Department for Education

TALIS Video Study and professional development

Research summary

November 2020

Jenni Ingram and Naomi Sani: University of Oxford and Education Development Trust

Government Social Research

Acknowledgments

The Teaching and Learning International Survey (TALIS) Video Study was led and managed in England by Education Development Trust in partnership with the Department of Education, University of Oxford. The England Team consisted of Elaine McCann, England TALIS Video Study National Project Manager, Naomi Sani, Education Development Trust Mathematics Expert, Anna Riggall, Education Development Trust Head of Research and TALIS Video Study Project Director, Jenni Ingram, University of Oxford Observation Expert, and Ariel Lindorff, University of Oxford Video Master Rater. Our thanks also extend to the team of Education Development Trust school recruiters, data collectors, videographers and data entry personnel, the team of University of Oxford video and artefact raters and analysts, and to Lowri Buckley, Data Collection Co-ordinator.

We would particularly like to thank the many mathematics teachers and their students who participated in the TALIS Video Study in England. They opened up their classrooms to us, enabling the better understanding of the nature of, and variation within, the teaching of mathematics in secondary schools.

The TALIS Video Study is an international study involving eight countries and economies working together. We are grateful to all the members of the TALIS Video Study International Consortium whose hard work and support led to the successful design and implementation of the study. We would also like to thank colleagues at the Organisation for Economic Co-operation and Development (OECD) for their constant support.

Disclaimer

Contents

List of figures

What is the TALIS Video Study?

The Teaching and Learning International Survey (TALIS) Video Study, run by the Organisation for Economic Co-operation and Development (OECD), provides new information on the teaching of mathematics in secondary schools across the eight participating countries/economies: Biobío, Metropolitana and Valparaíso (Chile), Colombia, England (UK), Germany[1], Kumagaya, Shizuoka and Toda (Japan), Madrid (Spain), Mexico, and Shanghai (China). Later in the report these regions are referred to by country name only. Data collection was conducted in England between October 2017 and October 2018. The TALIS Video Study complements the existing TALIS and Programme for International Student Assessment (PISA) studies by providing additional evidence on classroom processes, drawing on direct measures of classroom teaching and instruction. By looking directly into the classroom through video-recorded observation and lesson artefact collection, the TALIS Video Study addresses some of the limitations of using teacher self-reported data. The study aims to provide new and rich information about classroom processes and practices and contributes to current understanding of how they are related to student learning and other outcomes. The Department for Education (DfE) commissioned Education Development Trust and the University of Oxford to conduct the TALIS Video Study in England.

[1] Germany refers to a convenience sample of volunteer schools.

What did the study involve?

The TALIS Video Study required two lessons from the quadratic equations unit of work to be filmed. One of the lessons occurred during the first half of the unit and the second lesson occurred later in the unit.

The TALIS Video Study also required all participating teachers and students to complete two questionnaires, one at the beginning of the unit, and one at the end.

Students took a pre-test which focused on their general mathematics knowledge two weeks before the start of the unit of work that included quadratic equations. They then took a post-test within two weeks of the conclusion of the unit of work. The post-test had a narrower focus than the pre-test, to provide more precise measures of students' knowledge and understanding of quadratic equations.

In addition to the tests, questionnaires, and the videos of lessons, artefacts from those lessons, and the lessons that followed were also collected. These artefacts included lesson plans, handouts and worksheets, textbook pages, visual materials such as the projected slides shown, and/or any homework set, where they were available.

The videos of teaching were analysed by a team of raters, trained to look for comparable aspects of teaching – so they were all looking for the same behaviours and practices no matter what country the videos were from. Their ratings (of the video and artefact data within the study) were measured on a scale of 1 to 3 or 4, where a higher rating represented higher quality or higher prevalence of particular behaviours. A framework was designed specifically for the study to guide this. It focused on practices known from previous research to be related to student achievement as well as practices that were highly valued by the Mathematics Experts in each of the participating countries.

After the ratings were complete, analyses were possible. These analyses explored the variation, frequency, and prevalence of teaching practices and the relationships between teaching practices, student outcomes, and teachers and students' perceptions of learning quadratic equations. The analysis focused on six themes (or domains) of teaching:

- Classroom management
- Social-emotional support
- Discourse
- Quality of subject matter
- Student cognitive engagement
- Assessment of and responses to student understanding

These themes were analysed individually and in different combinations that focused on teaching practices specific to mathematics, or teaching practices relevant to all teaching.

Documenting what has been learned from the TALIS Video Study

The OECD is releasing data from the TALIS Video Study as part of two international reports. One is a policy-focused report[2] entitled *Global Teaching InSights A Video Study of Teaching*[3] documenting the findings from all eight participating countries. The other is a technical report detailing how the study was undertaken[4].

There are also three DfE published reports:

- The first report focuses on the findings specifically from the England study, complementing the OECD reports by providing a more focused and detailed analysis of the results in England and analysing differences within England across teachers and classes[5].
- The second report documents the findings from a qualitative analysis of the videos, focusing on interesting practices measured by the study in England[6].
- The third report is a technical report detailing how the study was conducted within England following the study protocol as set by the international consortium and noting the approved deviations[7].

A suite of reports

All reports are written with the intention of reaching a wide audience, including policy makers, schools, and practitioners. A suite of research summaries based on the full reports have also been produced specifically with schools and practitioners in mind. These reports contain the same material condensed and packaged for faster reading.

This report is one of three produced specifically for practitioners. It sits alongside the full England report, offering a short version of the summary points and key findings. There are also similar summary reports documenting the findings from the OECD's international policy report[8] and a summary of the England country analysis[9].

[2] OECD (2020a)
[3] Also referred to as the TALIS Video Study
[4] OECD (2020b)
[5] Ingram & Lindorff (2020)
[6] Ingram & Gorgen (2020)
[7] McCann et al. (2020)
[8] Riggall et al. (2020)
[9] Ingram et al (2020)

Table 1: Summary of available reports

Report	Publisher
Global Teaching InSights: Policy Report	OECD
Global Teaching InSights: Technical Report	OECD
TALIS Video Study: National Report	DfE
TALIS Video Study: Technical Report	DfE
TALIS Video Study: Case studies of mathematics teaching practices	DfE
TALIS Video Study: National Summary Report	DfE
TALIS Video Study: International Summary Report	DfE
TALIS Video Study and Professional Development	DfE

Source: Education Development Trust

What the TALIS Video Study means for teacher professional development

Teachers of mathematics, and especially novice teachers, are often influenced by their school's department "style" in which they are immersed, and can easily become moulded, or "cloned" in this style[10]. Mathematics teachers are influenced by their school and department contexts which generally leads to a dominant approach towards teaching and learning mathematics within a school[11]. This can be particularly problematic within a mathematics department if there is a shortage of specialist knowledge and expertise.

This report is intended to be a catalyst to promote reflection on teaching and learning approaches in schools, faculties and departments. It presents some of the insights gained from the TALIS Video Study with a view to:

- Stimulating reflection on practice both at an individual and departmental level
- Offering alternative approaches that can be used in the teaching of mathematics
- Challenging assumptions about how mathematics is taught and what works

This report invites teachers, heads of departments, educators and school leaders to reflect on:

- Understanding the mathematics
- Making connections to the real world, to other mathematics topics, and within the topic of quadratic equations
- The use and role of technology in the classroom and in teaching the topic of quadratic equations
- The role of multiple approaches to and perspectives on reasoning within the topic
- Engaging students in cognitively demanding material
- The use of patterns and generalisations to aid student understanding
- Encouraging students to persist through error
- Teaching students with different levels of prior attainment

