sup> National Audit Office (2016) *Training new teachers*, <https://www.nao.org.uk/wp-content/uploads/2016/02/Training-new-teachers.pdf>

<sup>176</sup> HM Treasury (2016) *Treasury Minutes. Government responses to the Committee of Public Accounts on the Thirty Seventh and the Thirty Ninth reports from Session 2015-16; and the First to the Thirteenth reports from Session 2016-17*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/565426/57448\\_Cm\\_935\\_1\\_Treasury\\_Minutes\\_Accessible.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/565426/57448_Cm_935_1_Treasury_Minutes_Accessible.pdf)

<sup>177</sup> Department for Education (2016) *Local analysis of teacher workforce: 2010 to 2015*, <https://www.gov.uk/government/statistics/local-analysis-of-teacher-workforce-2010-to-2015>

<sup>178</sup> Department for Education (2016) *Analysis of 'specialist' and 'non-specialist' teaching in England*, <https://www.gov.uk/government/publications/analysis-of-specialist-and-non-specialist-teaching-in-england>

available for schools and it cannot be assumed that results for sixth form and FE colleges are similar.

192. The shortfall of teachers of mathematics is not as large as some subjects, but demand for mathematics teachers is likely to rise over the coming years – secondary school pupil numbers are forecast to rise by 10.3 per cent by 2020,<sup>179</sup> and demand for mathematics and related graduates in the labour market is likely to continue to create pressure on graduate supply.

193. The graduate pool for mathematics teachers is smaller than for many other subjects. To meet its 2014/15 target for history teacher trainees, the government needed to attract 1 in 25 history graduates, compared to 1 in every 5 mathematics graduates.<sup>180</sup> Graduates from other mathematical STEM specialisms are also in high demand from industry.

### **How does the Teacher Supply Model calculate the need for ITT places?**

The TSM is used by the Department for Education to estimate the number of postgraduate initial teacher training trainees required in England for each academic subject and school phase.<sup>181</sup> The model uses specific policy assumptions (amongst others) on increased English Baccalaureate entry, core maths, the Further Mathematics Support programme and the new mathematics GCSE. The three most recent models forecast an increase in postgraduate mathematics ITT places from 2,581 in 2015/16 to 3,102 in 2016/17 and 3,102 in 2017/2018.

The scope of the TSM presently includes only English state-funded primary and secondary schools, along with attached nurseries and sixth-forms. Hence, the TSM does not directly estimate teacher need in FE colleges, sixth-form colleges or the independent sector. However, the TSM indirectly accounts for such teacher supply requirements, using assumptions about the proportion of ITT graduates and existing qualified teachers that enter these sectors.

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<sup>179</sup> Department for Education (2016) *National pupil projections: July 2016, Main Tables, Table 1*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/536870/SFR25Projns\\_2016\\_Tables.xlsx](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/536870/SFR25Projns_2016_Tables.xlsx)

<sup>180</sup> National Audit Office (2016) *ibid.*

<sup>181</sup> Department for Education (2017) *2017/2018 Teacher Supply Model Methodological Annex*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/613026/201718\\_Teacher\\_Supply\\_Model\\_Methodological\\_Annex.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/613026/201718_Teacher_Supply_Model_Methodological_Annex.pdf)

## Measures to improve teacher supply in schools focus on retraining non-specialists as well as recruiting specialists

194. A number of policies are in place to incentivise entry to mathematics teaching. Significant bursaries are on offer for graduates entering training for secondary mathematics. Students entering with a first or second class degree receive £25,000 tax free. Only physics trainees, who get £30,000 if they have a first class degree, receive more. The best graduates entering a PGCE or unsalaried School Direct are able to apply for scholarships worth £27,500. They give extra training and support as well as membership of professional bodies.<sup>182</sup>

195. In March 2015, the Department for Education launched a series of new initiatives costing £67 million to improve teacher supply in mathematics and physics.<sup>183</sup> The new investment will provide training for 15,000 existing teachers and attract up to 2,500 additional specialist mathematics and physics teachers up to 2020. They aim to bring teachers into the profession from previously untapped routes. They are:

- Teacher Subject Specialism Training (TSST) which provides extra subject knowledge to non-specialists who have completed ITT and those returning to the profession.
- Support for returners to come back to teaching.
- International recruitment.
- Undergraduate courses with qualified teacher status – to support universities to develop new courses that allow mathematics students to train to teach alongside their academic studies.
- Paid internships that give students the opportunity to experience teaching before they commit to it as a career.
- The Maths Chairs programme which provides additional incentives for graduates with PhDs to teach in schools and provide subject support to their colleagues.

196. Teacher Subject Specialist Training (TSST) has a critical role to play in addressing specialist teaching gaps. While the overwhelming majority of A level mathematics teachers have a relevant post A level qualification, this is not true of all secondary teaching. Teachers' subject background and content knowledge has an

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<sup>182</sup> Department for Education, *Get Into Teaching*, <https://getintoteaching.education.gov.uk/funding-and-salary/overview>

<sup>183</sup> National College for Teaching and Leadership (2015) *New plans to attract and train maths and physics teachers*, <https://www.gov.uk/government/news/new-plans-to-attract-and-train-maths-and-physics-teachers>

influence on effectiveness in terms of pupil achievement, but a range of factors influence this beyond prior qualifications.<sup>184</sup>

197. The Department for Education has commissioned the National Foundation for Educational Research (NFER) to conduct an evaluation and the findings are due to be published later in 2017. In the context of this review, many stakeholders have raised the issue of financial incentives for retention in relation to mathematics teachers. It seems unlikely that any single recruitment measure will address teacher supply challenges, but the Department should continue to identify and test innovative approaches to improving teacher supply.

## **Less is known about teachers of mathematics and numeracy in FE colleges**

198. Recruitment in further education and sixth form colleges is managed through a devolved process by institutions themselves, and has no link to the system that governs recruitment for schools. Though there are a number of routes available for entry to teaching in these institutions, including formal training through HE or other training providers, there are no national targets for entry to training.

