Department for Education

Statutory guidance

National curriculum in England: mathematics programmes of study

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Purpose of study

Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the basis for ingenuity in science, technology, and engineering. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality education in mathematics supports pupils' fluency, reasoning and problem-solving ability, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that they develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically, by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proving mathematical ideas
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasingly sophisticated unfamiliar contexts, making connections across mathematical ideas, and persevering in seeking solutions

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. Mastery of mathematics means that the necessary, organised into appropriately distinct domains, but pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study of broadly the same content at broadly the same time, but schools should also ensure that lessons are planned to be sufficiently flexible - understanding and their readiness to progress to the next topic. Teaching should be supplemented by additional support through teaching assistants and sustained intervention before any occasions when pupils may need to be withdrawn from lessons. Pupils who are not sufficiently fluent with earlier mastered mathematical content could be given extra time and support to ensure they are ready to access new content.

Information and communication technology (ICT)

Calculators should not be used as a substitute for pupils' acquisition of and fluency in fundamentals. They should be used to explore and understand the place of mathematics in society, to support conceptual understanding and exploration of more complex number problems, in order to gain more insight into mathematics, and to do ordinary calculations when appropriate. Teachers should use their judgement about when ICT tools should be used.

Spoken language

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum - cognitively, socially and linguistically. The quality and quantity of the spoken language that pupils hear and learn to develop should be planned and sequenced to support their understanding and mastery of mathematics, and to develop their ability to reason and to communicate mathematically, through being reflective and accurate in their use of mathematical vocabulary to express their ideas. Teachers should ensure that pupils are given sufficient opportunities to develop their mathematical understanding and reasoning, including through articulating and justifying their mathematical ideas and their reasoning, using the precise language of mathematics. Teachers should also ensure that pupils are given opportunities to develop their ability to formulate mathematical ideas in precise language, and to refine and develop their mathematical reasoning.

School curriculum

The programmes of study for mathematics are set out year by year for key stages 1 and 2. Schools, we, however, only expect teachers to teach the relevant programme of study by the end of the specified key stage. Within each key stage, schools have flexibility to choose when to introduce certain content and to decide how they will organise and sequence it, although schools can introduce key stage content during an earlier key stage. If appropriate, they can also introduce content from a programme of study for later years in mathematics on a year-by-year basis and make this information available online.

Attainment targets

By the end of each key stage, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

Schools are not required by law to teach the example content in (square brackets) in the content indicated as being 'non-statutory'.

Key stage 1 - years 1 and 2

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should include recognising patterns in the world and how they can be described, practical measures (including money and time), and simple geometry.

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and their properties, including those in the environment, using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in their use of mathematical language. Also, they should be able to draw and write simple sentences.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at this stage.

Year 1 programme of study

Number - number and place value

Pupils should be taught to:

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s
- given a number, identify one and ten more
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in numerals and words

Notes and guidance (non-statutory)

Pupils practise counting (1, 2, 3…), ordering (1st, 2nd, 3rd…), counting to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number, including reading and writing numbers to 100 in numerals, and they are fluent.

Pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations.

They practise counting and recording numbers and counting as unvarying objects, and counting in 2s, 5s and 10s from different multiples to develop their understanding of place value. They recognise the tens and ones in two-digit numbers, including teens and these numbers through increasingly challenging questions.

They recognise and create repeating patterns with objects and with shapes.

Number - addition and subtraction

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one digit and two digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ▢ − 9

Notes and guidance (non-statutory)

Pupils memorise and recall with mastery numbers to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 − 7 = 9; 7 = 16 − 9). They should notice the effect of adding or subtracting 0. This establishes addition and subtraction as reverse operations.

Pupils combine and increase numbers, counting forwards and backwards.

They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance, difference, difference between, more and less. They use the language of: equal to, more than, less (fewer), most, least and are exposed to the three operations flexibly.

Number - multiplication and division

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Notes and guidance (non-statutory)

Through posing and solving small problems, pupils begin to understand multiplication and division, recording and expressing in pictures, and learning multiplication tables, doubling and halving.

They make connections between arrays, number patterns, and counting in 2s, 5s and 10s.

Number - fractions

Pupils should be taught to:

- recognise, find and name a half as 1 of 2 equal parts of an object, shape or quantity
- recognise, find and name a quarter as 1 of 4 equal parts of an object, shape or quantity

Notes and guidance (non-statutory)

Pupils are taught half and quarter as 'fractions of' discrete and continuous quantities by recognising, comparing and describing fractions of objects, for examples, they could recognise and find half a length, quantity, set of objects or a shape. They should observe and be able to describe how the whole has been divided into halves and to measure, as well as recognising and combining halves and quarters as parts of a whole.

Measurement

Pupils should be taught to:

- compare, describe and solve practical problems for:
  - lengths and heights (for example, long/short, longer/shorter, tall/short, double/half)
  - mass/weight (for example, heavy/light, heavier than, lighter than)
  - capacity and volume (for example, full/empty, more than, less than, half full/full, quarter)
  - time (for example, quicker, slower, earlier, later)
- measure and begin to record the following:
  - lengths and heights
  - mass/weight
  - capacity and volume
  - time (hours, minutes, seconds)
- recognise and know the value of different denominations of coins and notes
- sequence events in chronological order using language (for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening)
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times

Notes and guidance (non-statutory)

The pairs of terms: mass and weight, volume and capacity, are used interchangeably at this stage.

Pupils move from using and comparing different types of quantities and measures using non-standard units, to using standard units, initially kilograms, litres and metres. They should be familiar with the vocabulary of comparison and common standard units.

In order to become proficient with standard measures, pupils begin to use measuring tools such as ruler, weighing scales and containers.

Pupils use the language of time, including telling the time throughout the day, first using a clock and then half past.

Geometry - properties of shapes

Pupils should be taught to:

- recognise and name common 2-D and 3-D shapes, including:
  - 2-D shapes (for example, rectangles (including squares), circles and triangles)
  - 3-D shapes (for example, cuboids (including cubes), pyramids and spheres)

Pupils handle common 2-D and 3-D shapes, naming these and related everyday objects fluently. They recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids are not always upright to each other.

Geometry - position and direction

Pupils should be taught to:

- describe position, direction and movement, including whole, half, quarter and three-quarter turns

Notes and guidance (non-statutory)

Pupils are taught to describe position, direction and motion, including left and right, towards, away from, between, around, and near, and to describe movement, turns and distance, in simple linear and non-linear contexts, such as on the floor, on a grid, on a map, and in a playground.

