



Secondary
National Strategy
for school improvement

Guidance

Curriculum
and Standards

Mathematics at Key Stage 4: developing your scheme of work

Mathematics
subject leaders

Status: Recommended

Date of issue: 05-2007

Ref: 00049-2007BKT-EN

Planning handbook summer 2007

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

department for
education and skills

Creating Opportunity
Releasing Potential
Achieving Excellence



How to use this planning handbook

This planning handbook is designed to support subject leaders as they work with their departments to develop an effective scheme of work in mathematics at Key Stage 4. It consists of a collection of linked tasks, to help subject leaders prioritise, plan and implement a manageable development programme that involves the department working collaboratively.

The handbook is intended to be flexible enough for schools to use in the context of a changing curriculum at Key Stage 4. In particular, it can be used to assist planning for a two-tier GCSE.

The handbook introduces the *Mathematics planning toolkit: Key Stage 4* through a sequence of 'bite-sized' tasks. It addresses the heart of the planning process:

- grouping objectives into teaching units so that:
 - pupils are given opportunities to consider key concepts, interconnections and applications of mathematics;
 - teachers can build effectively on pupils' existing knowledge;
- incorporating rich classroom tasks that:
 - engage pupils actively in learning mathematics and developing the skills needed to use and apply mathematics;
 - assist teachers' planning, for example by providing contexts where objectives can be linked and taught together.

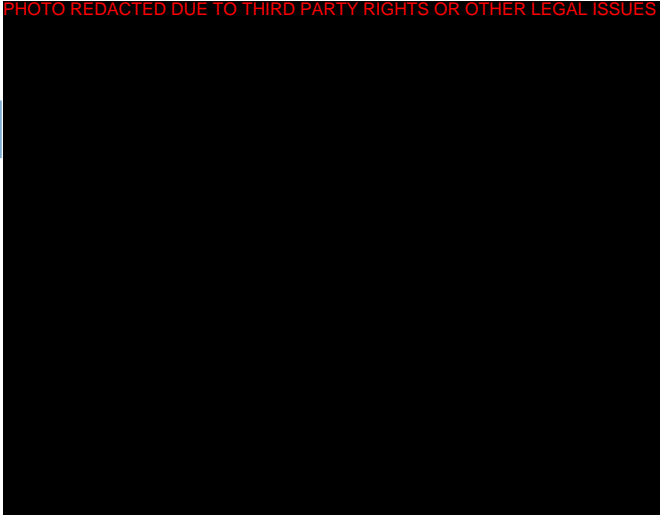
The handbook addresses the interplay between grouping objectives and selecting rich tasks that is the key to developing effective units of work. It encourages departments to take a realistic view of developments, moving between the 'big picture' and the detail of particular units, built up gradually over time. The planning toolkit is flexible enough to allow further elements to be added at a later stage, as schemes of work are developed in more detail. This might include, for example, key lesson plans, assessment tasks and examination questions.

Mathematics at Key Stage 4: developing your scheme of work

Contents

Making a start	3
Grouping objectives	9
The teaching calendar	16
Organising resources	18
Rich tasks	19
Unit planning	24
Ongoing developments	30
Appendix: Some underlying principles	32

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Making a start

Effective schemes of work

An effective scheme of work supports teachers' planning and helps to ensure that pupils have positive experiences of mathematics and make good progress in the subject. A good scheme of work will:

- set out a teaching programme that ensures balanced coverage of the curriculum and a clear progression, taking account of the opportunities and constraints of the school calendar and resources;
- outline the intended development within each unit of work, helping teachers to plan and to manage the transition from lesson to lesson, keeping the need for lesson notes to a minimum;
- incorporate a variety of rich learning tasks that:
 - involve pupils in their learning and promote mathematical talk for deeper understanding;
 - assist teachers in building on pupil's prior learning, differentiating according to need.

Leading the development of the department's scheme of work is central to the subject leader's role. It should be a rewarding, collaborative activity, providing a context for talking about teaching and learning mathematics, sharing good practice and enhancing the professional expertise of all teachers.

Time spent developing the scheme of work together supports the planning of all colleagues, reducing time spent in individual preparation and making it more effective. The notes can be revised when appropriate and remain available for when a unit is next taught. Such departmental discussions offer longer-term advantages, with positive impact on teaching and learning more generally.

What is a scheme of work?

There is a commonly accepted pattern to a teaching programme.

- The *programme of study* is divided into teaching units in such a way that related mathematical concepts and skills can be taught coherently over a sequence of lessons. Units are usually sequenced so that all strands of the subject are kept moving forward and concepts are revisited over time. (This pattern will vary in the case of modular courses.)
- Each *unit* is structured to achieve progression in learning within the identified area of mathematics, aiming to build on what pupils already know and to lead to observable progress. Often a unit is planned at more than one level in order to meet the needs of different pupils within a year group.
- Individual *lessons* are sequenced within the overall unit plan, working towards the intended outcomes of the unit. Teachers work to a common unit plan, but lessons may unfold differently in each classroom as teachers adapt to the responses and needs of their classes.

An effective scheme of work will address the first two of the aspects listed above, dividing the curriculum into teaching units and outlining the objectives and activities in each unit (perhaps occasionally including a key lesson plan). Lesson planning is made very much easier for individual teachers when the structure and content of a unit has been discussed and agreed.

In summary, key elements of your scheme of work are as follows.

- The curriculum map and teaching calendar

The map shows how the curriculum will be divided into teaching units and the calendar shows how units will be fitted into the school year. The map is likely to be revised as you develop units in more detail and the calendar will change according to the schedule of school events.

- Unit plans

Plans for each unit of work set out the sequence and content of the unit to support teachers' day-to-day planning. As a minimum, it will include the objectives for the unit and indicate key resources that will be used. More detailed plans, perhaps starting with one or two key units, will set out the structure of the unit in phases and provide more information and advice on key activities.

Developing your scheme of work: deciding where to start

This planning handbook sets out manageable tasks to help you and your colleagues develop or revise your Key Stage 4 scheme of work in mathematics. It is not suggested that you abandon your present scheme of work and start from scratch. It is preferable to set up an ongoing process of development and refinement, rather than aiming for a 'product' to be completed by a certain date.

The five main sections of the planning handbook are outlined below. They are not intended to be tackled sequentially. You will find yourself moving between the different sections to make adjustments or additions, as your scheme of work develops.

- Grouping objectives

Clustering objectives into teaching units, storing the units and creating a curriculum map

- The teaching calendar

Using the curriculum map to help sequence units of work to fit the school calendar

- Organising resources

Mapping resources and teaching ideas to units of work

- Rich tasks

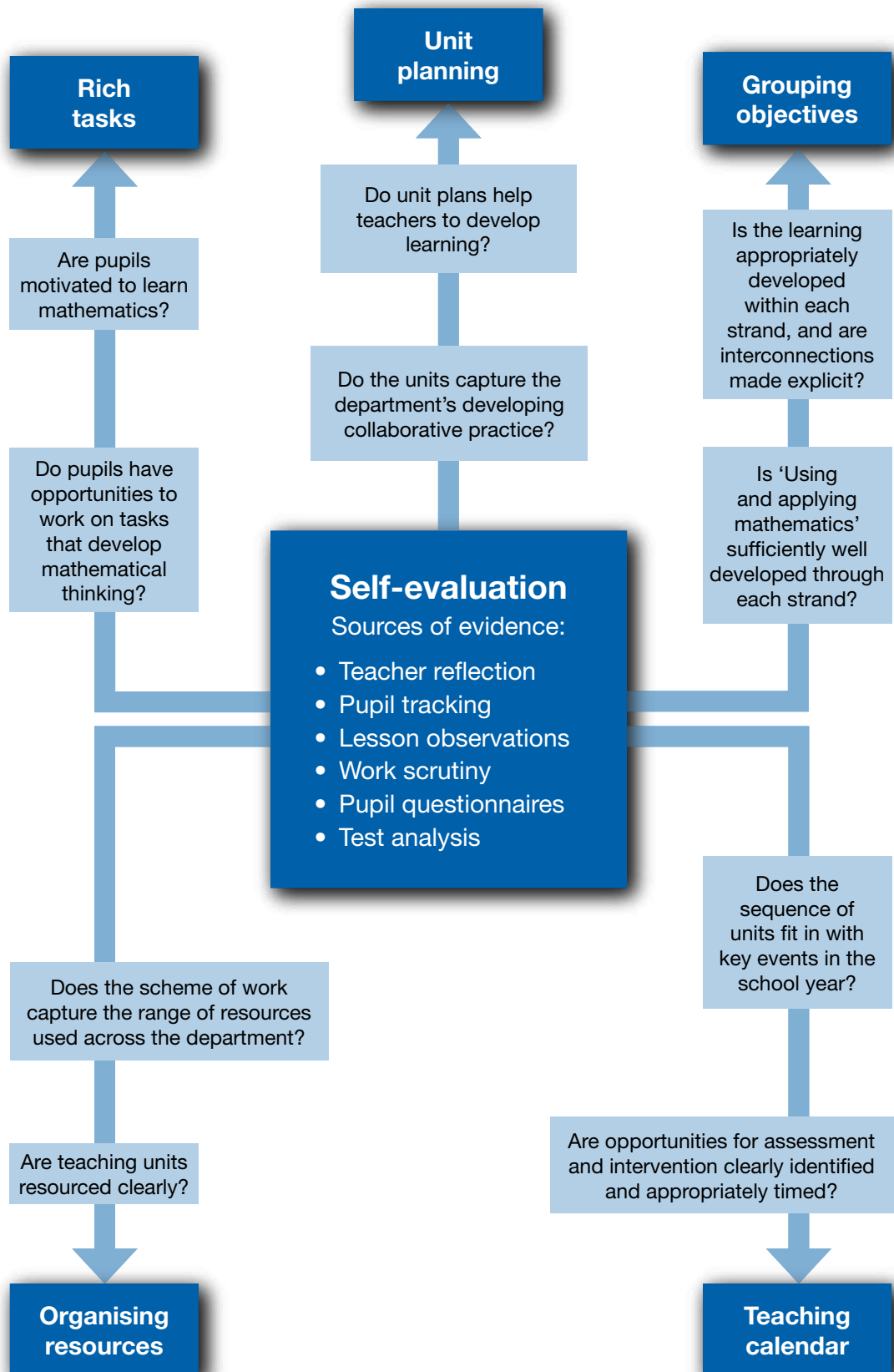
Exploring rich mathematical tasks and their potential for improving pupils' learning