199. This approach means that the data available about the need for mathematics teachers in FE and supply of new teachers are not as good as they are for secondary schools, where data are recorded in the School Workforce Census. Colleges that have submitted evidence to this review have spoken about challenges in recruiting mathematics teachers but a clear comprehensive system-wide picture of need does not exist. The data available are gathered, analysed and disseminated by the Education and Training Foundation, which also provides a recruitment advice service for FE colleges, funded by the Department for Education.

200. New mathematics teachers taking specialist teaching qualifications in FE colleges can claim bursaries equivalent to those for trainee teachers entering schools or sixth form colleges. Some colleges raised concerns, however, that they have found it difficult to recruit high quality teachers of GCSE mathematics to teach the large numbers of students resitting mathematics GCSE. This new and significant demand for teachers creates pressures and further increases the need for new teachers throughout the system.

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<sup>184</sup> Hill H, Charalambous C Y and Chin M (2016) *Teacher Characteristics and Student Learning: A Comprehensive Assessment*, Presented at the 2015 American Education Research Association (AERA) Annual Meeting, Chicago, IL, [http://cepr.harvard.edu/files/cepr/files/hill\\_charalambous\\_chin\\_teacher\\_characteristics\\_aera2015.pdf](http://cepr.harvard.edu/files/cepr/files/hill_charalambous_chin_teacher_characteristics_aera2015.pdf)

**Recommendation 7: The Department for Education should improve the evidence base on the FE workforce teaching mathematics and quantitative skills in order to assess supply, teaching quality and the effectiveness of current recruitment measures.**

## **Centrally-funded support programmes have a critical role to play**

201. There is a considerable amount of centrally-funded support available for teachers of mathematics. The government has invested £30 million in FE workforce improvements through the sector-owned improvement body, the Education and Training Foundation, which has funded training for thousands of teachers in the FE sector to help them improve their teaching of mathematics and English. The low A\*-C pass rates for GCSE resits demonstrates a significant ongoing need for better teaching and training. There is substantial variation in the results that providers achieve when entering students for GCSE mathematics resits.<sup>185</sup> Some of this difference is likely to be due to the differing prior attainment of the cohorts of pupils entered, but the quality of teaching may also vary from institution to institution.

202. The Education Endowment Foundation (EEF) has recently launched a new £5 million programme focused on 16-18 mathematics and English that will help to provide evidence on the most successful interventions for students.<sup>186</sup>

203. While there has been consideration of organisational approaches and strategies to increasing and improving mathematics and English provision in FE settings – such as case studies from the Institute of Education<sup>187</sup> and Ofsted<sup>188</sup> – there is an evidence gap on effective delivery and teaching of mathematics and English to those without GCSE A\*-C. This is confirmed by a recent literature

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<sup>185</sup> Department for Education (2016) A level and other 16 to 18 results: 2015 to 2016 (provisional). Maths and English tables 16b

<https://www.gov.uk/government/statistics/a-level-and-other-level-3-results-2015-to-2016-provisional>

<sup>186</sup> Education Endowment Foundation (2016) *New £5m fund to improve maths and English results for young people without good GCSEs*, <https://educationendowmentfoundation.org.uk/news/new-5m-fund-to-improve-maths-and-english-results-for-young-people-without-g/>

<sup>187</sup> UCL Institute of Education (2016) *Five Case Studies: the delivery of GCSE English and maths to the 16-19 cohort*, <http://www.excellencegateway.org.uk/content/etf2021>

<sup>188</sup> Department of Education (2015) *Increasing provision in English and mathematics through planning*, <https://www.gov.uk/government/publications/increasing-provision-in-english-and-mathematics-through-planning>

review<sup>189</sup> by the EEF, which found that the evidence about which interventions may work is limited.

204. The Department for Education has commissioned a research project to uncover and document effective practice in planning, engaging and teaching mathematics and English in non-selective FE settings. The research will help to improve understanding of the reasons for the disparities in outcomes between providers. Findings are due to be published in 2017.

**Recommendation 8: The Department for Education should expand its support to develop excellence in GCSE mathematics teaching across the FE sector. This should be informed by evidence of effective pedagogy for students who have not succeeded in the subject within secondary education and emerging evidence about the needs of the workforce.**

205. The Core Maths Support Programme (CMSP) has been set up to provide front line support to schools and colleges to offer and teach the new core maths qualifications. The CMSP has provided support and professional development to teachers, developed and shared teaching resources and raised the profile of the qualifications amongst universities, employers and other stakeholders. Until now this has been broadly led by demand from schools and colleges. Continued government funding for a support programme for core maths is essential. Significant numbers of teachers, including specialists in other quantitative subjects, should be trained to teach core maths, and the qualification needs to gain wide reach, covering all local areas and types of schools and colleges.

#### **Upskilling teachers and building core maths teaching capacity**

Harrogate Grammar School is a non-selective school, which has taught core maths successfully since September 2014. Teachers shared experiences of core maths teaching at professional development workshops and the teaching of core maths is now a default aspect of mathematics teacher CPD.

The school recognised the relevance of other subjects, and has made core maths part of its Teacher Subject Specialism Training (TSST). Teachers with specialisms in other mathematical and quantitative subjects, such as business and finance, have been upskilled in mathematics and statistics to enable them to contribute to core maths teaching with confidence.

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<sup>189</sup> Education Endowment Foundation (2016) *Improving Level 2 English and maths outcomes for 16-18 year olds Literature review*, [https://educationendowmentfoundation.org.uk/public/files/Publications/16-18\\_Literature\\_Review.pdf](https://educationendowmentfoundation.org.uk/public/files/Publications/16-18_Literature_Review.pdf)

**Recommendation 9: The Department for Education should continue to fund a central core maths programme until the qualification becomes embedded and enhance this to upskill large numbers of teachers of other quantitative subjects to teach core maths.**

206. The Further Maths Support Programme (FMSP) targets state schools and colleges that have no or few entrants to further mathematics and works with them to increase the number of students who can access mathematical courses at leading universities. Where schools or colleges cannot offer further mathematics with FMSP support, the programme provides direct face-to-face and online lessons and tuition to students. Additionally, if schools and colleges cannot support preparation for STEP, AEA and MAT examinations (leading university entrance exams), the programme provides direct tuition and support for students.