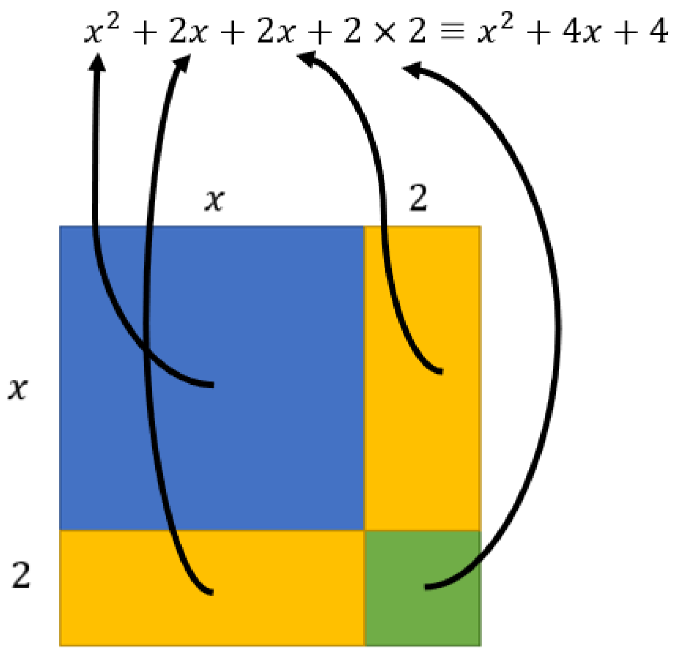
[10] Burghes & Robinson (2010), Xu & Clarke (2018)
[11] Geiger et al. (2017)

Understanding the mathematics

One focus of the analysis of the videos in the TALIS Video Study was on students' understanding of the rationale underlying procedures and processes. That is, whether students understand *why* or *how* a procedure works, or what makes that procedure or process appropriate in particular situations. It is more than the students knowing what steps are involved in a procedure, which is understanding *what* the procedure is.

There are lots of different ways in which students' understanding can be observed during mathematics lessons. One way is through the questions that students ask, such as asking why particular steps are needed, or why one solution strategy (such as completing the square) is more appropriate than another (such as factorising) for some types of quadratic equation. Another way is through the explanations and reasons students give that underpin the processes or procedures they have used when working on a task. This understanding of the rationale can also be seen when students make connections between visual and algebraic representations of the same equation, when students can not only identify the connections between the parts of the image or graph and the relevant parts of the algebraic representation, but also how these parts are related to the process or procedure as a whole. For example, Figure 1 illustrates some of the connections a student might describe or make by pointing and gesturing, that would suggest that the student was understanding how the algebraic and the geometric representations were connected.

Figure 1: Demonstrating the rationale by making connections between representations



Understanding the mathematics is more than students' listening to a teacher explaining the rationale, or students' remembering a procedure or process; there needs to be some evidence that the students themselves were engaging with *why* or *how*. In the study, one way in which this could be seen was through the idea-based discussions that students and teachers engaged with in the videos. One example was where teachers had asked students to work on a common problem in different ways and the discussion focused on comparing and contrasting the different processes the students had used and why they had made the steps they had made. Examples from the videos of these types of activity can be found in the accompanying case studies[12].

One way in which teachers can support students in engaging with the rationale behind a process or procedure is by asking students to think about or reflect upon their own thinking, or through metacognitive activities. In the TALIS Video Study, teachers would ask students to think about why they had taken particular steps when working through a procedure, or why they had chosen a particular solution strategy for a particular question. Teachers can also model this metacognition by talking through their thinking as they work through an example on the board.

In Section 6 of the case studies, some examples are given of how students attended to the rationale behind their work on quadratic equations. These case studies illustrate another way in which teachers in England provided opportunities for students to focus on

[12] Ingram & Gorgen (2020)

how and *why*, which was through identifying errors in examples of other students' work, or their own work. By asking students to not only identify what errors have been made but also to work out why these errors might have been made, teachers can support students in making sense of the reasoning behind the process being used.

Making connections

The TALIS Video Study looked at how mathematics teachers made connections in a variety of ways. In analysing the videos of teaching, the focus was on connections between ideas, procedures, perspectives, representations, or equations. The topic of quadratic equations (and mathematics in general) is rich in potential connections. Many of the teachers made connections between solutions of quadratic equations and roots of quadratic functions, and some teachers also used a graphical representation to then develop the cases where a quadratic equation may have two distinct roots, one repeated root, or no real roots, and connecting this to the discriminant in the quadratic formula. However, other types of connection were far less common and less explicit in the videos of mathematics teaching from England.

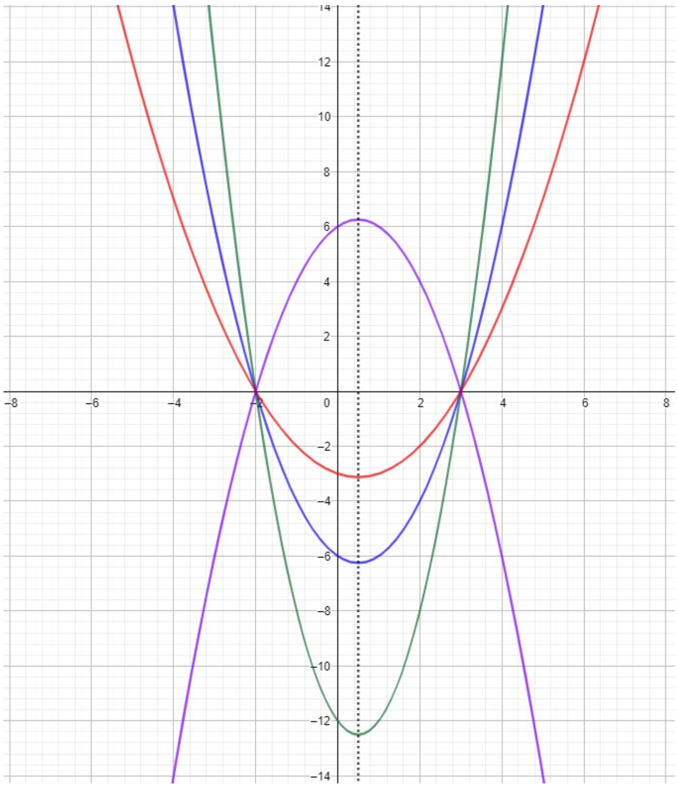
Connections between quadratic functions and quadratic equations were relatively common in English classrooms compared to other participating countries in the TALIS Video Study (further details on participating countries can be found in the International Summary and OECD policy report)[13]. A relatively common type of connection, though still rare, was between quadratic equations and the areas of rectilinear shapes or circles. The case studies[14] offer some examples of how finding areas of rectangles was used for making connections to factorising quadratic expressions, or to solving contextual problems where students needed to solve a quadratic equation in order to work out the length and width of a rectangle when given its area. These area models were also often used by teachers when introducing completing the square as a solution method (see below).

However, there were examples of other connections that were made even less often. One connection that was made was between quadratic equations and linear equations where students' knowledge of solving linear equations was explicitly built upon when considering quadratic equations. This included comparing and contrasting the similarities and differences in the algebraic manipulations involved. Other teachers used this connection to emphasise the idea that linear and quadratic equations are both examples of equations, or more generally polynomials (that is, emphasising that quadratic equations are polynomials of degree two). Another connection some teachers made was between other types of polynomials, such as between quadratic and cubic equations. Other teachers made a connection by considering one particular quadratic equation as an example of a member of a family of quadratic functions – for example, by drawing several parabolas with the same two roots and the same line of symmetry to show that there are several quadratic functions that could have these properties. That is, where

[13] Riggall et al. (2020), OECD (2020a)
[14] Ingram & Gorgen (2020)

$f(x) = n(x - a)(x - b)$, which has roots $x = a$ and $x = b$ for different values of n as in Figure 2**.**

Figure 2: Family of quadratic functions with the same roots and line of symmetry



Connections can also be made between different solution approaches. This can be using the limitations of solving by factorising to introduce a strategy that works for all quadratic equations, such as by completing the square. Or it could be a more substantial and cognitively demanding connection such as using the process of completing the square to derive the quadratic formula.

Making connections between mathematical representations, procedures, and concepts can support all students in developing a rich network of mathematical knowledge. This helps develop depths of understanding and is relevant for students of any prior attainment level. It is not sufficient to include a range of representations, procedures, or concepts; appropriate connections need to be made and used between them[15]. In the TALIS Video Study, connections were only measured if they were made explicit by either the teacher or the students. Yet in many classrooms, connections like those discussed above and in the case studies[16] were left implicit, and many students need support in making many of these connections themselves. When students are able to make clear, explicit and specific connections between different aspects of the mathematics, they can

[15] Nunes, Bryant, & Watson (2009)
[16] Ingram & Gorgen (2020)

develop deeper understanding[17]. Although it is possible to teach almost any subject or topic as a series of disconnected facts, theorems, procedures and processes, very few students would ever then make much sense of the mathematics.

