A new approach to mathematical and data education

THE ROYAL SOCIETY

A new approach to mathematical and data education
Issued: September 2024 DES8939
ISBN: 978-1-78252-728-2

To view the full report and to find out more about the Society's work in this area, scan the QR code or visit: **royalsociety.org/mathematical-futures**

Cover image: Infographic showing the total number of goals scored by each country who took part in the UEFA European Football Championship, mapped to their location. An example of how data can be used in everyday life. Data source: uefa.com/euro2024/statistics

Contents

Foreword

In today's world, mass illiteracy is largely – and thankfully – a thing of the past. However, we are faced with a new challenge – data illiteracy – which risks excluding millions of people from the national conversation and from an increasing number of better-paid jobs. A nation of citizens who lack quantitative literacy represents a potential modern-day threat to democracy and prospects for national renewal.

In our daily lives, we are continually faced with an avalanche of data, figures, numbers and statistics. Digital and data-based technology is also transforming the future of jobs, from finance to engineering and many more sectors.

We need to provide a mathematical and data education that better prepares all young people for their futures, whether for jobs in these sectors or to be equipped as citizens to play active roles in wider society.

Confidence in mathematics, in its broadest sense, is essential if we want our citizens, society, and economy to thrive. But we also have a major culture problem around mathematics. As a nation, we need to be more imaginative in how we talk about and teach mathematics – mathematics should not be a frightening word.

We need to work with teachers to create a new curriculum that combines mathematics as we know it with data, computing and AI and that places maths in real world contexts. Core Maths is an excellent, ready-made stepping stone to achieving this. We must also take a serious look at the current assessment system – the psychological toll of resits, for example, is a huge burden on young people.

I am under no illusion – this will not be an easy task. Reforming the education system will take time and major investment. However, if we do not start now, we risk today's young people being ill-prepared for the future, and the exacerbation of existing regional, gender and socio-economic inequalities.

We find ourselves with a once-in-a-generation opportunity to effect change and must begin to build a cross-party approach with support from teachers, students, parents, and employers. This matters too much to be a political football that could be punctured by the ebb and flow of politics.

Sir Adrian Smith,
President of the Royal Society

Sir Adrian Smith
President of the Royal Society.

Introduction

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS O

Sir Martin Taylor FRS
Chair of the Mathematical Futures Board and the Royal Society Advisory Committee on Mathematics Education (RS ACME).

Mathematical and data sciences are everywhere and their influence is growing rapidly. They increasingly support thinking and decisions in government, industry, finance and business, and in academic disciplines. They influence the day-to-day lives of individuals as employees, citizens and consumers of information. The massive increase in the use and availability of data through digital technologies means that this influence can only grow. For all our sakes, our education system must adapt to this rapid change.

In our report, we set out the case for a new approach to mathematical and data education. Throughout autumn 2023, we consulted widely with stakeholders from the teaching profession, universities, employers and business, and with thought leaders and policy makers. The responses demonstrated a high level of agreement that change was needed, and a high level of support for our suggested reforms.

We believe this new approach will equip future citizens with the capabilities, skills, adaptability, and resilience they need to flourish in a fast-changing, data-rich world. We also believe it will result in many more young people wanting to continue with the study of mathematics and data science, leading to a more mathematically-skilled labour force.

The mathematical and data skills needed in this new world will drive some of the biggest public policy solutions of the age, from tackling the climate and biosphere crises to transformative innovation in public service efficiencies, particularly in healthcare. These skills will be an engine of change for tackling persistent inequalities and can re-animate the productivity growth needed to rescue the UK from persistent economic malaise.

However, as we start this journey we face some serious disadvantages. Almost half of all adults have only the numeracy expected of an 11-year-old. A third of pupils in England effectively fail maths at 16 every year, and few improve when they retake the exams. Almost a quarter of UK 15-year-olds fail to achieve PISA Level 2 (eg carrying out a currency conversion), compared to only 15% in Singapore. Only 11% of UK students achieve the higher Level 5 (eg modelling complex situations) compared to 41% in Singapore. Behind all this lie issues of equity. Socio-economic status is strongly correlated with mathematical performance and the gap between bottom and top quintiles has not improved in 10 years.

None of this bodes well for equipping citizens with the skills to evaluate health risks, appraise the likely real costs of payday loans, compete for skilled jobs, start new high-growth companies, or plan well for old age. For citizens, for the economy, and for UK competitiveness, the situation must change.

This report explores how the educational system can respond to the explosion in data availability and the way that the exploitation of data through technologies is pervading every aspect of life. As the world has changed and is increasingly supported on a data-driven architecture, so the teaching of mathematics and data needs to change to equip citizens with the mathematical foundations to thrive as the fourth Industrial Revolution continues to unfold, and the UK grows its economy amidst intense international competition.

Whilst traditional approaches to UK mathematics education had a rationale that fitted the context of the times, that context has dramatically changed. This report explores how a more relevant and resilient future approach can be developed, but it offers no magic bullet or quick win. Success in reforming mathematics education is a decadal project where the new ideas and concepts we describe will need thorough testing by deeply knowledgeable stakeholders from across the education system, and where the strengthened approach that emerges will require sustained investment and considerable commitment.

Our view is that the benefits to individuals and to the nation of succeeding in reform will be profound, and that the policy option to do so is one with no regrets.

The pace of change is fast and is accelerating. Now is the time to act.

Sir Martin Taylor FRS, Chair of the Mathematical Futures Board and the Royal Society Advisory Committee on Mathematics Education (RS ACME)

Executive summary

The case for change – a new approach to mathematical and data education In the 21st century, the scope and application of mathematics have undergone a remarkable expansion, partly driven by an unprecedented surge in data availability, computing capabilities, and statistical methodologies. Data now plays a pivotal role in both employment and everyday life.

With increased demand for communication, collaboration and problem-solving skills, there is greater need for mathematically- and data-educated people. Mathematical and data literacy has become fundamental for daily life, but too many of our citizens have poor numeracy and too few are trained to the high levels of mathematical and data competence that will be needed in the future. This has serious implications for the future health of the UK economy. The rise of big data, machine learning and AI demands a shift towards statistics, data science and computing.

The scope of mathematical education needs to change from 'mathematics' to what we have called mathematical and data education (MDE); a combination of mathematics, statistics and data science, underpinned by computational tools. MDE has three interconnected elements which, taken together, provide the skills and competences that individuals and society need to thrive:

- **Foundational and advanced mathematics**
  An evolution of the maths currently taught in school, with greater emphasis on data, technology and computing. It will continue to reflect the subject as a canon of knowledge that can be studied to the highest level.

- **General quantitative literacy**
  Addressing the need for all students to confidently apply their mathematical and data skills to common, real-world, quantitative problems in a range of educational, employment and everyday contexts.

- **Domain-specific competences**
  Recognition that mathematical and data skills are increasingly used within the classroom and beyond, in job or domain-specific contexts.

All citizens need foundational mathematics skills and general quantitative literacy for their daily lives. Individuals in vocational and technical roles often need domain-specific competences particular to those occupations, while roles traditionally seen as non-quantitative now require increased mathematical and data skills. At the same time, the demand for employees with advanced levels of mathematical and data skills is already high and is certain to increase substantially.

Mathematical and data education – where are we now?

Across the UK, mathematics education has made progress over the past twenty years. It serves some students well, but lets too many down. There are wide gaps between the lowest and highest achievers, with a long tail of underachievement, linked to economic disadvantage.

Children's progress in mathematics slows as they transition from primary to secondary school, and their attitudes towards mathematics decline. Gaps between high and low achievers widen in key stage 3 (ages 11 – 14). This particularly impacts pupils from disadvantaged backgrounds.

In England, the qualifications and assessment system means that around a third of students leave compulsory education without a GCSE pass in Maths and with an enduring sense of failure, while for many who do achieve a higher GCSE grade there are few appropriate opportunities to continue their mathematical education.

Modern data and computational concepts and tools are largely absent from mathematical education as it is currently practised, while problem solving and application of mathematical learning in meaningful contexts are not given high priority.

There is a long-standing shortage of qualified mathematics teachers, which is most evident in lower secondary years (ages 11 – 14).

The future: mathematical and data education (MDE)

Mathematics is at the heart of mathematical and data education and its importance as a discipline cannot be overstated. Future citizens in all walks of life will need generic, transferrable mathematical competence to underpin their occupation-specific techniques and skills, and to support daily life.

Expert mathematicians

MDE will encourage and nurture future expert mathematical scientists. Students should learn to appreciate that mathematics is one of humanity's great intellectual achievements. For some, the encounter with the fundamental ideas of mathematics will be a gateway to pursuing advanced mathematics – this opportunity should be broadly accessible.

Quantitative literacy

MDE has a strong focus on general quantitative literacy for all learners, at every stage and level, both within the MDE curriculum and across other subjects and qualifications. General quantitative literacy includes real world problem solving where understanding the problem, data collection, mathematisation, performing calculations, and communication of results are all important.

Problems posed in general quantitative literacy do not always have a single 'correct' answer. This is markedly different from current teaching and assessment in mathematics, which typically prioritises mathematical fluency and / or speed. This has implications for teaching and assessment.

Whole-school approaches to mathematical and data education

MDE is embedded in many school subjects, and as students progress, they increasingly need domain-specific competences. These are needed to support different subjects, but also to help understand important, substantive issues such as climate change and sustainability. Consistency of terminology and approach is needed to benefit students, teachers and curriculum planners. Model whole-school MDE policies to underpin consistency and coherence between subjects and phases should be developed. Developing and implementing these policies will involve staff from a variety of relevant subjects, supported by mathematics staff.

Computational concepts, technology and tools

There is a striking difference between mathematics as done in school, and the way it is done beyond school. In employment, higher and further education, and adult life, computation is largely undertaken using digital technology, not by hand. This discrepancy has an effect on the real and perceived relevance of school mathematics. It needs to be addressed both by educational technology that offers tailored tutoring and immediate feedback, and by using computational tools routinely for learning, doing and exploring mathematics.

The interdependencies between computing and mathematics are rich and complex, and they apply in the school context. Mathematics and computing are nonetheless separate subjects, in life and in school, and we expect this to continue. We acknowledge the differences and expect the interdependencies will be fruitfully explored in the future design of MDE.

Stages and structures

In the future, mathematical and data education (MDE) will be a necessary part of everyone's lives. It must therefore provide an appropriate education for all students; one that acknowledges different needs, and start and end points, in a way that our present system does not.

This includes learners with Special Educational Needs, of whom there are around 1.6 million in schools in England. Special Educational Needs can refer to temporary or long-term learning difficulties or disabilities due to a wide range of factors, including communication, cognition and learning, social, emotional, and mental health, and sensory or physical needs. A move to MDE has the potential to benefit many of these pupils, whether it be through the increased use of assistive technologies or through a wider focus on applications and the use of numeracy in life skills. If the aspiration that MDE should be for all students is to be met, then plans for the development of MDE should include explicit consideration of the requirements of Special Educational Needs pupils from the outset.

Early years

High-quality early years mathematical education is critically important for the development of MDE. In the future, there should be greater emphasis on conceptual understanding and a stronger focus on spatial reasoning, which plays a key role in the later development of number, measure, data and geometry skills.

Primary (5 – 11)

Primary teachers are familiar with techniques which develop pupils' knowledge of maths, but the emphasis will have to move to enabling pupils to think, reason and apply their knowledge to solve problems. At present, the curriculum over-emphasises the performance of arithmetical operations but it is the structure of the number system and number relationships that are foundational for future study.

An MDE curriculum would encourage the use of calculators and other computational tools at the appropriate stages to support children's exploration of number, and to enhance problem solving and investigating.

11 – 14

The lack of progress in maths shown by many students during key stage 3 could be addressed by establishing a new assessment around age 14. This assessment would be low-stakes, ie not part of school accountability measures. It would assess competence in fundamental ideas and applying concepts in meaningful contexts.

Students aged 11 – 14 suffer particularly from the shortage of mathematics teachers. MDE should be taught by skilled and knowledgeable practitioners, so creative deployment of expertise already available from teachers of other subjects will be important, as will enhancing these teachers' expertise in mathematical and data education.

14 – 18

From the age of 14, as elements of choice, diverging education pathways, and qualifications with high-stakes assessments are introduced, so the challenge of cross-system coordination of MDE increases. To continue effective mathematical and data education from 14 – 18 requires a coherent structure that recognises different starting points and aspirations.

With the exception of those with the highest grades, GCSE performance currently provides little information about what learners can and cannot do. An essential element of a successful MDE structure will be qualifications and assessment methods that reliably describe the competences of learners.

The curriculum and qualifications structure needs reform to incorporate and recognise all three strands of MDE – foundational and advanced mathematics, general quantitative literacy and domain-specific competences – in different ways and to different extents.

Reimagining qualifications pathways for MDE lies outside the scope of this report, but we suggest some starting points and principles:

- **Foundational and advanced mathematics**
  Requires a structure similar to the one that has existed for many years. The foundational and advanced mathematics strand would foreground a range of mathematical concepts and skills, building sequentially to prepare for transition to undergraduate study of subjects with high mathematical demands, including mathematics itself.
- **General quantitative literacy**
  We envisage a second set of qualifications, designed to develop and assess the ability to use and apply mathematical concepts and use digital tools to address real-world quantitative problems. The existing Core Maths qualifications should be extended and developed as the basis of this strand.
- **Domain-specific competences**
  These already exist in vocational and technical qualifications such as BTECs and T levels and in recently reformed A levels. We expect this to grow and develop.

The developments in general quantitative literacy and domain-specific competences will involve teachers from disciplines other than mathematics. In time, and with appropriate support, general quantitative literacy could be taught by a range of teachers, not just by mathematics teachers.

Teachers

The long-term plans for MDE require investment – financially, politically and culturally – in teacher professionalism. They will depend heavily on a positively disposed teaching workforce across all stages and subjects and must offer teachers sufficient agency to embrace the undoubted challenges that reform will bring.

A new approach to mathematical and data education will require a steadily evolving cadre of teachers who embrace change, not as passive recipients, but as co-creators of a shared, more relevant, ambitious and exciting array of learning experiences, for both students and their teachers.

Next steps

The reforms outlined in this report cannot be developed by limited short-term measures; they will take 10 – 15 years to implement fully and will need planned and coordinated progress on four fronts: curriculum, qualifications, resources and professional development. They will need serious investment and careful planning, design, implementation, and evaluation. They will require collaboration between the stakeholders involved, cross-party support and determination to stay the course.

At the same time, the direction and shape of the long-term changes that are needed are already clear. There are significant risks in delay, and the process should begin as soon as possible.

Recommendations

AREA FOR ACTION 1: IMPLEMENTATION

Education is a complex system that shapes young people's lives and livelihoods. System change must be independent of political ideology and grounded in evidence. Meaningful change takes time, thoughtful planning and steadfastness.

RECOMMENDATION

- The government should sponsor an independent task force to plan for long-term system changes and implement the recommendations from this report. This task force should include relevant government departments such as Department for Education, Department for Science, Innovation and Technology, Department for Business and Trade, and the Treasury. It should also involve senior figures from key stakeholder bodies and should consult with devolved nations. A sufficient budget should be provided to commission exploratory and developmental projects. (See section 6.1 Next steps.)

AREA FOR ACTION 2: CURRICULUM

What young people learn shapes their perspective on the world. Lessons from past experience and other countries show that curriculum change must be coherent across age ranges and collaborative across subject specialisms.

RECOMMENDATIONS

- Design and implement a curriculum that integrates appropriate data, statistics, and computational tools coherently with mathematics. (See section 5.1 Overview.)
- Review the early years and primary curriculum to provide strong foundations, strengthening key areas such as spatial reasoning. (See section 5.3 Primary.)

AREA FOR ACTION 3: QUALIFICATIONS AND ASSESSMENT

Assessment is a measure of progress and qualifications are ‘passports’ to future opportunities. For students, for their future educators and employers, both must recognise what students can do with their learning.

RECOMMENDATIONS

- Develop a single MDE qualifications framework which enables all students to continue to study MDE to 18. Design the framework around parallel and complementary foundational and advanced mathematics and general quantitative literacy strands, with recognition of domain-specific competences acquired in vocational and technical routes. Base the general quantitative literacy strand on development of the existing Core Maths qualifications. (See section 5.5 MDE from 14 – 18.)
- Develop a low-stakes competency assessment, to be taken by all students around the end of key stage 3 (age 14), to enable individual learners to demonstrate mastery of the foundational MDE concepts and skills necessary for confident and engaged citizenship. (See section 5.4 Rethinking MDE from 11 – 14.)
- Develop assessment methods that identify and communicate what students know and can do. (See section 5.6 Assessment.)
- Develop new methods of assessment for general quantitative literacy that reflect how it is used in practice, including the use of digital technologies to analyse data sets. (See section 3.2.2 Implications of GQL for teaching and assessment.)
- Standardise MDE terminology and level of detail expected in all school and college courses and require awarding organisations to be consistent in how they describe MDE competences in their programmes of study and assessment criteria. Begin this process by carrying out a study into how MDE competences are currently described and used in existing high-stakes qualifications in non-maths subjects. (See section 3.3.2 Supporting teaching.)
- Develop online assessment methods that can grow as needed, enabling the use of MDE-specific digital tools and benefitting from lower costs and improved operations. (See section 5.4 Rethinking MDE from 11 – 14.)

AREA FOR ACTION 4: COMPUTATIONAL TOOLS AND TECHNOLOGY

Technology and data increasingly shape our world. Students should leave school or college with an understanding of how to use computational tools to interact with the world and understand it better.

RECOMMENDATIONS

- Computational tools and technologies, such as spreadsheets, apps and programming platforms, should be well-embedded at suitable stages within MDE learning. Curriculum designers should ensure these technologies are incorporated to meet the new MDE objectives. (See section 4.1 Tools for mathematical and statistical thinking.)
- The Department for Education should carry out a dedicated research programme on the potential impact of AI on MDE learning, identifying new approaches for students at all stages of their education. (See section 4.1 Tools for mathematical and statistical thinking.)
- Strengthen the links between MDE and computing by: including problems in MDE that draw on pupils' computing knowledge and skills, including programming; and providing rich, motivating MDE contexts for programming and other skills in computing lessons. (See section 4.3 MDE and computing concepts.)
- Ensure advanced MDE students (post-16) develop programming skills and learn the use of computational tools common in mathematically-demanding undergraduate programmes. (See section 4.3 MDE and computing concepts.)

AREA FOR ACTION 5: TEACHERS

Teachers are at the heart of a new approach to mathematical and data education and must be central to creation and development of all aspects of this new approach. The successful implementation of MDE will rely on a positive, confident, valued teaching workforce with the agency to drive forward the necessary changes.

RECOMMENDATIONS

- The government should prioritise funding over several years to support a major programme of professional development, including initial teacher training, early career training and continuous professional development to support the implementation of MDE. This should be designed into the implementation plan from the outset and sustained over time. (See section 6.2 Investment in teacher professionalism.)
- Develop professional development programmes, with supporting classroom resources, to enable current teachers to become expert teachers of MDE at key stage 3, and to encourage new routes into teaching. (See section 5.3 Primary.)
- Develop ways for mathematics and subject departments in schools and colleges to work with each other to support consistency and coherence between subjects and phases in the teaching of MDE across all subjects; for example, by developing whole-school MDE curriculum guidance documents. (See section 3.3.2 Supporting teaching.)

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Chapter one

Mathematical and data education – the case for change

Left
Mathematical information is needed to make everyday decisions such as energy consumption within the home.

Mathematical and data education – the case for change

1.1 Why change? Mathematics and data in the changing world

In 2010, human activities created captured copied and consumed about 2 zettabytes (2^{70}) of data. By 2020, this had grown to 64.2 zettabytes and is forecast to reach 181 zettabytes in 2025[1]. Thirty years ago Google, Facebook, Amazon and Tencent did not exist. Now they have combined revenues approaching $1 trillion, whilst data driven products and services from the tech giants have an almost continuous presence in every aspect of our lives.

This is an industrial and societal revolution at a blistering pace and scale. The World Economic Forum estimates that industry transformations driven by data technologies such as artificial intelligence (AI) and text, image and speech processing, will change almost a quarter of global jobs in the next five years[2]. Those leaving school will be facing a job market already quite different to the one that influenced their choice of GCSEs.

Alongside this data explosion, Moore's law has driven an exponential growth in analytical power, now given an additional great leap forward by powerful AI that can rapidly discern subtle patterns and solutions previously hidden. A stunning recent example is DeepMind's AlphaFold AI based programme. Sixty years of experiments have elucidated the structures of about 170,000 proteins. In the 6 years from AlphaFold 1 to now AlphaFold 3, DeepMind has now released 200 million predicted structures for almost every known protein.

Such extraordinary accelerations in analytical innovation have huge implications for drug discovery, understanding disease, and transforming the productivity of the health sector not just through better treatments, but also through the paradigm shift of understanding at a personalised level how to minimise disease risk in the first place and change fundamentally the nature of public health.

Such data-driven accelerated transformations are sweeping through all fields of human endeavour, from predicting weather and climate risks; sifting the cosmos for the goldilocks planets that might harbour life; delivering personalised drugs for cancer; improving the efficiency of managing national energy systems; creating the means for viral ideas to go instantly global; increasing the ability of data driven just- in-time logistics companies to almost instantly provide us with high quality affordable food, goods, news and entertainment; accelerating the advent of nuclear fusion through powerful simulations; to giving a city like London the significant productivity gains from millions of citizens having an accurate sense of when the next bus will arrive.

1 Taylor P, 2023. Volume of data/information created, captured, copied and consumed worldwide from 2010 to 2020, with forecasts from 2021 to 2025. See https://www.statista.com/statistics/871513/worldwide-data-created/ (accessed 14 June 2024).

2 World Economic Forum, The Future of Jobs Report 2023. See https://www.weforum.org/publications/the-future-of-jobs-report-2023/ (accessed 3 June 2024).

Technology driven change at the scale of entire economies is highly disruptive and creates new competitive fracture lines – winners and losers at every geography from local, regional, national, and international, alongside competition within and between sectors for new talent and advantage. The critical public policy questions now are: how to help those displaced to re-skill? How to prepare those being educated now to live and work in a world increasingly driven by data-based products and services where automated algorithmic decision processes are already all pervasive in mortgage, insurance, loan, and other markets? How to equip citizens with the numeracy skills to adapt and redeploy in employment markets where careers have a reducing half-life, and many current roles were not imagined even ten years ago. How to equip the nation that led the first industrial revolution, to innovate and compete successfully in the fourth?

1.1.1 The evolving role and nature of mathematics

The Mathematical Futures Programme commissioned an evidence review on the changing nature and importance of mathematics in the twenty first century[3]. The review focussed on three broad themes: the evolving role and nature of mathematics; mathematics and data in employment; and mathematics and data in society.

The review cites multiple sources which show how mathematics has a huge and diverse influence at the cutting edge of human activity. It illustrates how the mathematical sciences are becoming more connected, both within its subdisciplines and with other disciplines, and how other fields and sectors – in STEM, in data science, in social science and the humanities – have a growing need for mathematical expertise and skills.

An independent review commissioned recently by UKRI (United Kingdom Research and Innovation) shows the huge extent of the fields in which the demand for mathematical sciences is growing[4].

"Mathematical tools and techniques lie at the heart of numerous industries, ranging from financial services to the special effects and computer-generated imagery (CGI) used in the film industry, underpin much of the technology used in national security and defence, and are now essential in the life sciences. Medical advances increasingly rely on mathematical data analytics, machine learning and process modelling, while medical imagers such as magnetic resonance imaging (MRI) scanners use algorithms directly derived from mathematical methods. Engineering and material sciences remain heavy users of mathematical methods, allowing the UK to remain a leader in numerous advanced engineering fields such as aerospace and Formula 1 motorsport."[5].

3 Royal Society, 2021. An evidence review on the changing nature and importance of mathematics in the 21st century. See https://royalsociety.org/-/media/policy/projects/maths-futures/changing-nature-of-mathematics.pdf (accessed 10 June 2024).

4 Bond P, 2018. The era of mathematics: An independent review of knowledge exchange in the mathematical sciences. See https://www.ukri.org/wp-content/uploads/2022/07/EPSRC-050722-TheEraMathematics.pdf (accessed 30 July 2024).

5 *Ibid*.

BOX 1

Long-term strategies of the 2019 – 2024 government

The UK Digital Strategy[6]
The government's vision for harnessing digital transformation and building a more inclusive, competitive and innovative digital economy. The strategy argues that "Improving digital education in schools, and increasing undergraduate numbers in science, technology, engineering and mathematics (STEM subjects), will raise the base level of skills of the next generations to enter the workforce."

The National Data Strategy[7]
The UK's vision to harness the power of responsible data use to boost productivity; create new businesses and jobs; improve public services; support a fairer society; and drive scientific discovery. It states that "This data revolution has implications not only for experts with advanced analytical skills, but for the entire UK workforce. While we do not all need to become data scientists, everyone needs some level of data literacy in order to operate successfully in increasingly data–rich environments."

The National AI Strategy[8]
The ten year plan to make Britain a global AI superpower, states a key action for the short-term must be to "support the development of AI, data science and digital skills". Building a tech-savvy nation by supporting skills for the future is one of the government's ten tech priorities.

Implementation of a new approach to mathematical and data education would support ambitious long term government strategies.

6 DCMS, 2022. UK's Digital Strategy. See https://www.gov.uk/government/publications/uks-digital-strategy (accessed 7 March 2024).

7 DCMS, 2022. National Data Strategy. See https://www.gov.uk/government/publications/uk-national-data-strategy/national-data-strategy (accessed 7 March 2024)

8 GOV.UK, 2022. National AI Strategy. See https://www.gov.uk/government/publications/national-ai-strategy/national-ai-strategy-html-version (accessed 7 March 2024).

These changes are evident not just in terms of the level of demand, but also in the nature of the contributions made by mathematical sciences. "Mathematics is a discipline that enables us to understand and generate patterns and structures and develops powerful tools for working with them … The tools created by mathematicians can be converted to practical use, often in the form of models or algorithms, which are embedded within just about every modern technology and socio-technical system."[9].