207. Reformed AS/A levels in mathematics and further mathematics will be introduced for first teaching in 2017. Changes have been substantial and schools will require significant further support to teach the new qualifications well.

208. The Further Maths Support Programme provides an essential source of ongoing professional development, support and teaching resources for teachers of A level mathematics. Among other work, the programme supports state schools to teach higher-level mathematical thinking and problem solving expected of university mathematical study.

**Recommendation 10: The Department for Education should continue to fund centrally-delivered professional development programmes for teachers of AS/A level mathematics and further mathematics at least at their current level.**

## **University support for mathematics teaching**

209. Universities are significant sources of potential expertise and support for 16-18 teaching. In September 2016 the Department for Education published its green paper *Schools that Work for Everyone*,<sup>190</sup> which proposed that universities seeking to charge higher fees should play a direct role in improving school quality and pupil attainment. The Department says it was encouraged by the way that, as part of the consultation, higher education institutions were willing to think afresh about what more they could do to raise attainment in state schools, in recognition of their responsibility both to their own local communities and widening participation. The

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<sup>190</sup> Department for Education (2016) *Schools that Work for Everyone*, [https://consult.education.gov.uk/school-frameworks/schools-that-work-for-everyone/supporting\\_documents/SCHOOLS%20THAT%20WORK%20FOR%20EVERYONE%20%20FINAL.PDF](https://consult.education.gov.uk/school-frameworks/schools-that-work-for-everyone/supporting_documents/SCHOOLS%20THAT%20WORK%20FOR%20EVERYONE%20%20FINAL.PDF)

Department hopes and expects more universities will now come forward to be involved in school sponsorship and founding free schools, including more Maths Schools, as well as other partnerships with schools to raise pupil attainment.

- King's College London Maths School is a selective sixth form in south London which opened in September 2014. It is a small school with a capacity of 140 pupils on roll. The school provides an academic curriculum which strongly emphasises mathematics. Students take A levels in mathematics and further mathematics and the Sixth Term Examination Papers (STEP) alongside A levels in other related subjects. The school carries out outreach work with other schools in south London, including enrichment programmes for GCSE students from numerous partner schools and support for teachers of further mathematics.
- Jointly sponsored by the University of Exeter and Exeter College, Exeter Mathematics School operates a similar model, taking 120 students from across the South West. Boarding facilities are available for those travelling further than an hour to get to the school and means-tested financial support is available. The collaboration ensures that students benefit from the academic rigour and insights of University academics whilst enjoying the social aspects, support services and broader curriculum available through affiliation with the College. The school also delivers a variety of different outreach and engagement initiatives to support talent and interest in mathematics and physics. Examples include: primary and secondary masterclasses, an annual residential programme for 50 pupils, Teacher Subject Specialist Training (TSST) for those without specialist degrees in the subject, mathematics teacher network meetings and direct school support.

210. These schools represent an effective model for delivering highly specialised 16-18 mathematical education, drawing on the expertise of university departments.

211. Universities also have the potential, however, to make a significant contribution to improving 16-18 mathematics in other ways. Examples of projects with a strong focus on improving teaching and pupil attainment, which play to universities' subject strengths include:

- Underground Maths<sup>191</sup> – a University of Cambridge project funded by the Department for Education to develop curriculum and teaching materials across A level mathematics with the aim of fostering connections between elements of mathematics and deepening understanding of central concepts. The project

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<sup>191</sup> Underground Mathematics, <https://undergroundmathematics.org/>



worked with 45 partner schools and colleges to develop curriculum resources and now works with 600 affiliate schools and colleges to embed and evaluate the use of these resources.

- Loughborough University has an award winning Mathematics Education Centre.<sup>192</sup> The centre conducts research into mathematical thinking and teaching at all educational levels, which has led to the development and testing of resources to help students make effective use of independent study. Working closely with the Further Maths Support Programme, it offers enrichment days for students in 16-18 education and professional development for teachers, and works with over 200 schools.

212. The involvement of higher education in projects of these kinds deserves to be recognised and encouraged further by government. There is also a good case for universities to consider providing support for other subjects – for example, computing and modern foreign languages.

**Recommendation 11: The Department for Education, in issuing any new guidance to the independent Director of Fair Access to Higher Education, should continue to encourage universities to support 16-18 mathematics education in the context of access for and success of students from disadvantaged backgrounds.**

**Recommendation 12: The Department for Education, in supporting the Prime Minister’s desire for higher education to engage more with schools, should seek ways to encourage universities to consider specialism in 16-18 mathematics if establishing new schools, sponsoring existing schools or providing other support to schools, particularly in local areas where level 3 mathematics participation and achievement is poor.**

**Support to improve progression in mathematics should also be targeted at areas of most need**

213. Continued national level support is essential but regional and sub-regional differences suggest areas where more targeted and intensive action is needed. The Further Maths Support Programme targets schools that have not entered pupils for AS/A level further mathematics and has achieved significant success in driving up entries in targeted schools. There remain large numbers of schools that have not entered students and not all schools targeted have the capacity to take advantage of the support offered.

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<sup>192</sup> Mathematics Education Centre, <http://www.lboro.ac.uk/departments/mec/>

### **Local action to improve mathematics – the Haringey STEM commission**

Some local authorities have already developed their own programmes of support to improve mathematics skills. The Haringey Council 2015 STEM Commission looked at ways to raise attainment in mathematics alongside other STEM subjects.<sup>193</sup>

At the time the report was drafted, the number of 19 year olds qualified to Level 2 with English and mathematics in Haringey were lower than national, London and statistical neighbour averages, driven by poor teaching.

