Department for Education

School Efficiency Metric

A technical note on the definition and calculation of school efficiency

September 2018

Contents

Summary

About this document

This document provides technical detail on how school efficiency has been defined and calculated in the School Efficiency Metric. It has been produced so that users of the tool can understand in more detail:

- How school efficiency has been defined;
- What data has been used to calculate school efficiency;
- How the School Efficiency Metric has been calculated;
- How school efficiency is presented in the metric spreadsheet publication and how it can be interpreted.

This is a **technical note** that describes the methodology of the metric in full detail. It is intended for people with a good general level of statistical knowledge. Our guide provides an overview of the calculation of the metric in less technical terms.

If you are looking for **more information on why the department has published the School Efficiency Metric** and how you can use it, we recommend reading our guide.

Review date

The next review will take place ahead of the next publication of the School Efficiency Metric covering the 2017/18 school year.

Defining and measuring school efficiency

Definition of efficiency

Efficiency is generally defined as getting the maximum possible output from the inputs used, or producing a given level of output with the minimum of inputs. The most efficient decision-making unit (DMU) can be defined in two ways: theoretically (showing that, in principle, better results are not obtainable); and empirically (showing that no other organisation performs better). The School Efficiency Metric that the department has developed is a relative empirical measure that emphasises how each school's efficiency compares to similar schools.

There are two major components of efficiency:

- The **output** we are trying to maximise or the objective we are trying to achieve. In a car factory, the output might be the number of cars produced each month.
- The **inputs** that are required or used in the production of the output. In the car factory, this could be the cost of parts and equipment, the running costs of the factory and the number of workers or cost of labour.

Measuring school efficiency in England

Establishing a practical definition of school efficiency has been a key element of the department's work to develop a School Efficiency Metric. This section considers the two major components of efficiency in the context of schools in England, and explains the department's choices when defining and measuring school efficiency.

Output

In the first version of the Efficiency Metric, covering the 2013/14 year, published in January 2016, the Best 8 Value Added (VA) measure was used as an output measure for secondary schools and the Key Stage 1 to Key Stage 2 value added measure was used for primary schools. As of 2016, a new accountability system was introduced and this update of the Efficiency Metric covering 2016/17 reflects these changes using Progress 8 as an output measure for secondary phase and primary progress measures as an output measure for primary phase. The section below outlines why these measures were chosen as the outputs and the movement from the old value added measures to the progress measures.

It is sensible to choose attainment as the output when measuring school efficiency given that the objective of schools is to raise attainment to the highest possible level for all of their pupils. The challenge, therefore, is to choose the most appropriate measure of

attainment. At the time of the publication of the first Efficiency Metric at key stage 4, there were three options:

1. Percentage of pupils achieving 5A*-C at GCSE: This measure does not take into account prior attainment, though modelling techniques could control for this. The major objection to using this measure is that it only recognises achievement over a particular threshold and so does not discriminate well. It does not recognise all levels of achievement and progress: only attainment at a C grade or above is counted, and all grades from A* to C grades are considered equal.

2. Key stage 4 Average Points Scores (APS): This measure assigns points to each grade students achieve in every subject they take. The measure takes the average of the best 8 grades. It therefore overcomes, to some extent, the main objection to the 5A*-C measure. APS, however, also suffers from an absence of information on prior attainment, so any modelling would have to control for that separately.

3. Best 8 Value Added (VA) scores: this measure of attainment calculates how much progress a pupil has made compared to their expected progress, which is estimated based on their prior attainment and the relative progress of their cohort. A pupil's VA is, roughly, the difference between expected and achieved attainment. A school's VA is the average of their pupils' VA figures. The best 8 GCSE results are counted when calculating progress from the key stage 2 baseline.

Value Added was considered the most appropriate measure of education output for use in assessing school efficiency as it accounted for prior attainment and takes a pupil's performance across a significant number of subjects (and all grades within those subjects) into account. For primary schools, the output measure used was Key Stage 1 to Key Stage 2 value added. For secondary schools, the output used was Key Stage 2 to Key Stage 4 'Best 8 value added'.