Effective teaching emphasises connections between and within different aspects of mathematics[18]. Providing all students with the opportunities to make connections within and across topics in mathematics, to think mathematically, and to have access to high quality mathematics, are foundational aspects of high-quality instruction. Effective mathematics teaching is therefore considered to be that which includes encouraging cognitive reasoning, using rich collaborative tasks, and creating connections between topics both within and beyond mathematics[19].

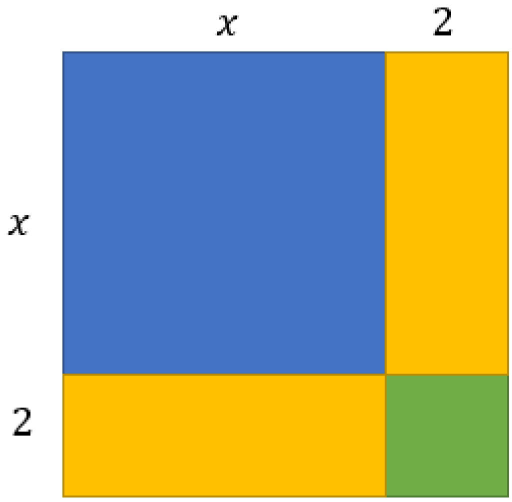
Connections between representations

Representations can be used to expose the mathematical structures that underpin work with quadratic equations. Factorising and expanding quadratic expressions and equations shifts between two different algebraic representations of the same expression or equation. A quadratic function can be expressed algebraically or presented visually as a graph. These different representations offer opportunities for mathematics teachers to make connections between the representations and the mathematical ideas they are representing[20]. Different representations can highlight some aspects more clearly than others, but some representations can also limit what can be seen or the connections that can be made.

Teachers frequently used images of squares when introducing solving quadratic equations by completing the square, making a connection between the geometric representation of a square, and the algebraic representation of x^2 or $(x+2)^2$ illustrated in Figure 3.

[17] Henderson et al. (2018)
[18] Askew et al. (1997), Hodgen et al. (2019)
[19] Askew et al (1997), Boaler (2009), Carter (2015), Mason & Spence (1999)
[20] Ingram & Gorgen (2020)

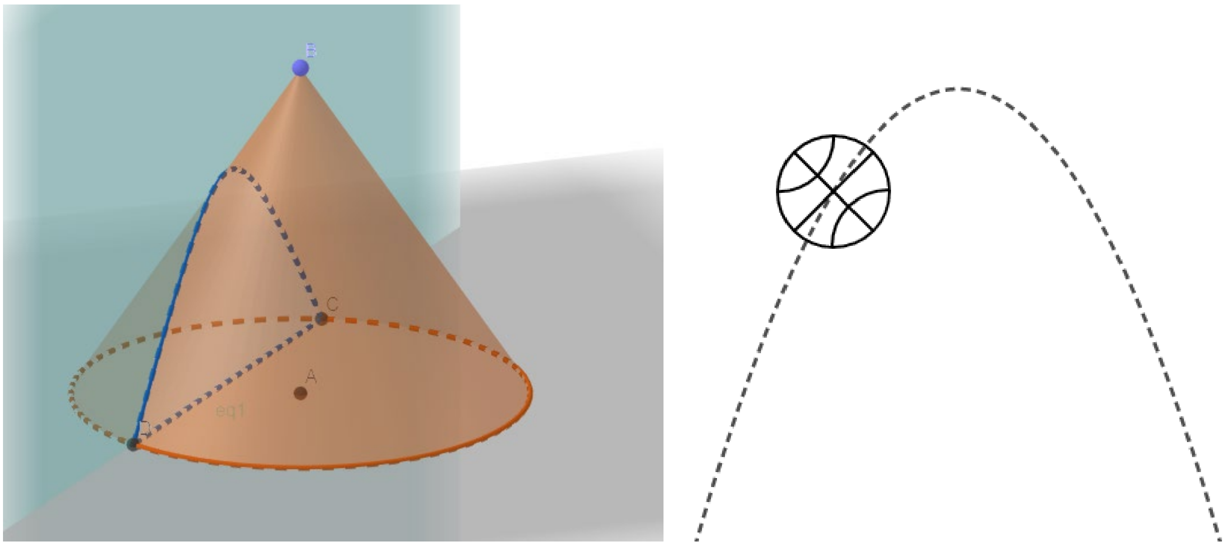
Figure 3: Geometrical representation of $(x + 2)^2$



Physical objects or models were very rarely used in any of the participating countries. In contrast, connections between different representations of quadratic equations, including tables, graphs, and equations, were more often seen in classrooms in England, particularly connections between the equations of quadratic functions and their graphs. Less common, and seen in fewer than half of the lessons, were connections to other mathematical topics, such as quadratic sequences. Students were also asked through questionnaires about whether their mathematics teacher connected or related quadratic equations to other topics or to their prior knowledge. In general, students reported that this happened less often than their teachers did.

In all countries, teachers and students rarely made connections among different aspects of mathematics. Similarly, students had limited opportunities to connect their learning to real-world contexts. Many of the more obvious connections to real-world contexts are generally not encountered until later in students' mathematics education, such as when considering quadratic functions as best-fit models for real-world data or considering quadratic curves as a type of conic section, as illustrated in Figure 4.

Figure 4: Contexts for quadratic functions:
A parabola as a type of conic section and the path of a thrown ball



In many cases, opportunities for students to connect classroom mathematics to real-world contexts can enhance their understanding[21], yet there were no or almost no real-world connections in the lessons observed during this study – with England the least likely country to include these.

Technology

Technology can help promote the use of connections as it can provide an opportune portal – a gateway to access relevant content and contexts. Teachers were seen to use technology in the TALIS Video Study but generally solely for communication purposes, as a whiteboard or blackboard might be used. In general, students in the typical classroom in all countries almost never used technology (non-graphing calculators, graphing calculators, tablets, cell phones, and computers) and only a very small proportion of classrooms in England (8%) ever used software such as that designed to carry out simulations or interactive graphing tasks. More ambitious use of technology could provide opportunity for greater connections to be foregrounded and enhance cognitive demand and engagement[22]. Examples of mathematics teachers using technology to make connections or to generalise from the mathematics given can be found in the case studies[23].

When making the connection between solving quadratic equations and the graphs of quadratic functions, several teachers used dynamic geometry software. This software

[21] Boaler et al (2000), Vappula & Clausen-May (2006)
[22] Clark-Wilson & Hoyles (2017)
[23] Ingram & Gorgen (2020)

enabled them to vary the coefficients or the constant term in a quadratic equation or function to show how this affects the solutions. This dynamic representation also enables students to "see" that coefficients do not necessarily need to be integers. Translations of quadratic functions were also used to illustrate the relationships between the values in a quadratic equation given in completed square form and specific features of the graph, such as the minimum or maximum point or the location of the line of symmetry. Treating $x^2 + 4x + 3 = 0$ as the intersection of two curves, $y = x^2 + 4x + 3$ and $y = 0$ can also lead students to understanding solutions of a broader range of equations from graphs such as solving $x^2 + 3x + 4 = 4x + 5$ by drawing $y = x^2 + 3x + 4$ and $y = 4x + 5$ and identifying the points of intersection. Technology-enhanced learning can benefit all learners – regardless of prior attainment level. In fact, the power of technology can make the mathematics more accessible for *all* students, by allowing the technology to complete some of the more routine aspects of calculations for example.

Many calculators, websites, and widely available software can solve quadratic equations. Using these facilities can shift the focus from the solutions to these equations to the processes involved. For example, the most recent version of OneNote will both solve a quadratic equation and show the steps involved in the process using different possible methods, as shown in Figure 5.