Mathematics-specific recommendations included establishing centres of excellence to improve teaching quality, improving local partnerships to provide better careers advice and promoting mathematical curricular activities.

These recommendations were embedded within a much wider framework of recommendations designed to build a framework of STEM skills that all pupils should have by age 16.

214. The government's new Opportunity Areas programme will see local partnerships formed to access a wider support to improve the prospects of disadvantaged young people. The Secretary of State announced a teaching and leadership innovation fund worth almost £75 million over 3 years focused on supporting teachers and school leaders in challenging areas to develop. There are already good examples of where concerted local efforts and strengthened partnerships can drive improvement to 16-18 mathematics and STEM provision.

215. The government announced six opportunity areas in October 2016 and a further six were announced in January 2017.<sup>194</sup> Opportunity areas are selected on the basis of a number of factors linked to poor social mobility. Given the value that advanced mathematics qualifications can bring, improving attainment and progression should be considered a priority in these areas. Some of the Opportunity Areas selected, including Blackpool and Derby, are among those with the lowest levels of post-16 learners participating in level 3 mathematics,<sup>195</sup> but as discussed in chapter 4, a number of other areas also have poor levels of level 3 mathematics participation.

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<sup>193</sup> Haringey STEM Commission, <http://stemcommission.org.uk/>

<sup>194</sup> Department for Education (2017) *Education Secretary announces 6 new opportunity areas*, <https://www.gov.uk/government/news/education-secretary-announces-6-new-opportunity-areas>

<sup>195</sup> Department for Education (2017) *Ad-hoc notice: Post 16 maths participation in 2015/16* (accompanying statistical release to this report), <https://www.gov.uk/government/publications/post-16-maths-participation>

### **The Northern Powerhouse Schools Strategy**

Work in this review of 16-18 mathematics has focused on the specific challenges and action needed to improve the study of mathematics. Sir Nick Weller recently completed The Northern Powerhouse Schools Strategy Review,<sup>196</sup> which made a number of recommendations for improving education in the north of England. Several Northern Powerhouse recommendations are relevant to the challenges identified in this review, including the need for government to understand local teacher supply issues in more detail and on a subject-by-subject basis. The report also highlighted the important role that strong multi-academy trusts and other support structures such as Teaching School Alliances play in supporting school improvement.

In response, the government has committed to design, fund and test a range of approaches such as effective professional development to attract and retain high-quality teachers in the North. It will also invest in developing the capacity of multi-academy trusts.

**Recommendation 13: The Department for Education should commission and fund interventions in local areas with low level 3 mathematics participation, coordinating local work to provide the best training and support for schools and colleges to build capacity at GCSE and level 3. This should include developing partnerships between schools and local and national sources of support.**

### **The role of technology in addressing teaching capacity issues**

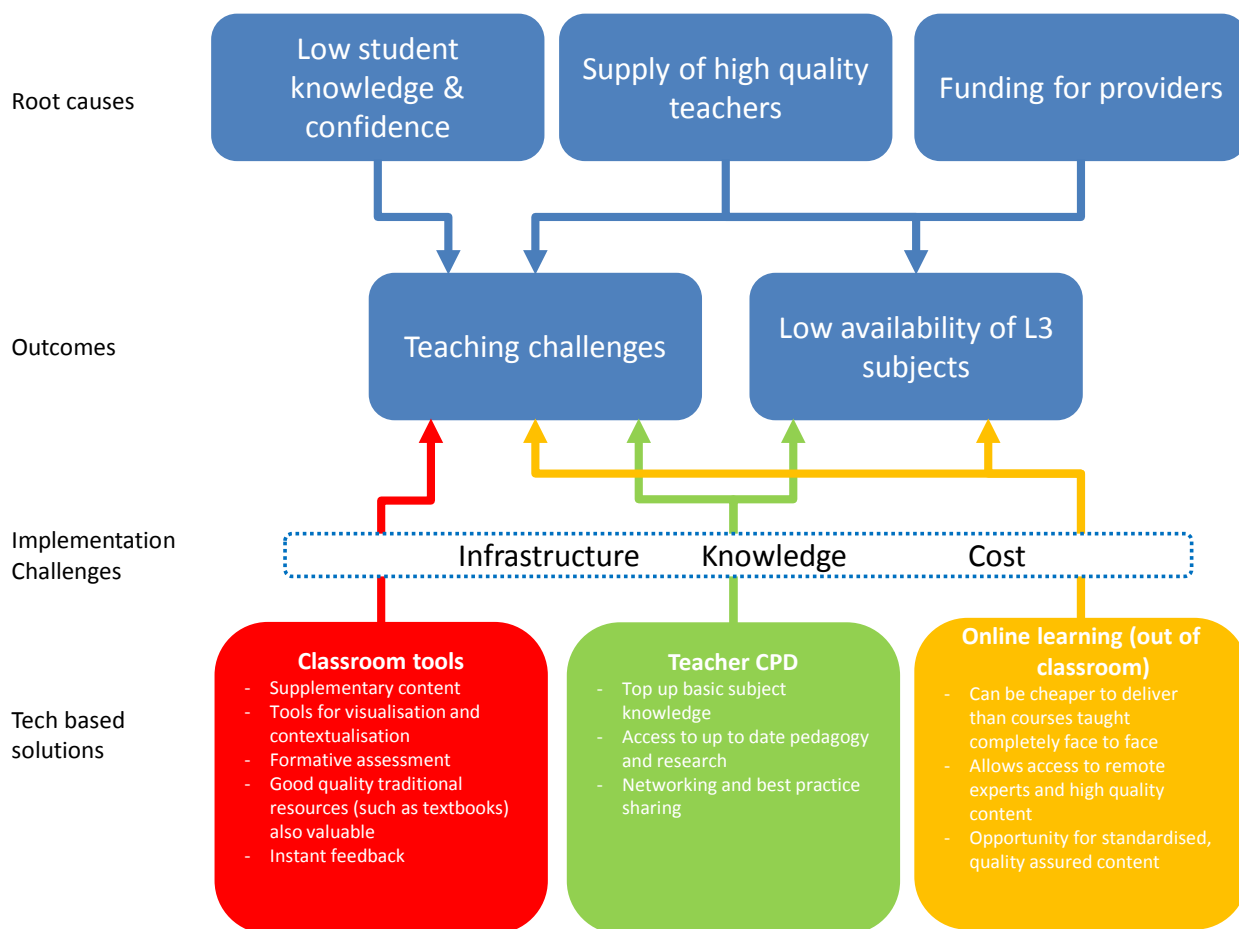
216. Technology may offer solutions to some of the challenges outlined above, including ways to improve the efficacy of teaching and manage the need for teachers through learning online. Several stakeholders consulted in this review highlighted the value that technology adds to 16-18 mathematics, but some advised caution in recommending technology as a solution to teaching capacity issues in mathematics – there are no simple technological solutions to the issues highlighted in this review.