1.1.2 Mathematics and data in employment

There is a substantial literature relating to the role of mathematical and data skills in the workplace, and how these are changing[10,11]. In all areas, and at all levels of employment, we need, and will increasingly need in the future, mathematically educated people who are equipped to deal with the opportunities and challenges of varied and ever-changing work roles, who know how to ask the right questions and how to play their role in finding the answers.

CASE STUDY 1

"Maths is the basis of everything we do."
Brice Michoud, co-founder of Ncam Technologies

Ncam are the creators of Ncam Reality, one of the most advanced real-time camera trackers in the world. Ncam Reality is used throughout the broadcast, film, and live events industries to visualise photorealistic graphics in real time.

When asked what mathematics would be useful for young people to know when thinking about joining the film and media industry, Brice stated: "Maths is the basis of everything we do. It is part of everything I do, every day. They need to understand:

- how to represent a point in 3D space (using matrices);
- how to transform that point, using translation, rotation, scaling as well as central projection (perspective);
- sampling – in statistics;
- algebra – using basic transformations;
- vector spaces."

He said, "Mathematics is common as we are simulating everything in the real world on the computer."

In his industry he looks for people with an ability to think logically and to solve problems.

In relation to problem solving skills, he explains people need to have the ability to break a big problem into smaller manageable problems, for example, in simulations and computer graphics. When creating models, he described the importance of being able to make predictions and then corrections.

"When solving a problem, designers need to start and then iterate, set a strategy, and check their strategy. These approaches help designers to reduce risk and understand what is happening – rather than simply being lucky."

9 *Op. cit.*, note 4.

10 Hoyles C, *et al*, 2013. Mathematics in the workplace: issues and challenges. See https://link.springer.com/chapter/10.1007/978-3-319-02270-3_4 (accessed 30 July 2024).

11 Straesser R, 2015. 'Numeracy at work': a discussion of terms and results from empirical studies, ZDM Mathematics Education 47, (pp665 – 674). See https://doi.org/10.1007/s11858-015-0689-0 (accessed 30 July 2024).

Globally, and in the UK in particular, there is a steady movement away from manual and low- skill jobs towards those requiring higher levels of expertise and problem-solving skills, many of which are mathematical in nature and require data skills[12,13].

A recent report published by the Department for Education and the Skills and Productivity Board found that digital literacy is already an essential requirement and that: "... across roles, skills around understanding and use of data will only increase in importance in future as responsibilities for data handling and data security are shared across organisations."[14].

A report published as part of the Skills imperative 2035 programme finds that "new skills that are projected to enter the top 20 in 2035 for the first time include Interacting with Computers, which was at rank 35 in 2010 and rank 24 in 2020 and is forecast to be the 14th most significant skill in 2035. Likewise, Analysing Data or Information rises from rank 33 in 2010, to rank 27 in 2020 and is projected to be rank 16 in 2035. These new skills appearing in the top 20 are consistent with the expected increasing significance of Processing Information which enters the top 10 skills for the first time in 2035."[15].

Working with big data has become increasingly important, while AI is fast becoming an essential requirement. "While AI and big data ranks only 15th as a core skill for mass employment today, it is the number three priority in company training strategies from now until 2027, and number one priority for companies with more than 50,000 employees. AI and big data is also the most strongly prioritised skill in the Insurance and Pensions; Management, Media, Entertainment and Sports; Information and Technology Services; Telecommunications; Business Support and Premises Maintenance Services; and Electronics industries."[16].

12 World Economic Forum, 2023. The future of jobs report 2023. See https://www.weforum.org/publications/the-future-of-jobs-report-2023/in-full/ (accessed 30 July 2024).

13 CIPD, 2021, Technology, AI and the future of work. See https://www.cipd.org/uk/knowledge/factsheets/emerging-future-work-factsheet/ (accessed 30 July 2024).

14 Department for Education, 2022. Skills needs in selected occupations over the next 5 – 10 years. See https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1095989/Skills_needs_in_selected_occupations_over_the_next_5-10_years_.pdf

15 Dickerson A *et al*, 2023. An analysis of the demand for skills in the labour market in 2035 (p56). See: https://www.nfer.ac.uk/publications/the-skills-imperative-2035-an-analysis-of-the-demand-for-skills-in-the-labour-market-in-2035 (accessed 16 August 2024)

16 World Economic Forum, 2023. The future of jobs report 2023 (p46).

The changing needs of the workplace are illuminated by the concept of Techno-Mathematical Literacies (TML)[17]. The ubiquitous use of ICT in all sectors changes the nature of the mathematical skills that are required, while not reducing the need for mathematics. Techno-Mathematical Literacies are "combinations of mathematical, ICT and workplace-specific competencies that demand an ability to deal with mathematical models and make decisions based on the interpretation of abstract information."[18]. These are complex skills which are grounded in data and the context of specific work situations. "Context and activity in which mathematical understanding takes place are … essential, and employees must make mathematical sense of situations that are quite different from their formal mathematical education."[19].

Dealing with mathematical models and taking decisions based on abstract information have always been tasks of highly trained employees, but the availability of technology means that an increasing number of people engage in these systems, which brings a new complexity to the workplace[20,21]. An important finding from research carried out by the University of London Institute of Education was that: "Effective learning of TML can follow from engagement in authentic activities that embed work process models made more visible and manipulable through interactive software tools."[22].

"We live in a world where mathematics is becoming increasingly important. you're not going to follow the debate on climate change if you do not understand exponentials, you're not going to be dealing with the evolution of a pandemic if you're not able to think in probabilistic terms. Maths is everywhere, and it's not just for the few people who become engineers, but to be able to participate in our economies, our societies, you need to have a really good understanding of fundamental mathematical concepts."

Andreas Schleicher, Director for Education and Skills, OECD

17 Hoyles C *et al*, 2010. Improving mathematics at work: the need for techno-mathematical literacies.

18 *ibid*

19 Bakker A *et al*, 2011. The use, nature and purposes of measurement in intermediate-level occupations, (pp737 – 746). See https://doi.org/10.1007/s11858-011-0328-3 (accessed 30 July 2024).

20 Kent P *et al*, 2007. Characterizing the use of mathematical knowledge in boundary-crossing situations at work. Mind, Culture, and Activity, 14(1 – 2), (pp64 – 82). See https://doi.org/10.1080/10749030701307747 (accessed 30 July 2024).

21 Van der Wal N J *et al,* (2017) Which techno-mathematical literacies are essential for future engineers? International Journal of Science and Mathematics Education, 15(1), (pp87 – 104), (p90). See https://link.springer.com/article/10.1007/s10763-017-9810-x (accessed 30 July 2024).

22 Hoyles, C, Noss, R, Kent, P, Bakker, A and Bhinder, C, 2007. Techno-mathematical literacies in the workplace: a critical skills gap.

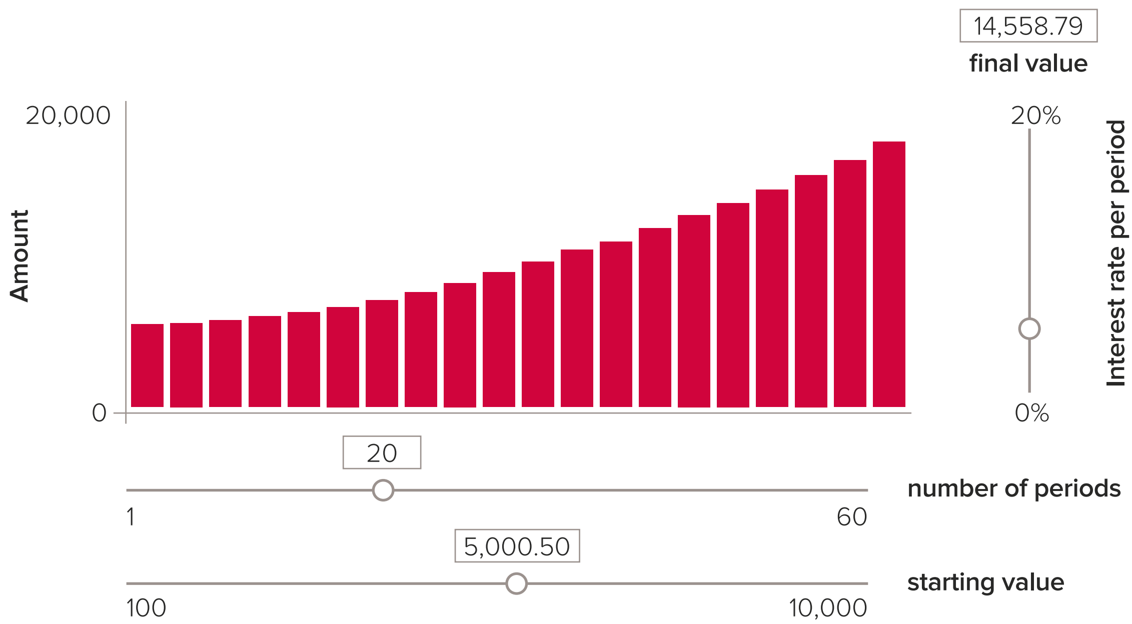
The complete model will not necessarily be visible, (because it would be too complex), but the software tools allow the manipulation of the key variables in the model, so their individual and combined effects can be explored dynamically[23].

Figure 1 illustrates this using the relatively simple example of compound interest.

FIGURE 1

Calculating compound interest[24]

There are 3 sliders that the user can vary: the interest rate (on right), the starting value and the number of periods (at bottom). By changing these 3 variables, the final value of the sum invested is computed (top right) but also the users 'sees' how it accumulates.



Source: Techno-mathematical Literacies in the Workplace (Teaching and Learning Programme, TLRP, ESRC), 2003.

23 Celia Hoyles. Personal communication.

24 Kent P *et al*, 2003. Techno-mathematical Literacies in the Workplace: Making visible the mathematics of working practice. Institute of Education, University of London.

1.1.3 Mathematics and data in society – navigating our data-rich world

All citizens, regardless of their employment, need a range of quantitative skills to be able to manage their daily lives, to evaluate numerical claims, to play their role in civic society, and generally to increase and embrace the opportunities available to them.

Adult numeracy

Numeracy is a fundamental necessity of modern life and the costs of innumeracy are high[25]. As National Numeracy says, "we use maths in every aspect of our lives at work and in practical everyday activities at home and beyond. We use maths when we go shopping or plan a holiday, decide on a mortgage or decorate a room. Good numeracy is essential to us as parents helping our children learn, as patients understanding health information, as citizens making sense of statistics and economic news. Decisions in life are so often based on numerical information; to make the best choices, we need to be numerate."[26].

While few would dispute this, there is still within the UK a widespread feeling that it is "OK to be bad at maths". A National Numeracy poll found that only half of adults (52%) would feel embarrassed to say they were bad at numbers and maths, whereas many more (67%) agreed they would feel embarrassed to say they were no good at reading and writing[27].

The level of adult numeracy is poor, with a large degree of regional variation[28]. According to a 2022 YouGov survey, 54% of the UK's working age population has low numeracy, with nearly half (45%) of respondents demonstrating a level of numeracy equivalent to that expected of children when they leave primary school[29]. International surveys show that this level of numeracy is significantly below many other OECD countries[30] (see Table 1).

England and Northern Ireland have some of the highest proportions of adults scoring at or below level 1 in numeracy; 24.1% of adults, around 8.5 million people, score at that level compared to the OECD survey average of 19.0%. The picture is not improving and it seems likely that the cultural acceptability of innumeracy must help perpetuate the problem.

25 Every Child A Chance Trust, 2018, The long-term costs of numeracy difficulties. See https://www.numicon.co.nz/uploads/66441/files/Numicon_research_ECC_paper.pdf (accessed 30 July 2024).

26 National Numeracy, 2024, Why is numeracy important? See https://www.nationalnumeracy.org.uk/what-numeracy/why-numeracy-important (accessed 2 April 2024).

27 National Numeracy, 2022. Talking about maths is the UK's last taboo. See https://www.nationalnumeracy.org.uk/news/talking-about-maths-uks-last-taboo (accessed 4 June 2024).

28 National Numeracy, 2022. The UK Numeracy Index. See https://www.nationalnumeracy.org.uk/what-numeracy/numeracy-index (accessed 4 June 2024)

29 National Numeracy, 2022. New survey shows Brits are not as good at maths as they think. See https://www.nationalnumeracy.org.uk/news/new-survey-uk-numeracy#:~:text=Nearly%20half%EE%80%80%20(45%)%EE%80%81%20of (accessed 15 August 2024).

30 OECD, 2012. The International Survey of Adult Skills 2012: Adult literacy, numeracy and problem solving skills in England, p50. See https://assets.publishing.service.gov.uk/media/5a7ce2d640f0b6629523c657/bis-13-1221-international-survey-of-adult-skills-2012.pdf (accessed 30 July 2024).

There is evidence that low levels of numeracy are harmful to the economy. A 2014 report for National Numeracy from Pro Bono Economics estimated the overall cost to the UK economy of outcomes associated with low levels of numeracy at around £20.2 billion per year[31]. Individuals with low levels of numeracy are likely to have reduced earning potential and poorer health, with knock-on negative impacts on social wellbeing and their economic prosperity[32].

Conversely, research shows that there are significant economic returns to people who study mathematics to higher levels. These findings have been repeated in many countries and have not changed over time[33,34]. Financial literacy is an area that has attracted particular attention, because of the complexity of financial products, the lack of confidence that many people have in managing money, their susceptibility to fraud and bad conduct, and the potentially high cost of bad decisions[35].

Awareness of the issue, and the provision of funding to address it, has historically been low, although the previous government's recent Multiply initiative has provided significant funding to support a nationwide programme[36]. The reforms proposed in this report are aimed at school and college level education and therefore addressing adult numeracy directly lies outside our scope. However, we agree with National Numeracy that this is a problem that needs urgent attention[37].

31 Pro Bono Economics, 2014. Report for National Numeracy: Cost of outcomes associated with low levels of adult numeracy in the UK, See https://www.probonoeconomics.com/national-numeracy-calculating-the-cost-of-low-adult-numeracy (accessed 4 June 2024).

32 Cherry G & Vignoles A, 2020. What is the economic value of literacy and numeracy? IZA World of Labor. See http://dx.doi.org/10.15185/izawol.229.v2 (accessed 30 July 2024).

33 Dolton P J & Vignoles A, 2002. The return on post–compulsory school mathematics study, Economica, 69(273), (pp113 – 142). See https://onlinelibrary.wiley.com/doi/abs/10.1111/1468-0335.00273 (accessed 30 July 2024).

34 Adkins M & Noyes A, 2016. Reassessing the economic value of advanced level mathematics, British Educational Research Journal 42(1), (pp93 – 116). See https://bera-journals.onlinelibrary.wiley.com/doi/abs/10.1002/berj.3219 (accessed 30 July 2024).

35 Financial Conduct Authority, 2021. Financial Lives 2020 Survey: the impact of coronavirus. See https://www.fca.org.uk/publications/financial-lives/financial-lives-2020-survey-impact-coronavirus (accessed 30 July 2024).

36 Department for Education, 2022. Multiplying maths skills for adults. See https://www.gov.uk/government/news/multiplying-maths-skills-for-adults (accessed 30 July 2024).

37 National Numeracy, 2019. Building a numerate nation: confidence, belief and skills. See http://www.nationalnumeracy.org.uk/sites/default/files/documents/Building_a_numerate_nation/building_a_numerate_nation_report.pdf (accessed 30 July 2024).

TABLE 1

Numeracy scores in OECD countries

	Country	Mean score
Countries outperforming England in numeracy	Japan	288 (0.7)
	Finland	282 (0.7)
	Flanders (Belgium)	280 (0.8)
	The Netherlands	280 (0.7)
	Sweden	279 (0.8)
	Norway	278 (0.8)
	Denmark	278 (0.7)
	Slovak Republic	276 (0.8)
	Czech Republic	276 (0.9)
	Austria	275 (0.9)
	Estonia	273 (0.5)
	Germany	272 (1.0)
	OECD average	269 (0.2)
	Australia	268 (0.9)
	Canada	265 (0.7)
	Cyprus	265 (0.8)
	Korea	263 (0.7)
Countries not significantly different from England in numeracy	England	262 (1.1)
	Poland	260 (0.8)
	Northern Ireland	259 (1.8)
Countries significantly below England in numeracy	Republic of Ireland	256 (1.0)
	France	254 (0.6)
	United States	253 (1.2)
	Italy	247 (1.1)
	Spain	246 (0.6)

Source: PIAAC (2012). Statistical significances are calculated at the 5% level. Standard errors appear in parentheses ( ).

The lives of all future citizens are shaped by what happens to them in school[42], so the design of the mathematical and data education children receive will have a huge impact on their individual lives and the future of our society. If everyone left school with the level of fluency we aspire to, that would bring immeasurable benefits for society, for innovation, for the economy and for individual flourishing. That is our ambition.

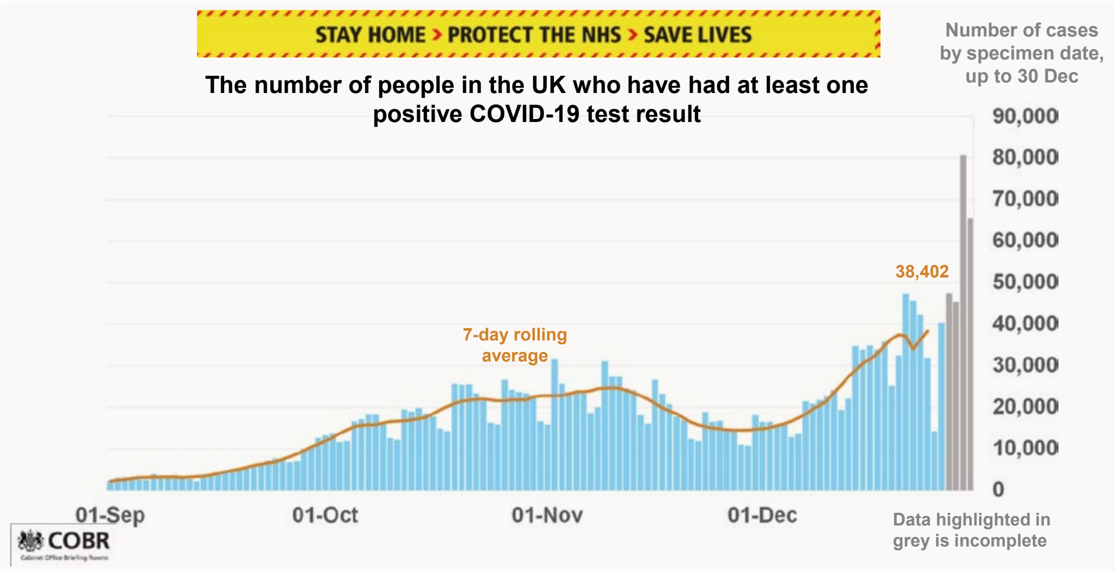
Citizenship and critical thinking

Numeracy and financial literacy are only a part of the picture[38]. The benefits of what we are calling general quantitative literacy go beyond the ability to participate in the workforce and the ability to manage one's money – they shape a person's life as a citizen and community member. In the modern world, citizens are increasingly required to use new technologies to find and become critical consumers of information, for their own fulfilment and to become engaged participants in social and political life.

To take a recent case, there was massive use and interest in data throughout the COVID-19 pandemic[39,40]. The news bulletins and government briefings were full of graphs of changing rates of infections, hospitalisations, and deaths, while the R number and exponential growth were common topics on the media and in general conversation. As an example, Figure 2 shows one of the slides shown at Prime Minister Boris Johnson's address to the nation on 4 January 2021. Such slides were subsequently given wide coverage in the media. It illustrates the kind of information that citizens were routinely expected to understand.

FIGURE 2

Slide shown at then Prime Minister Boris Johnson's address to the nation on 4 January 2021[41]



38 UK Parliament, 2024. Written evidence submitted by the Royal Society to the Education Committee inquiry on Financial Education. See https://committees.parliament.uk/writtenevidence/128796/default/ (accessed 30 July 2024).

39 Muñiz-Rodríguez L, *et al*, 2020, Deficits in the statistical and probabilistic literacy of citizens: effects in a world in crisis, *Mathematics*, 8(11). See https://doi.org/10.3390/math8111872 (accessed 30 July 2024).

40 Braund M, 2021. Critical STEM Literacy and the COVID-19 Pandemic, Canadian Journal of Science, Mathematics and Technology Education, 21, 339 – 356. See https://doi.org/10.1007/s42330-021-00150-w (accessed 30 July 2024).

41 GOV.UK, 2021. The number of people in the UK who have had at least one positive COVID-19 test result. See https://www.gov.uk/government/publications/slides-and-datasets-to-accompany-pms-coronavirus-address-4-january-2021 (accessed 4 June 2024).

Statistical estimates of vaccine effectiveness were widely discussed, as well as issues around the accuracy of diagnostic tests, the role of false-positives and the capacity of the health and care services. People were not just consumers of numerical information – every day, the government Coronavirus dashboard was visited more than a million times, with many people taking advantage of the ease with which data could be downloaded. Numerous personal analyses were generated and spread on social media.

Statistics from randomised trials allowed rigorous assessment of both vaccines and treatments, identifying both effective and ineffective interventions and saving tens of thousands of lives. Unfortunately, the pandemic also showed that data could be misused to make exaggerated claims about, for example, vaccine harms. Citizens need to be able to critically appraise claims based on data.

1.1.4 Mathematics and data in education

The lives of all future citizens are shaped by what happens to them in school[42], so the design of the mathematical and data education children receive will have a huge impact on their individual lives and the future of our society. If everyone left school with the level of fluency we aspire to, that would bring immeasurable benefits for society, for innovation, for the economy and for individual flourishing. That is our ambition.

While much of our focus is on data, quantitative thinking is not the only important dimension. Symbolic reasoning, geometric and spatial analysis, algorithmic thinking, and deductive argument are equally important elements of mathematical thinking. Presented in appropriate ways they should be integral to mathematical education from the earliest stages. And for all its economic and social importance, the place of mathematics in a young person's education is not simply one of instrumental value. Mathematics is one of humanity's greatest achievements and all pupils should have the opportunity to engage with some of its key ideas, just as they do for literature, history, and music.

We are not alone in thinking about these questions. Around the world countries are asking what skills and competences their citizens will need in the future, and how their education systems can provide them. Research commissioned from Sheffield Hallam University[43] on international policy initiatives gives numerous examples of how different countries are responding, as does a report from the Open University on developments in mathematical and data literacy[44]. The research shows that the UK has been relatively slow off the mark. Box 3 illustrates a number of these developments.

Our aim must be to shape a mathematical and data education that serves everybody, that is fit for the future and that is agile enough to respond to what will be a continual process of change.

42 Association of Child Psychotherapists, 2020. Understanding childhood: the child's experience of primary school (accessed 11 April 2024). See https://childpsychotherapy.org.uk/resources-families/understanding-childhood/childs-experience-primary-school (accessed 30 July 2024).

43 Adams G & Boylan M, 2023. Landscaping mathematics education policy: horizon scanning of international policy initiatives. See https://royalsociety.org/-/media/policy/projects/maths-futures/landscaping-international-mathematics-education-policy.pdf (accessed 30 July 2024).

44 Smith C et al, 2023. Mathematical and data literacy: competencies and curriculum implications at the intersection of mathematics, data science, statistics and computing. See https://royalsociety.org/-/media/policy/projects/maths-futures/intersection-mathematics-data-statistics-computing.pdf (accessed 30 July 2024).

BOX 3

Innovation in mathematics and data education in other countries

New Zealand: statistics, data literacy and data science
Aotearoa New Zealand has a well- established 'Mathematics and Statistics' curriculum which has been informed by research and is seen as progressive by the statistics education community. The national curriculum is notable for taking a strong stance by positioning statistics alongside mathematics: "Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data. These two disciplines are related but different ways of thinking and of solving problems."[45].

Singapore: Digital technology
Policy developments regarding access to digital devices offer potential to capitalise on the opportunities afforded by developments in digital approaches to mathematics teaching and learning. In Singapore, as part of a wider digital literacy policy, all learners will own their own school-prescribed personal learning device[46].

Sweden: programming in mathematics
Sweden introduced a new national IT strategy for schools that includes, starting in 2018, programming as a core content of the curriculum for pupils of all ages, including in mathematics and technology subjects. Mathematics, along with other subjects, was 'digitally remodelled'. The professional development of teachers was required to support this change[47,48].

Ontario: curriculum pathways
Ontario offers an example of a curriculum with multiple pathways for older students and one where real-world applications are notably pervasive[49]. The 2005 Ontario curriculum (influenced by US Standards) heavily emphasises mathematical processes (problem solving; reasoning and proving; reflecting; connecting; communicating; representing; selecting tools and strategies) in the familiar content areas of number, algebra, measurement, geometry and statistics[50].

Germany: mathematical modelling
In Germany, mathematical modelling has been included as one of six general mathematical competences in the mandatory standards for mathematics in middle schools since 2003 and later added to the standards for primary and upper secondary schools[51,52].

45 New Zealand's Ministry of Education, 2014, The New Zealand curriculum: mathematics and statistics. See https://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum/Mathematics-and-statistics (accessed 30 July 2024).

46 *Op. cit.*, note 43.

47 Vinnervik P, 2020. Implementing programming in school mathematics and technology: teachers' intrinsic and extrinsic challenges, International journal of technology and design education, (pp1 – 30). See https://link.springer.com/article/10.1007/s10798-020-09602-0 (accessed 30 July 2024).

48 Heintz F *et al*, 2017. Introducing programming and digital competence in Swedish K-9 education. See https://www.diva-portal.org/smash/get/diva2:1148061/FULLTEXT02 (accessed 30 July 2024).

49 Smith C & Morgan C, 2016. Curricular orientations to real-world contexts in mathematics. See https://discovery.ucl.ac.uk/id/eprint/1474125/3/Smith_Morgan_CJ final.pdf (accessed 30 July 2024).

50 Ontario Ministry of Education, 2007. The Ontario curriculum: mathematics. See https://www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics (accessed 30 July 2024).