As of 2017, Progress 8 measures are used to calculate efficiency scores for secondary schools and Progress reading, writing and maths are used for primary schools. This change from VA to progress scores will not change the overall outputs of the efficiency metric as these measures are standardised before being used in the efficiency calculation, as described in the 'Calculating the school efficiency metric section'.

Inputs

Education is a complex process and many factors, or 'inputs', contribute to a pupil's achievement in school: family and social factors; early educational experiences; aptitude or ability; and every educator or educational establishment a pupil interacts with before the point at which they take a particular examination or test.

In a school efficiency context, we seek only to measure the inputs controlled by the school, and the extent to which the school has contributed to the level of pupil achievement. Ideally, we would strip out that portion of education output that is influenced by inputs outside of a school's control. We do this by focusing on progress made while a pupil is in the school. We make no adjustments for pupils that move between primary schools or secondary schools and attribute their progress to the school where they take their KS2 or KS4 exams.

At the highest level, the inputs within a school's control are all accounted for by total or per-pupil funding. Therefore, the School Efficiency Metric uses per-pupil funding as the measure of school inputs. The metric shows how a school performs relative to others, given a certain level of funding.

We include only the funding that schools receive from central government and/or their local authority.[1] We do not include any funding that schools receive from other sources, such as donations, or any self-generated income. Some schools earn extra revenue by, for example, leasing their sports facilities. We do not include this stream of income as it could be seen to discourage income generation by increasing the measured input. In this document, when we refer to a school's income we refer only to the central government and local authority funding that we include in the measure of inputs.

The School Efficiency Metric methodology uses only one year of per-pupil funding as modelling the interactions between funding and outcomes over multiple years is very complex.

[1] The funding items that we include are detailed in the 'Summary of data used in the School Efficiency Metric' section.

Calculating the School Efficiency Metric

There are three steps to calculating the Efficiency Metric.

In the first step, we **calculate an 'efficiency score' for each school** by dividing its progress by its income per pupil. As progress and income per pupil are measured on different scales, they are not directly comparable. To allow as fair a comparison between the measures as possible, we standardise both before doing the calculation.

In the second step, **we identify a school's unique group of 'most similar schools'**. These are the 49 statistically most similar schools in terms of the proportion of pupils with a statement of SEN or an education, health and care (EHC) plan (% SEN) and the proportion of pupils who have been eligible for free school meals at any point in the last six years (% Ever6 FSM). In addition, they are of the same phase and organisational type. If a school has a sixth form, its most similar schools are all schools with a sixth form; conversely the 'most similar schools' of schools without a sixth form are all schools without a sixth form. Although there are a number of ways in which schools may differ from, or be similar to, other schools, the statistical similarity of the matched schools declines as we add more criteria.

From the 2015/16 version of the metric onwards, we also compare small primary schools only to other small primary schools. The 20% smallest schools are considered small for the purposes of the calculation.

In the third step, **we calculate a school's 'efficiency decile'**. We do this by comparing its 'efficiency score' to those of its 'most similar schools'. In each 'most similar schools' group, we band schools into deciles (10 groups of five) based on their 'efficiency score'. A school's relative efficiency is reported in terms of which decile they are located in their unique 'most similar schools' group – we call this a school's 'score' in the metric. A school in decile 1 has one of the largest five 'efficiency scores' (and is therefore one of the five most efficient schools) in its 'most similar schools' group; a school in decile 10 has one of the smallest five scores (and is therefore one of the five least efficient schools) in its 'most similar schools' group.

Schools that have both a primary and secondary phase receive both a primary and secondary Efficiency Metric. As school income is allocated on a school level, we have had to make assumptions about how this income is divided between phases. Our method for assigning income to each phase of these schools is described in the annex to this note.

The three steps are now discussed in more detail.

First step: calculating an ‘efficiency score’

Firstly, we ‘standardise’ a school’s progress and income per pupil.[2] Progress and income per pupil are measured on different scales and, consequently, are not directly comparable. Without adjusting either progress or income per pupil, an ‘efficiency score’ that divides progress by income per pupil places a greater weight on income per pupil than progress: this is due to the higher average of per-pupil income and the wider standard deviation. This would mean a one-unit change in per-pupil funding would lead to a greater change in the ‘efficiency score’ than a one-unit change in progress.