217. Technology has a wide variety of applications, ranging from classroom tools, provision of professional development and access to teaching online. It is likely that all aspects have potential, where designed well, to support teaching and the

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<sup>196</sup> Weller N (2016) *Sir Nick Weller's independent review of education in northern England to inform the northern powerhouse schools strategy*, <https://www.gov.uk/government/publications/northern-powerhouse-schools-strategy-an-independent-review>

expansion of mathematics provision. Figure 3 below summarises the main ways in which technology can support the teaching of mathematics and the barriers identified by respondents to this review.



**Figure 3. Potential technology-based solutions to teaching capacity issues.**

218. The adoption of technology in 16-18 education to support teaching is based in the decisions made by schools and colleges, which are free to decide the best teaching approaches to take. This has, however, led to variation in approach, scale and effectiveness in the use of technology in 16-18 education. It is a cause for concern that, despite the obvious potential of technology, robust evidence is limited on its value in 16-18 mathematics teaching.

**Recommendation 14: The Department for Education should seek to improve the evidence base on the role and effectiveness of technology in the teaching of 16-18 mathematics.**

219. Much of this chapter has been devoted the challenge of improving access to high quality teaching that will be needed if we are to raise participation in 16-18 mathematics. Classroom tools have a role to play in supporting teaching, but the transformational potential for technology lies in helping grow the number of effective

teachers through online professional development and the numbers of students learning online in a way that reduces classroom hours needed for mathematics.

220. There is potential for MOOC-style courses aimed directly at students to help address this. Though relatively small in scale, there have been a small number of courses developed over the past few years, including:

- Citizen Maths: basic skills for adult learners – A free online course helping students learn basic mathematics skills.<sup>197</sup>
- The Further Mathematics Support Programme: live online lessons for students.<sup>198</sup>
- Isaac Physics MOOC for A level Physics, with a mathematics in physics focus.<sup>199</sup>

221. An overview of the potential for the use of MOOCs in secondary education commissioned by the Department for Education<sup>200</sup> found that standalone MOOCs had high rates of failure, however, and were likely to remain viable options only for highly motivated pupils. While there may be potential for MOOCs to be used in 16-18 mathematics education, this lies in targeted areas, as already happens with some further mathematics students, rather than replacing conventional GCSE or A level teaching at significant scale.

**Recommendation 15: The Department for Education, in conjunction with partners such as the Institute for Apprenticeships, should fund online professional development resources and materials aimed at increasing the numbers of teachers of mathematics and quantitative skills within new technical education routes and core maths.**

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<sup>197</sup> Citizen Maths, <https://www.citizenmaths.com/>

<sup>198</sup> The Further Mathematics Support Programme, <http://furthermaths.org.uk/>

<sup>199</sup> Isaac Physics, <https://isaacphysics.org/>

<sup>200</sup> Department for Education (2014) *MOOCs: opportunities for their use in compulsory-age education*, <https://www.gov.uk/government/publications/moocs-opportunities-for-their-use-in-compulsory-age-education>

## Chapter 7: System and policy issues

### Should mathematics become compulsory for all 16-18 students?

222. There is a strong case for higher uptake of 16-18 mathematics. Increased participation would be likely to deliver significant payback in terms of labour market skills, returns to individuals, increased productivity and longer-term economic benefits. There are broadly two approaches that government could take to this:

- make the study of mathematics compulsory for all or more 16-18 students through a funding requirement or similar mechanism; or
- get most or all students choosing the subject beyond age 16 by changing expectations and culture by ensuring a more appropriate range of pathways.

223. The government should set an ambition for 16-18 mathematics to become universal in 10 years. There is not a case at this stage, however, for making it compulsory: the appropriate range of pathways is not available universally, teacher supply challenges are significant and it is unclear when sufficient specialist capacity will be in place for universal mathematics to become a realistic proposition.

224. Participation should be guided by the principle that all students should study the mathematics they need for the future. If delivered in practice, this would result in a significant boost to the number of students taking mathematics at level 3.

225. There is a clear role for government to shape expectations on this. For example: encouraging students aiming for mathematical STEM study to study AS/A level further mathematics, and others aiming for other science-based courses, including subjects allied to medicine, to take AS/A level mathematics. Similarly, there is a role in signalling core maths as a valuable qualification for students aiming for university courses or higher technical education with a significant quantitative element, such as quantitative social sciences, business and finance, and numerate technical fields.

226. If such expectations were set out they could lead to a significant boost to numbers taking mathematics at level 3. At the moment, around 135,000 students in

England enter degree courses with a 'medium' mathematical demand,<sup>201</sup> only a quarter of whom have a level 3 mathematics qualification.<sup>202</sup> Around 33,000 students choose to study courses with a 'high' mathematical demand, around two-thirds of whom have a level 3 mathematics qualification. Over 100,000 more students would study mathematics to level 3 – more than doubling current numbers.