Pupils make whole, half, quarter and three-quarter turns to both directions and connect turning clockwise with movement on a clock face.

Year 2 programme of study

Number - number and place value

Pupils should be taught to:

- count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward
- recognise the place value of each digit in a two-digit number (10s, 1s)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use <, > and = signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems.

Notes and guidance (non-statutory)

Using materials and a range of representations, pupils practise counting, reading, writing and comparing numbers to at least 100 and understanding a range of relations problems to develop fluency. They count in multiples of 2s (support their later understanding of odd/even).

As they become more confident with numbers up to 100, pupils are introduced to the place value of two-digit numbers for the first time. They should work with number system and represent them in various ways, including representations.

Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction. They become fluent and apply their understanding of place value to ordinal numbers (first, second, third…) to emphasise the value of each digit in two-digit numbers. They begin to understand 0 as a place holder.

Number - addition and subtraction

Pupils should be taught to:

- solve problems with addition and subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and 1s
  - a two-digit number and 10s
  - two two-digit numbers
  - adding 3 one-digit numbers
- show that addition of 2 numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 − 7 = 3 and 7 = 10 − 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30. They extend their understanding by adding to/take away subtraction and adding numbers to a different number (for example, 18 + 3 = 21 and 21 − 3 = 18; 7 + 3 = 10; 10 − 3 = 7). This establishes commutativity and associativity of addition.

Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Number - multiplication and division

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of 2 numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 times tables including the related division facts to aid recall. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts in any order and use these facts to derive related division facts to perform written and mental calculations.

Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing. They begin to understand the inverse relationship between multiplication and division and use this to check calculations and solve missing number problems.

Number - fractions

Pupils should be taught to:

- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- write simple fractions, for example, $\frac{1}{2} + \frac{1}{2} = 1$ and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

Notes and guidance (non-statutory)

Pupils now find, name, write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of shapes and common quantities by solving problems using shapes, objects and quantities. They connect unit fractions to equal sharing and grouping, to numbers when they can be visualised, and to measurements. They begin to understand that the whole can be made from 'parts' or 'fractions'. Pupils find the whole of a given unit fraction.

Pupils should count in fractions up to 10, starting from any number and using the $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line (for example, $\frac{1}{2}$, $\frac{1}{1}$, $\frac{3}{2}$, $\frac{2}{2}$). This reinforces the concept of fractions as numbers and that they can add up to more than 1.

Measurement

Pupils should be taught to:

- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =
- recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value
- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times
- know the number of minutes in an hour and the number of hours in a day

Notes and guidance (non-statutory)

Pupils use standard units of measurement with increasing accuracy, using their knowledge of the number system. They use the appropriate language and record using standard abbreviations.

Comparing measures includes simple situations such as 'half as high / twice as wide'.

Pupils become fluent in telling the time on analogue clocks and recording it.

They become fluent in determining the value of different coins. They work with amounts of money confidently and use the symbols £ and p accurately, recording pounds and pence separately.

Geometry - properties of shapes

Pupils should be taught to:

- identify and describe the properties of 2-D shapes, including the number of sides, and line symmetry in a vertical line
- identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces
- identify 2-D shapes on the surface of 3-D shapes, (for example, a circle on a cylinder and a triangle on a pyramid)
- compare and sort common 2-D and 3-D shapes and everyday objects

Notes and guidance (non-statutory)

Pupils handle and name a wide variety of common 2-D and 3-D shapes, including: quadrilaterals and polygons and cuboids, prisms and cones, and identify the properties of each shape (for example, number of sides, number of faces, number of corners, lines of symmetry and right angles). They also recognise and use familiar examples, such as footballs, rectangular boxes, tins and cartons.

Pupils read and write names for shapes that are appropriate for their word reading and spelling.

Pupils draw lines and shapes using a straight edge.

Geometry - position and direction

Pupils should be taught to:

- order and arrange combinations of mathematical objects in patterns and sequences
- use mathematical vocabulary to describe position, direction and movement including movement in a straight line and distinguishing between rotation as turning in a circle and in terms of right angles for quarter, half and three-quarter turns (clockwise and anticlockwise)

Notes and guidance (non-statutory)

Pupils describe and order their world using mathematical language and begin to use simple grids to describe positions.

They move and describe the position of objects in 2-D shapes, such as on a chessboard.

They describe movements between positions as translations of a given unit to the left/right and up/down.

They describe turns as right angles, clockwise and anticlockwise and relate right angles to quarters, halves and three-quarters of a circle.

Notes and guidance (non-statutory)

Pupils should work with patterns of shapes, including those in different orientations.

Pupils use the concept and language of angles to describe 'turn' by applying rotations, including in practical contexts (for example, pupils themselves moving in turns, giving instructions to other pupils to do so, and programming robots using instructions given in right angles).

Notes and guidance (non-statutory)

Pupils record, interpret, collate, organise and compare information (for example, using trips to our compound/home in pictographs with simple scales (1:1, 1:2)).

Lower key stage 2 - years 3 and 4

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including mental fluency and the concept of place value. This should ensure that pupils develop efficient mathematical methods with confidence (calculators are not to be used in lower key stage 2 at any time).

At this stage, pupils should develop their ability to solve a range of problems, including word problems, by applying their mathematical knowledge to a variety of routine problems. Alongside this increasing accuracy and speed, they should be beginning to view the world through shapes and their properties, and confidently describe the relationships between them. They should be able to describe, interpret and compare data, and begin to identify patterns and connections between these and nature.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication tables in both directions and fluency in them and

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

Year 3 programme of study

Number - number and place value

Pupils should be taught to:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a 3-digit number (100s, 10s, 1s)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1 000 in numerals and in words
- solve number problems and practical problems involving these ideas.

Notes and guidance (non-statutory)

Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.

They use larger numbers to at least 1 000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146 = 100 + 40 + 6, 146 – 100 = 46).

Using a variety of representations, including those related to measure, pupils continue to count in 10s, 100s and 1000s, so that they become fluent in the order and place value of numbers to 1 000.

Notes and guidance (non-statutory)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the numbers could exceed 100.