Figure 5: The steps OneNote shows for solving $x^2 + 5x + 3 =$

Steps for Completing the Square

- Quadratic equations such as this one can be solved by completing the square. In order to complete the square, the equation must first be in the form $x^2 + bx = c$.
  $x^2 + 5x + 3 = 0$

- Subtract 3 from both sides of the equation.
  $x^2 + 5x + 3 - 3 = -3$

- Subtracting 3 from itself leaves 0.
  $x^2 + 5x = -3$

- Divide 5, the coefficient of the x term, by 2 to get $\frac{5}{2}$. Then add the square of $\frac{5}{2}$ to both sides of the equation. This step makes the left hand side of the equation a perfect square.
  $$x^2 + 5x + \left(\frac{5}{2}\right)^2 = -3 + \left(\frac{5}{2}\right)^2$$

- Square $\frac{5}{2}$ by squaring both the numerator and the denominator of the fraction.
  $$x^2 + 5x + \frac{25}{4} = -3 + \frac{25}{4}$$

- Add -3 to $\frac{25}{4}$.
  $$x^2 + 5x + \frac{25}{4} = \frac{13}{4}$$

- Factor $x^2 + 5x + \frac{25}{4}$. In general, when $x^2 + bx + c$ is a perfect square, it can always be factored as $\left(x + \frac{b}{2}\right)^2$.
  $$\left(x + \frac{5}{2}\right)^2 = \frac{13}{4}$$

- Take the square root of both sides of the equation.
  $$\sqrt{\left(x + \frac{5}{2}\right)^2} = \sqrt{\frac{13}{4}}$$

- Simplify.
  $$x + \frac{5}{2} = \frac{\sqrt{13}}{2}$$
  $$x + \frac{5}{2} = -\frac{\sqrt{13}}{2}$$

- Subtract $\frac{5}{2}$ from both sides of the equation.
  $$x = \frac{\sqrt{13} - 5}{2}$$
  $$x = \frac{-\sqrt{13} - 5}{2}$$

0

Steps Using the Quadratic Formula

- All equations of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. The quadratic formula gives two solutions, one when $\pm$ is addition and one when it is subtraction.
  $x^2 + 5x + 3 = 0$

- This equation is in standard form: $ax^2 + bx + c = 0$. Substitute 1 for a, 5 for b, and 3 for c in the quadratic formula, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
  $$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3}}{2}$$

- Square 5.
  $$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 3}}{2}$$

- Multiply -4 times 3.
  $$x = \frac{-5 \pm \sqrt{25 - 12}}{2}$$

- Add 25 to -12.
  $$x = \frac{-5 \pm \sqrt{13}}{2}$$

- Now solve the equation $x = \frac{-5\pm\sqrt{13}}{2}$ when $\pm$ is plus. Add -5 to $\sqrt{13}$.
  $$x = \frac{\sqrt{13} - 5}{2}$$

- Now solve the equation $x = \frac{-5\pm\sqrt{13}}{2}$ when $\pm$ is minus. Subtract $\sqrt{13}$ from -5.
  $$x = \frac{-\sqrt{13} - 5}{2}$$

- The equation is now solved.
  $$x = \frac{\sqrt{13} - 5}{2}$$
  $$x = \frac{-\sqrt{13} - 5}{2}$$

Summary

Mathematics is a coherent and connected subject and there are many opportunities to make connections between topics, representations and using technology. This section has identified several opportunities for connections to be made within the topic of quadratic equations. These types of connections are also relevant for much of the mathematics that is taught in schools. The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

- What connections can be made within the topic, and to other topics that students will have met previously?
- What connections could be made between different representations of the same mathematical idea?
- How could digital technology be used to make connections more visible?
- Which connections need to be made explicit and which connections need to be made by the students themselves?
- What authentic and meaningful real-world or cross-curricular connections could be used?

Multiple approaches to and perspectives on reasoning

Students need to be able to choose between different mathematical strategies[24] and this is particularly the case when solving quadratic equations, where most students are taught at least four different solution strategies. Factorising only works for some quadratic equations. Solving equations by completing the square or by using the quadratic formula works for all quadratic equations, but draws attention to different features and structures within these equations. Solving quadratic equations by finding the roots of the associated quadratic function can help students make the connections between quadratic equations and quadratic functions, and to see that not all equations have real solutions, and not all equations have integer (or even rational) solutions.

In England, students rarely used, or were *asked* to use, two or more procedures or reasoning approaches to solve problems, or types of problem. Even rarer still were discussions where students compared different solution strategies. In the case studies[25] there are a few examples from lessons in England where the students developed different ways of solving a problem which were shared and compared publicly, determining and examining that these different ways lead to the same solution.

Comparing different solutions strategies can support students in thinking about mathematics from different perspectives. For example, there are two common ways of thinking about how to solve quadratic equations by completing the square. For the equation $x^2 + 4x + 2 = 0$ the first steps when using the completing the square method could be $x^2 + 4x + 4 = 2$ then $(x + 2)^2 = 2$ which emphasises the idea of making one side a complete square, which is easily connected to the geometric representations discussed above. Alternatively, the first step could be $(x + 2)^2 - 4 + 2 = 0$ which focuses on working with the x^2 and x coefficients and algebraic manipulation. Discussing the similarities and differences between these approaches, and the contexts in which the different methods are useful, is another way in which connections between strategies, representations, and processes can be made.

Summary

Sharing, discussing, and contrasting multiple approaches or perspectives can support students in seeing the connections within the mathematics, and can also support students in understanding why a process or procedure involves particular steps and in which contexts these processes or procedures are useful. The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

[24] Henderson et al. (2018)
[25] Ingram & Gorgen (2020)

- What opportunities are there for students to compare and contrast different approaches or strategies to the same questions or problems?
- What representations could be used to support students to consider different perspectives on the reasoning involved within a topic?

Students engaging in cognitively demanding mathematics

One focus of the TALIS Video Study was how students were given opportunities to cognitively engage with the mathematics within the topic of quadratic equations. This included looking at what solution strategies and subtopics students were taught, how much emphasis was placed on each strategy or subtopic, and the nature of questions and tasks students were asked to engage with in the lesson videos and in the lesson materials.

In England, many classrooms, although quite similar to one another in many ways, differed considerably on levels of instruction, especially in terms of *cognitive engagement* and the *quality of subject matter,* that is, the nature of the mathematical tasks and activities with which the students were engaging. These findings suggest significant "within-country" variation with some teachers making more use of cognitively demanding tasks, patterns and generalisations and different perspectives than others. In only 8% of classes in England were students asked to engage with cognitively demanding tasks, that is, tasks that involved students engaging in analysis, creation, or evaluation work that was cognitively rich and required thoughtfulness. In particular, there was variation in the experiences of students with lower prior attainment, where some teachers used a range of cognitively demanding tasks within the topic with classes that had lower average prior attainment, but other teachers used none at all[26].

Examples include detailed examinations or explorations of the features and relationships among mathematical procedures, processes, or ideas, formulating or inventing a way to solve a problem, and determining the significance or conditions of a mathematical idea, topic, representation, or process. These tasks and activities go beyond recall or the rote application of procedures. The cognitive demand of the subject matter is based on how students are being asked to engage with the mathematics, rather than the difficulty of the mathematics itself. This could include activities such as students analysing the steps in a worked example and annotating the example with what these steps were and why they were made. Evaluating which solution strategy would be most suitable for solving a particular equation, or using two strategies and evaluating which one was more efficient, would also count as cognitively demanding activities.

In the case studies[27] there are examples of some of the cognitively demanding tasks used by teachers from England. One of these examples asks students to create their own equivalent expressions. This idea of asking students to create their own examples is easily extendable to other aspects of quadratic equations. For example, teachers can ask students to create an example of a quadratic equation that is not factorisable, or an

[26] Ingram & Lindorff (2020)
[27] Ingram & Gorgen (2020)

example of an equation that has one of its solutions as $x = 2$. These sorts of questions often generate different responses from different students, opening up the opportunity to look for patterns within these responses which can lead to useful generalisations, or to compare and contrast the different strategies or solutions that students generated.