Standardising is a statistical method of putting both measures on a more equal footing. In practice, it means roughly that having better progress than 75% of schools is just as important as having lower per-pupil income than 75% of schools. We standardise progress and income per pupil so that they have a mean value of 100 and standard deviation[3] of 15. These values were chosen to make the distribution of efficiency scores easier to work with.

The formulas for standardising progress and income per pupil are:

[2] The income per pupil figure used to calculate an ‘efficiency score’ removes any area cost uplift applied to a school’s funding. However, the income per pupil displayed in the Efficiency Metric spreadsheet publication includes any area cost uplift.

[3] Standard deviation is a measure that reflects how spread out a set of values is from the mean of the set. The smaller the standard deviation is, the closer the set of values is to its mean.

Formulas for standardised progress and standardised income per pupil

$$\text{Standardised progress} = 100 + \left[\frac{\left(\text{Progress of school} - \mu_{\text{Progress}}\right) \text{ x } 15}{\sigma_{\text{Progress}}}\right]$$

$$\text{Standardised income per pupil} = 100 + \left[\frac{\left(\text{Income per pupil of school} - \mu_{\text{Income}}\right) \text{ x } 15}{\sigma_{\text{Income}}}\right]$$

Variable	Description
μ_{Progress}	The (weighted)[4] mean progress for all schools of the same phase.
σ_{Progress}	The (weighted) standard deviation of progress for all schools of the same phase.
μ_{Income}	The (weighted) mean income per pupil for all schools of the same phase.
σ_{Income}	The (weighted) standard deviation of income per pupil for all schools of the same phase.

Secondly, we calculate the school's efficiency score. This is its 'standardised' progress divided by its 'standardised' income per pupil, multiplied by 100.

Formula for the school efficiency score

$$\text{Efficiency score} = 100 \times \left[\frac{\text{Standardised progress}}{\text{Standardised income per pupil}}\right]$$

[4] The means and standard deviations used to standardise progress and income per pupil are weighted by school level FTE. This is explained in more detail in the annex to this note.

Worked example: Primary school

Consider a primary school with a progress score of 1.2 (averaged across reading, writing and maths) and income per pupil of £4500. The (weighted) mean values and standard deviations of progress and income per pupil are:

(Weighted) mean progress = 0.09

(Weighted) mean income per pupil = £4350

(Weighted) standard deviation of progress = 2.2

(Weighted) standard deviation of income per pupil = 803

Therefore, this school's standardised progress and income per pupil are:

$$\text{Standardised progress} = 100 + \left[\frac{(1.2 - 0.09)\ x\ 15}{2.2}\right] = 108$$

$$\text{Standardised income per pupil} = 100 + \left[\frac{(4500 - 4350)\ x\ 15}{803}\right] = 103$$

Its 'efficiency score' is therefore:

$$'\text{Efficiency score}' = \left(\frac{108}{103}\right)\ x\ 100 = 105$$

Worked example: Secondary school

Consider a secondary school with a Progress 8 score of 0.25 and income per pupil of £5200. The (weighted) mean values and standard deviations of progress 8 and income per pupil are:

(Weighted) mean progress 8 = 0.025

(Weighted) mean income per pupil = £5530

(Weighted) standard deviation of progress 8 = 0.34

(Weighted) standard deviation of income per pupil =918

Therefore, this school's standardised progress and per pupil funding are:

$$'\text{Standardised}'\text{progress } 8 = 100 + \left[\frac{(0.25 - 0.025)\ x\ 15}{0.34}\right] = 110$$

$$'\text{Standardised}'\text{ income per pupil } = 100 + \left[\frac{(5200 - 5530)\ x\ 15}{918}\right] = 95$$

Its 'efficiency score' is therefore:

$$'\text{Efficiency score}' = \left(\frac{110}{95}\right)\ x\ 100 = 116$$

Second step: identifying a group of 'most similar schools'

Identifying a school's 'most similar schools' group requires two levels of grouping. Firstly, we group all schools with schools of the same phase and organisational type. For example, a secondary academy's most similar schools are all secondary academies. Furthermore, we group schools with sixth forms with other schools that have sixth forms and schools without a sixth form with other schools without a sixth form. We also group the 20% smallest primary schools only with other small primary schools.