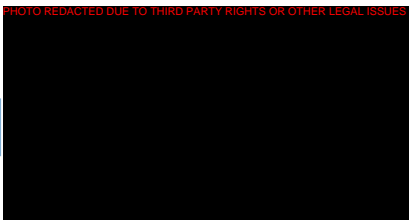
- Unit planning

Considering different ways of structuring a unit and using rich tasks to develop a unit in phases

Self-evaluation and development of the scheme of work

Diagram 1





The development of an effective scheme of work needs to start with a process of self-evaluation.

- If you have already carried out this process, and you feel ready to start work on your agreed priorities, go straight to 'The planning toolkit' on page 8.
- If you need to do further work on self-evaluation in order to decide your priorities, see **task MS 1** below. This provides some questions that you can use to stimulate discussion within the department and to help you decide the priorities for development. You will need to provide copies of diagram 1 for colleagues.

Note that **task MS 1**, a discussion based on **Diagram 1 Self-evaluation and development of the scheme of work**, is not intended to provide a comprehensive approach to departmental self-evaluation. A more detailed account is provided in the Secondary Strategy's *Middle leaders' self-evaluation guide* (2076-2005DCL-EN), and subject-specific guidance was provided in the mathematics subject leaders' development materials for spring 2006.

Task MS 1 Deciding your priorities

This task is intended to stimulate discussion among teachers. It is a suitable task for a departmental meeting.

Refer colleagues to *diagram 1*. This shows the various sections of this planning guide in relation to some key questions arising from the self-evaluation process.

Discussion prompt 1

Start by looking at the sources of evidence listed in the centre of the diagram.

- *Are these sources of evidence used effectively within your department?*
- *Which if any require further development?*

Discussion prompt 2

Consider each of the questions shown on the diagram. (In a large department, teachers could be asked to discuss individual questions, in pairs or small groups.) The diagram suggests priorities based on responses to the questions. Make a note of any areas that you think should be prioritised.

Draw the discussion together, aiming to reach agreement about the priorities for development of the scheme of work.

Deciding on your priorities is an important first step. However, there are some other steps that need to be taken before you can make the most effective use of this planning guide.

- You need a plan, setting out the steps to be taken in working towards your agreed priorities. This is a key part of your departmental improvement plan.
- You need a basic familiarity with the *Mathematics planning toolkit: Key Stage 4*. **Task MS 2** at the end of this section will get you started.

Developing your plan

Crucial aspects of developing a plan are setting out priorities on a realistic timescale, establishing a collaborative way of working and building up an electronic filing system that can evolve as your scheme of work develops.

It is important to recognise that the process is ongoing and to plan a manageable development schedule. Set out a plan over time, outlining what you want to achieve in the next term, the next year and beyond. Keep abreast of changes and respond positively:

- see opportunities rather than threats;
- anticipate change through the developmental work that you initiate.

Forward planning helps you to pace developments and manage change. If members of the department are aware of the plan then work moves ahead with a sense of direction and purpose, often with less conscious effort. The inevitable changes of plan can then happen in a more considered way, rather than as a reaction to events.

Agree with senior leaders and the department how you can manage time for collaborative work. Ensure that departmental meetings include development time and, where possible, find other ways of dealing with administrative matters. Clarify which tasks require collaborative work as a whole department, or perhaps the teachers teaching a particular year group, and which can be followed up by pairs of teachers, or individuals, reporting back later.

Make sure that you keep senior leaders informed and discuss the details of your plans with them. Their support will be invaluable. For example, they can help to facilitate arrangements for departmental planning time. The monitoring role that you jointly exercise will be better supported if they understand the role of unit plans in your scheme of work and how those plans inform the preparation of individual lessons.

It is important that developments are captured for the long term and become embedded in your scheme of work. The most flexible way of doing this is to set up an electronic filing system for your scheme of work and associated resources, thereby making it easier to incorporate changes and additions. Keep sufficient written notes to act as reminders for colleagues teaching the unit of work; these may be used in subsequent years and revised or expanded when appropriate.

The priorities you identified from the self-evaluation **task MS 1** will help you to formulate your plan. In some departments, establishing a collaborative way of working may be the highest priority and you should consider this when deciding where to begin. From small beginnings, the process is established and benefits are felt. It should then be possible to plan a rolling programme with an increasing pace of change.

The planning toolkit







The tasks in this planning handbook use resource files in the *Mathematics planning toolkit: Key Stage 4* on the CD-ROM, so that you can build up your scheme of work electronically. Before using the appropriate sections for your identified priorities, it is important to spend a few minutes becoming familiar with the planning toolkit.

The planning toolkit aims to make it easier to develop, record and revise your scheme of work over time. It consists of a collection of files grouped into folders. You can cut and paste from different files, to build up the different components of your scheme of work: curriculum map, calendar and unit plans. You can also add files from other sources that will be useful in your planning.

Task MS 2 Getting to know the planning toolkit

This task is initially for the subject leader, perhaps working with a colleague.

From the initial screen go to the folder **Guidance on planning**, noting that it contains this planning handbook and each of its sections as separate documents. Then explore the contents of other folders and their sub-folders.

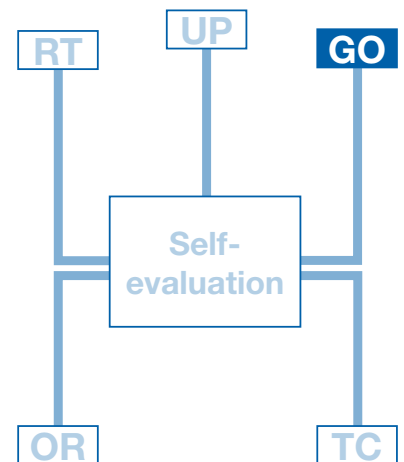
-  **Adaptable templates** – for your scheme of work
-  **Curriculum information** – the mathematics to be taught
-  **Ideas for rich tasks** – activities for the classroom
-  **Pedagogy and subject knowledge** – teaching approaches, guidance and self-study
-  **Assessment** – tracking pupils' progress (for future development)
-  **The developing scheme of work** – filing system for completed templates

After a few minutes' exploration, you should have a good idea of the purpose and content of each folder of the planning toolkit.

You should now be ready to move on to one of the following sections.

Grouping objectives

The curriculum map shows how the mathematics programme of study is divided into teaching units. Although this section describes how you might start the process from scratch (for example, when building up an electronic scheme of work for a two-tier GCSE course), you could use a similar process to create an overview of your existing scheme of work, or perhaps just one strand of the subject. Whichever approach you adopt, you will also need to revisit the curriculum map as you develop units in more detail.



Lists of objectives by strand

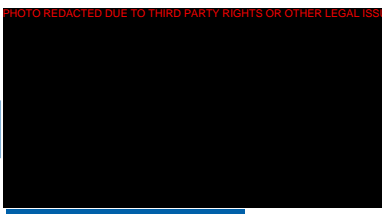
The process starts with the course specification or programme of study, divided into teaching objectives. It will be illustrated for the Higher tier of the new two-tier GCSE, but the process is equally applicable to the Foundation tier or to Key Stage 3. As a support for the process, the [Curriculum information](#) folder includes sets of objectives for the higher tier for each of the strands of number, algebra, shape, space and measures, and handling data. Each of these includes 'using and applying mathematics' objectives. Each strand has the same rationale.