Pupils use their understanding of place value and partitioning, and their knowledge of number facts and of the operations, to solve problems of increasing complexity. Pupils build on their learning of 3-digits to become fluent (see Mathematics appendix 1).

Number - multiplication and division

Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which objects are connected to objects.

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are solving mathematical problems in order to improve their fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and short division.

Pupils solve problems, including those involving the context of time. It operations to use and why. These include problems involving scaling (similar), the language of times as 3 times as big/tall/long, or times as big/tall/long and comparisons (for example, how many times taller is one building than another?). They also solve problems involving different units (2 metres stored equally between 4 children. A piece shared equally between 8 children).

Number - fractions

Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions with small denominators
- recognise, find and name a half as one of two equal parts of an object, shape or quantity
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above

Notes and guidance (non-statutory)

Pupils connect tenths to place value, decimal measures and to division by 10.

They begin to understand unit and non-unit fractions as numbers on the number line, and also begin to add fractions. They add and subtract fractions with the same denominator within one whole (for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$).

Pupils understand the relation between unit fractions as operators (fractions of), and division by tenths.

They continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and half and quarter as a division of quantities.

Pupils practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.

Measurement

Pupils should be taught to:

- measure, compare, add and subtract: lengths (in/centimetres); mass (kg/g); volume/capacity (l/ml)
- measure the perimeter of simple 2-D shapes
- add and subtract amounts of money to give change, using both £ and p in practical contexts
- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour clocks
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events (for example, to calculate the time taken by particular events or tasks)

Statistics

Pupils should be taught to:

- interpret and present data using bar charts, pictograms and tables
- solve one-step and two-step questions (for example, 'How many more?' and 'How many fewer?') using information presented in scaled bar charts and pictograms and tables.

They continue to interpret data presented in many contexts.

Year 4 programme of study

Number - number and place value

Pupils should be taught to:

- count in multiples of 6, 7, 9, 25 and 1 000
- find 1 000 more or less than a given number
- recognise the place value of each digit in four-digit numbers (1 000s, 100s, 10s, and 1s)
- order and compare numbers beyond 1 000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1 000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value

Notes and guidance (non-statutory)

Using variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1 000, including counting in 6s, 7s and 9s, and representing them in other multiples through varied and frequent practice.

They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they meet so far.

They connect estimation and rounding numbers to the use of measuring instruments.

Roman numerals should be put in their historical context so pupils understand that our number system evolved over time. Understanding that zero is a key important concepts of (and place value were introduced over a period of time.

Number - addition and subtraction

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columns addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and column addition and subtraction with increasing large numbers to aid fluency (see Mathematics appendix 1).

Number - multiplication and division

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and dividing, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and column addition and subtraction with increasing large numbers to aid fluency (see Mathematics appendix 1).

Number - multiplication and division

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.
- solve problems involving increasingly harder fractions to calculate quantities, and the quantity in which one whole is made up of a given number of parts (for example, 1/4 of a pizza weighs 200g, so the whole pizza weighs 800g)
- add and subtract fractions with the same denominator
- recognise and show, using diagrams, equivalent fractions with small denominators
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to 2 decimal places
- solve simple measure and money problems involving fractions and decimals to 2 decimal places

Measurement

Pupils should be taught to:

- convert between different units of measure (for example, kilometre to metre; hour to minute)
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes, minutes to seconds, years to months, weeks to days

Notes and guidance (non-statutory)

Pupils build on their understanding of place value and decimal notation to record metric measures, including money.

They use multiplication to convert from larger to smaller units.

Perimeter can be expressed algebraically as 2l + 2w where l and w are the dimensions of the same unit.

They relate area to arrays and multiplication.

Geometry - properties of shapes

Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to 2 right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.

Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram and rhombus, trapezium).

Pupils compare and order angles in preparation for calculating angles and continue to use the terms in preparation for calculating angles in year 5.

Pupils have opportunity to address a variety of needs to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a vertical or horizontal line, including where the line of symmetry does not divide the original shape.

Geometry - position and direction

Pupils should be taught to:

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon

Notes and guidance (non-statutory)

Pupils have a set of axes in one quadrant, with equal scales and integer labels. They read, write and plot pairs of coordinates, for example (2, 5), including using coordinates generated by rules.

Statistics

Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs
- solve comparison, sum and difference of problems using information presented in bar charts, pictograms, tables and other graphs.

Pupils understand and use a greater range of scales in their representations.

Pupils begin to relate the graphical representation of data to recording change over time.

Upper key stage 2 - years 5 and 6

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including mental fluency and the concept of place value. This should ensure that pupils develop efficient mathematical methods with confidence (calculators are not to be used in upper key stage 2 at any time).

At this stage, pupils should develop their ability to solve a range of problems, including word problems, by applying their mathematical knowledge to a variety of routine problems. Alongside this increasing accuracy and speed, they should be beginning to view the world through shapes and their properties, and confidently describe the relationships between them. They should be able to describe, interpret and compare data, and begin to identify patterns and connections between these and nature.

By the end of year 6, pupils should have memorised their multiplication tables up to and including the 12 multiplication tables in both directions and fluency in them and

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

Year 5 programme of study

Number - number and place value

Pupils should be taught to:

- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret the place value of each digit in a 7-digit number (1000s, 10s, 1s)
- compare and order numbers up to 1 000 000
- identify, represent and estimate numbers using different representations
- round any number up to 1 000 000 to the nearest 10, 100, 1 000, 10 000 and 100 000
- solve number problems and practical problems involving these ideas.

Notes and guidance (non-statutory)

Pupils now use multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 25 and 100.

They use larger numbers to at least 1 000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 4 (for example, 146 = 100 + 40 + 6, 146 – 100 = 46).

Using a variety of representations, including those related to measure, pupils continue to count in 10s, 100s and 1000s, so that they become fluent in the order and place value of numbers to 1 000.

Notes and guidance (non-statutory)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the numbers could exceed 100.

Pupils use their understanding of place value and partitioning, and their knowledge of number facts and of the operations, to solve problems of increasing complexity. Pupils build on their learning of 3-digits to become fluent (see Mathematics appendix 1).

Number - multiplication and division

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and dividing, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and column addition and subtraction with increasing large numbers to aid fluency (see Mathematics appendix 1).