227. In the medium term (5 years), strong provision should become established across all mathematics pathways, with core maths becoming widely available for students who can benefit and appropriate mathematics courses in place for technical routes. Other provision should be strengthened with the aim that success rates improve – notably for GCSE resits.

228. In the shorter term, high priority should be placed on the challenges of continuing to expand teaching capacity through recruitment and upskilling of teachers, including non-mathematics specialists, and on encouraging schools and colleges to offer mathematics options and encouraging students to take these.

229. The government should keep progress under close review given that changes to GCSE and A level mathematics are new, and reformed functional skills qualifications are yet to be designed. There are many risks to improving achievement and uptake of 16-18 mathematics which will need managing. For example, core maths could lose momentum; de-coupling of the AS may impact on A level numbers; CPD/training offers may not be taken up at a sufficient level.

## 16 to 19 funding

230. Current levels of 16-19 funding present considerable risk to any ambition to expand 16-18 mathematics provision. The national base rate of £4,000 per full-time student aged 16-17 is low, and is set to be maintained for the rest of the current Parliament, despite rising costs for schools and colleges. These include annual salary increases, the introduction of the national living wage from April 2016, the increase to employer national insurance contributions from April 2016, the increase

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<sup>201</sup> 'Medium' demand refers to subjects which have relatively high mathematical demands. In many cases, the primary need is for statistics and the ability to analyse and interpret data. Subject Groups: Agriculture and related subjects, Architecture, building and planning, Biological sciences, Business and administrative studies, Computer science, Education, Medicine and dentistry, Social studies, Subjects allied to medicine, Veterinary science.

<sup>202</sup> Hodgen J, Adkins M and Tomei A (2017) *The mathematical backgrounds of undergraduates: Interim report* (Nottingham: University of Nottingham),

<http://www.nottingham.ac.uk/research/groups/crme/projects/mathematical-backgrounds.aspx>

The study examined English-domiciled students who were age 16 in 2007/8, took a level 3 qualification between 2008/9 and 2010/11, and then entered university between 2010/11 and 2012/13. The final report, due in 2017, will form the basis of a resource which will enable universities to compare the mathematical backgrounds of entrants across institutions in all disciplines.

to employer pension contributions for the Teachers' Pension Scheme from September 2015, the introduction of the apprenticeship levy in April 2017, and general inflation.

231. 16-19 funding is significantly lower than the average amount received by schools for pre-16 pupils. The Sixth Form Colleges Association estimates 16-19 funding for sixth-formers in England is 20 per cent lower than funding for 11-16 year olds.<sup>203</sup> Consulted stakeholders reported that schools with sixth forms commonly subsidise 16-18 provision, which is heavily stretched.

### **Delivering English and mathematics for those who do not achieve an A\*-C at GCSE – the impact on an FE college**

Chichester College is an Ofsted outstanding rated FE college in West Sussex. It has 3,521 16-18 year old students, of which 1,374 are currently enrolled on either a GCSE mathematics or a functional skills course. There is an even split between the two qualifications.

Delivery of mathematics (and English) to 16-18 year olds has increased significantly over recent years. This has had a significant effect on staffing costs for the subjects which have risen from around £315,000 in 2013-2014 to £446,000 in 2015-2016. There have been costs associated with establishing a new department focused entirely on English and mathematics, purchasing of materials to support teaching and the subsequent extra exam entry costs. In addition, a large number of volunteer members of staff support students.

As a result of these costs and the wider funding challenges faced by post-16 providers, the college has had to reduce teaching time for students' main courses and reduce the budget available for staff CPD.

232. Funding rates for full-time study assume a minimum of 540 planned hours per year with an expected average of 600 hours. This is considerably less than the hours funded by many other countries. Funding per 16-19 year old in England, for example, pays for on average 20 hours per week assuming a 30 week year, compared to 28 hours per week in Norway.<sup>204</sup> In practice this limits the range of courses that can be provided for students, particularly in smaller schools. It is unlikely, for example, that core maths can be offered at scale if funding remains at current levels, despite three

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<sup>203</sup> Sixth Form Colleges Association (2016) *SFCA Funding Impact Survey 2016*, <http://www.sixthformcolleges.org/sfca-funding-impact-survey-2016-0>

<sup>204</sup> Independent Panel on Technical Education (2016) *Report of the independent panel on technical education*, [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/536046/Report\\_of\\_the\\_Independent\\_Panel\\_on\\_Technical\\_Education.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/536046/Report_of_the_Independent_Panel_on_Technical_Education.pdf)



A levels and core maths notionally being deliverable in 600 study hours. This should be considered a significant risk to any ambition to expand mathematics provision.

## **Pre-16 mathematics**

233. The scope of this review is limited to 16-18 mathematics, but inevitably problems in pre-16 mathematics have come into focus during the review as root causes of later issues. The poor performance of GCSE resit students, for example, often reflects fundamental issues at earlier stages of mathematics teaching. Numeracy can be weak, and these students often need to go back to basics. Similarly, a lack of confidence among students with good GCSE grades to continue the subject reflects problems at earlier stages, including a ‘teaching to the test’ culture at GCSE, which can lead to over-emphasis of memorisation strategies<sup>205</sup> and result in shallow understanding.

234. Due consideration should be given to how secondary mathematics teaching could be improved so that more students gain numeracy by age 16 and secure understanding of concepts taught at GCSE.

## **Culture and appreciation of mathematics in society**

235. The theme of society’s views towards mathematics emerged regularly in the course of this review. The issue extends beyond attitudes to mathematics and the frequently quoted observation that it is culturally acceptable for individuals to confess that they ‘can’t do maths’, whereas this would not be easily admitted for literacy. It includes wider considerations of the appreciation of mathematics and understanding of its role and value to society, which represent a backdrop for negative attitudes towards mathematics. Gender has a heavy influence on mathematics participation, reflecting deeply held cultural attitudes towards mathematics. While much good work has been done to tackle gender issues by organisations such as the Further Maths Support Programme, these problems are deep seated and enduring.