- The objectives are organised in three columns.
 - Those in the first column are intended to help pupils progress from level 5 at the end of Key Stage 3 to grade C at the end of Key Stage 4.
 - Those in the second column are intended to help pupils progress from level 6 to grade B.
 - Those in the third column are intended to help pupils progress from level 7 to grade A/A*.
- Objectives highlighted in **blue** are additional to those that have been taken from the *Framework for teaching mathematics: Years 7, 8 and 9*.
- Objectives highlighted in **bold** are key indicators of the target grade. These can be useful in tracking pupils' progress.

Some factors to consider before you can start

Before you can start the process of grouping objectives into teaching units, you need to make certain assumptions. These will be provisional and you may well make changes during the process, as each department will have a slightly different approach. A reasonable starting point might be to assume:

- five terms (or ten half-terms) across the two years for teaching the units. (There are about 11 half-terms, if part of the last half-term of Year 9 and some of the run-up to Year 11 examinations are included, but you need to allow teaching time to be given over to work experience, trips, examinations, revision and other contingencies.);
- five units per term (two or three per half-term);
- five to seven units per strand, giving from 20 to 28 units in total.




Remember that it is important not to overfill your programme and to leave space for assessment opportunities and appropriate intervention work.

A further issue is the need to adapt the scheme of work to meet the varying needs of pupils following the higher-tier programme of study and entering Key Stage 4 at any level from 5 to 8. An effective approach is to start grouping the first column of objectives (progression from level 5 to grade C) and allow this issue to surface as the second and third columns are brought into the discussion.

Grouping the objectives

It would be extremely valuable for the whole department to complete **task GO 1a** and the extension **task GO 1b** (both described below) together, drawing on experience of teaching in Year 9 when considering appropriate starting points and approaches in Year 10. This helps everyone to acquire a good overview of the curriculum, the key concepts and interconnections. It also gives teachers a sense of ownership and puts individuals more in control when they come to teaching the scheme of work.

Identify the strand on which you want to work: number, algebra, shape, space and measures or handling data. From the  **Curriculum information** folder, select and print copies of the relevant reference document, for example, *SSM KS4 higher-tier objectives and key indicators*, one for each member of the department. The task is to group these objectives into teaching units.

To prepare for the task, select the three related documents that separate the columns of the original (for example, *SSM KS4 higher level 5 to grade C*, *SSM KS4 higher level 6 to grade B* and *SSM KS4 higher level 7 to grade A/A**). Give the three documents to an administrative assistant, with the following instructions, so that the cut-up objectives are available for the task.

- Print each of the three documents on paper of a different colour.
- Cut up the sheets so that each objective is on a separate slip of paper.
- Provide some blank slips in each colour.

The blank slips can be used for objectives that are referenced in more than one teaching unit (especially likely for the 'using and applying mathematics' objectives).

Task GO 1a Grouping objectives into units for a selected strand

This task is for small groups of mathematics teachers (two or three people) to work on together.

Either involve all staff by getting them to work in smaller groups and then compare results **or** ask a small group to try it out beforehand and report back to a departmental meeting at which all colleagues participate in a discussion of the process and rationale.

Grouping objectives into units within a strand

Starting with those from the first column (progression from level 5), group the objectives into appropriate clusters, aiming for five to seven units in total. (NB the number of units may be different for different strands.) Look for key concepts and interconnections to help you group related objectives. Remember that it may be appropriate to revisit some objectives. Also, some objectives may be put aside or noted to include with objectives from another strand.

As you bring in objectives from the second and third columns, consider how you will differentiate for different groups of pupils. For example for Year 10 you might decide to plan at two levels within the higher tier:

- higher: plan from secure level 5 (column 1);
- upper higher: plan from secure level 6 (columns 2 and 3).

You may wish to keep the approach flexible. For example, early units might have some objectives in common, together with extension objectives for the upper higher level. Later units might require separate sets of objectives or even different headings.

Identify 'using and applying mathematics' objectives that you consider appropriate for each unit. Remember that this may be a provisional decision that you will need to reconsider later as you think about teaching approaches and tasks.

Save the results of sorting into outline units, for example, by pasting objectives into clusters and saving the clusters. (These will be required in **task GO 2.**)


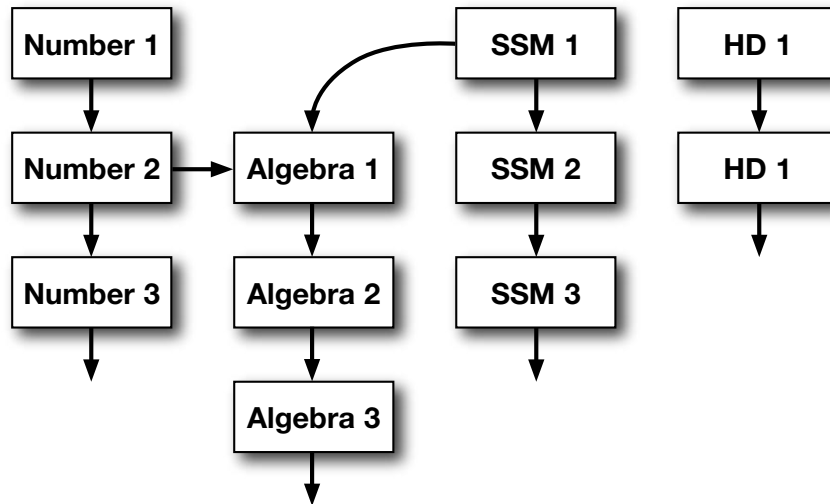
Task GO 1b follows immediately from **task GO 1a** as it involves giving your teaching units titles and putting them into a provisional teaching order. The details are then summarised on the curriculum map. From the  **Adaptable templates** folder the **Curriculum map template** enables you to incorporate all four strands. It provides space in each box for a unit code, a short descriptive title and an indication of the teaching time required. Diagram 2 illustrates the kind of arrangement that is involved.



Diagram 2



Curriculum map example with guidance is a partially completed illustration. As your scheme of work develops, you may wish to show a different number of units in the various columns, a cross-strand column or cross-strand units.

Task GO 1b Building up the curriculum map

This task is best completed by the department together, as an extension of task GO 1a.

Sequencing the units

Working in one strand of the curriculum, and having determined how you will group objectives into teaching units (about 5 to 7 units in total), the next task is to determine a teaching sequence. Consider:

- progression and order of difficulty in the mathematics;
- topic dependence, for example, work on surface area relies on knowledge of area of triangles, quadrilaterals and circles;
- the need to revisit areas of mathematics periodically.

Each unit can now be given a code, for example, *SSM1*, *SSM2*.

Building up the curriculum map

For each coded unit, such as *SSM3 upper higher*:

- give it a short title, for example, *Geometrical reasoning*;
- make a provisional estimate of how long it will take to teach, in terms of lessons or weeks.


Complete the appropriate section of the curriculum map.

At this point you have a choice about which task to do next.

- If you have extended departmental time together, you might choose to move on immediately to **task GO 3**, which repeats the above process for other strands of mathematics and completes the curriculum map.

- Alternatively, move on to **task GO 2** to capture details of the work so far and then complete the curriculum map, strand by strand, over several sessions.

The curriculum map gives an overview of the teaching units for the higher tier. Details of each unit are kept in a *Unit* plan and this is the focus for **task GO 2**. For this purpose some unit planning templates are offered (**Unit plan template 1**, **Unit plan template 2** and **Unit plan template 3**). For now, just choose the template that best suits your purposes. You can then save your outline units with clusters of objectives in your chosen blank template, to which you will add further details later.

To facilitate cutting and pasting electronically, the  **Adaptable templates** folder contains spreadsheets that set out the objectives for each strand, in alphabetical order. If you decide to use it, then select the one for your chosen strand (for example, *SSM KS4 higher tier objectives for copying*).

Task GO 2 Storing your outline units

This task is for the teacher who is taking responsibility for filing the scheme of work.

Filing the clustered objectives

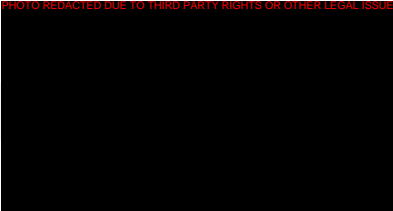
For each unit in turn copy the details from the curriculum map (code, title and provisional time allocation) into the appropriate sections of your chosen blank unit-planning template.

Next, refer to the clusters of objectives from **task GO 1a** and copy them to the appropriate section of the unit-planning template.

Finally, give the document an appropriate title such as *SSM3 upper higher* and save it in a folder for the appropriate strand, for example, *SSM unit plans*.

At this stage, you may want to think about the practical arrangements that will be necessary for storing and accessing the large number of electronic documents that your scheme of work will contain. See ‘A note about organising materials’ on page 15 at the end of this section.

Task GO 3 first repeats **tasks GO 1a** and **GO 1b** for the other strands of mathematics to complete the curriculum map. When this has been done, it is important to consider interconnections and dependencies across strands, as this might lead to some reordering of topics. A limitation of the process, as described, is that the content of each unit is confined to one strand of mathematics. At some stage in developing your scheme of work, it will be important to consider units that link strands together. For example, pupils could be helped to make sense of proportionality by a unit that addresses the concept across number, algebra and geometry. Not all units can be planned a long time in advance and a good scheme of work will allow contingency time for fresh units to be planned as part of targeted intervention for some groups around a particular topic.