Number - multiplication and division

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.
- solve problems involving increasingly harder fractions to calculate quantities, and the quantity in which one whole is made up of a given number of parts (for example, 1/4 of a pizza weighs 200g, so the whole pizza weighs 800g)
- add and subtract fractions with the same denominator
- compare and order the decimal equivalents of any number of tenths or hundredths
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- read, write, order and compare numbers with up to 3 decimal places
- round decimals with three decimal places to the nearest whole number
- compare numbers with the same number of decimal places up to 3 decimal places
- solve simple measure and money problems involving fractions and decimals to 3 decimal places

Measurement

Pupils should be taught to:

- convert between different units of measure (for example, kilometre to metre; hour to minute)
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days

Notes and guidance (non-statutory)

Pupils build on their understanding of place value and decimal notation to record metric measures, including money.

They use multiplication to convert from larger to smaller units.

Perimeter can be expressed algebraically as 2l + 2w where l and w are the dimensions of the same unit.

They relate area to arrays and multiplication.

Geometry - properties of shapes

Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to 2 right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.

Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram and rhombus, trapezium).

Pupils compare and order angles in preparation for calculating angles and continue to use the terms in preparation for calculating angles in year 5.

Pupils have opportunity to address a variety of needs to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a vertical or horizontal line, including where the line of symmetry does not divide the original shape.

Geometry - position and direction

Pupils should be taught to:

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon

Notes and guidance (non-statutory)

Pupils have a set of axes in one quadrant, with equal scales and integer labels. They read, write and plot pairs of coordinates, for example (2, 5), including using coordinates generated by rules.

Statistics

Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs
- solve comparison, sum and difference of problems using information presented in bar charts, pictograms, tables and other graphs.

Pupils understand and use a greater range of scales in their representations.

Pupils begin to relate the graphical representation of data to recording change over time.

Upper Key Stage 2 - Year 6 Curriculum

Pupils should be taught to: understand the relative size of numbers in relation to other numbers, recognise the place value of each digit in a four-digit number, compare and order numbers up to 10 000 000, and determine the value of each digit

Pupils should be taught to: round any whole number to a required degree of accuracy

Pupils should be taught to: use negative numbers in context, and calculate intervals across zero

Pupils should be taught to: solve number problems and problems that involve all of the above

Pupils should be taught to: read, write, order and compare numbers up to 1 000 000 and determine the value of each digit

Pupils should be taught to: count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000

Pupils should be taught to: interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

Pupils should be taught to: round any number up to 1 000 000 to the nearest 10, 100, 1 000, 10 000 and 100 000

Pupils should be taught to: solve number problems and practise problems that involve all of the above

Pupils should be taught to: read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Notes and guidance (non-statutory)

Pupils identify the place value in large whole numbers.

They continue to use number in context, including measurement. Pupils extend their understanding of the place value system to include decimals and understand the relationships that they have with the whole number system.

They should recognise and describe linear number sequences (for example, 3, 3 2, 4, 5 ...), including those involving fractions and decimals, and that the terms in such a sequence (for example, and ...).

Number - addition and subtraction

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columns and short multiplication)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Notes and guidance (non-statutory)

Pupils practise using the formal methods of column addition and subtraction with increasingly large numbers to aid fluency (see Mathematics exemplars).

They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2 300 = 10 162).

Number - multiplication and division

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics exemplars). They apply all multiplication tables and related division facts frequently, fluently and accurately in mental and written contexts to solve problems.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or as rounding the answer. 98 ÷ 4 = 24 r 2 or 24 + 2/4 or 24.5 (25).

Pupils use their understanding of place value to multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. They use their knowledge of multiplication and divide by 2, 4 and 8 by multiplying and dividing by halving and doubling as a strategy for multiplying and dividing by powers of 2 (200 is converting between units such as kilometres and metres.

Distributivity can be expressed as a(b + c) = ab + ac.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 25 = 25 x 4; 25, 50, 075 = 2 x 5 x 5 x 19 = 38 x 25).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example 13 + 24 = 12 + 25; 33 = 33; 33 = 33).

Number - fractions (including decimals and percentages)

Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, representated visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (for example, 2/5 + 4/5 = 6/5 = 1 1/5)
- add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions (for example, 0.71 = 71/100)
- recognise and use thousandths and relate them to tenths and hundredths and to one decimal place
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and in the decimal form
- solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those with a denominator of a multiple of 10 or 25

Notes and guidance (non-statutory)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

They extend their knowledge of fractions to thousandths and connect to decimals and measures.

Pupils connect equivalent fractions > 1 that simplify to integers with division to whole numbers. Pupils understand that three-quarters plus three-quarters is two other models, and have been familiar from Reception and Year 1 fractions.

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, and apply their understanding of fractions. They relate to scaling by simple fractions, including fractions > 1.

Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of fractions and apply their knowledge of fractions to decimals to two d.p. and understand percentages as 'parts of 100'.

Pupils should go beyond the measurement and money models of decimals, for example, by exploring decimals in measuring diagrams.

Pupils should make connections between percentages, fractions and decimals (for example, 100% represents a whole quantity and 1% is 1/100, 50% is 50/100 and 0.5 is half, so this is finding 1% of 50).

Measurement

Pupils should be taught to:

- convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- understand and use approximate equivalences between metric units and common imperial units such as inch, pound and pint
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm²) and square metres (m²), and estimate the area of irregular shapes
- estimate volume and capacity, using cm³ blocks to build cuboids (including cubes) and capacity by filling containers
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure (for example, length, mass, volume, money) using decimal notation, including scaling

Notes and guidance (non-statutory)

Pupils have prior knowledge of place value and multiplication and division to convert between standard units.

Pupils calculate the perimeter of rectangles and related composite shapes, including using the relationship of perimeter or area to find unknown lengths. Pupils calculate the area of shapes by applying their knowledge of the area of a rectangle. 4 (3+ 1)= 4(3) + 4(1) = 12 + 4 = 16 or 4(4) + 4(1) = 16 + 4 = 20 or 5(4) = 20 or 5(3+1) = 5(3) + 5(1) = 15 + 5 = 20

Pupils use pre-lunch and post-lunch examples to understand the relationship between 12-hour clocks and 24-hour clocks.

Pupils use their knowledge of place value and multiplication and division to convert between standard units.