51 Greefrath G & Vorhölter K, 2016. Teaching and learning mathematical modelling approaches and developments from German speaking countries, Springer International Publishing. See https://link.springer.com/book/10.1007/978-3-319-45004-9 (accessed 30 July 2024).

52 Kaiser G & Schwarz B, 2006. Mathematical modelling as bridge between school and university, ZDM, 38(2), (pp196 – 208). See https://link.springer.com/article/10.1007/BF02655889 (accessed 30 July 2024).

The recent storm of interest in generative AI serves as a reminder that children entering primary school today will leave school to take up roles and occupations that do not yet exist[53]. Their daily lives will be different from today's and will continue to change over their lifetimes. And as the content and goals of mathematical education change, so must the processes, to reflect the digital and technological environment into which children will grow. Our aim must be to shape a mathematical and data education that serves everybody, that is fit for the future and that is agile enough to respond to what will be a continual process of change[54].

As we explain in the next chapter, our present education system works well in many respects, but it leaves too many behind. For the sake of individual citizens and for the sake of society this must change. Any new approach must cater for all students, with every student having the opportunity to achieve the best they can. This objective is fundamental to our thinking and to the principles that are the foundations of the new mathematical and data education.

1.2 Mathematical and data education – the key ideas

The scope of the mathematical education that is needed is changing from 'mathematics' to a combination of mathematics, statistics and data science[55] underpinned by computational tools. We have called this mathematical and data education (MDE). MDE is built on a secure grasp of the basic toolbox of essential mathematics. It comprises three core elements that have complementary goals. Each of these core elements makes use of appropriate digital tools.

1.2.1 Foundational and advanced mathematics (FAM)

The first, which we call foundational and advanced mathematics (FAM), is an evolution of the current mathematics curriculum, with a greater emphasis on data and use of computational tools. It will continue to reflect the subject as a canon of knowledge that can be studied to the highest level and includes important technical elements to reach proficiency.

Foundational mathematics establishes essential capabilities for further mathematical study and for learning more generally. Advanced mathematics builds capacity for more demanding focused study and the application of mathematics and data science in subsequent learning. Together they incorporate the understanding of fundamental mathematical ideas, the development of more complex mathematical and data techniques, and skilled use of computational, data-analytic, and other tools at levels appropriate to the age of the learner. Examples include numerical and arithmetic skills, proportional understanding and reasoning, geometry, cartesian coordinates and elementary logic, probability, and descriptive statistics. At a more advanced level, algebra, and trigonometry, followed by calculus and mechanics, are needed for a huge range of mathematical applications, as are circular measure, vectors and matrices, while sampling theory and statistical distributions are essential for the study of statistical inference.

The scope of the mathematical education that is needed is changing from 'mathematics' to a combination of mathematics, statistics and data science underpinned by computational tools.

53 Galindo L, *et al*, 2021. An overview of national AI strategies and policies, OECD Publishing, Paris. See https://doi.org/10.1787/c05140d9-en (accessed 30 July 2024).

54 Department for Science, Innovation and Technology, Office for Artificial Intelligence, Department for Digital, Culture, Media & Sport, and Department for Business, Energy & Industrial Strategy, 2021. AI Roadmap. See https://www.gov.uk/government/publications/ai-roadmap (accessed 30 July 2024).

55 In broad terms, statistics is about describing the world, drawing inferences from data, and testing scientific hypotheses, often from planned collection of data. Data science uses a wider range of techniques and tools to discover patterns, extract insights, and make predictions from complex, real-world data.

1.2.2 General quantitative literacy (GQL)

The second, which we call general quantitative literacy (GQL) , addresses the growing and currently unmet need for all students to confidently apply their mathematical and data skills to the common, real-world, quantitative problems they are likely to face, and in a range of educational, employment and everyday contexts. These skills need to be developed beyond GCSE study.

The ability to use and apply mathematical concepts and use digital tools to address real-world quantitative problems. It is developed throughout education and is essential for operating effectively in daily life and work. Confidence and fluency in general arithmetic and proportional reasoning are its foundations, together with an understanding of what mathematics to use in different circumstances, and an appreciation of presenting and interpreting data. General quantitative literacy underpins the ability to take a critical view of arguments that are based on mathematics and data. Examples of general quantitative literacy include the ability to calculate the affordability of a payday loan or mortgage, or to critique quantitative claims made in advertising or the media.

1.2.3 Domain-specific competences (DSC)

The third, which we call domain-specific competences (DSC), recognises that mathematics and data skills are increasingly acquired and used outside of the mathematics classroom in a job or domain-specific context. These enable learners to use and apply mathematics and data skills in a range of other subjects and disciplines. They begin in primary school and are developed increasingly through the secondary years and beyond, using general digital technologies as well as those designed for applied contexts. Domain-specific competences may originate in mathematics lessons but are as likely to be acquired within the disciplines (for example in non-mathematics A levels and in technical and vocational qualifications such as T Levels). Examples include estimations in construction, procedures used by technicians in batch monitoring and the calculations used to plan sampling in biological fieldwork.

1.3 Mathematical and data education for all

In this section of the report, we explore the needs that all citizens will have in the future. We then discuss three different kinds of roles: vocational and technical occupations, traditionally non-quantitative roles and roles with high quantitative demands.

1.3.1 All citizens

All citizens need mathematical and data competence, and they need it for many reasons. Every day, most individuals will need to draw on more than one of the three core elements of mathematical and data education (MDE) to meet their needs, whether in education, work, or life generally (see Table 3).

TABLE 3

Foundational mathematics skills and general quantitative literacy (GQL) for all citizens

All citizens need foundational mathematics skills and general quantitative literacy (GQL) for daily life, including personal finance[56] and general employment[57]. GQL will enable them to critique quantitative claims with confidence in a variety of contexts.

Area of activity	Practical applications	Examples in daily life
Budgeting	Calculating income, expenses, and savings; using spreadsheets or budgeting apps to track and plan expenditure. Understanding utility bills and how they have been calculated.	Which is better, fixed price or variable rate electricity?
Health	Weighing the risks and benefits of choices about medical treatment.	Why are so many older people recommended to take statins?
Understanding mortgages, loans, insurance, pensions	Calculating monthly payments, interest rates and loan terms, and understanding risk.	I have some money available at the end of the month. Should I: pay down my credit card debt; invest in my pension; buy premium bonds?
Citizenship	Assessing the value of casting one's vote in a particular way in a marginal constituency.	How much should I believe opinion polls about future elections?
Daily life	Day-to-day decisions.	What does "20% chance of rain tomorrow" actually mean?
Shopping decisions	Comparing prices, calculating discounts, and determining best value for money while shopping.	If my supermarket has my usual brand of pasta on offer at 39p per 100g, but another brand of the same product is 50p per 100g with a 3 for 2 offer, which is better value for money?
Travel planning	Booking journeys, calculating travel distances and travel times, converting currencies and understanding time differences.	I want to get from London to Edinburgh as quickly / cheaply / sustainably as possible. Should I fly, drive, take the train or take the bus?
Climate	Understanding information about long-term trends. Understanding different perspectives.	Should I be worried if 2023 was the warmest year on record?
Probability in games	Making strategic decisions in games like poker, chess, or board games involves understanding probability and statistical outcomes.	What are my chances of winning the lottery?
Politics	Understanding competing claims about the efficacy of government policies.	When looking at trends in the economy, should we look at GDP or GDP per capita?
DIY, crafting and sewing	Working out material needs for home improvements. Measuring fabrics, calculating dimensions for sewing patterns, and understanding proportions for crafting projects.	How much will it cost me to redecorate my kitchen?

56 Lusardi A & Messy F, 2023. The importance of financial literacy and its impact on financial wellbeing, Journal of Financial Literacy and Wellbeing, 1(1), (pp1 – 11). See https://doi.org/10.1017/flw.2023.8 (accessed 30 July 2024).

57 Blotnicky K A, *et al*, A study of the correlation between STEM career knowledge, mathematics self-efficacy, career interests, and career activities on the likelihood of pursuing a STEM career among middle school students, International Journal of STEM Education, 5(22). See https://doi.org/10.1186/s40594-018-0118-3 (accessed 30 July 2024).

1.3.2 Vocational and technical occupations

People in vocational and technical occupations (eg electricians, technicians, nurses and other health care professionals) will often need domain-specific competences particular to their occupations. These may be of a very high order. They will be built on foundational or advanced mathematics and general quantitative literacy[58,59].

For example:

- **Chefs**
  Use of foundational mathematics skills to scale recipes and adjust ingredient quantities according to demand, measure ingredients accurately, convert measurements, determine costs per serving, budgets, and profit margins, and calculate cooking times based on weight or volume.

- **Carpenters**
  Utilise geometry and trigonometry as well as an understanding of forces and tensile strengths to design projects. They also use foundational mathematics to determine necessary quantities of materials, calculate costs, and estimate timelines.

- **Nurses**
  Use foundational maths for precise medication dosing and interpreting medical charts. Administering medications and managing intravenous fluids requires accuracy in proportions and measurements.

- **Heating Engineers**
  Utilise maths for comprehending thermodynamics and heat transfer, use geometry to fit heating systems. Calculating energy efficiency and load requirements also needs algebra and trigonometry.

- **Laboratory Technicians**
  Use statistics and probability to analyse data, ensuring accurate substance dilution through precise calculations. Understanding chemical reaction formulas and concentrations is crucial for their work.

1.3.3 Traditionally non-quantitative roles

These are roles that have traditionally been thought of as non-quantitative, but now require increased mathematical and data skills (eg administrators, lawyers, agricultural and land management professionals, journalists, civil servants, politicians). People in these roles will have moderate to advanced general quantitative literacy and, as appropriate, domain-specific competences, underpinned by foundational and advanced mathematics.

For example:

- **Lawyers**
  They may need to understand Financial Analysis, in cases involving financial disputes, such as business litigation, divorce settlements, or damages assessments; Interpreting Probability and Risk Assessment, to assess the likelihood of certain events occurring in insurance cases, criminal law (eg, probability of recidivism), or in assessing the likelihood of success in a case; using relative risks as a basis for determining causation in civil claims of harm from, say, exposure to carcinogens; Forensic evidence, particularly DNA profiles, requires understanding of random match probabilities and likelihood ratios.

---

58 Dalby D & Noyes A, 2015. Connecting Mathematics Teaching with Vocational Learning, Adults Learning Mathematics: An International Journal, 10(1), (pp40 – 49). See https://files.eric.ed.gov/fulltext/EJ1077715.pdf (accessed 30 July 2024).

59 International Labour Organization, 2001. Technical and vocational education and training for the twenty-first century: UNESCO and ILO Recommendations. See https://unesdoc.unesco.org/ark:/48223/pf0000220748.locale=en (accessed 30 July 2024).

- **Farmers**
  Crop planning and spacing; seed and fertiliser calculations; irrigation planning, to calculate irrigation requirements based on factors like soil type, crop water needs, and weather conditions; estimation of the potential yield of their crops using mathematical models based on factors such as plant population, growth rates, and environmental conditions; calculating subsidies.
- **Sales**
  Customer Relationship Management Systems rely on mathematical algorithms to manage customer data, track interactions, and provide insights into customer behaviour. Pricing strategies use mathematical models, considering factors like cost, competition, and market demand. Forecasting, to predict future sales, is based on historical data and market trends.
- **Policy Makers**
  Effective policy making is informed by evidence, hence policy makers may use mathematical and data skills such as: Data Analysis, to gather and interpret often complex information related to policy decisions; Quantitative Modelling, to use the outputs of models such as regressions and simulations to appreciate the potential long-term consequences of policy decisions; Financial and Economic Analysis, to understand budgets, resource allocation, and cost-benefit considerations which may be relevant to different stakeholders.
- **Journalists**
  They may require skills in Critical Thinking, to evaluate the quality of data sources and identify any biases or inconsistencies; Statistics, to interpret the results of medical studies, surveys and opinion polls; Data Literacy, to understand engagement analytics and interpret trends; Data Visualisation, to create and understand infographics and other charts to summarise complex information and communicate it to diverse audiences.
- **Care Home Managers**
  They employ a wide range of skills including Financial Management, to oversee budgets, control costs, and allocate resources including staff to ensure efficient daily and long-term operations; Risk Assessment, to balance the probability and severity of different potential risks to make decisions around appropriate mitigation strategies; Quantitative Assessment and Modelling, to use different assessment tools available to monitor both resident and staff health and wellbeing and the impact of any new or existing policies and approaches.

"Mathematics is, and will remain, an important, foundational component of an engineer's education. Digitalisation and working with data was one of two major trends impacting future engineering skills needs highlighted in the Royal Academy of Engineering's *Engineers 2030* literature review, which analysed and synthesised over 200 reports from the engineering community on future engineering skills. Digital and data skills are foundational, intersectional, and an increasingly pervasive requirement for engineers – especially when they are concerned with emerging technologies."

National Engineering Policy Centre, the Royal Academy of Engineering.

1.3.4 Roles with high quantitative demands

Demand for people with high levels of mathematical and data skills is already significant and it is certain to increase substantially in the coming years[60]. Mathematics graduates are already highly employable. Around 90% of maths graduates are in employment or further study fifteen months after graduation, and a very high proportion (85%) of these are employed in managerial and professional occupations. This figure is considerably higher than the sector average across subjects. On average, 76% of all mathematics graduates who are in employment are in these occupations[61].

The range of fields which need people trained to high levels is large and is growing fast. This is partly because of the growing demands in areas like physics and engineering[62], which have always required high levels of mathematical competence, but increasingly in domains like biology, genetics, finance and many of the social sciences, which are increasingly driven by big data, and which will require many more people with the requisite mathematical, statistical and data science capacities[63].

The 2018 Bond report, *The era of mathematics: An independent review of knowledge exchange in the mathematical sciences* noted that: "Impactful mathematics appears throughout the economy. Companies use statistics to plan and to manage risk, operational research is used to streamline operations, reducing costs, and improving productivity. Government statistics underpin effective evidence-based decision-making. Financial services, security, defence, health, manufacturing, transport, film-making, and many other sectors all make use of many fields within the mathematical sciences.

Developments in genomics, data science, economics, physics, quantum computing, biology, advanced engineering, epidemiology, zoology, sociology, geography, ecology, climate science, cybersecurity, social media analytics and numerous other fields all require the use not only of existing mathematical methods, but also the development of new, more powerful mathematical tools to continually spur advances and innovation."[64].

60 Royal Society, 2019. Dynamics of data science skills: how can all sectors benefit from data science talent? See https://royalsociety.org/-/media/policy/projects/dynamics-of-data-science/dynamics-of-data-science-skills-report.pdf (accessed 30 July 2024).

61 The Council for the Mathematical Sciences and the Heads of Departments of Mathematical Sciences, 2023. Mathematical Sciences recruitment and graduate outcomes – A diversity perspective of the landscape of Mathematical Sciences in UK universities. See https://www.hodoms.org.uk/wp-content/uploads/2024/04/CMS_HoDoMS_MathematicalSciencesRecruitmentDiversity_FINAL.pdf (accessed 31 July 2024).

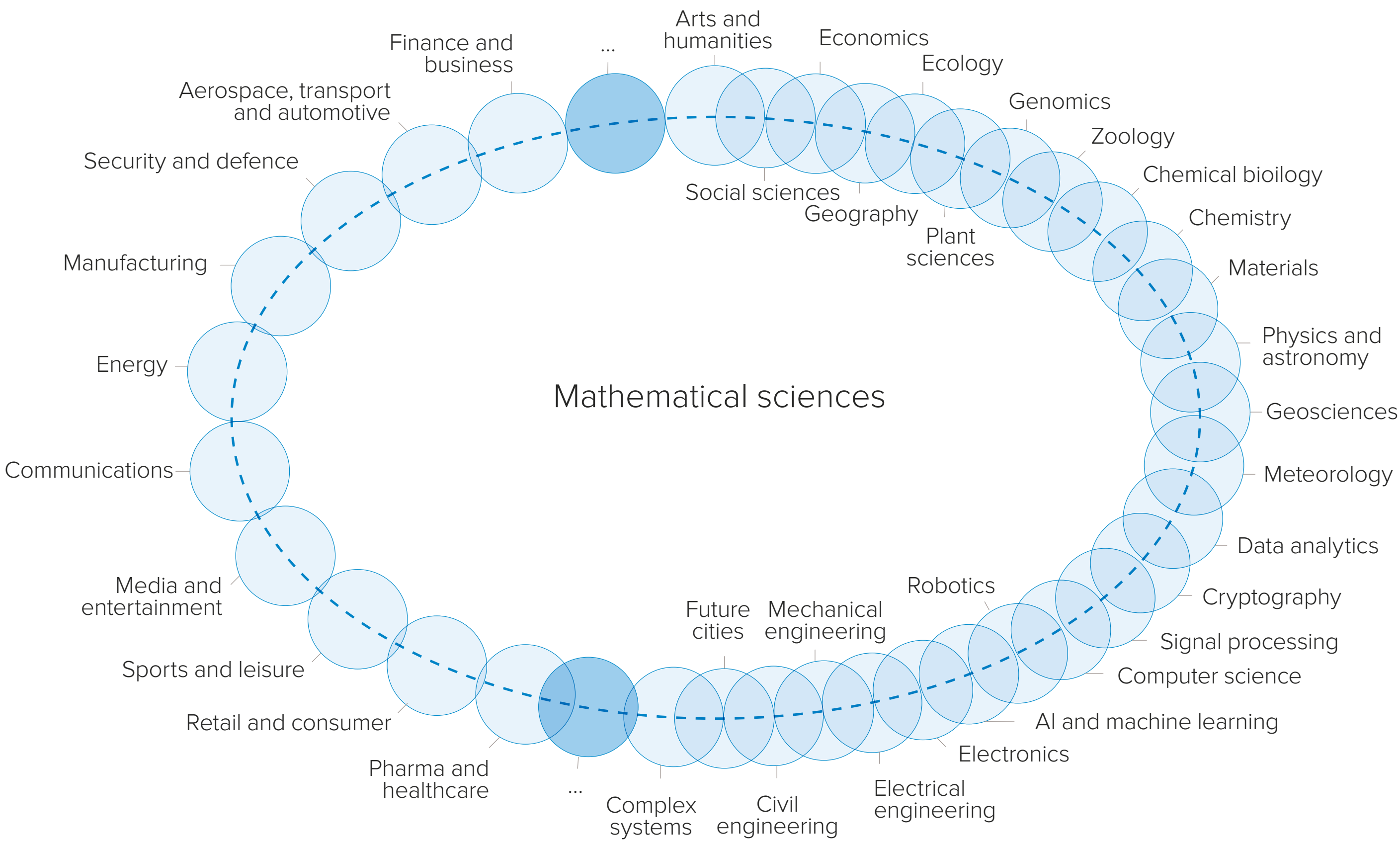
62 Institute of Physics, 2021. Physics in demand: the labour market for physics skills in the UK and Ireland. See https://www.iop.org/sites/default/files/2022-01/Physics-in-demand-labour-market-skills-uk-and-ireland.pdf (accessed 30 July 2024).

63 Brady H E, 2019. The challenge of big data and data science, Annual Review of Political Science, 22, (pp297 – 323). See https://www.annualreviews.org/content/journals/10.1146/annurev-polisci-090216-023229 (accessed 30 July 2024).

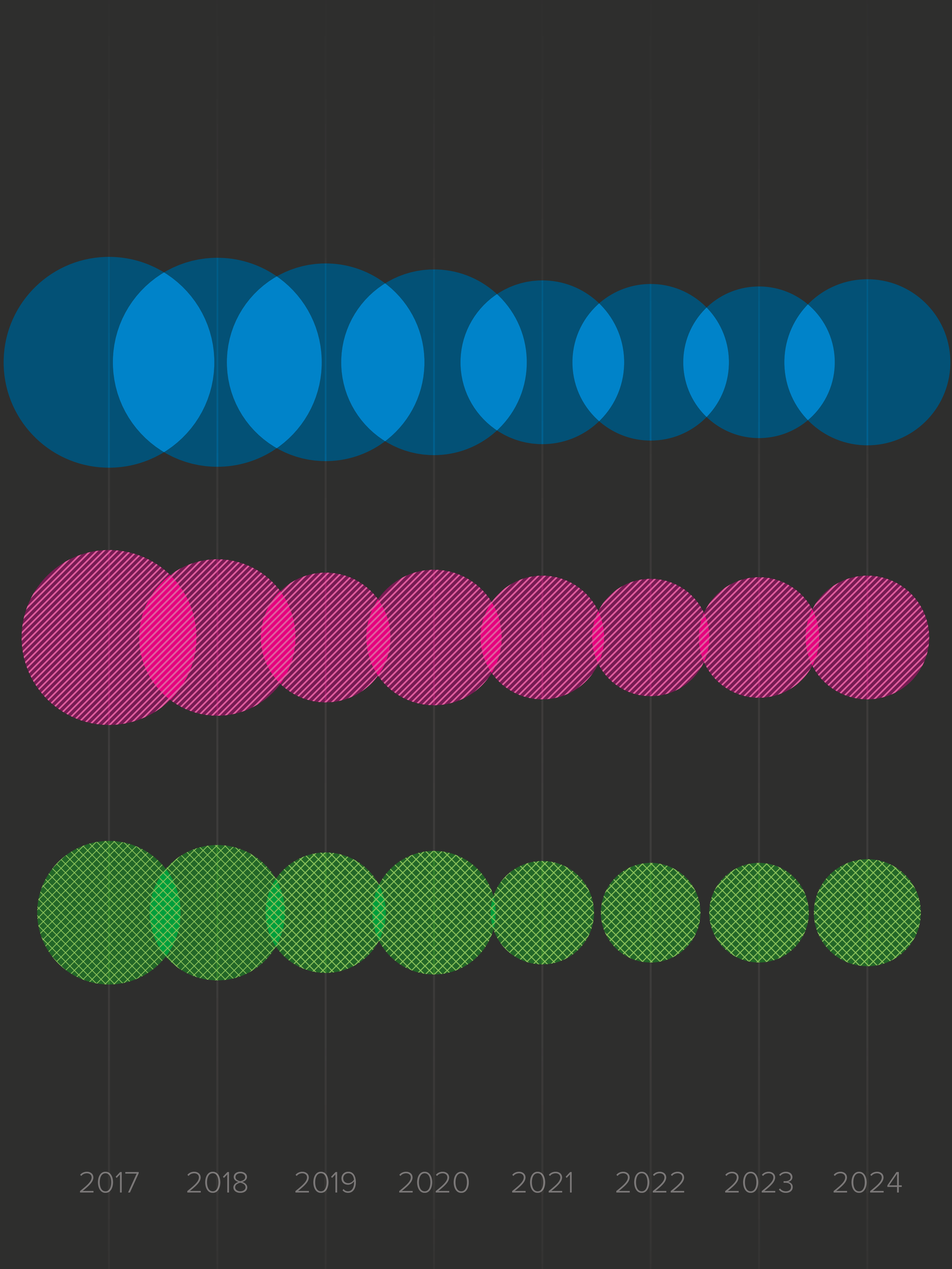
64 Op. cit., note 4.

FIGURE 3

Some linkages of the mathematical sciences to other academic fields and industry sectors[65]



65 Op. cit., note 4.


2017
2018
2019
2020
2021
2022
2023
2024

Chapter two

Mathematical and data education – where we are now

Left
Since 2007, the Electoral Commission has been monitoring public attitudes about aspects of elections and democracy in the UK. This image shows the trends in public perception of those surveyed who agree with the statements: (solid blue) Spending and funding is transparent; (pink diagonal) Authorities will take appropriate action; and (green crosshatch) I could easily find out how parties are funded. Data and full report available at electoralcommission.org.uk/research-reports-and-data/public-attitudes/public-attitudes-2024

Mathematical and data education – where we are now

2.1 Overview

Education in the UK is devolved. The four nations of England, Scotland, Wales and Northern Ireland each run their own education systems. England is much the largest with more than 85% of school age students. The systems in Wales and Northern Ireland are broadly similar to the English system, while Scotland differs in several respects.

In the United Kingdom the educational journey is divided into what are called 'key stages' (in England, Wales and Northern Ireland) or 'levels' (in Scotland). Broadly speaking there are three stages up to age 11 (including 'early years') and three from 11 to 18. The exact age bands and names of these levels vary between the countries.

Pupils in the UK perform reasonably well in international comparisons of mathematics education. In the 2022 PISA study of the performance of 15-year-olds, the UK as a whole scored 12th in mathematics out of the 80 participating education systems[66]. Six of the top seven countries were from East Asia (Singapore, Macao, Taiwan, Hong Kong, Japan and South Korea). As with many countries, the scores for all the UK countries showed a significant decline compared with the last PISA round in 2018 partly, but not entirely, attibuted to COVID-19.[67].

Behind these relatively healthy headline results lie some less welcome features: inequality and disadvantage, gender differences, maths anxiety, and educational consequences for adult numeracy.

2.1.1 Inequality and disadvantage

Across the UK there is a wide gap between the lowest and highest achievers, with a long tail of underachievement, ie a disproportionate number of low-attaining pupils[68]. There is a similarly wide gap between disadvantaged and advantaged pupils, with disadvantaged pupils much less likely to meet the expected standards at the end of all key stages[69]. Recent research from England, for example, examined the rate of progression to higher levels of mathematical study. As the chart in Figure 4 shows, children from wealthier backgrounds do better at the early stages, and this advantage remains unchanged throughout schooling[70].

66 Department for Education, 2023. PISA 2022: national report for England. See https://assets.publishing.service.gov.uk/media/656dc3321104cf0013fa742f/PISA_2022_England_National_Report.pdf (accessed 30 July 2024).
* England did not meet the sampling requirements. Higher performing pupils may have been over-represented and some of the results may therefore be higher than they would otherwise have been.

67 *Ibid*.

68 EPI, Benchmarking English Education. See https://epi.org.uk/research-area/benchmarking-english-education/ (accessed 4 June 2024).