Task GO 3 Completing the curriculum map

This task is for the staff of a mathematics department to work on together.

Completing the map for other strands

Repeat **tasks GO 1a, GO 1b** and **GO 2** for the other three strands of mathematics, so that you have a complete curriculum map and a set of outline teaching units.

Reviewing cross-strand links

Now, referring to your completed curriculum map, discuss links across the strands of number, algebra, SSM and handling data that may affect the ordering of topics. For example, it might be appropriate for work on fractions, percentages and decimals to precede probability in handling data. Make any adjustments needed to the order and highlight the links for future reference (for example, by drawing arrows to indicate where units in one strand must precede units in another).

Next, identify any cross-strand units in your existing scheme of work and make any adjustments needed to accommodate them in your revised scheme. Consider whether you want to develop such a unit or whether this is an issue to revisit at a future date.

Assessment and intervention

Finally, make sure that you consider how you will incorporate key assessment points into your scheme of work so that targeted intervention is included for appropriate groups of pupils.

The process of grouping objectives, as described in this section, should provide you with appropriately challenging programmes that build on pupils' existing knowledge, ensure progression and help all pupils to achieve their target grades.

Revisiting the curriculum map

Once you have grouped objectives and set up a curriculum map, you should only need to revisit the task occasionally. For example, a significant change in the syllabus you are following would affect your curriculum map.

The other main reason why you might need to make changes arises from teaching the units and developing them in more detail. You might need to reconsider:

- the **grouping of objectives** into units, for example, you may decide that it would help pupils' understanding if you linked different strands of mathematics into a cross-strand unit;
- the **relative time given to different units or the sequencing of them**, for example:
 - a critical area of weakness (perhaps emerging from analysis of test results) requires a new or revised unit of work that will take up more time;
 - there is too much repetition of work that pupils have covered before, which means that you only need to recap rather than re-teach the topic;

- the way in which you **differentiate for different teaching groups**, for example, you find that, at the start of Year 10, differentiating your scheme of work for the higher tier into two levels is sufficient but, later in the year, you need to introduce another level for the highest attaining pupils.

A note about organising materials


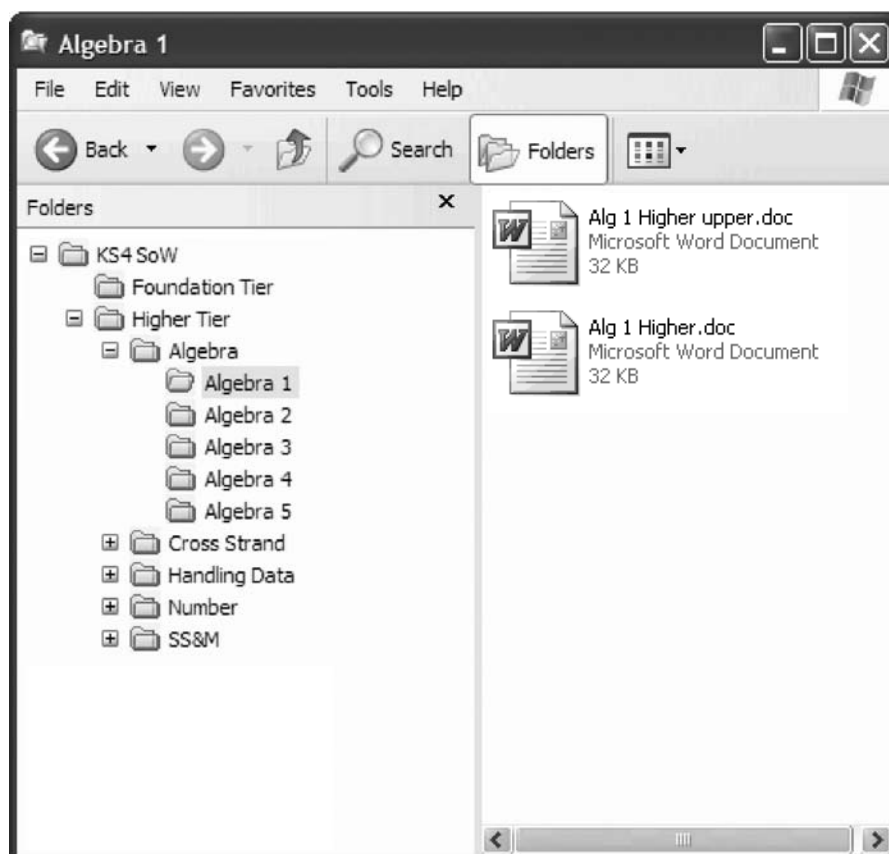
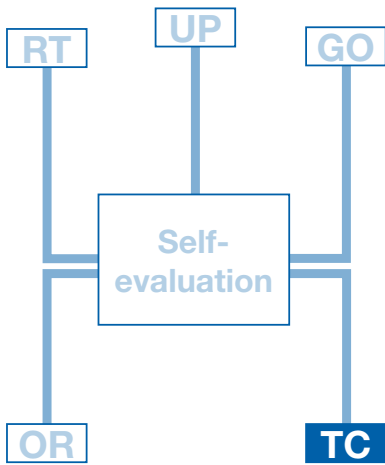
Schemes of work are inevitably rather complicated collections of documents. It is important to make sure that you organise all of the electronic resources for your scheme of work in such a way that they are easy to search and navigate. Perhaps most importantly, you should make sure that you have a consistent policy for naming and filing resources. The illustration below shows one way in which files could be organised. A suggested set of blank folders is provided in the folder  **The developing scheme of work**.

Diagram 3



You will also need to think about practical arrangements for organising resources.

- Make sure that there is a secure place on the school network where your scheme of work will be stored.
- Issue permissions or passwords to everybody who needs access to the materials.
- Ensure that somebody is responsible for making sure that all of the files are kept up to date and properly organised.



The teaching calendar

The calendar shows how teaching units will be sequenced and how they fit around school dates and events. The order of units will take account of interdependencies in the units, across as well as within strands, and the need for progression. It will also be affected by the nature of the assessment. In particular, a modular scheme will require a different order of topics from a linear scheme.

The **Adaptable templates** folder includes a **Teaching calendar template** with the following column headings.

Mathematics Department Teaching Calendar

Half term	Week beginning	Notes	Unit code	Mathematical content overview
Summer 2 Year 9	28/05/2007		HD1	Guided group tasks - Using & Applying. Emphasis on written explanations, justifying choices and interpreting diagrams
	04/06/2007			
	11/06/2007		SSM1	Practical tasks working in 2 and 3 dimensions including scale models, nets and constructions
	18/06/2007			
	25/06/2007	Analysis of test papers	Targets	Interactive tasks engaging pupils with individual and peer analysis of performance at KS3 and target setting for improvement at KS4
	02/07/2007	Collapsed timetable - business enterprise; careers-planning work experience		

The 'Notes' column provides space for capturing significant events and the overview column gives an 'at-a-glance' view of the content covered in each term. Before engaging with **task TC 1** you might find it helpful to refer to the **Teaching calendar example**, which is partially completed.

Task TC 1 Constructing a two-year calendar

This task is probably best completed by one or two teachers working together, perhaps the subject leader and Key Stage 4 coordinator.

Starting the calendar: school events

Record important school dates and events in the 'notes' column of the teaching calendar. Points to consider include:

- events that need to be interwoven with the teaching sequence, such as examinations and tests, including active examination preparation and feedback, extended projects and targeted intervention; also non-mathematical events such as work experience and excursions;
- events that affect scheduling or departmental organisation, such as access to resources, examination entry deadlines, analysis of test papers, departmental meetings and parents' evenings.

Completing the calendar: sequencing the units

Use the teaching order indicated in the curriculum map to determine how you will weave topics from different strands together in a sequence that will fit the calendar. Other factors to consider at this stage include:

- whether you are following a linear or modular scheme;
- the balance of topics in a term or half-term to ensure stimulus and variety;
- points that you have identified in the notes column, such as scheduling of examinations or availability of computer resources;
- other factors, such as the most suitable time of year, for example, loci in summer so an outdoor approach could be used.