**Recommendation 16: The Department for Education should commission a study, from pre-school onwards, into the cultural and other root causes of negative attitudes to mathematics, including gender and other sub-group effects.**

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<sup>205</sup> OECD (2016c) *Is memorisation a good strategy for learning mathematics?* <http://www.oecd-ilibrary.org/docserver/download/5jm29kw38mlq-en.pdf?expires=1480350865&id=id&accname=guest&checksum=6DF19F95870C3E4B5FC4E801B4C667BC>

## Information advice and guidance for students

236. Over the last few years, the Department for Education has strengthened requirements on schools to provide high quality, independent careers advice and guidance to young people. Alongside this, the Careers and Enterprise Company has been established to connect the world of work with schools and colleges through a network of advisers. The Company provides funding to projects to improve the interaction between young people and employers. Further to this, organisations work with schools to improve the advice and guidance about careers, particularly in relation to STEM careers, where the STEM Ambassadors programme plays a significant role.

237. Despite this, there remain significant challenges in getting good information to young people to inform their choices. Studies have highlighted the problem of complexity faced by students as they make choices at age 16. In particular, young people face a lack of clarity about where the choices they make will lead them.<sup>206</sup> Given the demand for mathematical skills in the economy it is essential that young people are provided with far sharper information about the mathematical needs of degree courses, technical fields and, importantly, their future careers.

**Recommendation 17: The Department for Education should, in any future work to improve careers provision and related advice, prioritise and make clear the importance of mathematics to a wide range of future careers.**

## Big data and data science

238. The increasing sophistication of technology is driving change to the economy and the nature of work. This is not only increasing the demand for mathematics and quantitative skills but is also changing the nature of required skillsets, in particular those relating to the analysis and use of ‘big data’.

**Recommendation 18: The Department for Education and the Department for Business, Energy & Industrial Strategy should commission a study into the long-term implications of the rise of data science as an academic and professional field, looking at skills required for the future and the specific implications for education and training in mathematics and quantitative skills.**

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<sup>206</sup> Hupkau C, McNally S, Ruiz-Valenzuela J and Ventura G (2016) *Post-Compulsory Education in England: Choices and Implications*, CVER Discussion Paper No. 1, <http://cver.lse.ac.uk/textonly/cver/pubs/cverdp001.pdf>

## Annex A: Timeline of key reforms

Autumn 2014	Core maths – first teaching
Sep 2015	GCSE mathematics (9-1) first teaching
Nov 2015 to Jul 2016	Sainsbury Panel Review of Technical Education
Jan to Apr 2016	Functional skills – consultation
Summer 2016	Core maths – first exams
Mar to Dec 2016	Schools national funding formula: government consultation response – stage 1
May to Sep 2016	Teaching Excellence Framework: year 2 – technical consultation
Sep to Dec 2016	Schools that work for Everyone – consultation
Apr 2017	Institute for Apprenticeships and Technical Education – operational
Easter 2017	Higher Education and Research Bill, final form, Royal Assent
Summer 2017	GCSE mathematics – first exams of reformed curriculum. 9-1 grades replace A*-G grades (in mathematics and English)
Sep 2017	A/AS level mathematics and further mathematics – first teaching of reformed qualifications – decoupled A and AS.
Summer 2018	A/AS level mathematics and further mathematics – first examination of reformed AS levels
Sep 2018	Office for Students – operational
Summer 2019	A/AS level mathematics and further mathematics – first examination of reformed A levels
Sep 2019	Functional skills – first teaching of reformed qualifications
Sep 2019	Post-16 technical education – first pathfinder routes
2022	Post-16 technical education – all 15 technical routes available
2022	Mathematics provision established across all post-16 education and training pathways, including core maths (this Review's proposal)

## Annex B: Consultees

### Individuals

Kenneth Baker, Edge Foundation  
Mick Blaylock, Core Maths Support Programme

Adrian Bowman, University of Glasgow

Tim Browning, University of Bristol  
Joseph Buckley, British Academy  
Simon Burgess, University of Bristol

Neil Carberry, Confederation of British Industry (CBI)

Richard Cochrane, Fine Art Maths Centre, University of the Arts, London

Kevan Collins, Education Endowment Foundation (EEF)

Paul Cook, King's College London

Rebecca Cotton-Barratt, University of Oxford

Richard Craster, Imperial College London

Ian Diamond, University of Aberdeen

Ann Dowling, Royal Academy of Engineering

Ian Dryden, University of Nottingham

Richard Earl, University of Oxford

Les Ebdon, Director of Fair Access to Higher Education

Jane Elliott, Economic and Social Research Council (ESRC)

Mike Ellicock, National Numeracy

Alun Evans, British Academy

Carolyn Fairbairn, Confederation of British Industry (CBI)

Ruth Fairclough, University of Wolverhampton

Jonathan Forster, University of Southampton

Simon Gallacher, Nuffield Foundation

Tim Gardam, Nuffield Foundation

Paul Glaister, University of Reading and Joint Mathematical Council

Philip Greenish, Royal Academy of Engineering

Alex Halliday, Royal Society and University of Oxford

Julia Hawkins, Millennium Mathematics Project, University of Cambridge

Josh Hillman, Nuffield Foundation

Jeremy Hodgen, Nottingham University

Celia Hoyles, University College London

Antony Jenkins, Institute for Apprenticeships

Samina Khan, University of Oxford

Biagio Lucini, Swansea University

John MacInnes, University of Edinburgh

Mary McAlinden, University of Greenwich

Andrew McGettigan, Fine Art Maths Centre, University of the Arts, London

Sandra McNally, London School of Economics

Rhys Morgan, Royal Academy of Engineering

Nick Weller, Dixons Academy Trust

Andrew Noyes, Nottingham University

Mary O'Mahony, King's College London

Emily Pitt, Queen Mary University of London

Luciano Rila, University College  
London  
David Russell, Education & Training  
Foundation (ETF)