Geometry - properties of shapes

Pupils should be taught to:

- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- know angles are measured in degrees, estimate and compare acute, obtuse and reflex angles
- draw shapes and angles, measure them in degrees (°)
- identify:
  - angles at a point and one turn (total 360°)
  - angles at a point on a straight line and half a turn (total 180°)
  - other multiples of 90°
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles

Notes and guidance (non-statutory)

Pupils become accurate in drawing lines with a ruler to the nearest millimetre and measuring with a protractor. They use conventional markings for parallel lines and right angles.

Pupils use the term diagonal and make conjectures about the angles formed between sides, diagonals and parallel and perpendicular lines, and begin reasoning of qualitatively. for example using dynamic geometry ICT tools.

Pupils use angle sum facts and other properties to make deductions about missing angles and isosceles lines to missing number problems.

Geometry - position and direction

Pupils should be taught to:

- describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed

Notes and guidance (non-statutory)

Pupils recognise and use reflection and translation in a variety of diagrams, including on a coordinate grid. They extend their knowledge of 2-D shapes. Reflection should be in lines that are parallel to the axes.

Statistics

Pupils should be taught to:

- solve comparison, sum and difference problems using information presented in a line graph
- complete, read and interpret information in tables, including timetables

Notes and guidance (non-statutory)

Pupils connect their work on coordinates and scales to their interpretation of time graphs.

They begin to decide which representations of data are most appropriate and why.

Year 6 programme of study

Number - number and place value

Pupils should be taught to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practise problems that involve all of the above

Notes and guidance (non-statutory)

Pupils use the whole number system, including saying, reading and writing numbers accurately.

Number - addition, subtraction, multiplication and division

Pupils should be taught to:

- add and subtract whole numbers with up to 4 digits by a two-digit whole number using the formal written methods (short multiplication)
- add and subtract numbers mentally with increasingly large numbers
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including using by simple fractions and problems involving simple rates

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics exemplars). They apply all multiplication tables and related division facts frequently, fluently and accurately in mental and written contexts to solve problems.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainder, as fractions, as decimals or as rounding the answer. 98 ÷ 4 = 24 r 2 or 24 + 2/4 or 24.5 (25).

Pupils use their understanding of place value to multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. They use their knowledge of multiplication and divide by 2, 4 and 8 by multiplying and dividing by halving and doubling as a strategy for multiplying and dividing by powers of 2 (200 is converting between units such as kilometres and metres.

Distributivity can be expressed as a(b + c) = ab + ac.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 25 = 25 x 4; 25, 50, 075 = 2 x 5 x 5 x 19 = 38 x 25).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example 13 + 24 = 12 + 25; 33 = 33; 33 = 33).

Number - fractions (including decimals and percentages)

Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, representated visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (for example, 2/5 + 4/5 = 6/5 = 1 1/5)
- add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions (for example, 0.71 = 71/100)
- recognise and use thousandths and relate them to tenths and hundredths and to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and in the decimal form
- solve problems which require converting between fractions, decimals and percentages (for example, 3/8 = 0.375 = 37.5%)

Notes and guidance (non-statutory)

Pupils should be taught throughout that percentages, decimals and fractions are different ways of expressing proportions.

They extend their knowledge of fractions to thousandths and connect to decimals and measures.

Pupils connect equivalent fractions > 1 that simplify to integers with division to whole numbers. Pupils understand that three-quarters plus three-quarters is two other models, and have been familiar from Reception and Year 1 fractions.

Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, and apply their understanding of fractions. They relate to scaling by simple fractions, including fractions > 1.

Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of fractions and apply their knowledge of fractions to decimals to two d.p. and understand percentages as 'parts of 100'.

Pupils should go beyond the measurement and money models of decimals, for example, by exploring decimals in measuring diagrams.

Pupils should make connections between percentages, fractions and decimals (for example, 100% represents a whole quantity and 1% is 1/100, 50% is 50/100 and 0.5 is half, so this is finding 1% of 50).

Measurement

Pupils should be taught to:

- convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- understand and use approximate equivalences between metric units and common imperial units such as inch, pound and pint
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), including using standard units, square centimetres (cm²) and square metres (m²), and estimate the area of irregular shapes
- estimate volume and capacity, using cm³ blocks to build cuboids (including cubes) and capacity by filling containers
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure (for example, length, mass, volume, money) using decimal notation, including scaling

Notes and guidance (non-statutory)

Pupils have prior knowledge of place value and multiplication and division to convert between standard units.

Pupils calculate the perimeter of rectangles and related composite shapes, including using the relationship of perimeter or area to find unknown lengths. Pupils calculate the area of shapes by applying their knowledge of the area of a rectangle. 4 (3+ 1)= 4(3) + 4(1) = 12 + 4 = 16 or 4(4) + 4(1) = 16 + 4 = 20 or 5(4) = 20 or 5(3+1) = 5(3) + 5(1) = 15 + 5 = 20

Pupils use pre-lunch and post-lunch examples to understand the relationship between 12-hour clocks and 24-hour clocks.

Pupils use their knowledge of place value and multiplication and division to convert between standard units.

Geometry - properties of shapes

Pupils should be taught to:

- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- know angles are measured in degrees, estimate and compare acute, obtuse and reflex angles
- draw shapes and angles, measure them in degrees (°)
- identify:
  - angles at a point and one turn (total 360°)
  - angles at a point on a straight line and half a turn (total 180°)
  - other multiples of 90°
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles

Notes and guidance (non-statutory)

Pupils become accurate in drawing lines with a ruler to the nearest millimetre and measuring with a protractor. They use conventional markings for parallel lines and right angles.

Pupils use the term diagonal and make conjectures about the angles formed between sides, diagonals and parallel and perpendicular lines, and begin reasoning of qualitatively. for example using dynamic geometry ICT tools.

Pupils use angle sum facts and other properties to make deductions about missing angles and isosceles lines to missing number problems.

Geometry - position and direction

Pupils should be taught to:

- describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed

Notes and guidance (non-statutory)

Pupils recognise and use reflection and translation in a variety of diagrams, including on a coordinate grid. They extend their knowledge of 2-D shapes. Reflection should be in lines that are parallel to the axes.

Statistics

Pupils should be taught to:

- solve comparison, sum and difference problems using information presented in a line graph
- complete, read and interpret information in tables, including timetables

Notes and guidance (non-statutory)

Pupils connect their work on coordinates and scales to their interpretation of time graphs.

They begin to decide which representations of data are most appropriate and why.