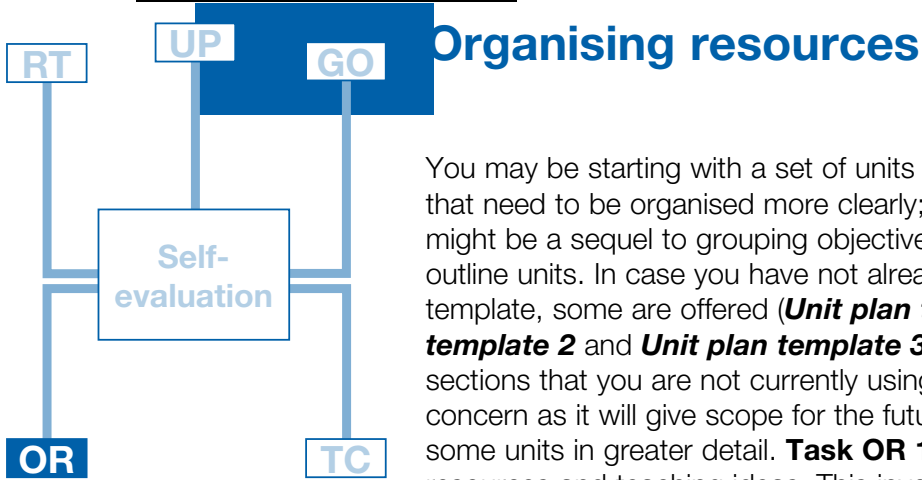
Reconsider the length of each unit, as you consider the time available in each term and the units that need to be covered, and record any changes onto the curriculum map.

Copy onto the calendar information about each unit (code, title and provisional time allocation) from the curriculum map. For completed units, drop brief details from the unit plan into the overview column. Save the completed calendar with an appropriate title, such as *KS4 calendar 2007/08 higher tier*.

Revisiting the calendar

You will need to revisit the calendar on occasions:

- as you develop your teaching units in greater detail and want to restructure or resequence them;
- to take account of in-school changes from year to year;
- to respond to developments in the 14–19 curriculum.



Organising resources

You may be starting with a set of units and teaching resources that need to be organised more clearly; alternatively, your task might be a sequel to grouping objectives, to create a set of outline units. In case you have not already chosen a unit-planning template, some are offered (**Unit plan template 1**, **Unit plan template 2** and **Unit plan template 3**). The template may have sections that you are not currently using. This is not a matter for concern as it will give scope for the future when you may develop some units in greater detail. **Task OR 1** is to match each unit to resources and teaching ideas. This involves:

- incorporating into the unit plan references to resources and any guidance on their use;
- making any appropriate adjustments to tasks or objectives to ensure a good match;
- filing any associated electronic resources in the same folder as the unit plan to which they relate.

You could complete this task in stages, perhaps half-term by half-term.

Task OR 1 Mapping resources onto the unit plans

This task could be shared among members of the department, with colleagues perhaps working in pairs. If you decide to do this you could provide an opportunity for pairs to bring back any issues to discuss at a department meeting. You will also need to ensure that modified units are collated electronically, in one place.

Work with the units one at a time:

- catalogue departmental resources, including equipment for practical tasks and use of ICT;
- add any guidance notes that are relevant to teaching the unit;
- check the provisional allocation of teaching time.

Look at the connection between the resources and tasks and the content objectives. If necessary, adjust the objectives or task so that there is a workable match, to ensure that the balance and coverage of objectives in the strand are maintained.

Reconsider the appropriateness of the ‘using and applying mathematics’ objectives in the light of the way the phases of the unit are developing.

Circulate the draft units or take them back to a departmental meeting. Ask colleagues for any other comments or helpful suggestions from their previous experience of using the activities and teaching the content. Incorporate any useful suggestions before finalising the units.


Save each unit plan in a folder by strand and coding to match a curriculum map or equivalent overview document. For example, *SSM 3 upper higher* is saved in a folder called **SSM unit plans**. Add any related electronic files to the folder, such as teacher or pupil resources.

Rich tasks

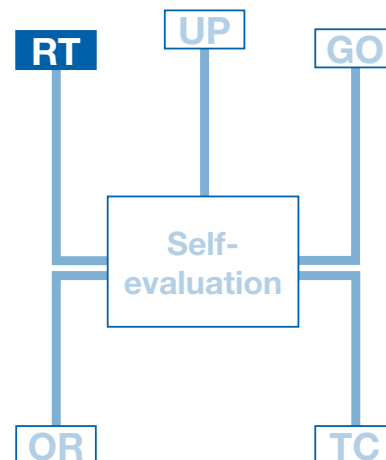
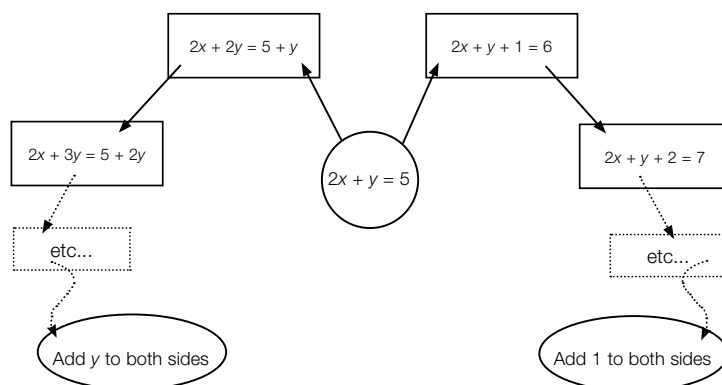
This section helps you to examine the nature of mathematical tasks that you use in the classroom, with a view to enriching the activities included in your unit plans.

Working together on a rich mathematical task

It is important to find new ways to stimulate mathematical thinking and talk in the classroom. This can be initiated by working together, as a department, on a piece of mathematics. It is too easy to assume that all teachers understand and explain concepts in mathematics in a similar way; this can affect the way they help pupils to learn mathematics. One way of opening up this discussion is to work together on a mathematical task, before trialling it in the classroom. You can then reflect on the richness of the task and its potential for promoting mathematical talk and enhancing pupils' learning.

To prepare for this session, you need to identify a rich mathematical task and how you will work on it with the department. The folder  **Ideas for rich tasks** contains many sources, for example, in the *Teaching mental mathematics* planning guides or the *Standards Unit Success for All* materials. You will also find examples in textbooks. Some sources give just the germ of an idea and how it might develop. In other sources the rich task is embedded in a more detailed programme of work. In this latter case, you may need to identify the starting point and consider ways of opening up the development from there. This may be necessary if you want to engage pupils in a deeper and more extended way, as you would, for example, in a mathematical investigation or project.

To illustrate the approach, an example would be 'Clouding the picture', from *Teaching mental mathematics from level 5: algebra*. This is an activity that involves pairs of pupils manipulating algebraic equations in a systematic way. The purpose of the activity is to help them to see how to generate equivalent forms. It is important to start with equations involving two variables such as $2x + y = 5$ so that pupils work with equivalent equations rather than rushing to a solution. Later, they might repeat for an equation in one variable, identifying which forms are easy to solve and which are difficult.



To stimulate a discussion in the department you will need to project the diagram onto a screen or sketch it on a flipchart and describe how to construct another branch from the centre circle.

Task RT 1a Exploring a rich collaborative task

This task is for a mathematics department to work on together.

Model the mathematical task and ask colleagues to work in pairs to generate other branches on the diagram and to generate other diagrams for more complex equations or inequalities. Emphasise the need to be systematic and to generalise what is happening along each branch.

Do not be tempted to discuss this task in general because this will defeat the aim of the session, which is to share mathematical thinking. Work on the wording of your questions and press colleagues for precise and carefully worded responses. Encourage them to consider their thinking carefully at some points during the task.

Following the mathematical discussion:

- reflect on the way in which each of you expressed your understanding and comment on any surprising aspects of the thinking that emerged;
- consider how the structure of the activity or paired work helped each of you to develop your thinking and to share your understanding.

Trialling the rich task

Two aspects of a rich task to consider are:

1. the nature of the task and the mathematics that it addresses;
2. how you present the task and engage the learners.

As you move on to consider the second aspect, it should become apparent that the task has the potential to address several objectives at the same time, in particular, 'using and applying mathematics' objectives. Additionally, the task might well be extended and developed over several lessons, perhaps to become a 'phase' in a unit of work. The appendix to this planning guide, **Some underlying principles**, describes (in section vi) features that characterise a rich learning task in mathematics. It would be helpful to distribute copies of this appendix to members of the department as pre-reading for the tasks.

Task RT 1b Preparing to trial the task with pupils

This task follows immediately from the previous task.

Identify some classes with whom it would be suitable to trial the task.

Sketch out a plan for the first lesson. You might find it helpful to refer to section vi of the appendix, **Some underlying principles**, which lists features of a rich task that you may wish to develop.

If you anticipate that the task will be extended over two or three lessons, discuss ideas as to how it might evolve, but keep plans flexible to allow for different responses among the various classes.

Agree dates and deadlines. Prepare resources and trial the lessons. If there is an opportunity to team-teach a lesson, that would be advantageous. Note down any particular points you want to bring to the review meeting.

Reviewing features of a rich task

Having worked on the mathematics of a task and trialed an approach in the classroom, it is important to reflect on what makes it a rich task and how you might develop its use further. **Task RT 2** refers again to the appendix, **Some underlying principles**.

Task RT 2 Features of a rich task

This task is for a mathematics department to work on together.

Refer to the list of features of a rich collaborative task, as set out in the bullet points of section vi of the appendix, **Some underlying principles**:

Which of these features were evident when you trialed the example of a rich task with pupils?