Completing the square is also widely considered to be a more cognitively demanding solution method; in England this was not taught in all the classes that participated. Instead, teachers largely focused on solving quadratic equations by factorising or by finding roots of quadratic functions. However, completing the square was more common in other countries such as Germany, Japan, and Shanghai (China).

A big focus for all students in England (both for higher and foundation tier) was quadratic functions with graphical representations, similarly so for Colombia and Germany. But in Japan and China graphical materials were almost non-existent[28]. This *could* challenge the long-held assumption that equations and functions need to be taught and learned together[29]. Conceptual reasoning is encouraged more in other countries than in England, especially in Chile, Spain and China. High-quality mathematics teaching was perceived by students to be linked with opportunities to engage in reasoning, for example to determine the number and kind of solutions of a given equation.

Summary

Cognitively demanding tasks and activities include those that ask students to be creative, to analyse the mathematics they are learning, and to evaluate the effectiveness of different procedures, processes and approaches. The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

- What opportunities are there for students to create their own examples?
- What tasks would generate different solution strategies that students could compare, contrast and evaluate?
- What cognitively demanding tasks or activities could be used when introducing a new concept or procedure?
- How can a teacher support students in analysing why the steps are made in a procedure, rather than just learning the steps needed?

[28] OECD (2020a)
[29] Leung et al. (2014)

Patterns and generalisations to aid student understanding

Patterns and generalisations were identified through the videos of the lessons and the lesson materials that accompanied these lessons. In the videos, the focus was on teachers or students looking for patterns, which sometimes focused on surface features of the mathematical ideas but at other times focused on deeper features. Students can develop strong understandings of mathematics by looking for patterns and making generalisations; it was, however, rare for the teacher or students to look for patterns and make generalisations across aspects of mathematics. Of the participating countries, only students in China tended to look for patterns and develop generalisations. In the TALIS Video Study, general rules did not count as generalisations, as generalisations are about the process of seeing structure and commonalities in patterns, procedures, or strategies[30]. This means that telling students that "multiplying something by zero will give you zero" was not included unless this was the conclusion reached after students had gone through the process of multiplying at least two different numbers or expressions by zero.

Teachers in the TALIS Video Study used patterns and generalisations in different ways and the case studies[31] illustrate some of the ways that teachers sequenced tasks or questions to create patterns that could be used for generalisation. In England, it was almost always the teacher that made the generalisation, with only a few examples where teachers asked students to generalise from a pattern. The principles of variation theory[32] are particularly useful in thinking about patterns which can highlight the underlying structure of a sequence of tasks or questions.

An example of how the sequencing of questions was used by a teacher in England to support students in noticing a pattern in the process of factorising a quadratic expression is given in Figure 6. Here, the structure of the expressions is kept invariant, while in the case of expanding the expressions the sign of the numbers varies, and in the case of factorising the x coefficient varies whilst the constant term and the x^2 coefficient are invariant. Having determined this, the teacher then supported the students in noticing what changed and what stayed the same in their answers to these questions, leading to a generalisation of the process of factorising and expanding quadratic expressions. In these situations, it is the structure of the sequence of questions as a whole, not the individual questions, that promotes mathematical sense-making[33].

[30] Kaput (1999)
[31] Ingram & Gorgen (2018)
[32] https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/
[33] Watson & Mason (2006)

Figure 6: Sequencing questions on factorising and expanding quadratic equations[34]

Expand

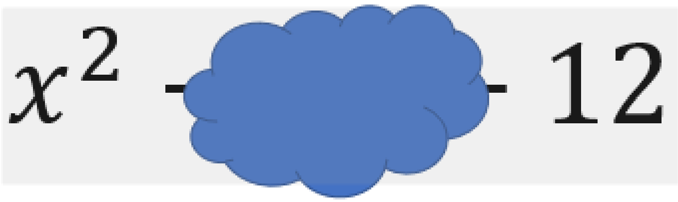
- $(x+2)(x+3)$
- $(x-2)(x+3)$
- $(x+2)(x-3)$
- $(x-2)(x-3)$

Factorise

- $x^2+8x+12$
- $x^2+7x+12$
- $x^2+13x+12$
- $x^2-13x+12$
- $x^2+4x-12$
- x^2+x-12
- $x^2+11x-12$

An alternative approach would be to ask students to generate their own examples through more cognitively demanding tasks, such as that shown in Figure 7. In this task, students are asked to identify the possible signs and coefficients of x which would lead to a factorisable quadratic expression.

Figure 7: Task where students need to identify a family of expressions with a particular structure



Students can be supported in generalising from these patterns by teachers drawing attention to the patterns through questions like “What’s the same?” to focus on the invariant structures or features, or “What’s different?” to focus on the variant features. This idea of distinguishing between what remains the same and what changes was used by some teachers to make connections to other types of polynomial, or to make distinctions between quadratic expressions, quadratic equations, and quadratic functions. These teachers would include both examples (including non-standard examples) and non-examples to contrast between linear and quadratic equations.

This use of examples and non-examples can also be used to distinguish between the meanings of equality and equivalence, as well as the distinction between letters being

[34] Taken from section 3.1 of the case study report (Ingram & Gorgen, 2020).

used to represent unknown quantities and letters being used to represent variables[35]. Dynamic geometry or other graphical software can also be used to highlight these distinctions. An expression like $(x + 2)(x - 3)$ is an object in itself – perhaps the answer to a question such as "find an equivalent expression to $x^2 - x - 6$" – and one of the key shifts students make when learning mathematics is moving to seeing expressions like this as objects rather than as processes for a specific calculation.

Digital technology allows teachers and students to efficiently explore a significant number of equations and graphs in different ways. Patterns and structures can emerge from these explorations, conjectures can be made, and relationships between quadratic equations and quadratic functions can be exposed. Elements of an equation can be independently investigated, for example by varying one variable and examining the effect of this change on the graph of the function. One example of how a teacher used dynamic geometry to generalise from the mathematics being considered is given in the case studies[36]. Using technology creatively can enhance the cognitive demands within a task. Many dynamic technology software packages are readily (and freely) available and are now supported by a wide range of resources and guides.

Algebra is used to express generalisation in mathematics, and is often introduced as a way of representing general relationships[37]. Students intuitively explore patterns (in a range of contexts: tile arrangements, geometric designs, simple number sequences, learning language), and algebraic thinking stems from these predictive relationships. In mathematics, students learn to express these generalisations in a symbolic form.

Summary

Patterns and generalisations underpin a lot of the mathematics taught in schools. The TALIS Video study revealed that it is largely the teachers who make the generalisations rather than the students. Working with patterns can also support students in making connections between different processes, perspectives or representations.

The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

- How could practice and sequencing questions or tasks be combined in ways that generate patterns?
- What are the underlying generalisations that students themselves could make?

[35] Küchemann (1978)
[36] Ingram & Gorgen (2020), section 3.3.
[37] Nunes, Bryant, & Watson (2009)

- Which patterns therefore need to be generated to support students in making these generalisations?

Developing students' persistence through errors

Encouraging students to learn from mistakes is a valuable cognitively demanding strategy and one which can help diminish the fear of failure[38]. In fact, although mistakes are an important opportunity for learning, many students interpret mistakes as evidence of their own inability – or "low ability". Students can even be reluctant to put pen to paper in case they make a mathematical mistake. Providing challenging and engaging mathematics may result in more mistakes being made – but these mistakes need to be viewed through the lens of a "growth mindset" – as an opportunity to learn and develop[39]. Student self-efficacy or lack thereof (and England has relatively low self-efficacy scores in the TALIS Video Study) may initially impact this approach – but welcoming the inclusion and exploration of misconceptions could boost both self-efficacy and understanding[40]. Unpacking and unravelling student misconceptions often requires deep knowledge of the subject with effective practitioners able to make sense of the students' alternative conceptions[41].