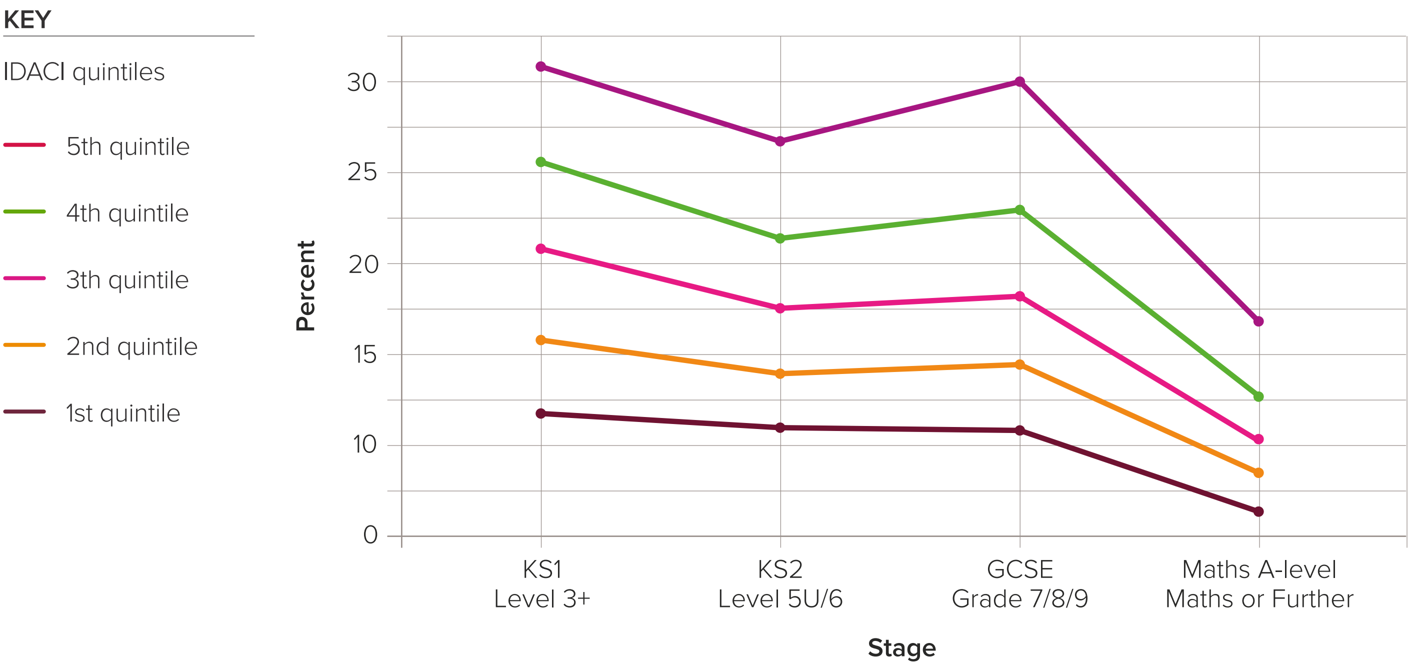
69 Ofsted, 2021. Research review series: mathematics. See https://www.gov.uk/government/publications/research-review-series-mathematics/research-review-series-mathematics (accessed 30 July 2024).

70 Noyes A *et al*, 2023. The Mathematics pipeline in England: patterns, interventions and excellence. See https://www.nottingham.ac.uk/research/groups/crme/documents/maths-pipeline-report.pdf (accessed 30 July 2024).

FIGURE 4

Inequality and attainment[71]

The percentage of students from each IDACI quintile in Cohort 1 achieving level 3 or above at the end of key stage 1, level 5U or above at the end of key stage 2, grade 7 or above at GCSE and completing at least one of A level mathematics or further mathematics. (IDACI = Income deprivation affecting children index, a measure of relative poverty).



There are also marked regional variations in educational attainment. These are stubbornly entrenched (see Box 4)[72].

Higher levels of education are linked with better health and higher levels of wealth both for individuals and for society. The relationships between educational attainment and factors such as socio-economic status, regional variation, ethnicity and gender are complex and are the focus of much research[73].

71 Tahir I, 2022. The UK education system preserves inequality. See https://ifs.org.uk/inequality/the-uk-education-system-preserves-inequality/ (accessed 4 June 2024)

72 SMF, 2016. Educational inequalities in England and Wales. See https://www.smf.co.uk/publications/educational-inequalities-in-england-and-wales/ (accessed 4 June 2024).

73 Tahir I, 2022. (see p41).

TABLE 3

Education in the UK

	England
Student numbers	
Primary and secondary state education (all ages excluding preschool)[74,75]	8,400,000[76]
Further education including part time, work-based study or college study (under age 19)[77]	840,000
Education systems	
Compulsory primary and secondary schooling (ages)	Primary (5 – 11), Secondary (11 – 16), Education leaving age[78] = 18
State curriculum	National Curriculum for England
Bodies setting the curriculum	The Department for Education (DfE)
Mainstream secondary qualifications	GCSEs, AS, and A levels
Bodies responsible for national assessment (excluding examination boards)	Standards and Testing Agency (STA) and Ofqual
Mainstream vocational qualifications	Apprenticeships , BTECs, Cambridge Nationals, Cambridge Technicals, NVQs, T Levels

74 Numbers for 2022 / 23, rounded to two significant figures.

75 Including middle, grammar, special, and hospital schools but excluding pupil referral units.

76 Department for Education, 2023. Schools, pupils and their characteristics. See https://explore-education-statistics.service.gov.uk/find-statistics/school-pupils-and-their-characteristics (accessed 30 July 2024).

77 Numbers for 2021 / 22, rounded to two significant figures; Department for Education, 2023. Education and training statistics for the UK. See https://explore-education-statistics.service.gov.uk/find-statistics/education-and-training-statistics-for-the-uk (accessed 30 July 2024).

78 While the school leaving age in England is 16, students must remain in official education or training until the age of 18.

Wales	Northern Ireland	Scotland
470,000[79]	330,000[80]	710,000[81]
41,000	19,000	110,000
Primary (5 – 11), Secondary (11 – 16)	Primary (4 – 11), Secondary (11 – 16)	Primary (5 – 12), Secondary (12 – 16)
National Curriculum for Wales (2008 – 2026) Curriculum for Wales (2022 –)	Northern Ireland Curriculum	Curriculum for Excellence
Department for Education and Skills (DfES)	Department of Education (DE) and Council for the Curriculum, Examinations and Assessment (CCEA)	Education Scotland
GCSEs, AS, and A levels, and Advanced Skills Baccalaureate Wales (from 2023)	GCSEs, AS, and A levels	National Qualifications (NQs), Highers, Advanced Highers, and Scottish Baccalaureate[82]
Qualifications Wales[83]	CCEA[84]	Scottish Qualifications Authority (SQA)
Apprenticeships, BTECs, NVQs, Cambridge Technicals, VCSEs (from 2027)	Apprenticeships, BTECs, Cambridge Technicals, NVQs	Apprenticeships, SVQs

79 Welsh Government, 2023 Schools' census results: January 2023. See https://www.gov.wales/schools-census-results-january-2023-html (accessed 30 July 2024).

80 Department of Education, Northern Ireland, 2022. Annual enrolments at grant-aided schools in Northern Ireland, 2022/23. See https://datavis.nisra.gov.uk/DEstatistics/annual-enrolments-at-grant-aided-schools-in-northern-ireland-202223.html#table-2-number-of-pupils-in-schools-and-children-in-funded-pre-school-education-by-type-of-establishment-attended-201819---202223 (accessed 30 July 2024).

81 Scottish Government, 2023. Summary statistics for schools in Scotland 2023. See https://www.gov.scot/publications/summary-statistics-for-schools-in-scotland-2023/pages/headline-school-and-early-learning-and-childcare-elc-statistics/ (accessed 30 July 2024).

82 The Scottish Baccalaureate consists of a coherent group of Higher and Advanced Higher qualifications alongside an Interdisciplinary Project, and is available in four subject areas (Science, Languages, Social Sciences, and Expressive Arts). NB Only 175 entries in 2023.

83 Some subjects accredited by Ofqual (the Regulator in England) are available in Wales where there is no Welsh-specific qualification developed to meet Qualifications Wales requirements. These are referred to as 'designated qualifications'.

84 GCSEs accredited by Ofqual (the Regulator in England) and A levels accredited by Ofqual and by Qualifications Wales (the Regulator in Wales) are also available in Northern Ireland, provided they are in line with educational policy set by the Northern Ireland Department of Education.

BOX 4

Headline findings: Inequalities by place[85]

Regional inequalities

- GCSE performance at age 16 across England and Wales reveals marked disparities between regions, with over 70% of pupils in London achieving 5 good GCSEs compared to 63% in Yorkshire and the Humber.
- Regional differences in attainment are already apparent by the end of primary school and they are observable even when you control for other factors such as ethnicity and income.
- Our analysis across different cohorts of children sitting exams at age 16 shows that regional inequalities have remained stubborn and in some cases worsened over the last three decades. Areas such as the North East, Yorkshire and the Humber, the West Midlands and the East Midlands have persistently underperformed, whilst London's performance has surged.
- Comparing the performance of 11-year-olds born in 2000 with those born in 1970 reveals that the geographic area a child comes from has become a more powerful predictive factor for those born in 2000 compared to 1970.

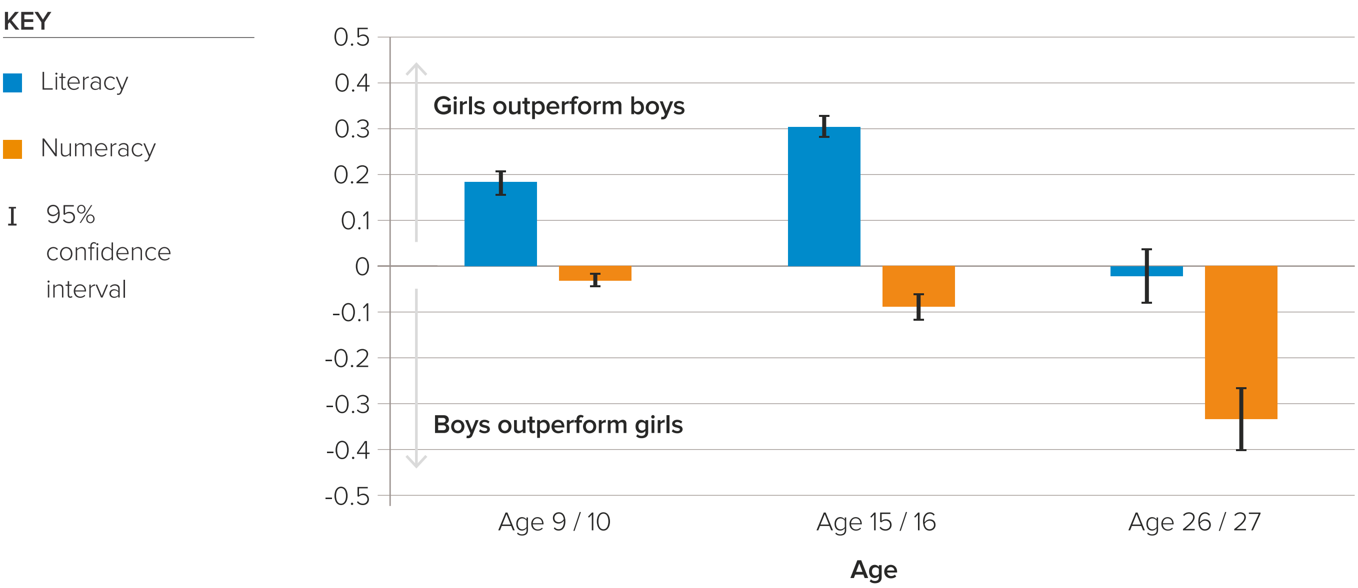
@SMFthinktank I smf.co.uk/education

85 Commission on Inequality in Education, 2016. Educational inequalities in England and Wales. See https://www.smf.co.uk/wp-content/uploads/2016/01/Publication-Commission-on-Inequality-in-Education-Initial-Findings-Slide-Pack-120116.pdf (accessed 10 June 2024).

FIGURE 5

The evolution of gender gaps in literacy and numeracy[86]

A gender gap is a measure of the effect of being female on the standardised test scores, given as a proportion of 1 standard deviation for the respective test population, where a + value shows females having higher scores. The gender gap measure is estimated by a regression analysis for each of the six tests (literacy and numeracy at each of the three ages).



86 The Evolution of Gender Gaps in Literacy and Numeracy. TIMSS (1995), PIRLS (2001), PISA (2000), PIAAC (2011 / 12).

2.1.2 Gender

Gender differences in all subjects, and mathematics in particular, have been the subject of extensive research worldwide. Across OECD countries in the recent PISA study of 15-year-old pupils the average mathematics score for boys was significantly higher than that for girls[87]. This holds true for all the UK nations. A recent study of mathematics education across all four devolved nations showed that: "… post- 16 participation in the UK at present remains disappointingly gender-biased, with significant, and likely increasing, implications for individual and for societal thriving."[88].

At the higher levels of performance boys outperform girls in mathematics at key stage 1 (5 – 7) and key stage 2 (7 – 11). At GCSE this difference disappears, but fewer girls than boys participate in A level, where five boys take part for every three girls. In terms of results, however, girls perform equally well[89].

There is evidence that gender gaps in numeracy and literacy evolve differently as young people mature. Data from large scale international assessments show that boys have a small advantage in numeracy at age 10, which grows considerably between age 15 and 27. The pattern for literacy is different; girls have an advantage at age 10. This grows by age 15 but virtually disappears at age 27[90] (see Figure 5).

Reflecting on possible causes the authors say: "Part of the growth in the gender gap in numeracy can be attributed to educational and career choices, as boys are more likely to pursue STEM-related careers. Unsurprisingly, those who pursue a STEM-related career engage more frequently with tasks requiring the use of numeracy skills. At the same time, they are as likely as people in non-STEM jobs to practice reading and writing, a finding that can contribute to explain why boys are able to catch up with girls and close the gender gap in literacy. The increase in the numeracy gender gap from age 15 to age 26/27 is plausibly related to choices concerning post-compulsory education.

We are indeed able to show that controlling for STEM-related careers reduces the size of the estimated gap by half. As far as literacy is concerned, there is evidence that the practice of reading and writing skills does not differ according to whether one pursues or not a career in STEM. In other words, literacy skills are transversal skills that are required and practiced in a much broader range of educational pathways and occupations than numeracy skills, which could explain why men have a strong incentive to develop them to succeed in the labour market and are then finally able to catch up with women."[91].

87 *Op. cit.*, note 66.

88 Golding J, 2022, UK Mathematics 14 – 19: the gender jigsaw (p4). See https://www.jmc.org.uk/wordpress-cms/wp-content/uploads/2022/03/UK-mathematics-14-19-gender-jigsaw-Report-Jan-22-final.pdf (accessed 30 July 2024).

89 Carroll M, 2023. Sex gaps in education in England. See https://www.cambridgeassessment.org.uk/Images/698454-sex-gaps-in-education-in-england.pdf (accessed 30 July 2024).

90 Borgonovi, F., Choi, A., & Paccagnella, M, 2021. The evolution of gender gaps in numeracy and literacy between childhood and young adulthood. Economics of Education Review, 82, 102119. P 5. See https://doi.org/10.1016/j.econedurev.2021.102119 (accessed 30 July 2024).

91 *Ibid*.

2.1.3 Maths anxiety

While the effects of socioeconomic background and gender have been known for many years, more recent research has focussed on the feelings of anxiety and apprehension felt by many children and adults when confronted by maths problems[92]. This is not simply a matter of cognitive demand; many children with high maths anxiety are normal to high achievers on curriculum maths tests[93]. There is evidence that maths anxiety can affect performance, and this can lead to a vicious circle, where maths anxiety leads to relatively poorer performance, which in turn increases maths anxiety[94]. Little is known about the causes of maths anxiety, although there appears to be a gender dimension, with girls more affected than boys[95]. There is also evidence that it can be transmitted, with teachers' and parents' own maths anxiety influencing that of pupils[96,97].

A recent review suggests that: "The cause-and-effect relations between mathematics achievement and mathematics anxiety are not fully understood, but generally difficulties (or perceived difficulties, independent of achievement) with mathematics precede the emergence of mathematics anxiety."[98]. This suggests that the maths anxiety problem can be dealt with in large part by preventing difficulties with maths in the first place.

2.2 Early years – the foundation stage

There are differences between the devolved nations and internationally, both in the age range considered to be early years, and in the predominant pedagogies. Internationally, Early Years are taken to be birth to 8 years whereas in England they are taken to be birth to 5. Internationally the prevailing pedagogy has moved towards a more play-based approach[99].

---

92 See, for example: Tes Magazine, 2024. Pisa: Most Scottish teenagers anxious about maths. See https://www.tes.com/magazine/news/secondary/maths-anxiety-schools-scotland-pisa (accessed 30 July 2024).

93 Carey E, *et al*, 2019. Understanding Mathematics Anxiety: Investigating the experiences of UK primary and secondary school students. See https://doi.org/10.17863/CAM.37744 (accessed 30 July 2024).

94 Gabriel F, *et al*, 2020. The impact of mathematics anxiety on self-regulated learning and mathematical literacy, Australian Journal of Education, 64(3), (p227 – 242). See https://doi.org/10.1177/0004944120947881 (accessed 30 July 2024).

95 Devine A, *et al*, 2012. Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety, Behavioural and Brain Functions 8(33). See https://doi.org/10.1186/1744-9081-8-33 (accessed 30 July 2024).

96 *Op. cit.* note 93.

97 Dove, J, Montague, J, and Hunt, T E, 2021. An exploration of primary school teachers' maths anxiety using interpretative phenomenological analysis. International Online Journal of Primary Education (IOJPE), 10(1), 32 – 49. See https://files.eric.ed.gov/fulltext/EJ1308555.pdf (accessed 30 July 2024).

98 Geary D C, 2024. Facing up to maths anxiety: how it affects achievement and what can be done about it. See https://www.cis.org.au/publication/facing-up-to-maths-anxiety-how-it-affects-achievement-and-what-can-be-done-about-it/ (accessed 4 June 2024).

99 Lunga P, *et al*, 2022. Play-based pedagogy: an approach to advance young children's holistic development, South African Journal of Childhood Education, 12(1). See https://doi.org/10.4102/sajce.v12i1.1133(accessed 30 July 2024).

In all countries of the UK there is recognition that early years settings are currently under-resourced, both in terms of finance and the education levels of the practitioners that work in them[100,101]. Recently announced changes to funding[102] have been criticised[103] as insufficient to enable an effective adult-to-child ratio.

In England, the early years foundation stage curriculum is defined by the statutory Educational Programme and the children work towards achieving Early Learning Goals. These summarise the knowledge, skills and understanding that all young children should have gained. Whilst the Educational Programme explicitly includes spatial reasoning, the Early Learning Goals do not, instead focussing on numbers to 10 and number patterns[104]. As in other stages of education, assessment goals drive classroom behaviour.

Also in England, the later Reception Baseline Assessment[105], typically taken during the first few weeks of a child's reception year in primary school, has been recognised as contributing to school accountability measures but is felt by many to be an invalid and inaccurate assessment of children's attainment, focusing only on number and pattern, and skewing what should be considered important in mathematics at that time in a child's education[106,107].

Navigating funding for training and professional development is confusing and currently there is no national pay scale for early years practitioners. Many practitioner trainees fail to achieve the GCSE maths requirement to undertake a level 3 Early Years Educator qualification, and as a result the government in England has scrapped this requirement[108].

---

100 House of Commons Library, 2024. Childcare funding in England. See https://researchbriefings.files.parliament.uk/documents/CBP-8052/CBP-8052.pdf (accessed 30 July 2024).

101 Early Education and Childcare Coalition, 2023, Pulse check: public attitudes towards early education and childcare. See https://static1.squarespace.com/static/646ca30371a2ef6a657e9309/t/64f6076e72efd906dd1a8cbb/1693845359851/EECC+Pulse+Check.pdf (accessed 30 July 2024).

102 Education & Skills Funding Agency, 2021. Early years 2022 to 2023 hourly funding rates for 2, 3 and 4-year-olds: technical note. See https://www.gov.uk/government/publications/early-years-funding-2022-to-2023/early-years-2022-to-2023-hourly-funding-rates-for-2-3-and-4-year-olds-technical-note (accessed 30 July 2024).

103 Early Years Alliance, 2022. Proposed early years funding changes likely to hit areas with low places hardest, analysis reveals. See https://www.eyalliance.org.uk/news/2022/08/proposed-early-years-funding-changes-likely-hit-areas-low-places-hardest-analysis (accessed 30 July 2024).

104 The Royal Society Advisory Committee on Mathematics Education (ACME), Primary and early years expert panel, 2024. Spatial reasoning. See https://royalsociety.org/-/media/policy/topics/education-skills/maths/perspective-spatial-reasoning.pdf (accessed 30 June 2024).

105 Standards & Testing Agency, 2020. Assessment framework: reception baseline assessment. See https://assets.publishing.service.gov.uk/media/5e550579e90e074dcf842ab3/2020_Assessment_Framework_Reception_Baseline_Assessment.pdf (accessed 30 July 2024).

106 British Educational Research Association, 2018. A baseline without basis: the validity and utility of the proposed reception baseline assessment in England. See https://www.bera.ac.uk/publication/a-baseline-without-basis (accessed 30 July 2024).

107 Roberts-Holmes G *et al*, 2020. Research into the 2019 pilot of reception baseline assessment (RBA). See https://discovery.ucl.ac.uk/id/eprint/10097328/ (accessed 30 July 2024).

108 Department for Education, 2023. Early years foundation stage (EYFS): regulatory changes. See https://www.gov.uk/government/consultations/early-years-foundation-stage-eyfs-regulatory-changes (accessed 30 July 2024).

2.3 The primary years

In primary schools the norm is for a class teacher to teach all subjects. This therefore offers an opportunity to connect MDE across the wider curriculum. In practice however the primary curriculum in England has moved away from project-based and cross-curricular approaches and mathematics, especially number, is taught in isolation[109].

The primary curriculum is broadly the same in all four countries. In England the last curriculum review, completed in 2014, emphasised fluent recall and practice[110]. That, now current, curriculum contains little statistics or data content and what there is emphasises representation of data rather than interpretation.

In England the pedagogy that has been promoted by the government is 'mastery'; whole-class interactive teaching where all pupils work together and progress at the same rate. Mastery is influenced by East Asian success in transnational assessments and an exchange programme between English and Shanghai teachers. The key principles include frequent formative assessment, and class- based instruction through small steps.

There is no prevailing pedagogy in the devolved nations, and in many cases, schools are free to design their own curriculum and pedagogy[111]. The use of calculators in English primary schools, especially at the early stages, is discouraged and Ministers banned the use of calculators in national maths tests for 11-year-olds in England from 2014[112]. There is no guidance on the choice or use of digital resources.

In primary school pupils are introduced to basic data representations, including being taught to "interpret and present data using bar charts, pictograms and tables"[113]. The primary science curriculum highlights the "gathering, recording, classifying and presenting data in a variety of ways to help in answering questions"[114], but there is a need for purposeful, relevant and engaging questions for data collection to focus and necessitate interpretation.

109 Department for Education, 2016. South Asian method of teaching maths to be rolled out in schools. See https://www.gov.uk/government/news/south-asian-method-of-teaching-maths-to-be-rolled-out-in-schools (accessed 30 July 2024).

110 Department for Education, 2014. National curriculum and assessment from September 2014: information for schools. See https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1152583/WITHDRAWN_-_NC_assessment_quals_factsheet_Sept_update.pdf (accessed 30 July 2024).

111 Welsh Government, 2024, Area of learning and experience: mathematics and numeracy. See https://hwb.gov.wales/curriculum-for-wales/mathematics-and-numeracy/designing-your-curriculum (accessed 3 April 2024).

112 Department for Education, 2014. Government bans calculators in tests for 11-year-olds. See https://www.gov.uk/government/news/government-bans-calculators-in-tests-for-11-year-olds (accessed 30 July 2024).

113 Department for Education, 2013. Mathematics programme of study: key stage 1 and 2. See https://assets.publishing.service.gov.uk/media/5a7da548ed915d2ac884cb07/PRIMARY_national_curriculum_-_Mathematics_220714.pdf (accessed 30 July 2024).

114 Department for Education, 2015. National curriculum in England: science programmes of study. See https://www.gov.uk/government/publications/national-curriculum-in-england-science-programmes-of-study/national-curriculum-in-england-science-programmes-of-study (accessed 30 July 2024).

There are also chronic problems with teacher recruitment. The latest analysis of the teacher workforce in England by the National Foundation for Educational Research warned that, based on applications as of February 2023, the government was on track to recruit just 79% of the primary teachers it needed for 2023/24[115]. The situation is similar in Northern Ireland, less acute in Scotland, and Wales has no recruitment issues.

Many primary teachers take a one-year postgraduate training course after their degree which includes between 15 and 30 hours to be introduced to the whole of the primary mathematics curriculum. This is followed by a two-year Induction for Early Career Teachers which may or may not include additional mathematics input.

2.4 Key stage 3 (ages 11 – 14)

Since the revised national curriculum was implemented in 2014, the school landscape in England has changed and many schools are now members of multi-academy trusts. Academies do not have to follow the national curriculum[116], although in practice, because of the national examination system, they follow the similar schemes of work set out by GCSE awarding organisations. These very often begin in year 9 (key stage 3) rather than in year 10 (key stage 4), with varying degrees of success[117].

There is currently no national assessment at the end of key stage 3 in England, and so schools' own assessments determine which GSCE tier is to be taken by which pupils. Key stage 3 teacher assessment in Wales is recorded, but not shared whilst there are national tests in Scotland and Northern Ireland.

There is a long-standing concern that for many children the transition from primary to secondary school is not well handled. Likewise, there is concern about the slow progress made by pupils in mathematics (and English) in the early years of secondary school[118]. This is a stage where attainment gaps open up. Gender differences also emerge here. The Aspires project[119] showed that girls are less likely than boys to aspire to STEM (science, technology, engineering and mathematics) careers, even though a higher percentage of girls than boys rate STEM as their favourite subjects.