Take each point in turn, remembering that how the task is presented and developed by the teacher is a critical factor.

Next, consider whether you could develop the task further by:


- adapting or opening it up to incorporate more features;
- addressing other underlying principles, such as the use of effective questioning or paired work.

Briefly discuss the implications of the subsequent paragraph, namely, that a rich task may present pupils with a degree of complexity and challenge to unravel. Colleagues may be able to give examples of exercises in which the questions are too simple and therefore pupils use intuitive methods rather than developing methods that are more generally applicable.

Note also the final sentence, 'Rich tasks also allow **all** learners to find something challenging and at an appropriate level to work on.' The self-adjusting nature of such tasks helps teachers in planning for a range of responses. It can result in unexpected contributions from pupils of various abilities.

In conclusion, it is worth noting that, as teachers, we spend a lot of time engaging pupils in doing mathematics, but perhaps more rarely spend time doing it ourselves! Rather than regarding it as an indulgence, departments that have worked together on some mathematics have found that it stimulates discussion about the subject and sharing of ideas as to how it might be taught. It is worth doing whenever you are looking at rich starting points for units of work.

Types of rich task

You will find many starting points in the folder  **Ideas for rich tasks**. However, it is also worth being aware of different activity types as this can suggest ways of developing your own ideas. Descriptions of some activity types can be found in a number of places, for example:

- Chapter 4 of *Improving learning in mathematics: challenges and strategies (Standards Unit Success for All materials)* describes and illustrates five types of activity (page 16 et al): classifying mathematical objects, interpreting multiple representations, evaluating mathematical statements, creating problems, analysing reasoning and solutions.
- The planning guides in the *Teaching mental mathematics* series describe types of task for each of the strands of mathematics, many examples being similar to those described above.
- The subject exemplification for *Leading in Learning: developing pupils' thinking skills in secondary schools* illustrates ten teaching strategies that can be used in a range of subjects to develop thinking and learning skills. They are: *advance organisers, analogies, audience and purpose, classifying, collective memory, living graphs and fortune lines, mysteries, reading images, relational diagrams and summarising*.

It would be worth drawing colleagues' attention to these and other sources to stimulate their thinking. Adapting an idea to suit their own purposes can be a creative and rewarding process for many teachers. On a suitable occasion, as part of a long-term plan, consider engaging your department in **task RT 3**.

Task RT 3 Types of rich task

The task is for a mathematics department to work on together.

Choose a **type** of task from whatever source is familiar to you. It might be:

- *Standards Unit Success for All materials;*
- *a Teaching mental mathematics planning guide;*
- *Leading in Learning: subject exemplification for mathematics.*

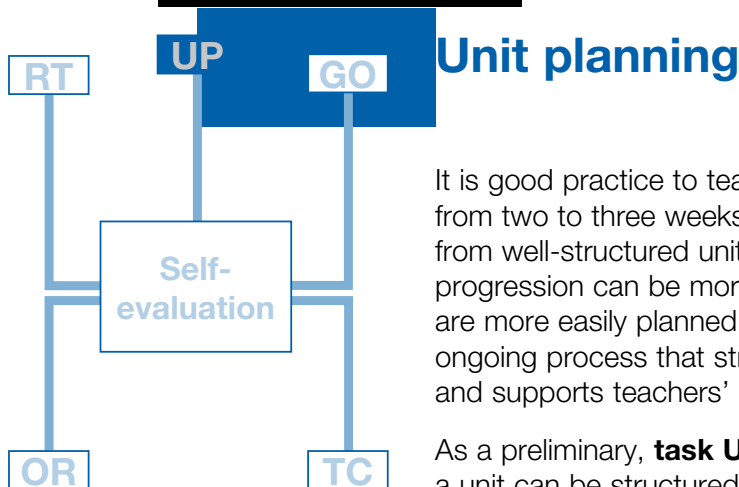
Discuss examples of the type of task that are given in the source, to clarify features of the type of task and what it has to offer.

Sketch out ideas for examples of your own, with teachers perhaps working in pairs for a few minutes. Share ideas and perhaps select one example to develop and trial in the classroom before building it into a unit plan.

Draw attention to other types of task illustrated in the source. These might be re-visited on a future occasion when you are looking to develop fresh ideas.

Revisiting rich tasks

A good unit plan will include one or more rich tasks. As suggested earlier, it is worthwhile identifying suitable tasks and exploring their mathematical potential whenever you are seeking to enhance, revise or develop a unit plan.




It is good practice to teach mathematics in units of work lasting from two to three weeks, or sometimes longer. Lessons planned from well-structured units are sharper and clearer and pupil progression can be more easily recognised. Also, the lessons are more easily planned. Developing effective unit plans is an ongoing process that strengthens the work of the department and supports teachers' professional development.

As a preliminary, **task UP 1** looks at different ways in which a unit can be structured and reshaped to make for effective learning. **Task UP 2** describes how to use rich tasks as the building blocks for structuring a unit of work into distinct phases or sequences of lessons. **Task UP 3** describes the process of evaluating and revising a unit plan and embedding it in your scheme of work.

Working together on the 'story' of a unit

Before starting to develop a particular unit, it helps to take an overview of the possible structures of a unit plan in order to build variety into the units making up the Key Stage 4 scheme of work. It is important to remember that the structure of units can be very different, depending on various factors such as pedagogic style, available resources and the topic to be taught. A scheme of work should contain examples of different unit structures, as this will help to ensure a variety of teaching methods to engage pupils, sustain interest and secure their learning.

Increasingly, pupil interviews are motivating and inspiring subject leaders to reconsider aspects of the scheme of work. Considering the 'story' of the mathematics in a unit of work and finding ways of describing these general structures, can enable colleagues to identify how mathematical topics are commonly taught and whether introducing a different structure for some aspects would better develop the learning.

For the following task you will need to provide copies of **Examples of the story of a unit** (from the folder  **Guidance on planning**) and some example units chosen from your scheme of work or a textbook that you use.

Task UP 1 The structure or 'story' of a unit of work

This task is for a mathematics department to work on together.

Talk about **Examples of the story of a unit**, so that colleagues have an idea about what is different about each of the types suggested. You may notice that the 'prepare–consolidate–apply' example is the story of the Year 9 *Geometric reasoning* mini-pack, which is a unit of work with supporting resources and guidance notes aimed at developing visualisation and proof with Year 9 pupils (DfES 0588/2002 G). This unit would be equally appropriate for the start of Year 10 and has been adapted by some departments to introduce circle theorems.

The structures on the handout are not the only ways of developing mathematical learning across a number of lessons.

- Ask colleagues to work in pairs, to consider a topic that is taught as a unit in your present scheme of work and try to summarise the story of how the mathematics is developed over several lessons.
- Alternatively, they could look at a chapter or chapters from a textbook and do the same.

Allow time for each pair to feed back and then consider what variety is emerging. Questions you might consider are:

- *Do effective units in your scheme of work have a clear structure (not necessarily one of the three types you considered earlier)?*
- *Are some units limited, for example, in building up skills by simple, step-by-step approaches that are not always effective for long-term learning?*
- *Would there be merit in developing a greater variety of units?*

Use the questions to gain a perspective on the structure of units in your scheme of work and how this might help you when developing units in more detail, as suggested in the next task.

Task UP 1 is essentially a preliminary or introductory task for the ongoing one of developing unit plans as described below. However, it is important to step back from the detail and consider the structure of any unit of work that you are developing or evaluating. You may find it helpful to return to the task at some future date in order to reconsider the variety of structures you are offering.

Developing a unit of work

The first step in planning a unit of work can be to examine objectives or choose a rich teaching activity (see *Rich tasks* section, page 19). Whichever comes first the other follows closely behind. Effective planning involves constant interplay between the two. As an activity is worked up and the teaching sequence becomes clearer, the objectives are refined and refocused. As the objectives are checked and the level of challenge becomes clearer, the teaching activity is adjusted, perhaps through ideas for modelling, scaffolding and questioning. This applies whether you are reshaping an existing unit or building up a new unit from scratch.



A very effective approach is to start by working together as a department on a rich mathematical task and then building a unit around it. The intention at this stage is to identify the main activities and sequences of teaching through the unit, not to produce a series of detailed lesson plans. A good starting point could be to choose to develop a unit on an aspect of mathematics that is thought to be 'difficult' or 'dull'. Initially, you might have in mind a particular aspect of mathematics (or aspects that you want to link) and only a general idea about the broad range of objectives that will be included.

There is no universal template for recording a unit plan. If you have an established structure in your school or department, use it. If not, you will need to devise one or adapt an example from the planning toolkit. Keep it simple to start with. Some elements are essential for providing a clear overview from which teachers can plan their lessons: the objectives, the activities and how these are sequenced and organised in terms of teaching approaches and resources. Other elements are helpful and can make the unit more effective but may need to evolve over time, perhaps following an initial plan–teach–review cycle. These could include more detailed and differentiated outcomes, guidance notes on particular features, assessment prompts including probing questions, key words, suggestions for extension and support.