Algebra

Pupils should be taught to:

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with 2 unknowns
- enumerate possibilities of combinations of 2 variables

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).

Pupils find percentages or 2dp/2dp or calculating angles of pie charts.

Pupils use their understanding of place value and multiplication and division to extend their knowledge of the ratio concept to include ratios involving fractions with common denominators.

Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3/8 = 0.375). For simple fractions with denominators of 2, 4, 5 and 10, they understand that these will convert to terminating decimals, and that other denominators will require rounding. Pupils can express the results of such conversions in different ways, for example, 3/8 = 0.375 or 0.38 or 0.4 or 3/8 = 37.5% and 3/8 = 0.375 or 3/8 = 3 parts out of every 8 parts.

Pupils are introduced to the division of decimal numbers by one-digit whole numbers, initially, including contexts involving money and measures. They recognise division calculations as the inverse of multiplication.

Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding to a specified degree of accuracy and checking the reasonableness of their answers.

Ratio and proportion

Pupils should be taught to:

- solve problems involving the relative sizes of 2 quantities where missing values can be found by using integer multiplication and divisions
- solve problems involving the calculation of percentages (for example, of measures and such as 15% of 360) and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).

Pupils find percentages or 2dp/2dp or calculating angles of pie charts.

Pupils use their understanding of place value and multiplication and division to extend their knowledge of the ratio concept to include ratios involving fractions with common denominators.

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- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).

Pupils find percentages or 2dp/2dp or calculating angles of pie charts.

Pupils use their understanding of place value and multiplication and division to extend their knowledge of the ratio concept to include ratios involving fractions with common denominators.

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Pupils are introduced to the division of decimal numbers by one-digit whole numbers, initially, including contexts involving money and measures. They recognise division calculations as the inverse of multiplication.

Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding to a specified degree of accuracy and checking the reasonableness of their answers.

Algebra

Pupils should be taught to:

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with 2 unknowns
- enumerate possibilities of combinations of 2 variables

Notes and guidance (non-statutory)

Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such:

- missing numbers, lengths, coordinates and angles
- formulae in mathematics and science
- equations with 2 unknowns (for example, x + 2y = 10), and establishing all solutions
- generalisations of number patterns
- number puzzles (for example, what 2 numbers can add up to)

Measurement

Pupils should be taught to:

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 dpwhere appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to larger unit, and vice versa, using decimal notation to up to 3 decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cuboids and cuboids using standard units, including cubic centimetres (cm³) and cubic metres (m³), and extending to other units (for example, cubic kilometres and cubic millimetres)

Geometry - properties of shapes

Pupils should be taught to:

- solve problems involving the relationship between angles in triangles and between angles at a point
- compute and order fractions, including fractions >1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalence
- multiply simple pairs of proper fractions, writing the answer in simplest form (for example, 1/4 × 1/2 = 1/8)
- divide proper fractions by whole numbers (for example, 1/3 ÷ 2 = 1/6)
- associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375 for a simple fraction such as 3/8)
- identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places
- multiply one-digit numbers with up to 2 decimal places by whole numbers
- use written division methods in cases where the answer has up to 2 decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Notes and guidance (non-statutory)

Pupils should practise, use and understand the addition and subtraction of fractions with different denominators and mixed numbers, using the concept of the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (for example, 1/5 + 3/10) and progress to varied and increasing complexity.

Pupils should use a variety of images to support their understanding of multiplication with fractions. This should include fractions of a set of objects, representation of fractions of shapes, and the use of bar models as well as pictorial representations.

Pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction. For example, if 3/4 of a length is 36 cm, then the whole length is 48 cm, then the whole length is 36 × 4 = 144cm.

They practise calculations with simple fractions and decimal fraction equivalents to aid fluency, including converting between fractions with common denominators.

Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3/8 = 0.375). For simple fractions with denominators of 2, 4, 5 and 10, they understand that these will convert to terminating decimals, and that other denominators will require rounding. Pupils can express the results of such conversions in different ways, for example, 3/8 = 0.375 or 0.38 or 0.4 or 3/8 = 37.5% and 3/8 = 0.375 or 3/8 = 3 parts out of every 8 parts.

Pupils are introduced to the division of decimal numbers by one-digit whole numbers, initially, including contexts involving money and measures. They recognise division calculations as the inverse of multiplication.

Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding to a specified degree of accuracy and checking the reasonableness of their answers.

Notes and guidance (non-statutory)

Pupils convert proportions between forms, from fractions to ratios) to a graphical representation as preparation for understanding linear/proportional graphs.

They know approximate conversions and are able to test if an answer is sensible.

Using the metric and imperial units, pupils will practise estimation and interpret changes for measures such as temperature.

They make the point that strategies to approximate and changes, for example, by inversion, are underpinned by their knowledge, understanding and using the formal properties of operations and equality.

Pupils have opportunities to compare and justify the speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.

Geometry - properties of shapes

Pupils should be taught to:

- draw 2-D shapes using given dimensions and angles
- recognize, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognize angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

Notes and guidance (non-statutory)

Pupils draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.

Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known properties.

These relationships might be expressed algebraically for example, $x + 2x = 180$ or $180 - 2x$.

Geometry - position and direction

Pupils should be taught to:

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Notes and guidance (non-statutory)

Pupils draw and plot a pair of axes in all 4 quadrants with equal scaling. This activity provides a context for the use of all 4 quadrants, including the use of negative numbers.

Pupils draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the 4 quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically for example, $(2, 3)$ and $(-2, 3)$ when $(2, 3)$ and $(-2, 3)$ are vertices of a rectangle with a vertical line of symmetry.

Statistics

Pupils should be taught to:

- interpret and construct charts and tables, including pie charts and line graphs to solve problems
- calculate and interpret the mean as an average

Notes and guidance (non-statutory)

Pupils construct their own bar charts, frequency diagrams, frequency polygons and simple pie charts.

Pupils interpret and draw graphs relating 2 variables, arising from their own data and from other sources.

They should recognize when it is more informative to relate or measure them to a graphical representation.

Pupils know when it is appropriate to find the mean of a data set.

Key stage 3

Overarching principles

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematics and ideas. The programme of study for key stage 3 builds on the knowledge, understanding and skills in the National Curriculum for mathematics at key stage 2 and lays the mathematical foundations for future mathematics learning, reasoning and competence in solving increasingly sophisticated problems. They should also be able to reason mathematically and express themselves with increasing precision and fluency.

Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next topic. Pupils who grasp concepts rapidly should be given deeper and broader work within the same curriculum content, including acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the relationship between integers, powers and roots, including fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- apply properties of numbers and mathematical relationships to aid calculation
- use algebraic to express information and solve problems
- substitute values in formulas, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, geometrical and diagrammatic representations (for example, equations, formulas, tables and diagrams, vectors, vectors and geometry)
- use standard and non-standard graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyze numbers, algebraic expressions, 2-D and 3-D shapes, probability and chance.

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems

Subject content

Number

Pupils should be taught to:

- understand and use place value for decimals, measures and integers of any size
- order positive and negative integers, decimals, and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥
- use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorization, including expression using indices and interpret prime factorization properly
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixtures, including those with a mixed operation
- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognize and use relationships between operations including inverse operations
- use integer powers and associated standard notation, including for 10 and its integer powers (1, 2, 3, 4 and distinguish between exact representations of roots and their approximate decimal forms
- interpret and compare numbers in standard form $A \times 10^n$, $1 \leq A < 10$, where n is a positive integer or 0
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express 1 quantity as a percentage of another, compare 2 quantities using percentages, and work with percentages greater than 100%
- solve problems involving percentage change
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express 1 quantity as a percentage of another, compare 2 quantities using percentages, and work with percentages greater than 100%
- solve problems involving percentage change
- understand and use decimals and decimal notation for properties, arrangements and quantities relevant to measurement, including money and scales
- use calculator and other technologies to calculate results accurately and then interpret them appropriately
- appreciate the infinite nature of the sets of integers, real and rational numbers

Algebra

Pupils should be taught to:

- use and interpret algebraic notation, including:
  - ab in place of $a \times b$
  - $3y$ in place of $y + y + y$ and $3 \times y$
  - a^2b in place of $a \times a \times b$, c^2d in place of $c \times c \times d$
  - $\frac{a}{b}$ in place of $a \div b$
  - coefficients written as fractions rather than as decimals
  - brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of 2 or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by reading the meaning of formulae or expressions
- plot and read from graphs, including those representing linear and quadratic functions and those representing simple rates of change
- generate terms of a sequence from either a term-to-term rule or a position-to-term rule, including quadratic sequences
- spot patterns in sequences, including those arising from properties of shapes
- lay out mathematical arguments, including those arising from properties of shapes, in a way that reflects the underlying mathematical structure
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions to simultaneous linear equations
- find approximate solutions to equations numerically using graphs, including those of the form: rectangular, equal lengths and perimeter, and rectangular, equal perimeter and area
- recognize and use the properties of arithmetic and geometric sequences
- recognize geometric sequences and appreciate other sequences that arise

Ratio, proportion and rate of change

Pupils should be taught to:

- change freely between related standard units (for example time, length, area, volume/capacity, mass)
- use scale factors, scale diagrams and maps
- express the relationship between 2 variables, when the factor is less than 1 and when it is more than 1
- use ratio notation, including reduction to and from the form $1:n$
- divide a given quantity into 2 parts in a given part:part or part:whole ratio; express the division as a fraction of the whole
- understand that a proportional relationship between 2 quantities can be expressed as a ratio or a fraction
- relate ratios and fractions and the associated calculations to the arithmetic of fractions and linear functions
- solve problems involving percentage change, including, percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including problems in rate-based contexts
- use compound units such as speed, unit pricing and density to solve problems

Geometry and measures

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapeziums, circles and composite shapes; volume of cuboids, including cubes and other prisms; surface area of cuboids, including cubes and other prisms; circumference of circles
- calculate and solve problems involving perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scales on various instruments
- derive and use the relationship between circumference and diameter, and between area of a circle and square of its radius; for a given the diameter or a given radius, calculate the circumference and area of a circle; for a given circumference or area, calculate the diameter and radius of a circle; for a given area, calculate the circumference of a circle
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other plane rectilinear shapes; and describe properties and definitions
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the properties of different triangles
- derive and use the sum of interior angles of a triangle, parallelogram, circle, and other plane figures (for example, equal lengths and angles) using appropriate language and technique
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles
- apply properties of angles at a point, angles at a point on a straight line, vertically opposite angles, and angles in a triangle to deduce and write down missing angles in any configuration
- understand that a translation is specified by a distance and direction
- describe translations as 2-D movements
- plane and use both standard and non-standard units of measure, including: metric and imperial units, and mixed units
- use the standard conventions for labelling and naming the sides of 2-D shapes, and identify and use their properties
- identify properties of the faces, surfaces, edges and vertices of cuboids, cubes, and other prisms, and use their properties to solve problems
- interpret and use 3-D coordinates in all octants
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other plane rectilinear shapes; and describe properties and definitions
- use the properties of rectangles to deduce and solve problems involving perimeters and areas of shapes and composite shapes made from rectangles
- recognize, describe, measure and calculate angles, including: vertically opposite; angles on a straight line; angles at a point; angles in a triangle; angles in a quadrilateral; angles in a regular polygon; interpreting and using bearings
- use properties of rectangles to deduce and solve problems involving perimeters and areas of shapes and composite shapes made from rectangles
- derive and use the sum of angles in a triangle and use it to find the sum of angles in any polygon, including in regular polygons
- derive and use the formulae for the perimeter and area of a parallelogram and a trapezium
- calculate, estimate, compare, and solve problems in: perimeter, area, and volume in context, including in the context of compound shapes
- recognize, describe, draw, complete and interpret properties of quadrilaterals to derive results about angles and sides, including properties of kites, trapeziums, and rhombuses
- know results to obtain simple probabilities
- know that the probability of an event occurring is between zero and one, inclusive
- understand that probability is the proportion of times the event would occur, in a very large number of trials
- use the properties of faces, surfaces and the associated calculations to the arithmetic of fractions and linear functions
- solve problems involving percentage change, including, percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including problems in rate-based contexts
- use compound units such as speed, unit pricing and density to solve problems