The TALIS Video Study shows students' thinking, contributions, struggles, and errors were sometimes used to inform the teaching approach but few teachers (8%) provided feedback that was complete and focused on why students' thinking was correct or incorrect. Ignoring students' errors or treating them superficially was commonly seen – although this may have been a limitation of the study, with very little "at desk" interaction captured with individual students. This could be connected to a teachers' level of contingency knowledge – that of being able to respond to students in real time, to answer their tangential questions, and to view their ideas as teaching opportunities: a teacher's readiness to react to situations that are difficult to anticipate[42]. Alternatively, pressures of a large curriculum may squeeze perceived time for teaching opportunities.

Developing cognitively demanding teaching and learning experiences, such as analysing, creating or evaluating mathematical material, requires thoughtfulness and skill. Cognitively engaging opportunities are essential for the development of all students and must not be the sole preserve of students with higher prior attainment. Examples include detailed examinations or explorations of the features and relationships among mathematical procedures, processes or ideas, formulating or inventing a way to solve a problem, and determining the significance or conditions of a mathematical idea, topic, representation, or process. These tasks and activities go beyond recall or the rote application of procedures. The cognitive demand of the subject matter is based on how students are being asked to engage with the mathematics, rather than the difficulty of the

[38] Swan (2005)
[39] Boaler (2013), Dweck (2006), Ingram et al. (2015)
[40] Smith (2017)
[41] Turner & Rowland (2006), Coe et al. (2014), Ball & Bass (2000)
[42] Turner & Rowland (2006), Petrou & Goulding (2011)

mathematics itself. The TALIS Video Study found that, generally, it was common for students to be asked to recall information, or to summarise and apply rules and procedures; being asked to contribute detailed thinking was far less common. Similarly, students rarely articulated the rationale for mathematical procedures and processes or engaged in other highly promoted practices for cognitive engagement. When explanations were given by teachers or students, the explanations were generally brief and/or superficial; lengthier, deeper explanations were observed in only about a quarter of classrooms (24%).

In all countries, students had frequent opportunities to develop mathematical fluency through repetitive practice. Practice is a vital part of learning mathematics but one of the principles behind the mastery programme in England is that this practice should be deliberate and intelligent, that is it should also aim to develop students' conceptual understanding, reasoning and mathematical thinking.

Summary

Productive struggle is important and significant for students of all attainment levels. All students should be given the opportunity to work on cognitively demanding mathematics and challenging tasks, and to know that it is an important aspect of the curriculum. To be denied the opportunity to struggle is to be denied the opportunity to think. Knowing when to support a student, and when to give them space to struggle is a delicate teaching decision, perhaps best summed up by Pólya: "The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work"[43].

The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

- What strategies can support students to persist through struggle?
- How can students be cognitively engaged in mathematical thinking and reasoning when they are finding a topic difficult?
- How does the support students need change depending on whether they have a misconception, are stuck with the mathematics, or have made a mistake?

[43] Pólya (1945, p.1)

Teaching students with different levels of prior attainment

In the English education system, the gap between the lowest-attaining students and the highest-attaining students has been narrowing over time in mathematics, but England continues to have one of the largest gaps between higher-attaining students and lower-attaining students across the developed world [44].

In England, it is common for students to be grouped or "set" into classes according to their prior attainment. This system (or structure) of "setting" students by prior attainment is not common in other countries, including those participating in the TALIS Video Study.

The TALIS Video Study highlighted some interesting findings associated with the pre-tests and post-tests. Students with identical pre-test scores performed quite differently in the post-test. Students from lower-attaining classes (the "lower sets") were likely to perform worse than their peers ***with identical pre-test scores*** from classes that had a higher average pre-test score.

Why do students perform worse depending on the "set" they are in? The deployment of teachers to different groups determines the potential progress of students within these groups[45]. In England, researchers have found that teachers with fewer mathematics qualifications are more likely to be allocated to lower-attaining sets[46], and classes containing disadvantaged students are more likely to have inexperienced teachers[41]. In contrast, "top sets" tend to be taught by well-qualified teachers[47]. The TALIS Video Study found that the differences in instructional quality between high- and low-performing classrooms was significant in England[48]. For example, classes with higher average prior attainment experienced classrooms where there were more likely to be opportunities for students to participate in classroom discourse, be asked questions that required them to analyse, synthesise, justify or conjecture, and experience explanations that focused on the deeper features of the mathematics. They also experienced more explicit connections to other topics, and had more opportunities to engage in cognitively demanding mathematics, work with multiple methods or approaches, and be asked to give detailed answers to questions that explained their thinking[49]. Yet there are examples of teachers using cognitively demanding tasks, asking students to think deeply about the mathematics, and asking questions that require students to analyse, synthesise, justify, or conjecture with classes across the attainment range in the case studies[50].

---

[44] Sizmur et al. (2019)
[45] Allen & Sims (2018a, 2018b)
[46] Moor et al. (2006)
[47] Wiliam & Bartholomew (2001)
[48] OECD (2020a)
[49] Ingram & Lindorff (2020)
[50] Ingram & Gorgen (2020)

While the TALIS Video Study found that classes were generally well managed, it was notable that some lower-attaining classes experienced less well-managed classrooms with more disruptions and fewer organised and efficient routines compared with higher-attaining classes.

Looking specifically at low attainment in mathematics, Hodgen and colleagues[51] found prior attainment in mathematics is the strongest predictor of future attainment. In other words, what students can learn appears to be largely predicted by what students already know. Raising students' current attainment should therefore be a primary focus, as other factors such as students' attitudes, behavioural tendencies, or home support are of lesser influence. Examining equity in education has been a focus of previous studies[52] and researchers have shown that unequal opportunities to learn subject matter may exacerbate achievement gaps between students[53]. The results from England appear to reflect this. It has been argued that unequal opportunity to learn mathematics is "one of the key factors driving inequality in schools"[54].

Opportunity to learn differences related to prior attainment

Organising students into attainment groups or 'sets' is relatively common in England. One very clear reason for this, is the GCSE examination system structure. The GCSE mathematics qualification is divided into two tiers: foundation and higher. Each tier is targeted towards a range of grades: 9 to 4 on the higher tier (with 9 being the highest) and 5 to 1 on the foundation tier. In England, students prepare for *either* the higher *or* the foundation tier. Only students studying for the higher tier are taught (and assessed on) the more advanced aspects of mathematics, such as solving quadratic equations by completing the square or using the quadratic formula. Learning is dependent on the implemented curriculum, that is, the *opportunity to learn*[55].

England implemented a curriculum reform with a reformed exam system in 2015 (with the "new" mathematics GCSE examined for the first time in summer 2017), shortly before the data for TALIS Video Study was collected (October 2017-October 2018). This reform had the goal of extending the opportunity to learn more topics to more students, and developing conceptual understanding for all students. At this time, it was impossible to predict how teachers and students would respond to the reformed, more demanding, exam syllabus and how this would impact on the split of student numbers entered for each (higher/foundation) tier. According to Ofqual, prior to the reformed GCSE, approximately 72% of students were prepared for the higher tier mathematics exam;

---

[51] Hodgen et al. (2020)
[52] OECD (2012)
[53] Kuger (2016), Patall et al. (2010)
[54] Schmidt et al. (2015, p.15)
[55] Burroughs et al. (2019), Klieme (2013), Schmidt & Maier (2009)

subsequently this figure dropped to around 58%. This figure is still in flux. An unintended consequence of the reformed exam system may be that fewer students are now preparing for the higher tier and so fewer students are studying the most conceptually demanding aspects of mathematics in the Key Stage 4 curriculum. Teaching and learning opportunities are potentially limited in lower-attaining classes which can result in unequitable experiences[56].

A growing body of evidence from around the world suggests grouping students by "ability" negatively impacts the achievement of students in "low" and "middle" groups[57]. Such practices can reinforce the belief that mathematical ability is innate, and that some students can do mathematics while others will never be able to[54]. This message can also persist into adulthood and impact on future outcomes. A longitudinal study considering contrasting grouping arrangements for students found the impact of being "set" at school in England was felt long after leaving school and well into adulthood: adults, who had experienced being grouped by "ability", were in "less professional" jobs and linked their limits in job prospects to the ability grouping used in school[58]. This may be particularly disconcerting when "setting" students by prior attainment which can be a rather arbitrary arrangement[54] that can disproportionately disadvantage students from lower socio-economic backgrounds[59].