As a way of gaining experience of this approach to unit planning, **task UP 2** is set up to stimulate the development of a unit which addresses a collection of algebra objectives in a new way with Year 10 pupils following the higher tier programme of study. It provides the support to a departmental discussion that could be used to sequence rich tasks, other activities and exercises into the unit.

Preparing for task UP 2

The notes to help to prepare for this task assume you are following the chosen example. You can adapt it for other rich tasks.

- Choose a planning template, perhaps adapted from one of those provided in the planning tool (**Unit planning template 1**, **Unit planning template 2** or **Unit planning template 3**).
- From the  **Guidance on planning** folder, choose between **'The gym problem' (rich task)** and **'Clouding the picture' (rich task)** and select just the rich task, not the complete unit.
- From the  **Curriculum information** folder, you will also need **Algebra KS4 higher tier objectives and key indicators**.
- Make available other tasks and resources used within the department to teach algebra in higher tier Key Stage 4. These ideas can be built around the rich activity and inform other phases of the unit, which together produce a coherent teaching sequence informing perhaps two weeks of teaching in the higher tier scheme of work.

Task UP 2 Developing a unit plan

This task is for a mathematics department to work on together.

- Use your chosen rich task (for example, **'The gym problem'** or **'Clouding the picture'**) to stimulate ideas for a unit plan in which this task would be a key part, forming a phase of the unit. Discuss where you might locate it in the unit and what the other activities and exercises might be.
- Pay careful attention to clustering of objectives that the sequence of teaching activities and resources could help to develop. Draw these objectives from the relevant strand or strands (for example, *Algebra KS4 higher objectives and key indicators*.) The outcome of these discussions should begin to form the essential elements of a unit plan, namely, teaching approaches centred on rich tasks and the associated objectives.
- Agree a template into which to drop the key elements that are the objectives, the activities and how these are sequenced and organised in terms of teaching approaches and resources.


Having agreed the outline of the unit, identify:

- who will type up the details on the unit plan template, including as a minimum:
 - agreed objectives;
 - the sequence of activities;
 - teaching and learning approaches;
 - resources and references;
- which teachers and classes will trial the unit;
- when you will meet to review the outcomes and make revisions.

Evaluating and revising your unit plan

Good developments are sometimes wasted because of pressures of time and the need to move on. It is important to guard against this by building in time to review your unit not too long after it has been trialed. This time will have been well-spent if you are able to make changes in the light of experience and then embed the unit in your scheme of work.

For the first one or two units that you develop, keep the evaluation process simple. Teachers need to know, in advance of the meeting, that they will be discussing how the unit developed in the classroom, any adaptations they made and how the pupils responded. The review could be informed by a range of evidence of impact, which becomes sharper as the process is repeated.

Unit plans vary from one school to another and, again, if this is new territory for the department, it is important to keep things simple. When you decide to involve your department in refining and improving the unit plan template, then the examples of complete units may be helpful to illustrate the approach, or in case additional ideas are needed. (They can be found in the  **Adaptable templates** folder (**Unit plan 2 example algebra 'Clouding the picture'** and

Unit plan 3 example algebra 'The gym problem'). However, remember that they are presented here to generate discussion and only represent the way in which a different department chose to shape the unit, based on their current practice and ideas stimulated by the rich tasks.

Task UP 3 Evaluating and revising your unit plan

This is a task for a mathematics department to work on together.

Allow time for overall reactions as to how trials of the unit went, then review each phase of the unit in turn. Ask teachers to comment on:

- how the teaching developed in the classroom;
- the engagement of pupils and observations about their progress.

Anecdotes should be supported by some evaluation about what provoked the reaction from pupils and, perhaps, that stimulates the need to add a few guidance notes to the unit plan.

Consider:

- any changes that you judge to be needed to the structure, objectives, content or emphasis of the unit;
- any helpful additions to the notes and, if appropriate at this stage, any additional sections you want to include on the unit plan;
- any adaptations needed to create a unit that would be suitable for pupils with different levels of prior attainment.

If working on Year 10 algebra, you may find it helpful to refer to the example units provided.

Decide how the detailed notes will be revised and how you will ensure that the unit is embedded in your scheme of work and will be used in the future.

Finally, reflect on the extent to which:

- the unit plan made lesson-planning simpler;
- the lessons were better-sequenced and the learning was developed in a more coherent way;
- teachers and pupils had a better sense of how the mathematics was developing and connecting.

Revisiting the process of planning and evaluating units of work

The aims of a first run-through of **tasks UP 2** and **UP 3** are to familiarise members of the department with the process of collaborative planning, to iron out some of the practical issues and to start to appreciate the benefits for teaching and learning. To this end, it might be helpful to repeat the process, perhaps in the following half-term. At this stage, your aim is for the department to become comfortable with the process; the choice of topic can be made with that in mind.

When you have established a way of working together it would be helpful to discuss and refine the ongoing process of developing unit plans. One factor to consider is the extent to which you need to differentiate your scheme of work at several levels. When they are built around rich tasks, unit plans can be used

more flexibly with different teaching groups. Pupils can respond at different levels and the teacher can adapt the unit in a more flexible way. This makes it easier to tailor the learning to the needs of pupils and the planning process becomes more manageable for teachers.

A second factor to consider when planning for Key Stage 4 is that most pupils have already encountered quite a wide range of mathematics. You might therefore think about helping pupils to:

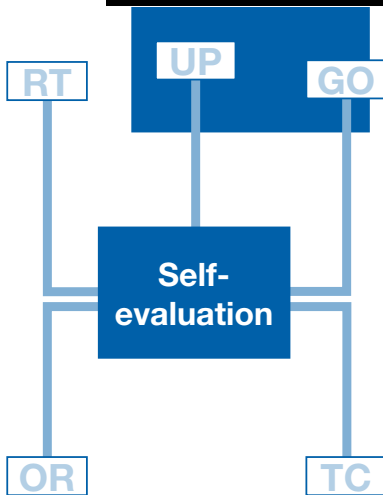
- *make sense of mathematics they have already encountered*, through tasks that encourage them to link ideas that they previously regarded as separate or were poorly understood, and to see the generality and wider applicability of their learning, rather than simply 're-teaching' the topic;
- move into *new areas of learning*.

A third factor is the extent to which you want to refine your unit plan, adapting your template or using blank sections. A basic plan will include objectives, the sequence of teaching activities, teaching and learning approaches, resources and references. To this you might want to add:

- more detailed and differentiated outcomes and links to other aspects of mathematics;
- guidance notes on particular features, suggestions for extension and support;
- key mathematical terms and notation;
- assessment tasks and probing questions;
- links to whole-school priorities such as cross-curricular development of thinking skills (*Leading in Learning*), ICT, literacy or a whole-school focus on assessment for learning.

Having decided on a topic or focus, you might take either of two possible approaches.

- Start with a broad aspect of mathematics you want to consider and identify a rich task (or tasks) that could form the heart of the unit. Explore the potential of these tasks before deciding on the precise objectives that the unit will address.
- Start with a unit for which the objectives are already grouped together, where you think a different approach is needed. Look for rich tasks and a structure that might help you to develop the unit, modifying the objectives if appropriate.



Ongoing developments

When you have tackled tasks in previous sections, take stock, reflect on what has been achieved and think about where to go next. Do not think just about the tasks completed but consider their impact. You might notice:

- a better response from pupils and more pace to the learning;
- a changing departmental culture with more discussion and a broader impact;
- lesson-planning becoming easier for units of work developed collaboratively.

It will be reassuring if you can see that small steps can make a big difference. Best practice in the department starts to surface and spread to other colleagues' lessons. This provides motivation for continuing a collaborative approach and setting out plans for the longer term, based around the self-evaluation diagram on page 5. The tasks outlined in the various sections of this planning handbook provide a sound basis from which to respond to external changes to the curriculum or issues raised through self-evaluation. As well as the judgements of colleagues, take account of the pupils' views, asking them what helps them to learn more effectively.

The main ongoing task is likely to be the development of more detailed unit plans, creating a dynamic scheme of work that continues to evolve. Core tasks are set out in **UP2 Developing a unit plan** and **UP3 Evaluating and revising your unit plan**. You may choose strategically units to redevelop by focusing on:

- the **pupils**:
 - analysing Year 9 performance to identify starting points for Year 10;
 - ensuring that unit plans provide opportunities for pupils to reveal what they know and to build on it;
 - close tracking of progress through the key stage, leading to additional intervention for some pupils to support specific curricular targets;
 - whole-school analysis of obstacles to learning, leading to shared foci, for example, thinking skills or peer and self-assessment;
- the **current scheme of work**:
 - analysing Year 11 pupils' performance to evaluate the relative effectiveness of elements of the existing scheme of work;
 - analysing performance of particular groups of pupils, for example, uptake of mathematics post-16;
- the **mathematics**:
 - analysing big ideas and interconnections in mathematics;
 - incorporating activities that promote mathematical thinking.