Geometry and measures

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapeziums, circles and composite shapes; volume of cuboids, including cubes and other prisms; surface area of cuboids, including cubes and other prisms; circumference of circles
- calculate and solve problems involving perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scales on various instruments
- derive and use the relationship between circumference and diameter, and between area of a circle and square of its radius; for a given the diameter or a given radius, calculate the circumference and area of a circle; for a given circumference or area, calculate the diameter and radius of a circle; for a given area, calculate the circumference of a circle
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other plane rectilinear shapes; and describe properties and definitions
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the properties of different triangles
- derive and use the sum of interior angles of a triangle, parallelogram, circle, and other plane figures (for example, equal lengths and angles) using appropriate language and technique
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles
- apply properties of angles at a point, angles at a point on a straight line, vertically opposite angles, and angles in a triangle to deduce and write down missing angles in any configuration
- understand that a translation is specified by a distance and direction
- describe translations as 2-D movements
- plane and use both standard and non-standard units of measure, including: metric and imperial units, and mixed units
- use the standard conventions for labelling and naming the sides of 2-D shapes, and identify and use their properties
- identify properties of the faces, surfaces, edges and vertices of cuboids, cubes, and other prisms, and use their properties to solve problems
- interpret and use 3-D coordinates in all octants
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other plane rectilinear shapes; and describe properties and definitions
- use the properties of rectangles to deduce and solve problems involving perimeters and areas of shapes and composite shapes made from rectangles
- recognize, describe, measure and calculate angles, including: vertically opposite; angles on a straight line; angles at a point; angles in a triangle; angles in a quadrilateral; angles in a regular polygon; interpreting and using bearings
- use properties of rectangles to deduce and solve problems involving perimeters and areas of shapes and composite shapes made from rectangles
- derive and use the sum of angles in a triangle and use it to find the sum of angles in any polygon, including in regular polygons
- derive and use the formulae for the perimeter and area of a parallelogram and a trapezium
- calculate, estimate, compare, and solve problems in: perimeter, area, and volume in context, including in the context of compound shapes
- recognize, describe, draw, complete and interpret properties of quadrilaterals to derive results about angles and sides, including properties of kites, trapeziums, and rhombuses
- know results to obtain simple probabilities
- know that the probability of an event occurring is between zero and one, inclusive
- understand that probability is the proportion of times the event would occur, in a very large number of trials
- use the properties of faces, surfaces and the associated calculations to the arithmetic of fractions and linear functions
- solve problems involving percentage change, including, percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including problems in rate-based contexts
- use compound units such as speed, unit pricing and density to solve problems

Probability

Pupils should be taught to:

- record, describe and analyze the frequency of outcomes of simple probability experiments involving chance, including those with unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that if they repeat an experiment, the outcome may be different
- enumerate sets and combinations of 2 sets of data in various ways, including in simple two way tables or Venn diagrams
- generate theoretical sample spaces for single and combined events with equally likely, including outcomes systems and use these to calculate theoretical probabilities

Statistics

Pupils should be taught to:

- describe, interpret and compare observed distributions of single variable through: appropriate graphical representation involving discrete, continuous and grouped data; appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped discrete numerical data
- describe simple mathematical relationships between 2 variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs

Key stage 4

Overarching principles

the mathematical content that should be taught to all pupils, in standard type additional mathematical content to be taught by more able mathematicians, in italics

Tables 1-2

Together, the mathematical content set out in the key stage 3 and key stage 4 programmes of study describes the mathematical knowledge, understanding and skills of the Mathematics qualification. Wherever it is appropriate, given pupils' security of understanding and readiness to progress, pupils should be taught the full content set out in this programme of study.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include surds, rounding to specified accuracy
- select and use appropriate calculation strategies to solve increasingly complex problems
- apply properties of number and mathematical relationships to aid calculation
- *change freely between standard form and ordinary numbers*
- *interpret and use standard mathematical formulae, including those from science and the workplace; rearrange formulae to change the subject*
- *assign variables to quantities, interpret expressions and formulae, including those with more than 2 variables*
- *interpret and use the 3 laws of indices*
- *factorize and simplify more complex expressions, including those with more than 2 terms*
- *factorize quadratic expressions of the form $x^2 + bx + c$ and $ax^2 + bx + c$*
- *simplify and manipulate algebraic fractions, including those with quadratic terms in the numerator and denominator*
- *solve equations and inequalities involving more than 2 terms, including those with brackets, those with fractions, those with unknowns on both sides, and those with one or more unknowns in the denominator*
- *solve quadratic equations by factorizing, by completing the square, by using the quadratic formula, and by trial and improvement, including those with no real solutions*
- *solve simultaneous linear equations in 2 variables (linear/linear or linear/quadratic), including interpreting the meaning of the solution in the context of the problem*
- *solve linear inequalities in 1 variable and represent the solution set on a number line*
- *solve linear inequalities in 2 variables and represent the solution set on a suitable graph*
- *solve simultaneous linear equations in 3 variables by elimination and substitution, including interpreting the meaning of the solution in the context of the problem*
- *solve linear inequalities in 2 variables (linear/linear or linear/quadratic), including identifying the region in which the solution lies*
- *solve simultaneous linear equations and inequalities graphically, including identifying the region in which the solution lies*
- *solve quadratic equations in 2 variables (linear/quadratic), including interpreting the meaning of the solution in the context of the problem*
- *solve linear and quadratic equations in 1 variable, including those with unknowns on both sides and those with one or more unknowns in the denominator*
- *solve quadratic inequalities in 1 variable and represent the solution set on a number line*
- *solve quadratic inequalities in 2 variables, including identifying the region in which the solution lies*
- *solve simultaneous linear equations in 2 variables by elimination and substitution, including interpreting the meaning of the solution in the context of the problem*
- *solve linear and quadratic equations in 1 variable, including those with unknowns on both sides and those with one or more unknowns in the denominator*
- *solve quadratic inequalities in 1 variable and represent the solution set on a number line*
- *solve quadratic inequalities in 2 variables, including identifying the region in which the solution lies*
- *solve simultaneous linear equations and inequalities graphically, including identifying the region in which the solution lies*
- *solve quadratic equations in 2 variables (linear/quadratic),

right-angled triangles {and, where possible, general triangles} in 2 {and 3} dimensional figures

- know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45°, 60°
- {know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles}
- {know and apply Area = $\frac{1}{2}$ ab sin C to calculate the area, sides or angles of any triangle}
- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}

Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to 1
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- {calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}

Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- {construct and interpret diagrams for grouped discrete data and continuous data, ie, histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
  - appropriate graphical representation involving discrete, continuous and grouped data, {including box plots}
  - appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}
- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

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