115 Headteacher Update, 2023. Where are all the teachers? Primary teacher training on track to significantly under-recruit. See https://www.headteacher-update.com/content/news/where-are-all-the-teachers-primary-teacher-training-on-track-to-significantly-under-recruit (accessed 4 April 2024).

116 House of Commons Library, 2019. FAQs: academies and free schools. See https://researchbriefings.files.parliament.uk/documents/SN07059/SN07059.pdf (accessed 30 July 2024).

117 Ofsted, 2023. Coordinating mathematical success: the mathematics subject report. See https://www.gov.uk/government/publications/subject-report-series-maths/coordinating-mathematical-success-the-mathematics-subject-report (accessed 30 July 2024).

118 Ofsted, 2015. Key stage 3: the wasted years? See https://www.gov.uk/government/publications/key-stage-3-the-wasted-years (accessed 30 July 2024).

119 Archer Ker, L, DeWitt, J, Osborne, J F, Dillon, J S, Wong, B, and Willis, B, 2013. ASPIRES Report: Young people's science and career aspirations, age 10 – 14. King's College London. See https://kclpure.kcl.ac.uk/portal/en/publications/aspires-report-young-peoples-science-and-career-aspirations-age-1 (accessed 30 July 2024).

This well-documented 'dip' whereby many students make little progress in the early years of secondary schooling impacts some more than others. Recent analysis of learner trajectories through secondary education shows that only half of high attaining 11-year-olds from disadvantaged backgrounds are still high attainers by age 16[120].

At key stage 3, the mathematics curriculum incorporates the use and interpretation of a range of graphical representations, tables, charts and diagrams and statistical measures[121]. Secondary science emphasises the centrality of the classic data cycle[122]. From this, if taught well, pupils can gain a good introduction to data and statistics. However, teachers will have varied pedagogical knowledge and experience and may require support, and there are risks to coherence. Recent research by the Joint Mathematical Council indicates that the use of technology in key stage 3 is at best patchy[123] with provision of hardware and software often an issue.

Opportunities are also missed for pupils to learn techniques for using and presenting data. Well- designed software can help pupils focus on important concepts in the analysis, presentation and interpretation of data. Common Online Data Analysis Platform (CoDAP)[124] is one such example: free-to-use software designed specifically for curriculum use.

The shortage of qualified mathematics teachers is most evident within key stage 3[125]. Recent research found that nearly half of secondary schools have used non-specialists to teach at least some maths lessons and that most schools with a shortage privilege key stage 4 teaching over key stage 3[126]. The percentage of key stage 3 maths lessons taught by those without a maths degree has been estimated to be 57%[127]. Year 7 classes and lower sets in other year groups were disproportionately likely to be taught by more than one teacher[128].

120 *Op. cit.*, note 70.

121 Department for Education, 2021. Mathematics guidance: key stage 3. See https://assets.publishing.service.gov.uk/media/621629ac8fa8f5490d52ee78/KS3_NonStatutory_Guidance_Sept_2021_FINAL_NCETM.pdf (accessed 30 July 2024).

122 *Op. cit.*, note 114.

123 Joint Mathematical Council of the UK, 2023. Mathematics education and digital technology. See https://www.jmc.org.uk/2023/07/05/mathematics-education-and-digital-technology-a-report-from-a-working-group-of-the-jmc/ (accessed 30 July 2024).

124 The Concord Consortium, 2024. About CODAP. See https://codap.concord.org/about/ (accessed 30 July 2024). (accessed 12 April 2024).

125 National Foundation for Educational Research, 2024. Teacher labour market in England annual report 2024. See https://www.nfer.ac.uk/publications/teacher-labour-market-in-england-annual-report-2024/ (accessed 30 July 2024).

126 Schools Week, 2023. Lack of specialist maths teachers 'massive injustice', says flagship scheme boss. See https://schoolsweek.co.uk/lack-of-specialist-maths-teachers-massive-injustice-says-flagship-scheme-boss/ (accessed 30 July 2024). (accessed 15 January 2024).

127 Allen R & Sims S, 2018. How do shortages of maths teachers affect the within-school allocation of maths teachers to pupils? See https://www.nuffieldfoundation.org/wp-content/uploads/2018/06/Within-school-allocations-of-maths-teachers-to-pupils_v_FINAL.pdf (accessed 30 July 2024).

128 *Op. cit.*, note 117.

2.5 Key stage 4 (ages 14 – 16)

Virtually all students in England, Wales and Northern Ireland take the GCSE in mathematics, nearly all at the end of year 11[129,130].

GCSE mathematics must cater for a very broad attainment range, from those with low levels of numeracy to those who will be studying undergraduate mathematics within 3 years. GCSEs in England are graded from 9 to 1. The minority of students (43%) take the higher tier exam, which gives access to grades 9 to 3. 57% take the foundation tier, which is based on a subset of the full specification and gives access to grades 5 to 1 only[131].

In England pupil performance at GCSE has been improving. Except for results in 2020 and 2021, which were affected by COVID-19, the proportion of pupils attaining a grade 4 or above has steadily increased over time. In 2019, 71.5% of pupils obtained a grade 4 or above, and 20.4% of pupils obtained a grade 7 or higher[132].

That is good news, but it is not the whole story. A recent Ofsted subject report warns that: "… a high quality of education leads to strong pupil outcomes, but this is not necessarily true in reverse. Strong exam outcomes do not, necessarily, indicate a high-quality mathematics education. That is because, in some schools, pupils are taught a narrowed curriculum that allows them to be successful in exams without securing the mathematical knowledge they need to be successful later."[133]. Many students therefore achieve a reasonable GCSE grade but lack a secure grasp of foundational mathematics[134].

Less able students are particularly impacted. To quote Ofsted again: "Pupils who are learning mathematics more slowly than their peers frequently receive a mathematics education that does not meet their needs. They are often rushed through the study of new content, in order to 'complete the course', without securely learning what they are studying. This frequently results in pupils repeating content in key stage 4 that they have already studied, but not learned, in key stage 3 (and 2). Often the curriculum for these pupils is narrowed with little teaching of how the facts and methods learned can be used to solve problems mathematically. Many of these pupils develop a negative view of mathematics."[135].

---

129 National Statistics, 2024. Key stage 4 performance: academic year 2022/23. See https://explore-education-statistics.service.gov.uk/find-statistics/key-stage-4-performance/2022-23 (accessed 30 July 2024).

130 In Scotland, mathematics is a core subject in the curriculum up to age 16. In S4 (equivalent to Year 11 in England, Wales and Northern Ireland), the majority of learners are presented at National 4 or National 5 level for mathematics and/or applications of mathematics. In 2023, just under half of the S4 cohort achieved an award at SCQF Level 5 (National 5) in either mathematics, applications of mathematics, or both. In general, over 70% of learners leave school with a qualification in Numeracy at SCQF Level 5 (roughly equivalent to GCSEs) or better. Based on a personal communication, John Neeson, Education Scotland.

131 OCR, 2023. GCSE mathematics: what proportion of students take each tier? See https://support.ocr.org.uk/hc/en-gb/articles/360038360392-GCSE-Mathematics-What-proportion-of-students-take-each-tier (accessed 30 July 2024).

132 Ofqual, 2023. GCSE outcomes in England. See https://analytics.ofqual.gov.uk/apps/GCSE/Outcomes_Link1/ (accessed 4 April 2024).

133 *Op. cit.*, note 117.

134 Pass marks can be very low. The Grade 4 mark for Higher tier in 2023 was 39/300.

135 *Op. cit.*, note 117.

The statistical content in GCSE mathematics consolidates what has been taught at key stage 3 and introduces important areas such as probability, distributions, samples and populations, trends and bivariate data. Importantly, GCSE mathematics includes statistics content at both foundation and higher tiers, offering all pupils a chance to gain a sound statistical understanding[136]. There is also a separate, freestanding, GCSE in statistics, taken by 27,000 students 2023[137]. Some teachers bring this statistical learning to life for pupils, linking teaching to statistical claims in the media, for example, and using real data. But these examples are rare[138].

The way in which the qualifications regulator Ofqual maintains standards over time, thereby providing confidence to stakeholders, is through "a combination of statistical evidence and expert judgement."[139]. Strictly speaking therefore GCSEs are neither wholly norm referenced nor wholly criterion referenced[140]. In reality, outcomes change little from year to year (with the exception of a system shock such as a pandemic) and a 'standard pass' threshold (grade 4) acts in a norm-referencing way, so that it is built into the system that a significant minority will fail to achieve this standard[141].

2.6 Post-16

After GCSE students choose from a range of different options. At Level 3, approximately 37% study only academic qualifications, mainly A levels. About 20% follow technical courses, mainly BTECs, with a small but growing number taking the new T level courses. A further 11% follow mixed academic and technical courses of study. The remainder are in apprenticeships (4%), employment or other training (4%), are following courses below Level 3 (17%) or are NEET (Not in Education, Employment or Training)[142].

---

136 Department for Education, 2013. GCSE mathematics: subject content and assessment objectives. See https://www.gov.uk/government/publications/gcse-mathematics-subject-content-and-assessment-objectives (accessed 30 July 2024).

137 Ofqual, 2023. Provisional entries for GCSE, AS and A level: summer 2023 exam series. See https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2023-exam-series (accessed 30 July 2024).

138 Davies N & Sheldon N, 2021. Teaching statistics and data science in England's schools. See https://doi.org/10.1111/test.12276 (accessed 30 July 2024).

139 National Foundation for Educational Research, 2013. Maintaining qualification and assessment standards: summary of international practice (p3). See https://assets.publishing.service.gov.uk/media/5a81e3c1e5274a2e87dc004e/0113_NewmanBurdett_Maintaining_qualification_and_assessment_standards_V4_FINAL.pdf (accessed 30 July 2024).

140 Criterion-referenced assessment measures an individual's performance against predetermined criteria or standards, whereas norm-referenced assessments compare individuals to each other. Criterion-referenced assessments focus on determining whether a person has achieved a particular level of competence or mastery. Examples are the driving test and graded tests in music.

141 *Op. cit.*, note 129.

142 Department for Education, 2024. Participation in education, training and employment age 16 to 18. See https://explore-education-statistics.service.gov.uk/find-statistics/participation-in-education-and-training-and-employment (accessed 10 June 2024).

In 2021 around 580,000 16-year-olds in England took the GCSE in mathematics. Each year around 30% of 16-year-olds (that is some 200,000 students[143]) attending state-funded schools in England fail to attain a grade 4 or better. These students are required to resit GCSE mathematics examinations or else to pursue a ‘pass’ in an equivalent functional skills qualification. The outcomes are not encouraging and there have been increasing calls for a rethink and transformation of the policy[144]. Two thirds of the GCSE resit students do not improve their performance by age 19[145], while others may resit multiple times before achieving the pass grade. This bears particularly on disadvantaged students and those with Special Educational Needs, who are both more likely to fall within the resit group and less likely to succeed.

For many of these students much of the content of the GCSE will not be relevant to their needs. Though foundation tier GCSE mathematics covers the quantitative skills adults should possess, it neglects essential aspects such as financial applications, data handling and the use of spreadsheets. Alternative courses that better meet the needs of resit students, for example by offering opportunities to use data and techniques that are common in the workplace, including spreadsheets and real data, have been designed. However, they cannot be offered in schools and colleges as they do not meet the regulatory requirement that GCSEs should cover the content specified by the national curriculum.

At the higher levels mathematics is the most popular A level subject. In 2023 in England, around 90,000 students, about 14% of 18-year-olds, took A level mathematics[146]. Mathematics A level is well regarded, both as a preparation for mathematically demanding courses in higher education (mathematics, science, engineering etc) and as a course of study in its own right. Some 15,000 of these students also took further mathematics. There is also an A level in statistics, taken by some 800 students in 2023.

Statistics is a compulsory strand of A level mathematics, incorporating the study of published large datasets (LDS). AS and A level specifications require students to become familiar with the dataset for the final assessment, use technology such as spreadsheets and statistical packages, interpret data in summary and statistical form, and investigate questions arising in real contexts. These modest expectations are not being met. A 2020 review of A level mathematics teaching found that some centres “chose not to cover this element of the course or only engage with it superficially, as they felt the time and resources to engage with it do not reflect the marks available.”[147].

143 National Statistics, 2023. Schools, pupils and their characteristics. See https://explore-education-statistics.service.gov.uk/find-statistics/school-pupils-and-their-characteristics (accessed 30 July 2024).

144 Education Policy Institute, 2024. Time for a resit reset? See https://epi.org.uk/publications-and-research/blog-time-for-a-resit-reset/ (accessed 4 April 2024)

145 Department for Education, 2020. A level and other 16 to 18 results: 2018 to 2019 (revised). See https://explore-education-statistics.service.gov.uk/find-statistics/a-level-and-other-16-to-18-results/2021-22 (accessed 30 July 2024).

146 *Op. cit.*, note 137.

147 Redmond B, *et al*, 2020. Teaching and learning for ‘moving goal-posts’: reformed A levels in mathematics (p5). See https://bsrlm.org.uk/wp-content/uploads/2020/05/BSRLM-CP-40-1-13.pdf (accessed 30 July 2024).

At this level, concepts and skills in data and statistics are essential. Data analysis and modelling represent the foundation of machine learning and AI. Data concepts and skills are a bridge to what's 'under the bonnet' of machine learning and AI, which are so central to the future economy and technology developments.

Virtually all A level mathematics students will have achieved a grade 7, 8, or 9 at GCSE. (The picture is different for other A level subjects, where students with grades 6 and 5 routinely take A level courses.) This leaves a very large group of around 250,000 students with middle grades 4, 5 and 6. For the great majority of these students there will be no qualification available to them to continue their mathematical education[148]. A small number will be offered one of the relatively new suite of qualifications called Core Maths, which was taken by 12,000 students in 2023[149].

The UK is unusual among developed countries in having such a low level of participation in mathematical study at upper secondary level. A 2012 report from the Nuffield Foundation showed that most of the 24 countries in the study had participation rates of 80% or more, with 8 having rates of 100%. In contrast, at the time of the research fewer than 20% of students in England, Wales and Northern Ireland studied any mathematics after the age of 16. Scotland fared slightly better, at about 30%[150]. Since 2014 the GCSE resits policy has increased the number of students in England studying maths post-16, but most of this increase is at lower, indeed remedial, levels.

Formal stand-alone qualifications are not the whole picture. Many students following vocational courses (for example BTECs in engineering) will study significant mathematical content, often to high levels. The new T levels (for ages 16 – 19) are underpinned by a framework of General Mathematical Competences ensuring that key mathematics and data skills will be developed as appropriate to that technical pathway[151]. And following the A level reforms of 2019 a number of A level specifications include explicit statements of the mathematical content that students are expected to have mastered, and the examinations are designed to test this knowledge[152]. Through these routes more 14 – 18 year olds are now involved in a wider range of mathematical, statistics and data activities than was the case ten years ago.

148 FFT Education Datalab, 2023. Could there be more demand for post-16 maths? See https://ffteducationdatalab.org.uk/2023/01/could-there-be-demand-for-more-post-16-maths/ (accessed 30 July 2024).

149 Mathematics in Education and Industry, 2023. Summary of Core maths entries and results August 2023 (UK). See https://mei.org.uk/summary-of-core-maths-entries-and-results-august-2023-uk/ (accessed 30 July 2024).

150 Nuffield Foundation, 2012. Is the UK an outlier in upper secondary maths education? See https://www.nuffieldfoundation.org/project/is-the-uk-an-outlier-in-upper-secondary-maths-education (accessed 30 July 2024).

151 Education and Skills Funding Agency, 2021. T Level resources for universities. See https://www.gov.uk/government/publications/t-level-resources-for-universities (accessed 30 July 2024).

152 Norris J & Noyes A, 2023. Mapping mathematical competences across subjects for advanced level qualifications in England. See https://doi.org/10.1002/curj.204 (accessed 30 July 2024).

In 2014, a new set of qualifications with the title 'Core Maths' was introduced. They focus on developing problem-solving and decision-making skills, as well as improving mathematical fluency and general quantitative literacy[153]. In terms of demand on the student they equate, more or less, to AS levels. Core Maths qualifications are offered by three exam boards. They vary in their content, but all include topics such as statistical analysis, mathematical modelling, financial mathematics, and critical analysis of data. Core Maths courses are intended to be more practical and applied than A level. They use real-world data and explore mathematical concepts in authentic contexts that are relevant to everyday life, business, or social issues. (See Appendix for sample examination questions.)

The number of students taking a Core Maths qualification has been modest, with around 12,000 students (fewer than 2% of 18-year-olds) taking the qualification in 2023. There are a number of possible causes. The effective removal of AS levels means that they do not fit easily alongside the standard three or four A level programme of study. And while they are accepted as equivalent to an AS level in terms of UCAS points many universities seem unaware of their existence and do not specify them in their entry requirements[154]. Issues of funding and staffing also play a role[155].

We believe Core Maths has great potential and that it is an important post-16 option for the future[156]. We discuss the possibilities further in chapter 5.

2.7 Computational technology and tools for learning mathematics

Mathematics, as currently taught in schools, is missing out on extensive opportunities to transform understanding and learning through use of computing technology and tools. This is despite the fact that computational tools are readily available that are specifically designed or suited to acquiring mathematical concepts and exercising them. These tools are distinct from the excellent but generic Edtech tools that apply broadly across subjects, such as Google Classroom, Khan Academy, interactive whiteboards, Kahoot, and videoconference software. Tools specifically suited for mathematical learning include: GeoGebra; Scratch for 2D geometric computing; Mathematica for algebra and calculus; Common Online Data Analysis Platform (CODAP) for statistical analysis; Autograph for 2D and 3D visualisation; and of course spreadsheets for modelling, and handling data at a scale big enough to be interesting and too big for pencil and paper. Tools like these encourage the application of computational thinking[157] to mathematical problem solving, and help develop mathematical concepts along the way.

153 Department for Education, 2014. Core maths qualifications: technical guidance. See https://assets.publishing.service.gov.uk/media/5a7eec7b40f0b6230268c64d/Core_Maths_Technical_Guidance_-_Consultation_response_July_2014_-_Amended__PT_.pdf (accessed 30 July 2024).

154 Nuffield Foundation, 2020. The early take-up of Core Mathematics. See https://www.nuffieldfoundation.org/project/the-early-take-up-of-core-mathematics (accessed 30 July 2024).

155 STEM Learning, 2019. Promoting Core Maths: background, evidence, plans, opportunities and challenges. See https://www.stem.org.uk/resources/elibrary/resource/417725/promoting-core-maths-background-evidence-plans-opportunities-and (accessed 30 July 2024).

156 The British Academy and The Royal Society, 2022. Joint statement on Core Maths Qualifications: the importance of promoting Core Maths as practical and valuable qualifications. See https://royalsociety.org/-/media/policy/Publications/2022/2022-01-26-core-maths-joint-statement.pdf (accessed 30 July 2024).

157 Computational thinking can be defined as automatable solving of a mathematical problem by means of abstraction, decomposition, pattern recognition, algorithmic thinking, modelling, logical and analytical thinking, generalisation and evaluation of solutions and strategies.

In July 2023 the Joint Mathematical Council of the UK published a report on digital technologies and mathematics education, revisiting and updating a report published in 2011. That report confirmed the picture also painted in a recent report[158] which concluded that "despite the promises of digital technology to enhance mathematics education, and the ongoing transformation of all aspects of modern society by technology, little has changed in the intervening years since the publication of the 2011 Report. Progress against its recommendations has been slow at best."[159].

The 2011 and 2023 reports remark on the fragmented nature of the implementation of digital technologies, with no overall coherence and little evidence of widespread impact. They contrast this with the situation in higher education where there has been a rapid shift in the use of digital technologies and: "genuine efforts to use the tools that professional mathematicians and users of mathematics will use in their employment futures." They note that: "This leaves a considerable gap between the use of digital technologies in school curricula and assessment and the kinds of uses that are made of these in other subjects and in the jump from compulsory education to higher education and work."[160].

The state of affairs in mathematics teaching now contrasts with the situation in computing education where a good deal has changed in the last decade or so. In 2010 the focus was firmly on ICT – routine exercising of application software such as word processors and spreadsheets. Partly in response to the Royal Society's 2012 report *Shut down or restart?* the government announced in 2012 that the national curriculum would be revised to include a much greater emphasis on programming and on the principles of computer science. The new curriculucm was published in 2013, came into force in September 2014 and was reviewed in July 2024[161].

By 2017, according to the Royal Society report *After the reboot*, the state of UK computing education was still "patchy and fragile". Since then, partly through the work of the DfE funded National Centre for Computing Education (NCCE), substantial progress has been achieved. As a result, the number of entries at GCSE has increased by 35% since 2017, and entries at A level have more than doubled. Admittedly the revolution is not yet complete – substantial issues remain with pupil gender balance, socio-economic inequality, allotted curriculum hours, and teacher recruitment. Nonetheless, the subject has modernised substantially over the course of the decade. There is surely a model here in the revision of the Computing curriculum, pointing to the possibility of the sort of thorough modernisation that might be achieved for mathematics teaching also, over the course of a decade or so.

158 Crisan C, Geraniou E and Hodgen J, 2023. Educational technologies in mathematics education, UCL Institute of Education. See https://royalsociety.org/-/media/policy/projects/maths-futures/educational-technology-mathematics-education.pdf (accessed 30 July 2024).

159 *Op. cit.*, note 123.

160 *Op. cit.*, note 123.

161 Government consultation on Proposed Subject Content for GCSE Computer Science (education.gov.uk), 2024. See https://consult.education.gov.uk/computing-policy-team/gcse-computer-science-subject-content-update/supporting_documents/Proposed%20Computer%20Science%20GCSE%20subject%20content%20Government%20Consultation.pdf (accessed 30 July 2024).

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Chapter three

Mathematical and data education – the future

Left
Flask framework is one way of using the computer programming language, Python.

Mathematical and data education – the future

3.1 Foundational and advanced mathematics

In the future, students and citizens in all walks of life will need generic, transferable mathematical and data competences to underpin occupation-specific applications, and in their daily lives. The acquisition of key mathematical and data concepts, techniques and reasoning skills underpins those competences.

Advanced and sophisticated applications of mathematics and statistics increasingly underpin much of the modern world, and the demand for expert mathematical scientists trained to high levels in mathematics and its applications is certain to grow[162,163,164,165]. The education of these highly skilled professionals begins in school, and it will likewise be a fundamental part of the role of MDE to encourage and nurture these young people and aid their transition to advanced and higher levels of study.

3.1.1 Mathematics as a discipline

For all its economic and social importance, the place of mathematics in a young person's education is not simply one of instrumental value. Students should learn to appreciate that mathematics is one of humanity's great intellectual achievements. They should learn that the discipline is underpinned by a set of ideas of extraordinary power and beauty such as prime numbers, the concept of a variable, the idea of a proof, and the key concepts of space and geometry. For some students, encounters with these ideas will be a gateway to pursuing advanced mathematics and a lifetime of interest and enjoyment in mathematics for its own sake. Not all students will follow this path, of course, but we believe that all students should have the opportunity.

3.1.2 Concepts and competences

Mathematics is, of course, at the heart of MDE and its importance as a discipline cannot be overstated. The best traditions of mathematics education that develop deep conceptual understanding, technical fluency and generic problem-solving skills will continue to be essential, as will be developing forms of logical, numerical, spatial, probabilistic, and statistical reasoning. This traditional mathematical grammar – the capacity to 'speak the language' – is complemented in our vision for MDE by increased attention to statistical and data- scientific concepts and techniques.

162 *Op. cit.*, note 4.

163 Smith A, 2004. Making mathematics count: the report of Professor Adrian Smith's inquiry into post-14 mathematics education. See https://dera.ioe.ac.uk/id/eprint/4873/ (accessed 30 July 2024).

164 *Op. cit.*, note 44.

165 The Council for the Mathematical Sciences and the Heads of Departments of Mathematical Sciences, 2023. Mathematical Sciences recruitment and graduate outcomes: a diversity perspective of the landscape of Mathematical Sciences in UK universities.

The foundational concepts, techniques and reasoning skills of mathematics have not changed over many years; they are the basic ingredients, the fundamental grammar, the essential tools for mathematical thinking. Neither has the challenge diminished of enabling young people to acquire these concepts and skills. Our vision of MDE builds on these foundations in two ways. Firstly, MDE incorporates greater emphasis on some of the core concepts and competences of statistical and data-scientific thinking. Secondly, the development of these foundational concepts and competences should be enhanced with the use of computational tools and technologies (see chapter 4).

Many of the foundational conceptual steps, which might seem straightforward from an expert's perspective, are among the most challenging for learners, and can prove to be deeply problematic in terms of the later MDE education on which they are built. Take, for example, acquiring a confident grasp of the structures of the number system, place value, percentages and ratios; or the principles of distributivity, associativity and commutativity that underpin algebraic thinking. The transformation from partial, weak or no understanding of such things to sound, confident understanding is profound and empowering.

The large numbers of young people who do not achieve a GCSE grade 4 by the age of 16 have missing or faulty foundational mathematical concepts and reasoning; they have been 'building on sand'. Even for many of those who do make the grade, there remain serious weaknesses in those foundations that impact upon future work and lives. The challenges of making these conceptual, technical and reasoning gains cannot be underestimated, and as we have described in chapters 1 and 2, the long-term impact of their delayed acquisition is a burden for individuals and society.

At the later stages of MDE the existence of three strands means that one qualification pathway will not have to cover everything. This should allow for a clearer focus on mathematical abstraction, generalisation and proof for advanced learners, helping to support transitions to higher studies in mathematics. All A level maths students currently encounter some statistics (and mechanics) but our formulation of MDE calls for a more central positioning of statistical and data-scientific concepts at this advanced level, together with wider and better use of the technologies and tools (eg coding) that can support advanced MDE.