In the second level of grouping, we identify the 49 other schools that have the most similar proportions of % Ever6 FSM and % SEN. These are a school's 'most similar schools'. These schools are identified using the Euclidean Distance Matching method. These statistically similar schools, as well as the school that we are grouping, form the 'most similar schools' group of 50.

Technical explanation: Euclidean Distance Matching method

This is a statistical technique used to calculate the distance between two data points across, theoretically, many dimensions. In creating 'most similar schools' groups, we use it to calculate the distance between two schools in terms of % SEN and % Ever6 FSM. The smaller this distance, known as the 'Euclidean distance', the more similar the schools are in these characteristics.

The calculation of the 'Euclidean distance' involves a comparison of % SEN and % Ever6 FSM. As with progress and income per pupil, these two characteristics are measured on different scales. % SEN = 20% is relatively large, whereas % Ever6 FSM = 20% is relatively small. To overcome the issues that this causes, these values are 'standardised'.

$$\text{Standardised value of \% SEN} = \frac{(\text{\% SEN of school} - \text{Mean value of \% SEN across phase})}{\text{Standard deviation of \% SEN across phase}}$$

$$\text{Standardised value of \% Ever6 FSM} = \frac{(\text{\% Ever6 FSM of school} - \text{Mean value of \% Ever6 FSM across phase})}{\text{Standard deviation of \% Ever6 FSM across phase}}$$

The Euclidean distance between two schools, school X and school Y, is calculated using the following formula.

$$\text{Euclidean Distance between schools X and Y} = \sqrt{(\text{SEN}_x - \text{SEN}_y)^2 + (\text{Ever6}_x - \text{Ever6}_y)^2}$$

Where

Variable	Description
SEN_x	Standardised value of % SEN of school X
SEN_y	Standardised value of % SEN of school Y
Ever6_x	Standardised value of % Ever6 FSM of school X
Ever6_y	Standardised value of % Ever6 FSM of school Y

We repeat these steps to find the Euclidean distance between school X and all other schools. The schools corresponding to the 49 smallest Euclidean Distances are the 49 statistically most similar to School "X" in terms of % SEN and % Ever6 FSM and become its 'most similar schools'. Each school has a unique 'most similar schools' group of 50 including the school itself and its 49 'most similar schools'.

There are 50 schools in each 'most similar schools' group. This achieves a balance between having a manageable number of comparator schools for schools to benchmark against, and

ensuring that most schools have at least one 'most similar school' within 25 miles. Group sizes of 50 ensure that almost 90% of schools have an 'most similar school' within 25 miles.

Worked example of Euclidean Matching

Consider **school A**. It is a secondary academy, with a sixth form. Its % SEN = 10% and its % Ever6 FSM is 30%.

Consider four potential 'most similar schools' for school A. These are:

School B: A primary academy. Its % SEN is 12% and its % Ever6 FSM is 27%.
School C: A secondary maintained school without a sixth form. Its % SEN is 8% and its % Ever6 FSM is 35%.
School D: A secondary academy with a sixth form. Its % SEN is 12% and its Ever6 FSM is 36%.
School E: A secondary academy with a sixth form. Its % SEN is 23% and its Ever6 FSM is 15%.

The first level of grouping is to identify the schools of the same phase and organisational type as school A. Also, as school A is a school with a sixth form, we also identify the schools that have sixth forms.

School B is a primary academy. Although it is of the same organisational type, it is in a different phase to school A. Therefore, it cannot be a 'most similar school' to school A.

Similarly, school C cannot be an 'most similar school' to school A as it is of a different organisational type – it is a maintained school whereas school A is an academy – and does not have a sixth form, whereas school A does.

Schools D and E can be 'most similar schools' to school A, as, like school A, they are secondary academies with sixth forms.