Many teachers will be familiar with the "Five Big Ideas" promoted by the National Centre for Excellence in the Teaching of Mathematics (NCETM), drawn from evidence and underpinning "teaching for mastery". The "Five Big Ideas" are: Mathematical thinking; Fluency; Variation; Representation and Structure; and Coherence. "Teaching for mastery" is a term that has come to describe a set of practices (combined with a coherent curriculum) to keep all students progressing at a similar rate, by promoting "low threshold high ceiling" activities which are accessible to *all*, with alternative "stepping-off" points and outcomes.

Achieving mastery suggests students knowing "*why*", as well as knowing "*that*" and knowing "*how*", and involves much more than students memorising procedures. A "mastery curriculum" is based on the principle of being accessible to *all* children, no matter what their prior attainment.

Differences in classroom experiences

Much like in other large-scale studies, the TALIS Video Study illustrates that the quality of classroom teaching and learning opportunities that students receive varies by socio-

[56] Boaler (2013)
[57] Nunes, Bryant, Sylva, & Barros (2009), Boaler (2000), Marshall (2013), Francis et al. (2019)
[58] Boaler (2005)
[59] Marshall (2013)

economic background[60]. Classes with a lower average socio-economic status were seen in the TALIS Video Study to have less opportunity to learn, with fewer connections made within and between topics, fewer approaches employed to develop student understanding, and fewer opportunities for discourse.

The idea that grouping by "ability" reduces achievement overall is based on two premises: that *opportunity to learn* mathematics is limited for some students, and the potential negative effect of lower expectations in terms of what students can learn and achieve in mathematics.

Some researchers believe that teachers who "foster positive student outcomes" do so through their "beliefs in the rights of all students to have access to mathematics education in a broad sense"[61]. This broad sense encompasses an understanding and appreciation of the big ideas of mathematics, a painting of the big picture as opposed to bite-size, atomised pieces, which are often the diet for lower-attaining students in England[62]. Findings from the TALIS Video Study show scope to improve teaching in most classrooms, but attention needs to be paid to lower-attaining classes in terms of their opportunity to access higher levels of instruction.

The TALIS Video Study shows students in England to have relatively low self-efficacy scores. Levels of self-efficacy – the extent to which students feel confident in their ability to perform tasks required of them – are important, both in terms of how these impact on future learning and in terms of current performance. In England, it is culturally acceptable to profess to being poor at mathematics[63]. Research has shown that teaching practices can contribute to students' own views about their mathematics ability being a fixed or malleable trait[64]. Research has also shown that highly effective teachers have high expectations of their students and believe that almost all students can succeed in mathematics[65]. Supporting students to feel confident in their ability to engage with the mathematical tasks asked of them can both raise achievement in mathematics and increase participation rates in future mathematics study[66].

Summary

Examination results are often used as the norm for judging the academic success of both students and institutions. However, the current head of OFSTED, Amanda Spielman[67],

---

60 OECD (2020a)
61 Anthony & Walshaw (2009, p.149)
62 Boaler et al. (2000)
63 Smith (2017)
64 Sun (2018)
65 Askew et al. (1997), Coe et al. (2014)
66 Smith (2017)
67 GOV.UK (2017)

has recently indicated that exam results alone are insufficient to ascertain whether students have “received rich and full knowledge from the curriculum”. Hattie and other researchers also report that it is engagement with school (and the number of years of education), not examination grades, that leads to better outcomes for individuals in later life[68]. Coe and colleagues also argue that “enhanced student outcomes” need not be limited to student academic attainment but should also include whatever is valued in education[69]. Consequently, measures of prior attainment that are used to group or set students may not accurately reflect students’ conceptual understanding of mathematics, and setting by prior attainment often only values academic attainment in mathematics and ignores other non-cognitive student outcomes such as self-efficacy in mathematics and students’ interest in mathematics.

The findings from the TALIS Video Study suggest that the following aspects of teaching could be considered further:

- What connections between topics or representations can support students of all levels of prior attainment to develop a deeper understanding of mathematics?
- How can lower-attaining students be cognitively engaged in mathematical thinking and reasoning?
- How can explanations focus on the deeper aspects of the mathematics while still being accessible to all students in a class?

[68] Hattie (2015), Boaler (2013), Noyes (2012)
[69] Coe et al (2014, p. 11)

Final comment

This report focuses on the professional development implications arising from the TALIS Video Study. With a focus on prompting reflection by teachers on those teaching practices that were a focus in the study, this report presented some of the findings and insights from the study which could offer alternative approaches to the teaching of quadratic equations, and more broadly challenge assumptions about the teaching of mathematics in England.

This report focused on the practices that are most relevant to mathematics teaching. These included practices that relate to understanding the mathematics, making connections both within mathematics and to real-world contexts, using multiple approaches and perspectives, engaging students in mathematically demanding activities, using patterns and generalisations, and supporting students to persist through errors or mathematical struggles.

In addition, many of the teaching practices discussed were less frequently observed with classes that had lower average prior attainment. Yet in the TALIS Video Study there were examples of teachers making use of these practices with some of the lowest-attaining classes, as illustrated in the case studies[70].

The TALIS Video Study offers a range of insights both from other classrooms in England, and from classrooms in other countries, that can be used to develop mathematics teaching in many classrooms. Details of how England is both similar to and different from other countries participating in the TALIS Video Study can be found in the OECD policy report[71]. Examples of how teachers in England made use of some of the rarer teaching practices found in the study are also available in the case studies[68].

[70] Ingram & Gorgen (2020)
[71] OECD (2020a)

References

Allen, R. & Sims, S. (2018a). *How do shortages of maths teachers affect the within-school allocation of maths teachers to pupils? Report for the Nuffield Foundation*. Retrieved from https://www.nuffieldfoundation.org/wp-content/uploads/2018/06/Within-school-allocations-of-maths-teachers-to-pupils_v_FINAL.pdf

Allen, R. & Sims, S. (2018b). Do pupils from low-income families get low quality teachers? Indirect evidence from English schools. *Oxford Review of Education*, 44(4), 441-458. http://dx.doi.org/10.1080/03054985.2017.1421152

Anthony, G. & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the west. *Journal of Mathematics Education*, 2(2), 147-164.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & William, D. (1997). *Effective teachers of numeracy: Report of a study carried out for the Teacher Training Agency.* London, United Kingdom: King's College, University of London.

Ball, D. L. & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex Publishing.

Boaler, J. (2005). The 'psychological prison' from which they never escaped: the role of ability grouping in reproducing social class inequalities. *FORUM*, 47(2-3), 135-144. http://dx.doi.org/10.2304/forum.2005.47.2.2

Boaler, J. (2009). *The elephant in the classroom: Helping children learn and love maths*. London, United Kingdom: Souvenir Press.

Boaler, J. (2013). Ability grouping in mathematics classrooms. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Heidelberg, Germany: Springer.

Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping – disaffection, polarization and the construction of failure. *British Educational Research Journa*l, 26(5), 631-648. http://dx.doi.org/10.1080/713651583

Burghes, D. & Robinson, D. (2010). *Lesson study: Enhancing mathematics teaching and learning*. Reading, United Kingdom: CfBT Education Trust.

Burroughs, N., Gardner, J., Lee, Y., Guo, S., Touitou, I., Jansen, K., & Schmidt, W. (2019). *Teaching for excellence and equity: Analyzing teacher characteristics, behaviors and student outcomes with TIMSS*. New York, NY: Springer.

Carter, A. (2015). *Carter review of initial teacher training (ITT)*. Retrieved from https://www.gov.uk/government/publications/carter-review-of-initial-teacher-training

Clark-Wilson, A. & Hoyles, C. (2017). *Dynamic digital technologies for dynamic mathematics: Implications for teachers' knowledge and practice. Final Report*. London, United Kingdom: UCL Institute of Education Press.

Coe, R., Aloisi, C., Higgins, S., & Major, L. M. (2014). *What makes great teaching?* Retrieved from http://www.suttontrust.com/wp-content/uploads/2014/10/What-Makes-Great-Teaching-REPORT.pdf

Dweck, C.S. (2006). *Mindset: the new psychology of success*. New York, NY: Ballantine Books.