All of this raises the question of curriculum balance, breadth and focus. In this report we do not set out a model curriculum but do note that the mathematics curriculum, and indeed the school curriculum more generally, is overcrowded. To fulfil the intention of MDE will require reorganisation and rationalisation of the curriculum. This might be achieved by moving some content into later stages, capitalising on connected content in allied subjects, and gaining time by increased use of computational tools. Our thinking on 14 – 18 mathematics pathways (see chapter 5, section 5.5) explores these problems of curriculum purpose and breadth.

3.1.3 Implications for Foundational and Advanced Mathematics for teaching and assessment

While this aspect of the MDE vision appears close to what is already in place in schools, there are a number of implications. Firstly, there is an ongoing need to improve the development of the foundational concepts and competences of MDE from early years and thereafter. Establishing strong conceptual foundations and good mathematical learning trajectories at this early stage is critically important.

Secondly, with the greater breadth of MDE, there is need for professional development and resources for teachers as new statistical and data elements of foundational and advanced mathematics emerge through the curriculum at every stage. And thirdly, the vision that all of this is supported with more embedded use of computational tools will likewise require new resources and carefully planned professional development over a number of years[166].

Likewise, there will be a need to rethink aspects of assessment. National assessment systems tend to focus greater reward on technical fluency than conceptual understanding or reasoning. The well-known WYTIWYG adage – 'what you test is what you get' – explains why such assessments tend to encourage classroom practices that prioritise mathematical doing over understanding. In addition, the lack of progress with online or computer-based assessments[167] means that there is little incentive to use computational tools in classrooms when they are not needed for high-stakes assessments. For all the proposals in this report, assessments are critical drivers of – or barriers to – change, and the improvement of foundational and advanced mathematics is no exception.

BOX 5

Toolkits for MDE

The report we commissioned from the Open University investigates the intersection of mathematics, data science, statistics and computing and suggests a framework of competencies for a curriculum at that intersection[168]. The report explores the idea of 'toolkits' in mathematics, computing and statistics / data science. These are not simply a collection of techniques, but the ways of thinking and reasoning, and the language, concepts and tools inherent to those disciplines. For example:

- The mathematical toolkit includes: Using quantities and methods; Analysing covariation; Reasoning mathematically; Using mathematical representations; Using mathematical aids and tools.
- The statistics / data science toolkit includes: Data stewardship; Handling data; Data representation; Statistical thinking and methods; Probabilistic reasoning; Using computational aids and tools.
- The computing toolkit includes: Computational thinking (including developing algorithms; Using algorithms; Programming; Representing and manipulating data; Safe use of technology; Using a range of aids and tools including emerging technologies[169].

166 *Op. cit.*, note 123.

167 *Op. cit.*, note 158

168 *Op. cit.*, note 44.

169 *Op. cit.*, note 44.

3.2 General quantitative literacy

As mathematics and data become increasingly embedded in society, so citizens and employees need the capabilities to make sense of their quantitatively saturated worlds and workplaces. Mathematical and data education (MDE) therefore has a strong focus on general quantitative literacy (GQL) for all learners, at every stage and level, both within the focused MDE curriculum and across all subjects and qualifications.

The major international comparison of mathematics education (OECD:PISA) foregrounds mathematical literacy (see Box 6)[170]. The ideas of mathematical literacy and quantitative literacy are used variously in different countries and contexts, and many countries are responding to growing calls for increased financial literacy[171]. Whilst proponents of different terms focus on their distinctions and particular demands, here we focus on what is common between them, ie literacy, by which we mean the capacity to read, to write and to communicate. We use general quantitative literacy rather than Mathematical Literacy in line with our broad MDE ambition; GQL incorporates financial and other mathematical and statistical literacies.

GQL extends much further than 'basic skills' and now, more than ever, the capacity to navigate the world with mathematics depends on a capacity to use computational tools. These include, at a minimum, the use of a basic calculator, but for increasingly large numbers of people the use of spreadsheets is becoming an essential skill for adult life.

BOX 6

OECD definition of mathematical literacy[172]

"...Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well- founded judgements and decisions needed by constructive, engaged and reflective citizens."

170 OECD, 2018. PISA 2022 mathematics framework (draft). See https://pisa2022-maths.oecd.org/files/PISA 2022 Mathematics Framework Draft.pdf (accessed 30 July 2024).

171 *Op. cit.*, note 43.

172 OECD, 2023. PISA 2022 Assessment and Analytical Framework, PISA, OECD Publishing, Paris. See https://doi.org/10.1787/dfe0bf9c-en (accessed 10 July 2024).

A key idea in the development of GQL, whatever the level of demand, is the focus of attention on realistic contexts or problems. The challenge of teaching GQL is not about finding contexts to use abstracted maths skills. These can be constructed, but often lead to 'pseudo-realistic learning', in which problems can be so contrived that they provoke scepticism[173]. Instead, teaching GQL begins with an interesting context and then draws appropriately on MDE concepts and methods to make sense of that context and act appropriately. For example,

- What is the best mobile phone deal for me?
- Why are young people not being offered COVID-19 vaccines?
- What happens to food prices now that inflation is falling?
- Is it worth buying a lottery ticket?
- Should I be worried about a 20% increase in risk of getting a rare cancer?
- Is a £60mn fund for improving mental health for young people a lot of money?
- Should I be worried about my children if 50 unspecified schools in England have concerns about their construction materials?

These kinds of questions can as easily be posed at primary level, for example

- What is the cheapest way we can all travel to the cinema?
- What is the quickest way to get to school?
- Plan a party with games, hats, food, and a budget.
- Should the school road be closed to traffic at school times?

3.2.1 Problem solving

GQL is akin to real world problem solving where what matters first is understanding the problem, formulating questions and planning investigations. Then come data collection, mathematisation, performing calculations, and communicating new insights. These in turn may inform reformulation of the questions, leading to what is known as the problem-solving cycle (see Figure 6).

Figure 6 reflects how statistics and data science are problem-solving disciplines, rooted in real-world challenges or problems. This provides a clear distinction from the abstraction of 'pure' mathematics, although of course mathematical insights can be essential in solving problems[174]. This means the educational focus in GQL shifts to solving problems and conducting enquiries, which is not typically part of mathematics education.

All investigations need a plan to decide what information to collect, and how to collect it. The plan may involve scientific experimentation, including ideas of randomised trials, or may be a survey, or draw on online resources, such as the deals being offered by mobile phone operators. Ideas such as representative samples and unbiased measurement become crucial and are best learned through active engagement. Then data needs to be collected and accurately recorded into, for example, a spreadsheet, with accompanying quality checks, and in a form ready for analysis – so called 'data wrangling'.

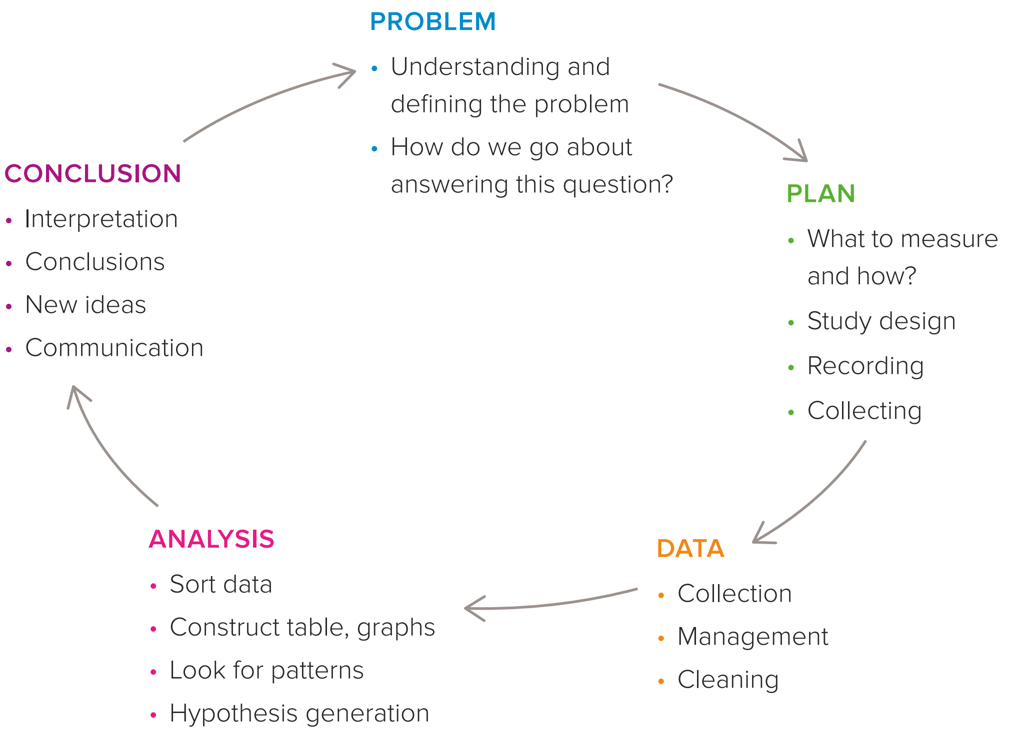
173 Stein S, *et al*, 2004. Incorporating authentic learning experiences within a university course, Studies in Higher Education, 29(2), (p239 – 258). See https://doi.org/10.1080/0307507042000190813 (accessed 30 July 2024).

174 Ukobizaba F, *et al*, 2021. Assessment strategies for enhancing students' mathematical problem-solving skills: a review of literature, EURASIA Journal of Mathematics, Science, and Technology Education, 17(3). See https://doi.org/10.29333/ejmste/9728 (accessed 30 July 2024).

Analysis can be broken into two broad phases. Exploratory summarising of data into tables and graphs can suggests possible patterns, and whereas simple descriptions may be adequate for earlier education, at a higher level it is natural to use this phase for generating hypotheses. This leads to the confirmatory stage, where foundational mathematics can come into its own, eventually leading to techniques such as hypothesis testing and interval estimation. Finally, these analyses are brought together into a conclusion, using judgement to draw appropriate interpretations, and communicating results through text, tables and graphics. This inevitably leads to more questions, and the cycle starts again.

FIGURE 6

The problem-solving cycle[175]



175 Problem-solving cycle, going from problem, plan, data, analysis to conclusion and communication, and starting again on another cycle, based on the New Zealand Census at School project. Spiegelhalter D, 2020. The Art of Statistics. See https://www.penguin.co.uk/books/294857/the-art-of-statistics-by-spiegelhalter-david/9780241258767 (accessed 30 August 2023).

The problem-solving cycle emphasises active investigation, which is appropriate education for many of the professional roles outlined in chapter 1 (see section 1.3). But it is also crucial that an informed citizenry can be skilled and critical consumers of the numbers that are thrown at them, whether on social media, advertising or elsewhere. The COVID-19 pandemic showed how some people were easily swayed by claims based on faulty interpretation of real data, supplemented by anecdote and supposition[176,177]. Many people seemed unable to judge the reliability of sources of evidence, and to weigh up competing claims.

By carrying out investigations at school, completing the problem-solving cycle and then critiquing each other's work, we may hope that future citizens will be better able to judge what 'good' evidence is.

3.2.2 Implications of GQL for teaching and assessment

The sort of problems posed within GQL, whether primarily statistical, data science or financial, will often not have a single 'correct' answer. Judgement may be involved at all stages of the cycle, particularly in the conclusions drawn. This is markedly different from standard teaching and assessment in mathematics. Related issues occur in computational thinking, which leans on the power of computational tools to complete any mathematical / statistical calculations in the problem-solving cycle (see Figure 6).

The emphasis on making sense of the context (using MDE) can be a challenge for traditional mathematics teaching, which typically prioritises mathematical fluency and / or speed with less concern for usefulness and realistic, relevant applications[178]. Such questions about the purposes for teaching MDE, and the emphasis on contextualised sense-making as opposed to technical fluency, would require support and continuing professional learning.

176 Kim S, *et al*, 2022. What predicts people's belief in COVID-19 misinformation? A retrospective study using a nationwide online survey among adults residing in the United States, BMC Public Health, 22(2114). See https://doi.org/10.1186/s12889-022-14431-y (accessed 30 July 2024).

177 Atuheirwe A, *et al*, 2024. Misinformation, knowledge and COVID-19 vaccine acceptance: a cross-sectional study among health care workers and the general population in Kampala, Uganda, BMC Public Health, 24(203). See https://doi.org/10.1186/s12889-024-17678-9 (accessed 30 July 2024).

178 Arthur Y D, 2018. Connecting Mathematics To Real Life Problems: A Teaching Quality That Improves Students' Mathematics Interest, IOSR Journal of Research & Method in Education, 8(4), (p65 – 71). See https://www.iosrjournals.org/iosr-jrme/papers/Vol-8 Issue-4/Version-2/J0804026571.pdf#:~:text=URL%3A https%3A%2F%2Fwww.iosrjournals.org%2Fiosr (accessed 30 July 2024).

GQL should be central in primary education and develop naturally in the early years of school. MDE's increased focus on GQL should be helpful for those pupils with special educational needs, for whom some level of competence in quantitative matters may be important for the development of their life skills[179,180].

Responsibility for the development of GQL widens throughout lower secondary as these competences get developed more widely across the curriculum, and where new contexts become increasingly important as the student encounters different subjects and issues. Thereafter further development of GQL complements both foundational and advanced mathematics and domain-specific competences. Upper secondary education presents opportunities to use GQL to explore issues that concern young people, and many might see more of their mathematical and data education focused on GQL than on advanced mathematical techniques.

Assessing GQL and problem solving, particularly in high stakes assessment, is challenging and needs improving. GQL in everyday life is nothing like that of a timed written exam, whether paper or computer-based, which comprises abstract and pseudo-realistic questions. Some examples of GQL assessment across the UK have explored alternative approaches[181] but further work is needed in this area to align the learning and assessment experience and make the GQL part of MDE more meaningful.

RECOMMENDATION

Area for action 3: qualifications and assessment

Develop new methods of assessment for general quantitative literacy that reflect how it is used in practice, including the use of digital technologies to analyse data sets.

---

179 Kroesbergen E H, 2003. Mathematics interventions for children with special educational needs: a meta-analysis, Remedial and Special Education, 24(2), (p97 – 114). See https://doi.org/10.1177/07419325030240020501 (accessed 30 July 2024).

180 Robbins B, 1991. Mathematics for All in Ashdown R, *et al*, The Curriculum Challenge, (p137 – 155). See https://www.taylorfrancis.com/chapters/edit/10.4324/9780429454738-9/mathematics-brian-robbins (accessed 30 July 2024).

181 Tout D, 2020, Evolution of adult numeracy from quantitative literacy to numeracy: Lessons learned from international assessments, International Review of Education, 66, (p183 – 209). See https://doi.org/10.1007/s11159-020-09831-4 (accessed 30 July 2024).

3.3 Domain-specific competences – MDE across the curriculum

3.3.1 Consistency and coherence

Mathematical and data education (MDE) is not just a matter for mathematics departments, it is embedded in many, arguably most, school subjects. This means that MDE should be an integral part of the teaching of those subjects, ideally in a way that is coherent across the whole-school curriculum. We have argued that there is potential for MDE in the curriculum at primary level, although that potential is not currently realised. It is not generally evident at secondary level, where education is organised around the teaching of individual subjects which historically have operated in isolation from each other. There is however great potential for using MDE to underpin the teaching of many topics and issues that are salient to students. For example, the knowledge and skills of MDE are essential to understanding climate and sustainability, an issue which is high on the list of issues that matter to all young people and can only grow in importance.

This is an area where progress has been made. In 2012, reports from SCORE[182], and the Nuffield Foundation[183] analysed the mathematics content of A levels in several subjects. Subsequently, Ofqual introduced requirements that the specifications for A levels in those subjects should explicitly highlight mathematical content and should include minimum weightings for mathematical skills[184]. A level geography, for example, requires statistical methods; translating information between text, graphical, and numerical form; interpreting and presenting charts and graphs; use of percentages, proportions and ratios and use of maps and scales; and the use and interpretation of spatial data through geographical information systems.

In the vocational and technical spheres, teachers of BTECs and other vocational qualifications have long accepted responsibility for teaching the mathematical content of their courses, while the framework for General Mathematical Competences[185] proposed by the Royal Society's Advisory Committee on Mathematics Education has been adopted as a basis for the design of the new T Levels, with a view to ensuring a level of consistency across those courses[186].

182 Science Community Representing Education, 2012. Mathematics within A level science 2010 examinations. See https://www.stem.org.uk/resources/elibrary/resource/25956/mathematics-within-level-science-2010-examinations (accessed 30 July 2024).

183 Nuffield Foundation, 2012. Mathematics in A level assessments. See https://www.nuffieldfoundation.org/sites/default/files/files/Maths_in_A_level_Assessments_Nuffield_Foundation_WEB.pdf (accessed 30 July 2024).

184 Ofqual, 2021. Functional skills mathematics: conditions and requirements. See https://assets.publishing.service.gov.uk/media/6286607ce90e071f5b6723f6/6775-4_Functional_Skills_Mathematics_Conditions_and_Requirements.pdf (accessed 30 July 2024).

185 The Royal Society, 2019. Mathematics for the T level qualifications: a rationale for General Mathematical Competences (GMCs). See https://royalsociety.org/-/media/policy/topics/education-skills/maths/mathematics-tlevels-gmc.pdf (accessed 30 July 2024).

186 *Op. cit.*, note 151.

At GCSE level some subjects include requirements for mathematics in their specifications. The assessment requirements for Combined science, for example, specify that 20% of total marks should be used to credit relevant mathematical skills[187]. The specifications for Geography include extensive references to graphical, numerical, and statistical skills, and to the use of quantitative and qualitative data. The specification for Physical Education requires that: "Students should develop knowledge and understanding of data analysis in relation to key areas of physical activity and sport", while the National Curriculum Programme of Study for Citizenship education includes several references to quantitative reasoning, for example: "income and expenditure, credit and debt, insurance, savings and pensions, financial products and services, and how public money is raised and spent"[188].

An internal report commissioned for the Mathematical Futures Programme examined how mathematics requirements were described across all relevant GCSE specifications in the UK. It found that the specifications for some subjects included explicit descriptions of required mathematical content but these were not consistent across subjects. In other subjects the requirements were not specified, but could be inferred from descriptive content. The study found that the Scottish Qualifications Authority specifications demonstrate a relatively more consistent approach, but they lack detail.

While the precise content of domain-specific competences will, by definition, vary from subject to subject, consistency of terminology and approach across MDE would be immensely helpful to students, teachers and curriculum planners. A good place to start building towards that goal would be to carry out a detailed study of how MDE is described in the specifications for all existing high-stakes qualifications, including both vocational and technical qualifications and the specifications for GCSE and A level. This should lead to recommendations for a consistent approach, use of terminology and level of detail across subjects. This was the thinking behind the General Mathematical Competences; their consistent adoption and application across the new T Levels was intended to facilitate cross-college discussions and professional development for staff developing those competences.

In 2016 the Association for Science Education published: "The Language of Mathematics in Science: A Guide for Teachers of 11 – 16 Science"[189], which gives guidance for teachers of 11 – 16 science on explanations of key ideas and terminology in maths, and good practice in applying mathematical ideas in science.

"Mathematics is particularly valuable when combined with the subject specialist approaches and knowledge of other disciplines. When connected with these 'real-world' disciplines, mathematics can enhance young peoples' ability to live fulfilling, rewarding and sustainable lives. For example, when drawn on within geography, mathematics enables young people to better understand how our climate has and will change, the risk presented by a 1:100 flood, or the numbers behind the UK's 606,000 net migration increase."

Royal Geographical Society.

187 Ofqual, 2020. GCSE subject level conditions and requirements for combined science (2021) (p15). See https://assets.publishing.service.gov.uk/media/5fa2c2f9d3bf7f03aef8124e/GCSE_Subject_Level_Conditions_and_Requirements_for_Combined_Science__2011_.pdf (accessed 30 July 2024).

188 House of Commons Library, 2023. Financial and enterprise education in schools (England). See https://researchbriefings.files.parliament.uk/documents/SN06156/SN06156.pdf (accessed 30 July 2024).

189 Association for Science Education, 2016. The language of mathematics in science. See https://www.ase.org.uk/mathsinscience (accessed 30 July 2024).

The Association for Science Education publication points out that "Consistency between mathematics and other subjects is clearly desirable wherever possible: it is unhelpful to have arbitrary differences in approaches and terminology between the subjects."[190]. This is evidently correct, but consistency does not mean uniformity. Different disciplines use quantitative methods in different ways. Understanding those differences is itself an important goal of learning.

3.3.2 Supporting teaching

In the long run, the aim should be an understanding that MDE is every teacher's concern. A helpful step towards this would be to develop model whole-school MDE policies, to illustrate how consistency and coherence between subjects and phases could be built into every student's educational journey.

Underpinning all this there will be a need for Professional Development, especially for subject teachers, and for the introduction of these ideas into initial teacher education. The Association for Science Education has produced a range of follow-up publications and discussions and has run courses aimed at helping teachers[191]. The Royal Geographical Society has run programmes that have trained over 1,000 teachers[192]. We think these are good models and we encourage other subject areas to consider similar initiatives.

A key to success is partnership between stakeholders, so that the case for the value of MDE is made from within subject communities, not urged on them from without. The Royal Geographical Society's work with the Advanced Mathematics Support Programme is a good example of how such partnerships can work.

Such whole-school guidance has been developed for the teaching of literacy in schools and that could provide a useful model[193]. Implementing such policies will require staff in mathematics departments to take a role in supporting their colleagues, but its creation will involve staff from all the appropriate subjects. We recognise that this is an ambitious objective and would take time to implement. We believe however that in the long term there would be substantial gains for learners.

190 *Op. cit.*, note 189.

191 For example, Cottle D, 2021. Improving students' mathematical skills in secondary science: ideas from mathematics pedagogy, School Science Review, 102(381), (p61 – 64). See https://www.ase.org.uk/resources/school-science-review/issue-381/improving-students-mathematical-skills-in-secondary (accessed 30 July 2024).

192 Royal Geographical Society, 2023. Geography and Core Maths. See https://www.rgs.org/schools/projects-and-partnerships/geography-and-core-maths (accessed 30 July 2024). (accessed 15 April 2024).

193 Education Endowment Foundation, 2021. Improving literacy in secondary schools. See https://educationendowmentfoundation.org.uk/education-evidence/guidance-reports/literacy-ks3-ks4 (accessed 30 July 2024).

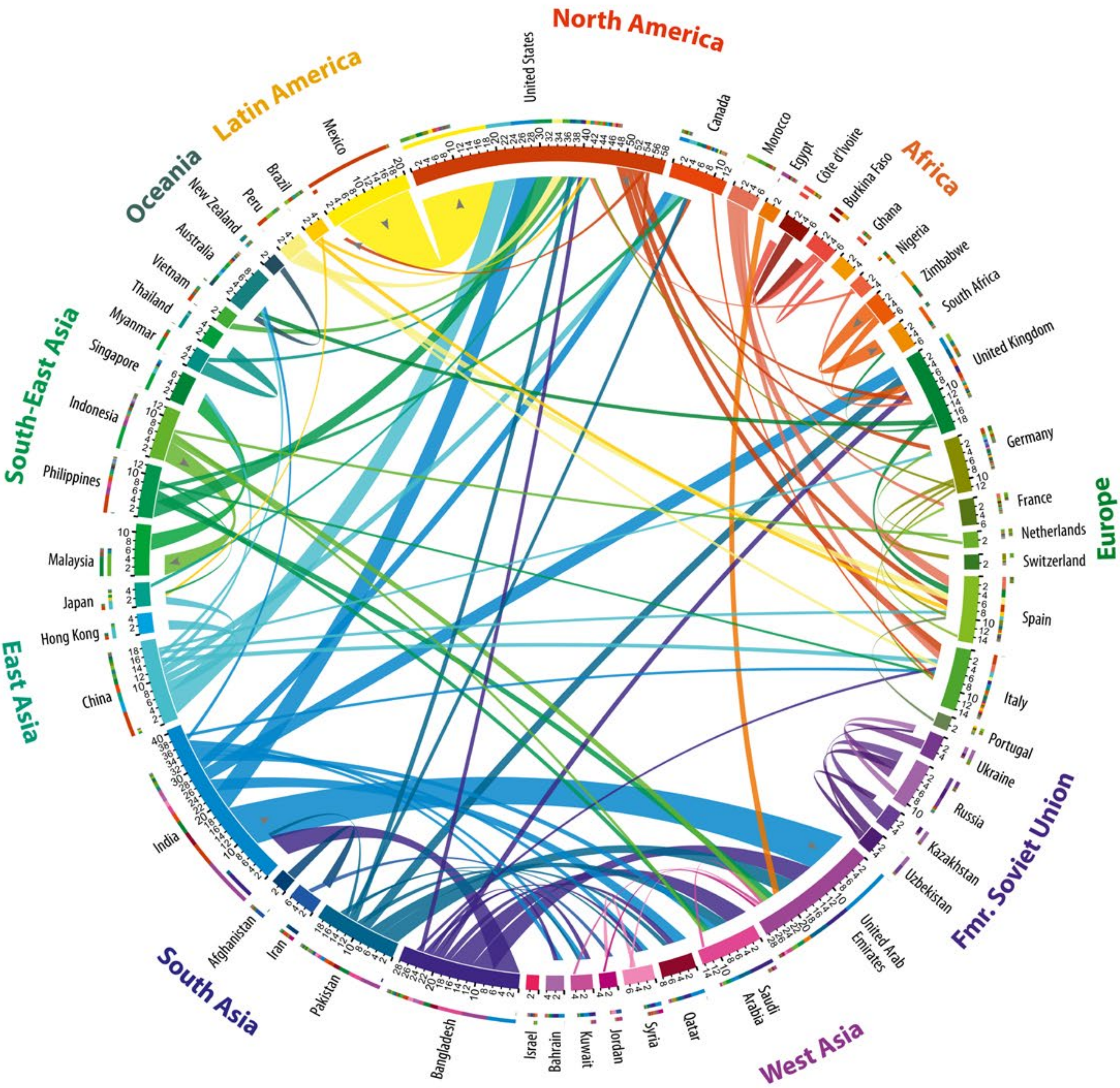
RECOMMENDATIONS

Area for action 1: implementation

Standardise MDE terminology and level of detail expected in all school and college courses and require awarding organisations to be consistent in how they describe MDE competences in their programmes of study and assessment criteria. Begin this process by carrying out a study into how MDE competences are currently depicted and used in existing high-stakes qualifications in non-maths subjects.