Consider which of these foci is the most appropriate to your current circumstances and is likely to help raise standards in mathematics. In the longer term, a good mathematics department will consider all three approaches. Finally, remember that the principles set out in the appendix should underlie all good teaching.

Some underlying principles

A lot is now known from research about what is effective in teaching and learning. The four following pages incorporate a facsimile of a section from the Standards Unit pack M Swann (2005), *Improving learning in mathematics: challenges and strategies*, pages 7 to 10.

The pack was developed initially to improve learning in mathematics in post-16 institutions and builds on existing successful practice by exploring approaches that encourage more active learning through the use of group work, discussion and open questioning. These principles equally underpin the Secondary National Strategy's approaches to improving mathematics pedagogy and can help to inform teachers' planning at any key stage.

Two copies of this multi-media resource are available to all maintained secondary schools. To request a copy visit www.ncetm.org.uk/ and complete the online order form.

References included in extract from *Improving learning in mathematics: challenges and strategies*.

(22) Swan M. (2000), *GCSE mathematics in further education: challenging beliefs and practices*, *The Curriculum Journal*, vol.11, no 2

(24) Swan M. and Green M. (2002), *Learning mathematics through discussion and reflection*, Learning and Skills Development Agency

(9) Black P. and Wiliam D. (1998), *Inside the black box: raising standards through classroom assessment*, King's College, University of London School of Education, London

(3) Askew M. and Wiliam D. (1995), *Recent research in mathematics education 5-16*, Ofsted Reviews of Research, HMSO, London ISBN: 0113500491

(1) Ahmed A. (1987), *Better mathematics: a curriculum development study*, HMSO 104, London

Extract from M. Swan (2005), *Improving learning in mathematics: challenges and strategies*, The Standards Unit, pages 7 to 10.

2 • Some underlying principles

"Lessons are now far more enjoyable for students. I would like to adapt the materials for all teaching sessions."

Helen Johnson
Solihull College

These resources, we believe, are designed to encourage 'best practice' – research-based and with proven effectiveness. The activities have undergone several rounds of observed trials in a variety of learning contexts: colleges, schools, prisons and work-based learning environments. The feedback has been used to refine the materials. We have also built on the research and development work of earlier research projects in further education [22, 24].

The resources by themselves, however, do not guarantee effective teaching. This is entirely dependent on how they are used. When using them, therefore, we suggest that you try to implement the following principles that should underlie all good teaching.

(i) Build on the knowledge learners bring to sessions

"I did not anticipate the levels of skill and understanding that I found."

Marian Ebrey
HMP Hewell Grange

Effective teaching assumes that learners do not arrive at sessions as 'blank slates', but as actively thinking people with a wide variety of skills and conceptions. Research shows that teaching is more effective when it assesses and uses prior learning to adapt to the needs of learners [9]. This prior learning may be uncovered through any activity that offers learners opportunities to express their understanding. It does not require more testing. For example, it can take the form of a single written question given at the beginning of a session to set the agenda for that session and elicit a range of explanations. These responses may then be used as a starting point for discussion.

(ii) Expose and discuss common misconceptions

Research has shown that teaching becomes more effective when common mistakes and misconceptions are systematically exposed, challenged and discussed [3]. The sessions described here typically begin with a challenge that exposes learners' existing ways of thinking. Cognitive conflicts occur when the learner recognises inconsistencies between existing beliefs and observed events. This happens, for example, when a learner completes a task using more than one method and arrives at conflicting answers. Activities are carefully designed so that such conflicts are likely to occur. Research has shown that such conflicts, when resolved through reflective discussion, lead to more permanent learning than conventional, incremental teaching methods, which seek to avoid learners making 'mistakes'.

"I always asked a lot of questions and thought they were really helpful. I now realise these sometimes closed discussion down or cut them off. Now I step back and let the discussion flow more. This is very hard to do."

Mandy Cave
Carmel College

(iii) Develop effective questioning

There have been many studies of teachers' questioning. Typically, most questions are low-level, testing the ability of learners to recall facts and procedures. Such questions are also called 'closed', meaning that they permit just one single correct response. Fewer questions promote higher-level reflective thinking, such as the ability to apply, synthesise or explain. Such questions are called 'open' because they invite a range of responses. The research evidence shows that a variety of lower-level and higher-level open questions is much more beneficial than a continuous diet of closed recall questions.

A second finding is the importance of allowing time for learners to think before offering help or moving on to ask a second learner. Studies have shown that many teachers wait for less than one second. Longer 'wait times' are associated with significantly improved achievement [3].

(iv) Use cooperative small group work

Many learners think that learning mathematics is a private activity. They frequently enter post-16 education under-confident and reluctant to discuss difficulties. It is therefore essential that a supportive and encouraging atmosphere is created in the learning environment. It is the teacher's responsibility to ensure that everyone feels able to participate in discussions and this is often easier in small group situations. It is interesting to consider why small group activities are used less often in mathematics than in other subject areas, where they are commonly used to good effect. One possible reason might be the lack of suitable resources. We hope that this resource will help to fulfil this need.

There is now general agreement in research that cooperative small group work has positive effects on learning, but that this is dependent on the existence of shared goals for the group and individual accountability for the attainment of these goals. It has also been seen to have a positive effect on social skills and self-esteem [3].

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

(v) Emphasise methods rather than answers

Often we find that learners focus more on obtaining a correct answer than on learning a powerful method. They often see their task as 'getting through' an exercise rather than working on an idea.

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Completion is seen as more important than comprehension. In these resources, we do not concern ourselves with whether or not learners complete every task, but instead we try to increase their power to explain and use mathematical ideas. Learners may work on fewer problems than in conventional texts, but they come to understand them more deeply as they tackle them using more than one method.

(vi) Use rich collaborative tasks

Rich tasks:

- are accessible and extendable;
- allow learners to make decisions;
- involve learners in testing, proving, explaining, reflecting, interpreting;
- promote discussion and communication;
- encourage originality and invention;
- encourage 'what if?' and 'what if not?' questions;
- are enjoyable and contain the opportunity for surprise. [1]

Textbooks often assume that we should begin topics by solving simple questions and then gradually move towards more complex questions. While this may appear natural, we find that learners tend to solve simple questions by intuitive methods that do not generalise to more complex problems. When the teacher insists that they use more generalisable methods, learners do not understand why they should do so when intuitive methods work so well. Simple tasks do not motivate a **need** to learn.

Rich tasks also allow **all** learners to find something challenging and at an appropriate level to work on.

"I used the teaching methods in other lessons. This proved very effective. Learners who find maths difficult benefit from their peers within the groups."

Steve Woodward

"Even those who sit back were drawn into the activities."

Sue Sealey

S&B Training Ltd.

(vii) Create connections between mathematical topics

A common complaint of teachers is that learners find it difficult to transfer what they learn to similar situations. Learning appears compartmentalised and closely related concepts and notations (such as division, fraction and ratio) remain unconnected in learners' minds. In this resource, we have therefore included 'linking activities' that are particularly designed to draw out connections across mathematical topics. The index refers to sessions as 'mostly number' or 'mostly algebra' in order to reflect these connections.

(viii) Use technology in appropriate ways

While new technologies have transformed our lives in many ways, they have had less impact inside most mathematics classrooms. They do offer us the opportunity to present mathematical concepts in dynamic, visually exciting ways that engage and motivate learners. In the sessions that follow, we have sought to illustrate some of this potential through the provision of a few computer 'applets'; these are small pieces of purpose-built software that are designed to be very easy to use.

This publication is available for download from:

www.standards.dfes.gov.uk

www.teachernet.gov.uk/publications

Copies may be available from:

DfES Publications

Tel: 0845 60 222 60
Fax: 0845 60 333 60
Textphone: 0845 60 555 60
e-mail: dfes@prolog.uk.com

Ref: 00049-2007BKT-EN

© Crown copyright 2007

Produced by the
Department for Education and Skills
www.dfes.gov.uk

The content of this publication may be reproduced free of charge by schools and local authorities provided that the material is acknowledged as Crown copyright, the publication title is specified, it is reproduced accurately and not used in a misleading context. Anyone else wishing to reuse part or all of the content of this publication should apply to OPSI for a core licence.

The permission to reproduce Crown copyright protected material does not extend to any material in this publication which is identified as being the copyright of a third party.

Applications to reproduce the material from this publication should be addressed to:

OPSI, Information Policy Team,
St Clements House,
2–16 Colegate, Norwich NR3 1BQ
Fax: 01603 723000
e-mail: hmsolicensing@cabinet-office.x.gsi.gov.uk

Disclaimer

The Department for Education and Skills wishes to make it clear that the Department and its agents accept no responsibility for the actual content of any materials suggested as information sources in this document, whether these are in the form of printed publications or on a website.

In these materials icons, logos, software products and websites are used for contextual and practical reasons. Their use should not be interpreted as an endorsement of particular companies or their products.

The websites referred to in these materials existed at the time of going to print.

Tutors should check all website references carefully to see if they have changed and substitute other references where appropriate.