David Sainsbury, Gatsby Charitable  
Foundation  
Steve Smith, University of Exeter  
Yvette Solomon, Manchester  
Metropolitan University  
Mark Steel, University of Warwick

Charlie Stripp, Mathematics in  
Education and Industry (MEI)  
and National Centre for  
Excellence in the Teaching of  
Mathematics (NCETM)

Anthony Tomei, King's College  
London

Michael Wilshaw, then HMCI, Ofsted  
Alison Wolf, King's College London  
Patrick Wolfe, University College  
London

## Organisations

Altrincham Grammar School for Boys  
Altrincham Grammar School for Girls  
AQA  
Ark

Association of Colleges (AoC)  
Association of Employment &  
Learning Providers (AELP)  
Association of School and College  
Leaders (ASCL)  
Association of Teachers and  
Lecturers (ATL)

Barnsley College  
Bennett Memorial Diocesan School  
British Academy  
British Army  
British Chambers of Commerce  
(BCC)  
British Psychological Society  
Bury College

Cambridge Assessment  
Cardinal Newman College  
Careers & Enterprise Company  
City and Islington College  
City of Stoke-on-Trent Sixth Form  
College  
College of NW London  
Conference of Heads of  
Departments of Mathematical  
Sciences (HoDoMS)

Conference of Heads of University  
Departments of Economics  
(CHUDE)

Council of Mathematical Sciences  
(CMS)

Cowley International College

Derby College

Edge Foundation  
Education & Training Foundation  
(ETF)

Education Endowment Foundation  
Education for Engineering (E4E)  
EEF (The Manufacturers'  
Organisation)

Farnborough Sixth Form College  
Federation of Small Businesses  
(FSB)

Further Maths Support Programme

Gatsby Charitable Foundation

Harlow College  
Harrogate Grammar School  
HegartyMaths

Institute for Employment Studies  
Institute of Directors (IoD)  
Institute of Mathematics and Its  
Applications (IMA)  
Institute of Physics

Institute of Structural Engineers  
Institution of Civil Engineers  
Isaac Physics  
Ivybridge Community College

Jisc

Kendrick School

London Mathematical Society (LMS)

Macmillan Academy  
Malmesbury School  
Middlesbrough College

National Association of Head  
Teachers (NAHT)  
National Centre for Excellence in the  
Teaching of Mathematics  
(NCETM)  
National Union of Teachers  
Newham Sixth Form College  
Nottingham Academy (Greenwood  
Academies Trust)  
Nuffield Foundation

OCR  
Outwood Grange Academies Trust

Physiological Society

Reading School, Reading  
Royal Academy of Engineering  
Royal Economic Society  
Royal Geographical Society  
Royal Society of Biology  
Royal Society of Chemistry  
Royal Statistical Society (RSS)  
Russell Group

Salford City College  
Shipley College  
Sir Isaac Newton Sixth Form  
Skills for Growth  
South Hunsley School  
Stoke-on-Trent College  
Sutton Trust

Trinity TSA  
Truro and Penwith Academy Trust

UCAS  
Ufi Charitable Trust  
UK Mathematics Trust  
Universities UK

Walsall College  
Winstanley College  
York College

## Annex C: Glossary

**ACME** A Royal Society committee which develops advice to influence mathematics education policy

**CMSP** Core Maths Support Programme – funded by the Department for Education, the CMSP helps schools and colleges to promote and implement core maths

**Decoupling** A levels are now a linear qualification with assessment at the end of two years' study. This means AS levels are now standalone qualifications and do not count towards the final A level grade

**FE** Further education includes any study after secondary education which is not part of higher education. FE Colleges offer courses and qualifications in a range of vocational and academic subjects at many levels

**FMSP** Further Maths Support Programme – supported by the Department for Education, the FMSP works with schools and colleges in England to support and promote the study of AS/A level Mathematics and Further Mathematics

### **GCSE Foundation and Higher**

Foundation tier: Students who sit this paper can achieve C-G grades. It focuses on core mathematical understanding and skills

Higher tier: students who sit this paper can achieve an A\*-C grade. Questions on this paper will stretch the most able

**HE** Higher education – university level education

**Institute of Apprenticeships** From April 2017 the Institute for Apprenticeships, an independent body, will help ensure employers get the skills and experience they need from the apprenticeships system by ensuring they respond to the needs of business.

**Key stage** A recognised stage of learning – from nursery education, through school and into further and higher education  
(Key stage 5: period of education for students aged 16-18)

**LA** Local authority – a council that has responsibility for providing local services to pupils of school age and adult education in its area

**Large programme uplift:** A 10 per cent increase in funding only available for high quality study programmes providing students with substantial stretch and challenge. It is paid for 2 years within academic years 12 and 13 only

**Level 3 qualifications** They include A levels, Advanced extension, GCEs in applied subjects, IB

**Level 2 qualifications** They include GCSEs, BTEC first diplomas and OCR nationals, and level 2 functional skills qualifications

**Linear A level** GCSE, AS and A level qualifications are now linear. Students sit all exams for each qualification at the end of the course

**OECD** The Organisation for Economic Co-operation and Development – promoting policies that will improve the economic and social well-being of people around the world

**OFFA** The Office for Fair Access – promoting and safeguarding fair access to higher education

**Ofsted** Office for Standards in Education, Children’s Services and Skills – education and young people’s care regulator

**Ofqual** Office of the Qualifications

and Examination Regulator – qualifications, exams and tests regulator

**Post 16 Skills Plan** The official response to the Sainsbury Review's recommendations on the future of Technical Education

**Programme for international assessment (PISA)** A triennial international survey which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students.

**Q step** A programme designed as a strategic response to the shortage of quantitatively-skilled social science graduates in the UK

**Quantitative skills** These include involve the ability to handle data and use numerical evidence systematically

**SFA** Skills Funding Agency – government agency which funds skills training for further education

**Trends in International Mathematics and Science Study (TIMSS)** Regular international comparative assessments of student achievement in mathematics and science

**UCAS** University and Colleges Admission Service



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