Area for action 5: teachers

Develop ways for mathematics and subject departments in schools and colleges to work with each other to support consistency and coherence between subjects and phases in the teaching of MDE across all subjects; for example, by developing whole-school MDE curriculum guidance documents.


North America
United States
Canada
Latin America
Mexico
Brazil
Peru
Oceania
New Zealand
Australia
South-East Asia
Vietnam
Thailand
Myanmar
Singapore
Indonesia
Philippines
Malaysia
East Asia
Japan
Hong Kong
China
India
South Asia
Afghanistan
Iran
Pakistan
Bangladesh
Israel
Bahrain
Kuwait
Jordan
Syria
Qatar
Saudi Arabia
United Arab Emirates
West Asia
Uzbekistan
Kazakhstan
Russia
Ukraine
Fmr. Soviet Union
Portugal
Italy
Spain
Switzerland
Netherlands
France
Germany
Europe
United Kingdom
Africa
Morocco
Egypt
Côte d'Ivoire
Burkina Faso
Ghana
Nigeria
Zimbabwe
South Africa

Chapter four

Computational concepts, technology and tools

Left
Global Migration Data Sheet 2005 – 2010: Unique estimates of migration flows between the top 50 sending and receiving countries, by the Wittgenstein Centre for Demography and Global Human Capital (IIASA,VID/ÖAW,WU)**.** Circular plots created with Circos, a software package for creating circular layouts of data and information (Krzywinski, M. *et al.* Circos: an Information Aesthetic for Comparative Genomics. Genome Res, 2009, 19:1639 – 1645). This work is licensed under CC-BY-NC-SA 3.0. Original image and supporting data available at: download.gsb.bund.de/BIB/global_flow/VID_Global_Migration_Datasheet_web.pdf

Computational concepts, technology and tools

4.1 Tools for mathematical and statistical thinking

This report addresses the potential of computing technology and tools to transform mathematical education, but of course they already figure prominently in the national computing curriculum. There are close connections between mathematics and computing. Computational thinking is a golden thread that shares with mathematics and MDE some key general concepts, especially the notions of algorithm and of problem abstraction and decomposition. We view the two school subjects as distinct and remaining so, with some key foundations in common: the concepts just mentioned, and practical grounding in the use of computers as a tool – basic IT skills and some familiarity with programming of various kinds and styles. Other aspects however are quite distinct. For instance, each subject carries a notion of literacy. However, we view digital literacy in computer science, and GQL – general quantitative literacy – in MDE, as two largely separate competences. Where there is some overlap however is with data literacy, and so a coherent approach across both MDE and computing is essential.

Computational tools come in different styles, for use at various levels. At elementary level, manipulatives – sets of physical objects for mathematical learning like Cuisenaire rods and Dienes apparatus – have a long history of helping pupils' understanding. Virtual manipulatives can also help scaffold pupils' mental models of properties of, and relationships between, mathematical objects. They have the added advantage that the class can share them on display screens or interactive whiteboards.

Electronic calculators have been used in schools since the 1980s. Often pupils learn manual methods first and only then use calculators to automate and extend the complexity of computations. However, use of calculators can provide access to more motivating and advanced problems and may help with mastery by focusing limited working memory on ideas, rather than on procedures, memorised step-by-step. In the modern era, the same sorts of trade-offs apply to the use of computers for learning.

Dynamic geometry and graphing software, supplementing teacher-led instruction, can help with learning geometrical and algebraic concepts. Pupils grasp a basic user interface to create dynamic sketches, and experiment to discover underlying structures.

Spreadsheets are particularly important. They are implicit in England's national curriculum for computing but rarely go beyond artificial business scenarios. We envisage them as a central component of MDE, for example capturing the mathematics of everyday life – household budgets or the impact of changes in interest rates or modelling physical systems. Pupils could model saving accounts, loan repayments and investments, and many other familiar and meaningful contexts. Once pupils have grasped the basic principles and the interface, spreadsheets help to introduce data science, including exploratory visual analysis and statistical tests.

MDE teaches the concepts and methods of data science, which also provide the foundations for understanding machine learning and AI. In their work on mathematical and data literacy for the Mathematical Futures Programme, Smith et al (2023)[194] highlighted the benefits of using authentic data, at sufficient scale, integrated with user-friendly, commercial-grade technology. They also highlighted the popularity of taught courses at level 3 (see Box 7).

RECOMMENDATION

Area for action 4: computational tools and technology

Computational tools and technologies, such as spreadsheets, apps and programming platforms, should be well embedded at suitable stages within MDE learning. Curriculum designers should ensure these technologies are incorporated to meet the new MDE objectives.

Computer algebra systems do symbolic manipulation and answer common questions in secondary algebra and calculus. Provided the mathematical principles are emphasised, supplementary use of algebraic manipulation software allows learners to focus on expressing problems or systems in mathematical terms and on the interpretation of results.

BOX 7

Resources for data science

Maths education charity, Mathematics in Education and Industry (MEI), has developed and introduced courses and teaching and learning resources in data science linked to level 3 maths study (AS / A level maths and Core Maths). These include:

- Introduction to Data Science: A short online self-study course for students of A level maths and Core Maths, introducing data science concepts and data analysis techniques using Python code. The course makes use of the large data sets (LDS) from A level maths[195].
- Data Science Taught Course: A 10-week curriculum enrichment course covering statistics, data analysis and machine learning of around 60 guided learning hours[196]. Students work with real datasets taken from a variety of contexts. They explore, analyse and visualise the data, and build and test models using Python. The course builds on GCSE and level 3 maths study, developing data skills and deepening students' understanding of statistics, data analysis and machine learning.

194 *Op. cit.*, note 44.

195 MEI, Introduction to Data Science. See https://mei.org.uk/introduction-to-data-science/ (accessed 7 July 2024).

196 MEI, Data Science Taught Course. See https://mei.org.uk/data-science-taught-course/ (accessed 7 July 2024).

Recent rapid advances in AI large language models (LLMs) such as Chat GPT and Gemini have wide ranging implications for education. Initial reactions from maths educators have been sceptical. Early LLMs were incapable of reasoning and were prone to 'hallucination'[197] even with relatively straightforward maths problems. More recently LLMs have become able to write and execute programs to tackle many problems in the MDE domain, at least as far as the end of school level. Pupils can learn thoughtful and critical use of these models, refining prompts to improve output or provide chain of thought reasoning. Access to LLMs does not substitute for mastery of MDE any more than access to calculators, spreadsheets or computer algebra systems would, but helps with asking good questions and evaluating answers, freed up from mechanical symbol manipulation.

RECOMMENDATION

Area for action 4: computational tools and technology

The Department for Education should carry out a dedicated research programme on the potential impact of AI on MDE learning, identifying new approaches for students at all stages of their education.

4.2 Educational technology

There are plenty of online tools for practising mathematical knowledge and skills. These platforms have engaging interfaces, go beyond simple 'drill and practice' and often include 'gamified' rewards that encourage engagement and progress, but they rarely allow pupils to explain their reasoning. There is good evidence for the benefit to pupils of regular engagement, and teachers can access detailed data on pupils' attainment and progress. Adaptive questioning builds on that, tailoring questions to pupils' level of attainment, and offering support where it is most needed. For more motivated, independent pupils, YouTube and TikTok offer tutorials and further practice questions.

Expert, 1:1 tutoring has a significant effect on pupils' outcomes but has been too expensive to be used at scale. Advances in AI will expand access to tutorial style support, albeit through dialogue with a computer system rather than a human tutor. It is too early to tell whether it will sustain pupil engagement and have a lasting impact on pupil outcomes. Teachers may struggle to reconcile this style with the traditional classroom model, particularly in a 'mastery' context in which pupils learn as a class. The shortage of maths teachers may nonetheless mandate that sort of individualised approach.

197 AI hallucination happens when a large language model (LLM) confidently produces outputs that are inaccurate.

Assistive technology can already do a lot to support pupils who would otherwise find it hard to participate. Speech to text and text to speech interfaces are common, and optical character recognition tools allow textbook pages to be converted to other formats. Live captions can be used in lessons, on the board or pupils' own devices. Material is best presented without unnecessary distraction or irrelevant context, and this is even more important for neurodiverse learners. These technologies will have increasing potential to help learners with a variety of special needs to engage with MDE.

4.3 MDE and computing concepts

Programming is a powerful tool for mathematics and data science and offers an effective means to develop understanding in MDE. The computational thinking that happens before coding is equivalent to the mathematical reasoning needed to structure solutions to problems in MDE, as both Pólya[198] and Wolfram[199] describe.

Visual programming languages, like Scratch, lend themselves to geometric thinking, with a Cartesian coordinate system and 'turtle' graphics. These draw geometric figures via primitive move and turn instructions, and trace paths with a virtual pencil. There is considerable potential for integrating them more tightly into mathematical education[200]. Text-based programming, such as Python, focuses on core programming constructs like sequence, selection, iteration, functions, variables, and lists. These skills could be applied in MDE to address some mathematical or data-based problems in a way that is closer to practice in the world of work.

Some jurisdictions have introduced programming into the mathematics curriculum but for England it would be most effective to build on the programming that is taught in the computing curriculum, to solve MDE problems. In turn, MDE could provide much needed rich, authentic contexts to enhance work in computing. Pupils progressing to university to study mathematically-demanding programmes should have further coding and computing integrated into their pre-university studies, to help the transition.

RECOMMENDATIONS

Area for action 4: computational tools and technology

Strengthen the links between MDE and computing through: including problems in MDE that draw on pupils' computing knowledge and skills, including programming; and providing rich, motivating MDE contexts for programming and other skills in computing lessons.

Area for action 4: computational tools and technology

Ensure advanced MDE students (post-16) develop programming skills and learn the use of computational tools common in mathematically demanding undergraduate programmes.

198 Pólya, George, 1945. How to Solve It. Princeton University Press.

199 Wolfram, C, 2020. The Math(s) Fix: An Education Blueprint for the AI Age. United States: Wolfram Media, Incorporated.

200 University College London, 2019, ScratchMaths. See https://www.ucl.ac.uk/ioe/research/projects/ucl-scratchmaths (accessed 30 July 2024).

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Chapter five

Stages and structures

Left
Interpreting information such as train times and planning journeys is an everyday use of general quantitative literacy. © iStock.com / Teamjackson.

Stages and structures

5.1 Overview

In the future, mathematical and data education (MDE) will be a necessary part of everyone's lives, so it must provide an appropriate education for all students, one that acknowledges different needs, start and end points, in a way that our present system fails to do.

This includes learners with Special Educational Needs, of whom there are around 1.6 million in schools in England[201]. Special Educational Needs can refer to temporary or long-term learning difficulties or disabilities due to a wide range of factors including communication, cognition and learning, social, emotional, and mental health, and sensory or physical needs.

A move to MDE has the potential to benefit many of these pupils, whether it be through the increased use of assistive technologies (see Chapter 4, section 4.2) or through a wider focus on applications and the use of numeracy in life skills. If the aspiration that MDE should be for all students is to be met, as we believe it should, then plans for the development of MDE should include explicit consideration of the needs of Special Educational Needs pupils from the outset.

Underpinning all this must be a curriculum based on coherent foundations.

RECOMMENDATION

Area for action 2: curriculum

Design and implement a curriculum that integrates appropriate data, statistics, and computational tools coherently with mathematics.

TABLE 4

Three phases of mathematical and data education (MDE)

Age	
3 / 5 – 11	MDE is integrated in generalist teacher practice; cross-curricular connections are seamless; manipulatives and digital technologies are used commonly and creatively; foundational and advanced mathematics, general quantitative literacy and domain-specific competences are naturally entwined and woven throughout the curriculum as appropriate.
11 – 14	Specialist (and non-specialist) teachers of mathematics focus more on the essential foundational and advanced mathematics and general quantitative literacy necessary for functioning in society, with additional foundational and advanced mathematics for most learners. The domain-specific competences are now developed more by other subject teachers. All teachers become teachers of MDE, supported by new thinking about how the components of MDE (mathematics, statistics, data and computing) cohere across the whole curriculum as the MDE strands (FAM / GQL / DSC).
14 – 18	With the arrival of learner choices, diverging education pathways, and qualifications with their high-stakes assessments, the challenge of cross-education coordination of MDE increases. A reimagining of MDE qualification pathways that are well suited to the education, work and life needs of varied learners will be needed.

201 GOV.UK, Special educational needs in England, Academic year 2023 / 24 – Explore education statistics. See https://explore-education-statistics.service.gov.uk/find-statistics/special-educational-needs-in-england (accessed 7 July 2024).

5.2 Secure beginnings

Research evidence shows that individual differences in mathematical attainment between children when they start primary school persist throughout schooling. Those who start behind tend to stay behind[202,203] and this effect is not particular to the UK. High quality early years mathematical education is critically important and is dependent on there being knowledgeable practitioners with good understanding of child development, and key mathematical ideas[204]. Such effective practitioners provide a rich blend of learning experiences which build good conceptual understanding of number, with connections between quantities, number words and digits. This establishes strong foundations for later arithmetic, and hence for MDE.

There is, therefore, an urgent need for more professional development of the least qualified and / or most maths-averse maths educators, as they have the greatest responsibility for affecting the maths success and life chances of the most disadvantaged young children at school entry.

The current Early Learning Goals place insufficient emphasis on deep conceptual understanding; the emphasis on recall of abstract number facts does not provide strong foundational thinking. The current curriculum's weak focus on spatial reasoning[205], following the removal of Shape and Space from the Early Learning Goals, is also problematic given the evidence of the relationship between spatial reasoning and mathematical relationships developed in number, measure, data and geometry[206,207]. This is especially important for girls, and those from lower socio-economic backgrounds, who historically have been offered fewer opportunities to engage with spatial activities[208].

Effective early years settings, including childminders, private and voluntary provision, nurseries, nursery units attached to schools and reception classes, recognise the importance of working with parents and carers, as the home environment is a key element in attitudes to, and achievement in, mathematics[209]. A future MDE, and the transition to it, with its emphasis on application, will aim to break unhelpful cycles in which parents feel poorly equipped to support their children's early MDE development.

202 Duncan G J *et al*, 2007. School readiness and later achievement. See https://psycnet.apa.org/record/2007-16709-012 (accessed 30 July 2024).

203 Early Intervention Foundation, 2018, Realising the potential of early intervention. See https://www.eif.org.uk/report/realising-the-potential-of-early-intervention (accessed 30 July 2024).

204 Executive function is a set of mental skills that include working memory, flexible thinking and self-control.

205 *Op. cit.*, note 104.

206 Hawes Z & Ansari D, 2020. What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior, Psychonomic Bulletin & Review, 27, (p465 – 482). See https://pubmed.ncbi.nlm.nih.gov/31965485/ (accessed 30 July 2024).

207 Education Endowment Fund, 2023. Pilot evaluation of Spatial Cognition to Enhance mathematical learning. See https://d2tic4wvo1iusb.cloudfront.net/production/documents/projects/SPACE-pilot-evaluation-plan-AMENDED-091123_clean.pdf?v=1712092276 (accessed 30 July 2024).

208 Newcombe N S, *et al*, 2019. Spatial skills, reasoning, and mathematics, in Dunlosky J, The Cambridge Handbook of Cognition and Education, (p100 – 123).

209 University of Surrey, 2023. Spatial reasoning: briefing for policymakers. See https://www.surrey.ac.uk/spatial-reasoning (accessed 30 July 2024).

5.3 Primary

Primary teachers typically teach all subjects to their class. They are familiar with techniques which develop pupils' knowledge of maths, but the emphasis will have to move to enabling pupils to think, reason and apply their knowledge to solve problems. This would be a good basis for MDE. Cross-curricular contexts include broadening opportunities for learning mathematics in age-appropriate ways (for example, with practical design and technology, and physical activity outdoors including geography) so gaining more time and greater motivation. Transition to MDE would provide an opportunity to rebalance content across the years, creating space to develop deeper understanding of key concepts.

At present, the curriculum over-emphasises the performance of arithmetical operations, but it is the structure of the number system and number relationships that are foundational for future study. This begins in the early years, where addition and subtraction statements go hand in hand; in mental methods, where decomposing numbers allow a sense of size, approximation and rapid calculation; and in calculation methods, where distributivity, associativity and commutativity underlie a method.

In parallel, the curriculum under-emphasises the beginning of algebraic and other types of reasoning such as logical, spatial and statistical. These should become more sophisticated as students move through key stages, contributing to later content within the MDE curriculum.

An MDE curriculum would encourage the use of calculators and other computational tools at the appropriate stages to support children's exploration of number[210], and enhance problem solving and investigating. It would be an opportunity to rethink the purpose of timed multiplication table tests[211], which have been criticised as skewing activity in year 4 by focussing on recall rather than automaticity, promoting an image of maths as focussing on memorising rather than thinking, and engendering negative attitudes because of the focus on speed[212].

In summary, a move to MDE would be an opportunity to reprioritise, with some changes of emphasis. The changes we propose would, we believe, be welcomed by early years and primary teachers, most of whom would adapt without difficulty over an appropriate timescale and given sufficient support in the way of resources. In terms of implementation, the network of maths hubs provides a well-tried mechanism for the necessary training, although the current capacity and strategy for supporting professional change would need to be reviewed if the full ambition for MDE is to be realised.

RECOMMENDATION

Area for action 2: curriculum

Review the early years and primary curriculum to provide strong foundations, strengthening key areas such as spatial reasoning.

210 Mycroft-Smith L, 2021. The calculator in maths curriculum: research and UK policy. See https://my.chartered.college/research-hub/the-calculator-in-maths-curriculum-research-and-uk-policy/ (accessed 30 July 2024).

211 Tes Magazine, 2019. Why times-tables check does not reflect pupils' ability. See https://www.tes.com/magazine/archive/why-times-tables-check-does-not-reflect-pupils-ability (accessed 3 April 2024).

212 Association of Teachers of Mathematics & Mathematical Association, 2021, The teaching and learning of multiplication bonds: a position statement. See https://atm.org.uk/News/the-teaching-and-learning-of-multiplication-bonds-a-position-statement (accessed 30 July 2024).

5.4 Rethinking MDE from 11 – 14

The so-called key stage 3 'dip', whereby many students aged 11 – 14 make little progress in the early years of secondary schooling, has been well documented for many decades[213]. Our vision for MDE offers an opportunity for rethinking this important stage of education, one that has been largely overlooked in the swings between prioritisation of post-16 and early years since the 1990s. We propose that consideration be given to establishing a newly designed low-stakes assessment at the end of the key stage (age 14) which focuses on fundamental ideas and concepts applied in meaningful contexts.

Its purpose would be to act as a competency assessment for students to demonstrate their mastery of the key MDE content which is essential for functioning in everyday contexts. The content would include, for example, meeting various contexts for the key concept of proportional thinking, as students move from additive to multiplicative ways of working.

Any such qualification would be a single national assessment, similar to the national key stage 2 assessments (SATs), taken in Year 6, but different in the sense that it would be low-stakes, ie not part of school accountability measures, thereby reducing the need to teach-to-the-test. The pass threshold would be high, in recognition that these concepts and skills are essential for everyone to be able to function in society, as well as being a secure foundation for the more sophisticated MDE learning that will follow. Results would be reported to individual students, and for many students the results would be a useful formative assessment point in advance of the decisions about the next steps for their MDE education, within the framework set out below.

RECOMMENDATION

Area for action 3: qualifications and assessment

Develop a low-stakes competency assessment, to be taken by all students around the end of key stage 3 (age 14), to enable individual learners to demonstrate mastery of the foundational MDE concepts and skills necessary for confident and engaged citizenship.

We propose that such an assessment should be available online and on demand towards the end of year 9. Students not ready to take it in year 9 could take the test later and whilst some would be ready to take it earlier there would be no need for them to do so as it would not be reported in any school-level measures. Developing online assessment would allow for the integration of standard education technologies useful for MDE, though this would need careful design from the outset[214].

RECOMMENDATION

Area for action 3: qualifications and assessment

Develop online assessment methods that can grow as needed, enabling the use of MDE-specific digital tools and benefitting from lower costs and improved operations.

We are aware that there are already discussions around similar competency assessments in literacy, digital skills, and mathematics[215], and would expect that joint conversations would prove fruitful.

---

213 *Op. cit.*, note 118.

214 *Op. cit.*, note 123.

215 Department for Education, 2022. Opportunity for all: strong schools with great teachers for your child. See https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1063602/Opportunity_for_all_strong_schools_with_great_teachers_for_your_child__print_version_.pdf (accessed 30 July 2024).

It is widely acknowledged that at lower secondary level there is an issue with the supply and knowledge base of mathematics teachers (see chapter 2 section 2.3). Ensuring that MDE is taught by skilled and knowledgeable practitioners means looking more widely to enhance expertise already available from teachers of other subjects and building on it. The inclusion of an 'essentials' curriculum (as described above) may well prove attractive to such teachers.

We believe that in the long run the adoption of MDE will lead to a virtuous circle: broadening the scope from mathematics to MDE at this stage needs good general quantitative literacy and good subject knowledge, but not necessarily more advanced mathematics. As more students want and are able to study MDE at advanced levels the supply of teachers (and not just teachers of MDE) able to teach these competencies to future generations will improve.

In the meantime, there is an unavoidable tension between the aspiration to have all pupils taught by teachers with high (ie graduate level) qualifications in mathematics and the reality of the limited availability and increasing demand for such people. Teachers who are already confident in their own understanding of the relevant mathematical and data concepts would benefit from additional support in understanding appropriate pedagogies for this content so that they teach for understanding as well as fluency. At the same time, the different demands of MDE open up the possibility of new routes to recruitment and of a strong focus on high quality teaching in the middle years of key stage 3 (age 11 – 14).

RECOMMENDATION

Area for action 5: teachers

Develop professional development programmes, with supporting classroom resources, to enable current teachers to become expert teachers of MDE at key stage 3 (age 11 – 14), and to encourage new routes into teaching.

5.5 MDE from 14 – 18

It is already the case that some students continue with their MDE education as a natural part of their post-16 education, whether through foundational and advanced mathematics, general quantitative literacy, domain-specific competences, or a combination of all three. Consider, for example, a student taking Core Maths alongside A levels in mathematics and a science subject. Our vision is one in which all students continue to study MDE in some form until the end of their compulsory education, with MDE forming an organic part of every learner's education, just as literacy does now, from the earliest years through to higher education, employment and citizenship. As such, our vision for MDE is a good fit with the wider call for a broad and balanced post-16 curriculum[216].

Much has been written about 14 – 18 mathematics pathways over the last two decades[217]. Given the vision for MDE, with its three strands and broader domain, we find the complementary notion of a portfolio useful. As learners progress through secondary education and increasingly receive their MDE across the curriculum, and in diverse qualifications from 14 – 18, they will be able to acquire and apply their developing knowledge and skills in different contexts, for example as part of vocational and technical qualifications.

Taken together these can form a portfolio of qualifications which give an overall picture of the learner's mathematical and data capabilities. So, rather than thinking of maths to 18 in binary terms as has happened in the past, it might be conceptualised as a portfolio that accrues from the many combinations of qualifications taken at that age. This is already the case to some degree, although not always conceptualised that way[218].

A curriculum and qualifications structure for MDE will need to incorporate and recognise all three elements – foundational and advanced mathematics (FAM), general quantitative literacy (GQL) and domain-specific competences (DSC) – in different ways and to different extents, depending on the needs of different kinds of learners. Many of the components already exist, to varying degrees, for example:

- Advanced mathematics exists in AS and A level mathematics and further mathematics.
- General quantitative literacy exists, although in a less well-developed way, in Core Maths.
- Domain-specific competences exist in many vocational and technical qualifications such as BTECs. Since 2019, many reformed A levels explicitly include domain-specific competences, as do the new T Levels.

Our vision is one in which all students continue to study MDE in some form until the end of their compulsory education, with MDE forming an organic part of every learner's education, just as literacy does now, from the earliest years through to higher education, employment and citizenship.

216 Finegold P, 2023. Why the next government should commit to a fundamental review of the education system. See https://royalsociety.org/blog/2023/08/government-review-education-system/ (accessed 30 July 2024).

217 *Op. cit.*, note 163.

218 *Op. cit.*, note 152.

To support MDE from 14 – 18 these strands will need to be developed and brought together in a structure that is coherent and that recognises the different starting points and aspirations of students. This is a substantial task and we recommend that it should be carried out by an expert and authoritative body, working to a timetable that allows ample time for consultation, development and research. That task lies outside the scope of this report, but we are able to suggest some starting points.

For foundational and advanced mathematics there should be a strand that resembles the tiered structure that has existed for many years, namely GCSE (foundation), GCSE (higher), AS maths and A level maths. There would however be some important modifications and the content range and demand are not necessarily equivalent to their current, analogous, qualifications. These qualifications would typically take one year, although there could be a 'thin' two-year option, rather like the 'majors' and 'minors' in the proposed Advanced British Standard[219]. They would foreground a range of foundational and advanced mathematical concepts and skills and would build sequentially to levels appropriate for transition to undergraduate study of mathematically demanding subjects, including physics and engineering as well as mathematics itself.

To strengthen the development of general quantitative literacy we envisage a second set of qualifications, designed to develop and assess the ability to use and apply mathematical concepts, to use digital tools to address real-world quantitative problems and to develop students skills and techniques with data (see the Smith *et al* report for exemplification of this last point)[220].

These qualifications will resemble and build on the existing Core Maths qualifications. These qualifications are not simply 'applied maths', but develop the whole range of MDE skills, including critical thinking, problem formulation, communication and so on (see Appendix for sample examination questions). We think their assessment should be innovative and reflect more closely the ways in which people use general quantitative literacy. Students could start at any level as prerequisites are likely to be lower level foundational and advanced mathematics courses.