Francis, R., Taylor, R., & Tereshchenko, A. (2019). *Reassessing 'ability' grouping: improving practice for equity and attainment*. London, United Kingdom: Routledge.

Geiger, V., Anderson, J., & Hurrell, D. (2017). A case study of effective practice in mathematics teaching and learning informed by Valsiner's zone theory. *Mathematics Education Research Journal*, 29**,** 143-161. https://doi.org/10.1007/s13394-017-0191-9

GOV.UK (2017). HMCI commentary: Ofsted's Chief Inspector, Amanda Spielman, discusses findings from recent research into the primary and secondary curriculum. Retrieved 12 October 2017 from http://www.gov.uk/government/collections/hmcis-monthly-commentaries

Hattie, J. (2015). *What works best in education: The politics of collaborative expertise* London, United Kingdom: Pearson.

Henderson, P., Hodgen, J., Foster, C., & Küchemann, D. (2018). *Improving mathematics in key stages two and three*. London, United Kingdom: Education Endowment Foundation.

Hodgen, J., Coe, R., Foster, C., Brown, M., Higgins, S., & Küchemann, D. (2020). *Low attainment in mathematics: An investigation focusing on Year 9 students in England. Final Report.* London, United Kingdom: UCL Institute of Education.

Ingram, J. & Gorgen, K. (2020). *TALIS Video Study case studies.* London, United Kingdom: DfE.

Ingram, J. & Lindorff, A. (2020). *TALIS Video Study National Report.* London, United Kingdom: DfE.

Ingram, J., Lindorff, A., Sani, N., McCann, E. & Riggall, A. (2020). *TALIS Video Study national report summary*. London, United Kingdom: DfE.

Ingram, J., Pitt, A. & Baldry, F. (2015). Handling errors as they arise in whole-class interactions. *Research in Mathematics Education*, 17(3), 183-197.

Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema and T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 145–168). New York, NY: Routledge.

Klieme, E. (2013). The role of large-scale assessments in research on educational effectiveness and school development. In: M. von Davier, E. Gonzalez, I. Kirsch, & K. Yamamoto (Eds.), *The role of international large-scale assessments: Perspectives from technology, economy, and educational research*. Dordrecht, The Netherlands: Springer.

Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in School*, 7(4), 23-36.

Kuger, S. (2016). Curriculum and learning time in international school achievement studies. In S. Kuger, E. Klieme, N. Jude & D. Kaplan (eds.), *Assessing contexts of learning. An international perspective*. Cham, Switzerland: Springer.

Leung, F., Clarke, D., Holton, D., & Park, K. (Eds.) (2014). How is algebra taught around the world? In *Algebra teaching around the world*. Rotterdam, The Netherlands: Sense Publishing. http://dx.doi.org/10.1007/978-94-6209-707-0

Marshall, P. (Ed.) (2013). *The tail*: *How England's schools fail one child in five – and what can be done.* London, United Kingdom: Profile Books Ltd.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics 38*, 135–161.

McCann, E., Riggall, A., Ingram, J., & Lindorff, A. (2020). *TALIS Video Study: Technical report*. London, United Kingdom: DfE.

Moor, H., Jones, M., Johnson, F., Martin, K., Cowell, E. Bojke, C,. & Merit, P. (2006). *Mathematics and science in secondary schools: The deployment of teachers and support staff to deliver the curriculum*. The Department of Education and Skills Report RR708. Nottingham, United Kingdom: DFS Publications.

National Centre for Excellence in the Teaching of Mathematics (n.d.). Five big ideas in teaching for Mastery. Retrieved 25 October 2020 from https://www.ncetm.org.uk/teaching-for-mastery/mastery-explained/five-big-ideas-in-teaching-for-mastery/

Noyes, A. (2012). It matters which class you are in: student-centred teaching and the enjoyment of learning mathematics. *Research in Mathematics Education*, 14(3), 273-290.

Nunes, T., Bryant, P., & Watson, A. (2009). *Key understandings in learning mathematics*. London, United Kingdom: Nuffield Foundation

Nunes, T., Bryant, P., Sylva, K., & Barros, R. (2009). *Development of maths capabilities and confidence in primary school* (Research Report DCSF-RR118). London, United Kingdom: Department for Children, Schools and Families.

OECD (2020a). *Global Teaching InSights A Video Study of Teaching*. Paris: OECD http://www.oecd.org/education/school/global-teaching-insights.htm

OECD (2020b). *Global Teaching InSights: Technical Report.* Paris: OECD http://www.oecd.org/education/school/global-teaching-insights.htm

OECD (2012). *Equity and quality in education: Supporting disadvantaged students and schools*. Paris, France: OECD Publishing. http://dx.doi.org/10.1787/9789264130852-en

Patall, E., Cooper, H. & Allen, A. (2010). Extending the school day or school year: A systematic review of research (1985-2009). *Review of Educational Research*, http://dx.doi.org/10.3102/0034654310377086.

Petrou, M. & Goulding. M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching, Mathematics Education Library vol. 50* (pp. 9-25). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-90-481-9766-8_2

Pólya, G. (1945). *How to solve it*. Princeton, NJ: Princeton University Press.

Riggall, A., Sani, N. and McCann, E. (2020). *TALIS Video Study international summary*. London, United Kingdom: DfE.

Reynolds, D. (2008). Schools learning from their best (the within school variation project), Nottingham, United Kingdom, National College for School Leadership. Retrieved 26 October 2020 from https://dera.ioe.ac.uk//7381/

Schmidt, W., Burroughs, N., Zoido, P., & Houang, R. (2015). The role of schooling in perpetuating educational inequality: An international perspective. *Educational Researcher,* 44(7), 371-386. https://doi.org/10.3102/0013189X15603982

Schmidt, W. & Maier, A. (2009). Opportunity to learn. In G. Sykes, B. Schneider, & D. N. Plank (Eds.), *Handbook of education policy research* (pp. 541-559). New York, NY: Routledge.

Sizmur, J., Ager, R., Bradshaw, J., Classick, R., Galvis, M., Packer, J., Thomas, D., and Wheater, R. (2019). *Achievement of 15-year-olds in England: PISA 2018 results*. Department for Education Research report. Accessed 15/04/2020 from https://www.gov.uk/government/publications/pisa-2018-national-report-for-england

Smith, A. (2017). *Smith review of post-16 mathematics.* London, United Kingdom: Department for Education.

Sun, K. (2018). The role of mathematics teaching in fostering student growth mindset. *Journal for Research in Mathematics Education*, 49(3), 330-355.

Swan, M. (2005). Improving learning in mathematics: challenges and strategies. London, United Kingdom: Department for Education and Skills.

Turner, F. & Rowland. T. (2006). *The knowledge quartet: A means of developing and deepening mathematical knowledge In teaching?* Cambridge, United Kingdom: University of Cambridge.

Vappula, H. & Clausen-May, T. (2006). Context in maths test questions: Does it make a difference? *Research in Mathematics Education*, 8(1), 99–115.

Watson, A. (2009). Algebraic reasoning. In T. Nunes, P. Bryant and A. Watson (Eds.), *Key understandings in learning mathematics*. London, United Kingdom: Nuffield Foundation.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: using variation to structure sense-making. *Mathematical Teaching and Learning*, 8(2), 91–111.

William, D. and Bartholomew, H. (2001). It's not which school but which set you're in that matters: the influence of ability-grouping practice on student progress in mathematics. *British Educational Research Journal*, 30(2), 279-293.

Xu, L. & Clarke, D. (2018). Validity and comparability in cross-cultural video studies of classrooms. In L. Xu, G. Aranda, W. Widjaja & Clarke, D. (Eds.), *Video-based* *research in education: Cross-disciplinary perspectives.* Abingdon, United Kingdom: Routledge.

Department for Education

Reference: DFE-RR109

ISBN: 978-1-83870-211-3

For any enquiries regarding this publication, contact us at: alice.gallimore-roberts@education.gov.uk or www.education.gov.uk/contactus

This document is available for download at www.gov.uk/government/publications