In the second level of grouping, we calculate the Euclidean Distances between school A and the schools that pass the first level of grouping (schools D and E), in terms of % SEN and % Ever6 FSM. Their school level characteristics in standardised form are:

School A: Standardised SEN = 0.10. Standardised Ever6 FSM = -0.20
School D: Standardised SEN = 0.11. Standardised Ever6 FSM = -0.10
School E: Standardised SEN = 0.18. Standardised Ever6 FSM = -0.40

Therefore:

$$\text{Euclidean Distance between schools A and D} = \sqrt{(0.10 - 0.11)^2 + (-0.20 - 0.10)^2} = 0.10$$

$$\text{Euclidean Distance between schools A and E} = \sqrt{(0.10 - 0.18)^2 + (-0.20 - -0.40)^2} = 0.22$$

Worked example of Euclidean Matching

The “Euclidean distance” between schools A and D is smaller than between schools A and E. This means that school D is statistically more similar to school A than school E is, in terms of % SEN and % Ever6 FSM.

If these are amongst the 49 smallest “Euclidean distances” between school A and all potential ‘most similar schools’, school D and school E will be amongst school A’s unique ‘most similar schools’ group.

Third step: calculating a school’s ‘efficiency score’

A school’s relative efficiency is reported in terms of in which ‘efficiency decile’ they located within their unique ‘most similar schools’ group. Every ‘most similar schools’ group of 50 is split into deciles (10 groups of five) that are based on schools’ ‘efficiency scores’. How a school’s ‘efficiency score’ translates into its ‘efficiency decile’ is shown in the following table.

‘Efficiency decile’	‘Efficiency score’ in ‘most similar schools’ group
1	Largest to 5th largest
2	6th largest to 10th largest
3	11th largest to 15th largest
4	16th largest to 20th largest
5	21st largest to 25th largest
6	26th largest to 30th largest
7	31st largest to 35th largest
8	36th largest to 40th largest

‘Efficiency decile’	‘Efficiency score’ in ‘most similar schools’ group
9	41st largest to 45th largest
10	46th largest to 50th largest

Summary of data used in the School Efficiency Metric

This section summarises the data used in calculating the School Efficiency Metric.

Progress

For the primary phase, we used an average of the key stage 2 progress reading, progress writing and progress maths for the 2016/17 cohort taking the tests. This measures the progress made between key stage 1 and key stage 2.

For the secondary phase, we use the Progress 8 measure for the 2016/17 exam-taking cohort. This measures the progress made by pupils between key stage 2 and key stage 4 in the subjects that make up their Attainment 8 GCSE (or equivalent), including English and Maths. Progress 8 compares each pupils' achievement (based on Attainment 8) with the average attainment of all pupils nationally who had a similar starting point.

Income per pupil

We include only the funding that comes from central government and/or their local authority in the measure of income per pupil. Where applicable, we include:

- Schools block funding (which schools receive from the dedicated schools grant given to local authorities);
- 16-19 funding;
- Pupil premium funding – including deprivation and service child pupil premium;
- Year 7 literacy and numeracy catch-up premium

These funding items were received by maintained schools during the financial year 2016-17 and by academies during the academic year 2016/17. In calculating the Efficiency Metric, we remove the area cost uplift (i.e. the Hybrid Area Cost Adjustment used in school funding calculations) applied to funding received during the period.
This means that schools in high cost areas are not in a low 'efficiency decile' simply because of their location.

We don't include any funding that schools receive from other organisations, such as donations. Neither do we include any self-generated income. We do not include funding awarded for designated reasons – such as academy start-up grants, which are awarded to counteract cost-pressures associated with starting a new school that are not present elsewhere. Counting these would penalise new schools in the metric.

School characteristics

Schools' names, phase and organisational type are correct as at January 2018 taken from Get information about schools.

We characterise a school as having a sixth form as per its sixth form status as per Get information about schools, correct at 2018 January.

A school's overall FTE is as reported from the School Census of January 2017, and so is correct as of this date.

A school's SEN information is as reported from the School Census January 2017, and so is correct as of this date.

A school's Ever 6 FSM information is as reported from the School Census of January 2017, and so is correct as of this date.

Annex

Assigning income per pupil to each phase of schools with primary and secondary phases

For schools with only one phase, calculating income per pupil is relatively simple. This becomes more complicated when considering the income per pupil of schools with both primary and secondary phases.

One possible method for calculating the income per pupil in these cases is to use the same method used to calculate it for schools with only one phase, which is to divide total income by total number of pupils, and assign the resulting figure to each phase. However, schools tend to receive higher funding per pupil for their secondary pupils than for their primary pupils. If we were to use this method, the calculated income per pupil figure is likely to be an overestimate of the figure received for the school's primary pupils, and an underestimate of the figure received for their secondary pupils. This becomes clear in the worked example below.

Therefore, we have devised another method for assigning income per pupil to each phase of such schools. The method is described in the steps below. This method assigns a more realistic income-per-pupil figure to each phase than the alternative method of splitting funding according to pupil numbers.

The steps used to assign phase-specific funding are:

Method for assigning income per pupil to each phase of schools with more than one phase for maintained schools and academies.

1) **Calculate total income of the school.** This is the sum of the individual funding items described in the 'Summary of data used in the school efficiency metric' calculation.
2) **Remove the area cost uplift applied to funding.**
3) **Calculate local authority age weighted pupil units (LA AWPUs) as an implied percentage.** One of the funding factors used by local authorities (LAs) when allocating funding to schools is a "basic entitlement" factor. In allocating the funding under this factor, LAs decide on an amount per pupil to give to primary and secondary pupils. This per pupil amount is the age weighted pupil unit (AWPU). Using this, we calculate a phase-specific LA AWPU %, using the following formula.

$$\text{Primary LA AWPU \%} = \left(\frac{\text{Primary AWPU of LA}}{\text{Primary AWPU of LA} + \text{Secondary AWPU of LA}}\right)$$

Method for assigning income per pupil to each phase of schools with more than one phase for maintained schools and academies.

$$\text{Secondary LA AWPU \%} = \left(\frac{\text{Secondary AWPU of LA}^{5}}{\text{Primary AWPU of LA} + \text{Secondary AWPU of LA}}\right)$$

4) **Calculate phase specific "units".**

$$\text{School primary units} = \text{Primary LA AWPU \% of school's LA x School's primary FTE}$$

$$\text{School secondary units} = \text{Secondary LA AWPU \% of school's LA x School's secondary FTE}$$

5) **Calculate total funding per unit.**

$$\text{School total funding per unit} = \frac{\text{School's total funding}}{\text{School primary units} + \text{School secondary units}}$$

6) **Calculate implied total phase funding of school.**

$$\text{School implied primary funding} = \text{School total funding per unit x school primary units}$$

$$\text{School implied secondary funding} = \text{School total funding per unit x school secondary units}$$

7) **Calculate implied phase funding per pupil of school.**

$$\text{School implied primary funding per pupil} = \frac{\text{School's implied primary funding}}{\text{School's primary FTE}}$$

$$\text{School implied secondary funding per pupil} = \frac{\text{School's implied secondary funding}}{\text{School's secondary FTE}}$$

[5] Local authorities have specific primary, key stage 3 and key stage 4 AWPUs. The secondary AWPU used here is an average of key stage 3 and 4 AWPUs, weighted by pupil numbers in the key stages across the local authority.

Worked example: All through school

Consider an all through school with the following information.

$$\text{Primary FTE} = 300$$

$$\text{Secondary FTE} = 700$$

$$\text{Total funding} = £4{,}000{,}000$$

$$\text{Local authority area cost uplift} = 1$$

$$\text{Total funding after removing the area cost uplift} = £4{,}000{,}000$$

$$\text{Primary AWPU of school's LA} = £4{,}000$$

$$\text{Secondary AWPU of school's LA} = £6{,}000$$

The LA AWPUs as an implied percentage for this school are therefore:

$$\text{Primary LA AWPU \%} = \left(\frac{4{,}000}{4{,}000 + 6{,}000}\right) = 0.4 = 40\%$$

$$\text{Secondary LA AWPU \%} = \left(\frac{6{,}000}{4{,}000 + 6{,}000}\right) = 0.6 = 60\%$$

Using these percentages, the school's phase units are:

$$\text{School primary units} = 40\% \text{ x } 300 = 120$$

$$\text{School secondary units} = 60\% \text{ x } 700 = 420$$

The school's total funding per unit is:

$$\text{School total funding per unit} = \frac{£4{,}000{,}000}{120 + 420} = £7407$$

Using this total funding per unit, the school's implied phase funding per unit is:

$$\text{School implied primary funding} = £7407 \text{ x } 120 = £888{,}889$$

$$\text{School implied secondary funding} = £7407 \text{ x } 420 = £3{,}111{,}111$$

These implied phase funding levels are then used to calculate an implied phase funding per pupil of the school.

$$\text{School implied primary funding per pupil} = \frac{£888{,}889}{300} = £2{,}963$$

$$\text{School implied secondary funding per pupil} = \frac{£3{,}111{,}111}{700} = £4{,}444$$

If we were to use the alternative method of dividing total income by total number of pupils, the funding figure assigned to each phase would be:

$$\text{School alternative primary and secondary funding per pupil} = \frac{£4{,}000{,}000}{1000} = £4{,}000$$

This alternative income per pupil figure of £4,000 is larger than the implied primary figure of £2,963, and smaller than the implied secondary figure of £4,444. As schools tend to receive more funding per pupil for their secondary pupils than their primary pupils, the implied primary and secondary income per pupil figures are likely to be a more accurate reflection of what this school received than the alternative income per pupil figure. Therefore, we use these income-per-pupil figures to calculate the 'efficiency scores' for this school.

Weighted mean and standard deviation formulas

In calculating a school's 'efficiency score', we standardise its progress score and income per pupil using their 'weighted' means and standard deviations. These are weighted by schools' FTE, as in the following formulas.

Formulas for weighted mean and standard deviations

$$\textbf{Weighted mean of progress} = \frac{\sum_{i=1}^{n}(\mathrm{FTE_i\ x\ progress_i})}{\sum_{i=1}^{n}(\mathrm{FTE_i})}$$

$$= \frac{(\mathrm{FTE_1 x\ progress_1}) + (\mathrm{FTE_2 x\ progress_2}) + \cdots + (\mathrm{FTE_n x\ progress_n})}{\mathrm{FTE_1} + \mathrm{FTE_2} + \cdots + \mathrm{FTE_n}}$$

$$\textbf{Weighted mean of income per pupil} = \frac{\sum_{i=1}^{n}(\mathrm{FTE_i\ x\ Income_i})}{\sum_{i=1}^{n}(\mathrm{FTE_i})}$$

$$= \frac{(\mathrm{FTE_1 x\ Income_1}) + (\mathrm{FTE_2 x\ Income_2}) + \cdots + (\mathrm{FTE_n x\ Income_n})}{\mathrm{FTE_1} + \mathrm{FTE_2} + \cdots + \mathrm{FTE_n}}$$

$$\textbf{Weighted standard deviation of progress} = \sqrt{\frac{\sum_{i=1}^{n}\mathrm{FTE_i\ x\ (progress_i} - \overline{\mathrm{progress}})^2}{\frac{M-1}{M}\ \mathrm{x}\ \sum_{i=1}^{n}\mathrm{FTE_i}}}$$

$$\textbf{Weighted standard deviation of income per pupil} = \sqrt{\frac{\sum_{i=1}^{n}\mathrm{FTE_i\ x\ (Income_i} - \overline{\mathrm{Income}})^2}{\frac{M-1}{M}\ \mathrm{x}\ \sum_{i=1}^{n}\mathrm{FTE_i}}}$$

$\mathrm{FTE_i}$ = Number of full time equivalent pupils in school i.

$\mathrm{Progress_i}$ = progress in school i.*

$\mathrm{Income_i}$ = Income per pupil in school i.

$\overline{\mathrm{Progress}}$ = weighted mean of progress

$\overline{\mathrm{Income}}$ = weighted mean of income per pupil

M = number of non zero values of $\mathrm{FTE_i}$

*Progress as measured by Progress 8 for secondary phase and the average across progress reading, writing and maths for primary phase.

Further information

Other relevant departmental guides

- Guide on progress measures for primary schools
- Guide on Progress 8 for secondary schools

Links to efficiency tools and resources

- School financial benchmarking
- School resource management webpages
- Education Endowment Fund Teaching and Learning Toolkit

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