Many students would take a combination of FAM and GQL qualifications, either in parallel or sequentially. As for the FAM stream, the GQL options could be one- or two-year. We have in mind a set of levelled options. GQL level 3 would be similar to the existing Core Maths qualifications. GQL levels 1 and 2 are new and would need to be developed from scratch, although level 2 would have some similarity with the GCSE Use of Mathematics that was piloted from 2006 – 2010 (or the original functional maths qualifications from the same era)[221]. Level 2 would be valued in its own right, since it is predominantly assessing GQL, and different from the FAM that is foregrounded in FAM level 2. GQL level 4 would be a challenging qualification, drawing on FAM level 3 (ie equivalent to existing AS level maths.) A combination of FAM level 3 / GQL level 4 would be an excellent preparation for many MDE-demanding programmes in higher education.

219 Department for Education, 2023. A world-class education system: the Advanced British Standard consultation. See https://www.gov.uk/government/consultations/a-world-class-education-system-the-advanced-british-standard (accessed 30 July 2024).

220 *Op. cit.*, note 44.

221 Noyes A, 2011. Evaluating mathematics pathways: final report. See https://assets.publishing.service.gov.uk/media/5a7a40ff40f0b66a2fc0101e/DFE-RR143.pdf (accessed 30 July 2024).

The two strands of qualifications – prioritising FAM and GQL respectively – would allow for reduced content in 'maths' compared to the current A level (which includes statistics, modelling, problem solving, etc) and therefore create space for more advanced mathematics skills and thinking. FAM level 4 would provide a better foundation for transition to undergraduate mathematics than the current A level, with genuine and sustained engagement with, for example, proof[222].

As noted above, many of the components of domain-specific competences already exist in various stages of development, in vocational courses at all levels and in the recently revised A levels. The key issues here are transparency, coherence and coordination, and the need to reflect the broader remit of MDE. Vocational and subject teachers have a clear idea what MDE competences their students need to acquire. For them, the issue is to be confident that their students have the necessary mathematics, data and statistical skills that they need to build on. A well-designed qualifications structure would provide that confidence.

At key stages 4 and 5 we see opportunities for teachers from many disciplines to become involved, teaching both general quantitative literacy and domain-specific competences. With appropriate training the GQL qualifications could be taught by a wide range of teachers, not just by mathematics teachers. At present, these would be mathematically well-qualified subject specialists who have experience of using quantitative methods within their discipline and/or have confidence and skills with general quantitative literacy. Importantly, the design of any such GQL qualification should be collaborative, drawing on expertise from across the curriculum. This would, in turn, make teaching these more appealing to non-maths teachers.

RECOMMENDATION

Area for action 3: qualifications and assessment

Develop a single MDE qualifications framework which enables all students to continue to study MDE to 18. Design the framework around parallel and complementary foundational and advanced mathematics and general quantitative literacy strands, with recognition of domain-specific competences acquired in vocational and technical routes. Base the general quantitative literacy strand on redevelopment of the existing Core Maths qualifications.

222 Lyakhova S and Neate A, 2021. Further Mathematics, student choice and transition to university: part 2 – non-mathematics STEM degrees, Teaching Mathematics and its Applications, 40(3), (p210 – 233). See https://academic.oup.com/teamat/article/40/3/210/6180123 (accessed 30 July 2024).

We believe that an essential element of a successful MDE structure will be qualifications that reliably describe the competences of learners. This will be important if the idea of a portfolio of qualifications is to have real meaning.

5.6 Assessment

None of this can happen unless it is supported by a well-designed assessment system. Assessment plays an important and necessary role in school accountability. However, high-stakes assessments have been overemphasised in recent years, with damaging effects on learners caused by teaching to the test. As already noted, this was highlighted in the recent Ofsted subject report for mathematics[223]. The WYTIWYG adage still stands – What You Test Is What You Get – so our aspirations for a new kind of MDE need to be encouraged and supported by appropriate modes of assessment.

We have described how the design of the GCSE system means that a significant number of students are destined to fail GCSE mathematics and will leave school with no record of their achievement[224]. That GCSE mathematics is not criterion-referenced is also unhelpful for learners and other stakeholders. In practice, those attaining the highest grades demonstrate that they have mastered the majority of the curriculum, whilst those with the lowest grades have mastered very little of it. For those with the middling 'good grades' that are considered key to entering university and many professional and vocational roles, GCSE provides little information about what those learners can and cannot do. We believe that an essential element of a successful MDE structure will be qualifications that reliably describe the competences of learners. This will be important if the idea of a portfolio of qualifications is to have real meaning.

Ofsted itself has recognised the intrinsic weaknesses in GCSE mathematics as it is currently constituted. In its recent subject report, it recommends that: "The Department for Education, Ofqual and Awarding bodies should: .... explore whether the current design of the mathematics GCSE, including the tiers of entry offered and current typical grade thresholds, contribute to practices in schools that are not in pupils' best interests"[225]. We support this recommendation. Important lessons can be learned from previous projects that have developed methods of criterion-referenced assessment in mathematics, for example, the Graded Assessment in Mathematics (GAIM)[226] project developed at King's College London, and we suggest that these should be revisited.

RECOMMENDATION

Area for action 3: qualifications and assessment

Develop assessment methods that identify and communicate what students know and can do.

223 *Op. cit.*, note 117.

224 We have not made a recommendation about GCSE resits despite the many serious concerns about this policy (see, for example, the recent paper from MEI: https://mei.org.uk/app/uploads/2024/07/Addressing-GCSE-maths-resit-failure-v3.pdf). Our focus is on the future whereas this is an issue that clearly warrants attention now. The Royal Society Advisory Committee on Mathematics Education's previous work, *GCSE Mathematics resits,* offers suggestions and ways forward that align with the spirit of this report. In the longer term we believe the wider range of recommendations made within this report would be the best way to address the problem.

225 *Op. cit.*, note 117.

226 STEM Learning, 1999. Graded assessment in mathematics (GAIM). See https://www.stem.org.uk/cx4qw (accessed 30 July 2024).

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Chapter six

The way forward

Left
A Global Positioning System (GPS) enabled smartphone knows your location within about 5 metres. Connected to a network of satellites, GPS is underpinned by mathematical methods. © iStock.com / Miguel Angel Partido Garcia.

The way forward

6.1 Next steps

The reforms we seek cannot be developed by limited short-term measures; they will take 10 – 15 years to implement fully. They will need serious investment and careful planning, design, implementation, and evaluation. They will require collaboration between the stakeholders involved, cross-party support and determination to stay the course. The responses to our stakeholder consultation strongly supported this view. At the same time respondents supported the fundamental premise of this report; that the direction and shape of the long-term changes that are needed are already clear, that there are significant risks in delay and that the process should begin as soon as possible.

RECOMMENDATION

Area for action 1: implementation

The government should sponsor an independent task force to plan for long-term system changes and implement recommendations from this report. This task force should include relevant government departments such as Department for Education, Department for Science, Innovation and Technology, Department for Business and Trade, and the Treasury. It should also involve senior figures from key stakeholder bodies and consult with devolved nations. A sufficient budget should be provided to commission exploratory and developmental projects.

The role of the task force would be to begin the process of long-term planning of developments in curriculum, qualifications, assessment, and teacher recruitment and support, and to commission the developmental and exploratory projects suggested in our recommendations. The report we commissioned from Sheffield Hallam University demonstrates how many educational policy initiatives, both in the UK and in other countries, have not lived up to their promise because they have focused on a single issue and have not considered sufficiently the interrelationships with other parts of the complex educational jigsaw.

We suggest that the task force should learn from this history and draw on the experience of past large-scale educational developments, both in the UK and in other countries. Particular attention should be paid to piloting new initiatives before wider changes are made, to sequencing of professional development and to evaluation[227] (see Box 8).

Many of the changes we seek can be achieved within existing educational frameworks. Nevertheless, like all significant educational changes they are complex and involve many interlocking parts. If they are to succeed, they will need careful strategic planning and systemic alignment between a number of educational and other stakeholders (including employers, parents and students themselves). We suggest that at an early stage the group should recruit groups of expert teachers, senior curriculum leaders and strategic thinkers to advise and support the planning and the exploratory projects. These teachers will in due course form an important element of the training force that will be needed.

227 *Op. cit.*, note 43.

BOX 8

Features associated with successful large-scale educational policy developments[228]

Purpose
Clear vision of policy purpose, rooted in an articulated purpose and nature of mathematics education.

Consensus
Through dialogue that involves stakeholders, particularly around ensuring concerns are addressed about the role of basic mathematical skills in relation to mathematical thinking.

Feasibility
This is particularly relevant to transnational policy movements – what is often referred to as policy borrowing or contextualised adaptation where feasibility might be appropriate in each context.

Coherence
Multiple simultaneous innovations can lead to a lack of coherence between innovations in mathematics education and other educational policies, or between mathematical educational innovations.

Systemic alignment
Alignment has two aspects:

- between curriculum, pedagogy, assessment, and teacher professional development and how lack of such alignment can stifle innovation;
- with wider system issues such as teacher professional conditions, accountability measures and marketisation.

Piloting and sequencing
Piloting an initiative, depending on scale and governance structures, before wider changes. Professional development is taking place in parallel with or even leading changes to curriculum, pedagogy, qualification, and assessment.

Sustained attention
High-performing systems tend to have longer policy cycles in recognition that change takes time.

Collaboration and relationships
This relates to the importance of creating opportunities for dialogue between policymakers, mathematics education researchers, teachers, and other stakeholders. There is a growing acknowledgement that recognising teachers as important actors in policy development and enactment brings multiple benefits, including greater ownership of policy.

228 Adams G and Boylan M. Sheffield Hallam University. Landscaping Mathematics Education Policy: Horizon scanning of international policy initiatives, 2023, p33 – 34. See https://royalsociety.org/-/media/policy/projects/maths-futures/landscaping-international-mathematics-education-policy.pdf (accessed 30 July 2024)

Drawing on advice received during our consultation process we suggest that at an early stage the task force should commission an independent expert body to begin the task of planning the curriculum and qualifications developments that will be needed. We also suggest that it should consider the role that supporting national strategies, in particular for educational technology and for teacher professional development and recruitment, will play, as possible enablers or barriers.

6.2 Investment in teacher professionalism

The reforms recommended in this report cannot be achieved by decree, nor can they be implemented simply through manipulating curriculum structure, content or changes to the nature of the qualifications. As the US educational reformer, Ted Sizer, has said: "Things remain the same because it is impossible to change very much without changing most of everything."[229]. Based on the widely held consensus that change is needed, the 'no regrets' starting position lies with drawing on and enhancing teacher professionalism.

UK reformer Sir Michael Barber argues that the quality of an education system cannot exceed the quality of its teachers[230]. Reform to mathematical education is therefore reliant on investment – financially, politically and culturally in teacher professionalism. The long-term plans for MDE will depend heavily on a positively disposed teaching workforce across all phases and subjects, with sufficient agency to embrace the undoubted challenge that reform will bring.

A challenge lies in how to overturn the general sense that all or most educational policy initiatives are a drain on teachers' already stretched professional, physical and emotional reserves. A new approach to mathematical and data education will require coopting a steadily evolving cadre of primary / secondary / FE, science / social science / humanities and, yes, maths teachers, to embrace change as a positive. Not as passive recipients of change but as co-creators of a shared, more relevant, ambitious and exciting array of learning experiences, both for students and their teachers too.

In 2023, Ofsted reported that the overall picture of mathematics education in England was broadly healthy, stating how this did not arise through chance but was (in part) a consequence of the many teachers in receipt of high quality, subject-specific Professional Development[231]. MDE's novel approach will require significant upfront professional learning both for traditional maths specialists and non-maths teachers alike. Reflecting the cross-disciplinary nature of its implementation and the challenge for learning in a rapidly evolving subject, MDE will require dedicated teacher professional learning from initial training, in early career and beyond, including through an expansion of the flourishing formal and informal networks of teaching professionals praised in the Ofsted report.

229 Sizer T, 1983. Phi Delta Kappan, June edition (p674).

230 Barber M & Mourshed M, 2007. How the world's best-performing school systems come out on top. See https://www.mckinsey.com/industries/education/our-insights/how-the-worlds-best-performing-school-systems-come-out-on-top (accessed 30 July 2024).

231 *Op. cit.*, note 117.

The additional financial investment in teachers of MDE should be considered in the context of the longer-term economic gains and the further benefits of ensuring that all future citizens have experienced mathematical and data education that is sufficiently challenging, engaging and valued.

RECOMMENDATION

Area for action 5: teachers

The government should prioritise making funding available over several years to support a major programme of professional development, including initial teacher training, early careers training and continuous professional development to support the implementation of MDE. This should be designed into the implementation plan from the outset and sustained over time.

6.3 Timetable

Our task has been to look to the future and to take a long view. The world of education is large and complex. It involves many interconnected strands and long term change requires planned and coordinated progress on four fronts: curriculum, qualifications, resources and professional development[232]. The history of educational development, here and in other countries, shows that unless attention is paid to all four strands failure is likely[233]. In devising our recommendations we have been very aware of this, and of the way in which they depend on and feed into each other. Except for the first recommendation none of them is free standing.

Since we began our work the appetite for change has developed remarkably. The previous Prime Minister's announcement in January 2023 about mathematics for all to 18 was followed by the plan for the Advanced British Standard. Since then, the Labour government, elected in July 2024, announced in the same month its intention to undertake a wide-ranging review of curriculum and assessment. Alongside all this is awareness that developments in AI are going to have serious consequences for education, and mathematics education in particular, and that these need consideration rather urgently. Meanwhile the response to our discussion paper last autumn revealed strong support for the direction of change we are advocating, along with a desire to begin the process quickly.

It is, in short, time to seize the moment. Our prime recommendation is that government should establish an independent task force, with a brief to plan for the long-term system changes that will be required and the means to take forward the detailed recommendations set out in this report. With a change of government and the opportunity for new thinking that presents, we think this should be set up by early 2025. One of the task force's first jobs will be to begin the detailed planning, taking into account the co-dependencies we have described. It is not our role to pre-empt that planning, but we expect that detailed development could begin within a year after it begins working, with work starting on some of our recommendations within six months.

232 Oates T, 2010. Could do better: Using international comparisons to refine the National Curriculum in England. See https://www.cambridgeassessment.org.uk/images/112281-could-do-better-using-international-comparisons-to-refine-the-national-curriculum-in-england.pdf (accessed 30 July 2024).

233 *Op. cit.*, note 43.

PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES

Appendices

Left
MRI scanning uses advanced mathematics to allow medical experts to understand the workings of the brain and to identify potential problems. © iStock.com / semnic.

APPENDIX 1

About the programme

The Mathematical Futures programme was launched in February 2020. It set out to answer three core questions:

- What mathematical competencies will be needed for citizens and society to thrive in the future?
- How should education systems develop these mathematical competencies?
- What changes should be put in place to move towards that future?

The programme was overseen by the Mathematical Futures Board and the Royal Society's Advisory Committee on Mathematics Education. The Board met 19 times during the project.

The programme was supported through donor funding from three industry partners: ARM, Google and GSK, and received ongoing support from the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society. We gratefully acknowledge their support.

Scope

The purpose of the Mathematical Futures programme is to look ahead. Our focus has been on future needs and how they might be met, rather than the reform of existing structures.

Our report focusses on learners from the ages of 4 – 19, although the many destinations of learners after formal education and training, whether into higher, vocational and technical education and employment, are a crucial part of the picture.

The scope of the programme and of this report is broad. Mathematics is a multifaceted subject which touches on all aspects of everyday life and work. Likewise, the educational structures in which learning occurs are large and complex.

Our focus on the big picture means that there are a number of important areas where our discussion is necessarily limited. Among these are Special Educational Needs and Disabilities (SEND), Adult Numeracy, gender differences and the myriad issues that arise from socioeconomic disadvantage.

Education is a devolved matter in the UK, and the four nations run their own systems. While much of the focus of Chapter 2 is on the English system we understand there are significant differences across the nations, particularly in Scotland. We believe that the arguments in the report have general validity and will apply across all the nations of the UK (and indeed more widely). We believe we have much to learn from each other and hope that the report will act as a spur to constructive discussions.

APPENDIX 2

Methodology

Call for views

An initial call for views ran from October 2020 to January 2021. We received 191 responses largely from across the mathematics, computing, mathematics-user and mathematics education communities, and workshop responses from three cross-stakeholder validating webinars (nine focus groups) held in April 2021.

Commissioned research

The Board commissioned six research reports, as detailed below. The papers are available at **royalsociety.org/mathematical-futures**

Title	Authors
An evidence review on the changing nature and importance of mathematics in the 21st century.	Henry Lane and Andy Martin, Firetail
Educational Technologies in Mathematics Education.	Dr Cosette Crisan, Dr Eirini Geraniou and Professor Jeremy Hodgen, Institute of Education, University College London.
Scenarios for the future of mathematics.	Henry Lane and Andy Martin, Firetail.
Mathematical and Data Literacy: Competences and curriculum implications at the intersection of mathematics, data science, statistics and computing.	Professor Cathy Smith, Vinay Kathotia, Dr Robert Ward-Penny, Oli Howson and Dr Michel Wermelinger, The Open University.
Landscaping Mathematics Education Policy: Horizon scanning of international policy initiatives.	Dr Gill Adams and Professor Mark Boylan, Sheffield Hallam University.
Landscaping national mathematics education policy.	Professor Mark Boylan, Dr Gill Adams and Amy Birkhead, Sheffield Hallam University.

Discussion paper and consultation

In September 2023 the Programme published a discussion paper which set out an outline of the thinking to date. The publication of the paper marked the beginning of a period of consultation that ran until the end of the year. The paper was widely circulated: it was available via the Royal Society website and was directly distributed to over 1,000 individuals from a wide range of organisations and sectors. It was also shared widely on social media.

The paper invited readers to respond to the following questions:

1. Do you support our vision for the future of mathematics and data education?
2. Is the vision appropriate for all students?
3. Are there any areas of our vision that need further development?
4. What are the first steps needed to begin the process of change?

We received 58 written responses and held follow-up meetings involving many of the organisations listed in the acknowledgements in Appendix 4. The responses were, without exception, thoughtful and considered, and we thank the many people who took part for the time and effort they took to respond.

APPENDIX 3

Examples of Core Maths examination questions

Example 1[234]

The table shows some information from a survey of 500 people about the waiting time, in days, between making an appointment to see a doctor at a health centre and the date of the appointment.

Waiting time (t days)	Frequency
$0 \leq t < 1$	124
$1 \leq t < 7$	192
$7 \leq t < 14$	112
$14 \leq t < 28$	56
$28 \leq t$	16

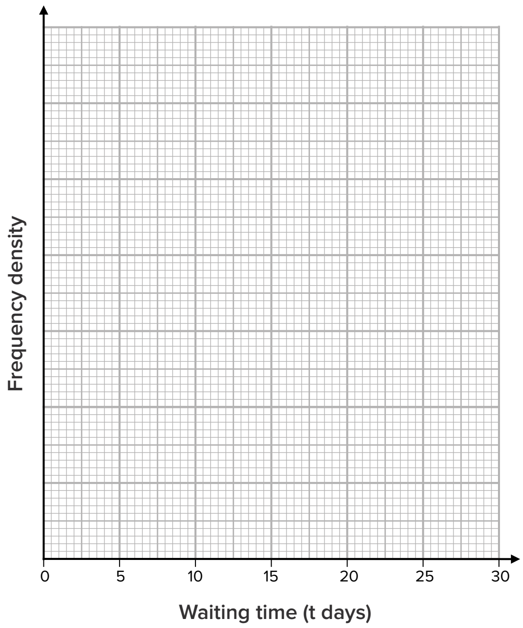
An additional 80 people make an appointment to see a doctor at the health centre.

a. Using the information in the table, work out an estimate for the number of the additional 80 people who will have a waiting time of less than one day.

16 of the 500 people who completed the survey waited at least 28 days between making an appointment and the date of the appointment.

b. Explain why the interval $28 \leqslant t$ used to represent these people cannot be shown on the histogram.

c. On the grid below, draw a histogram to represent the information for all the people who have a waiting time of less than 28 days.



A newspaper reports that the average waiting time in the UK between making an appointment to see a doctor and the date of the appointment has recently exceeded 2 weeks for the first time.

d. By finding an estimate for the median, compare the waiting time at the health centre with the average waiting time in the UK.

The health centre aims to provide people with an appointment with a waiting time of less than 10 days.

e. Work out an estimate for the number of people, from the 500 people surveyed, who had a waiting time of less than 10 days.

234 Pearson Edexcel Core Maths. Mathematics in Context sample question, 2022. See https://qualifications.pearson.com/content/dam/pdf/mathematics-in-context/Mathematics/2014/Exam-materials/7mc0-02-que-20220615.pdf (accessed 10 April 2024).

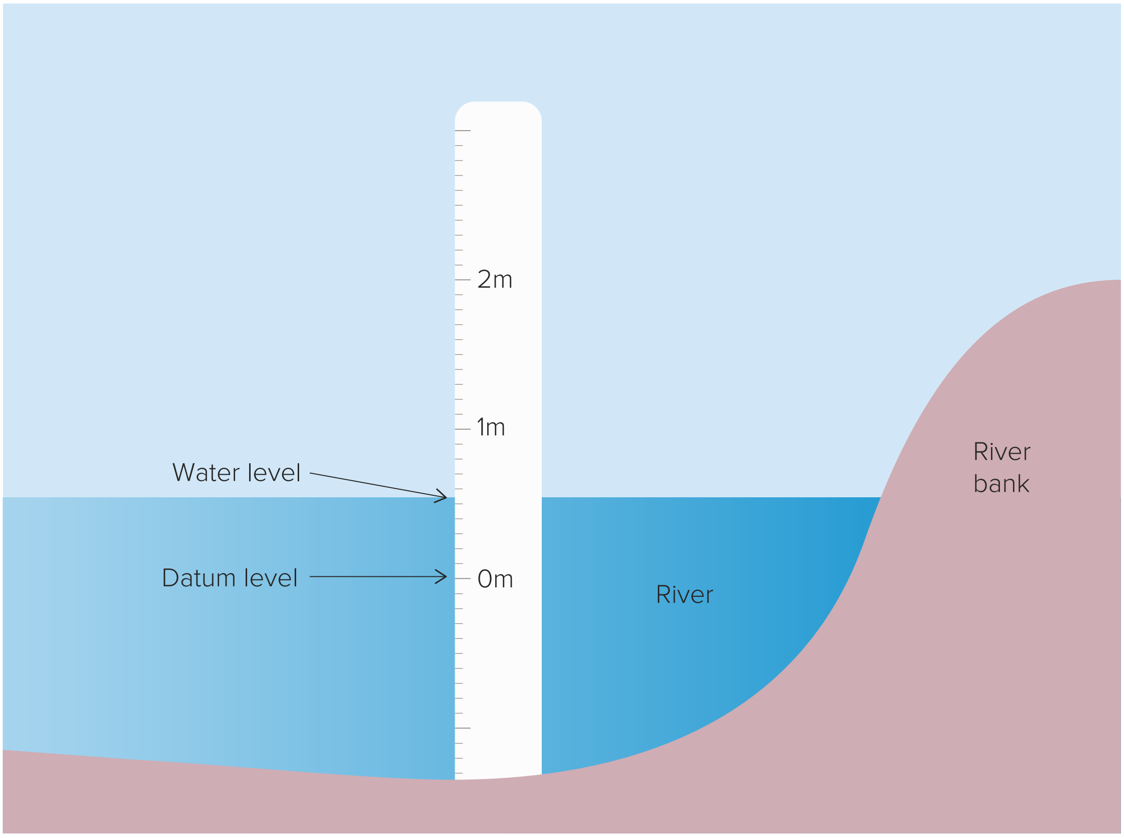
Example 2[235]

The residents of a small town find the cost of their house insurance has gone up. The insurance companies say that their risk of flooding is high. A river flows through the town. If the river level rises by more than 2 metres above a given datum level, the town will flood.

The residents decide to investigate the situation so that either they can refute the insurance companies' argument, or they can claim funding for flood defences.

The town's archives have records going back 120 years giving the greatest height of the water above the datum level each year. The mean of these heights is 0.61m with standard deviation 0.48m. One of the residents tries using the Normal distribution to model this situation.

Show that using the Normal distribution as a model suggests that the flood risk in this town means that a flood can be described as a "Once in 500 years event".



235 OCR Level 3 Certificate in Core Maths B (MEI), Statistical Problem Solving, sample paper 2020. See https://www.ocr.org.uk/Images/174104-unit-h869-02-statistical-problem-solving-sample-assessment-material.pdf (accessed 30 July 2024).

APPENDIX 4

Acknowledgements

Mathematical Futures Board members

The members of the Mathematical Futures Board acted in an individual and not a representative capacity and declared any potential conflicts of interest.

Board members
Chair: Sir Martin Taylor FRS, Professor of Mathematics and former Physical Secretary and Vice-President of the Royal Society.
Vice-Chair: Anthony Tomei CBE, past Director, Nuffield Foundation, and member of the Royal Society Advisory Committee on Mathematics Education.
Professor Miles Berry, Professor of Computing Education, University of Roehampton and Chair, NCCE Academic Board.
Professor Andrew Blake FREng FRS, Consultant in AI and Vice-President, Clare Hall, University of Cambridge.
Karen Giles (until Summer 2022), Headteacher, Barham Primary School (London).
Nuno Guarda (until March 2022), Head of Corporate Affairs UK & Ireland, Cisco Systems.
Graham Keniston-Cooper, Chair of Development Board, Isaac Newton Institute and Trustee, National Numeracy.
Lynne McClure OBE, Head of Mathematics Solutions, Cambridge Partnership for Education; Trustee and Education Lead, Academy for Mathematical Sciences; and Trustee, National Numeracy and MathsworldUK.
Professor Andrew Noyes, Professor of Education and founding Director of the Observatory for Mathematical Education, University of Nottingham, and Chair of the Joint Mathematical Council (JMC).
Dr Vanessa Pittard, Deputy Chief Executive, MEI (Mathematics in Education and Industry).
Sir David Spiegelhalter OBE FRS, Emeritus Professor of Statistics, Centre for Mathematical Sciences, University of Cambridge.

Royal Society staff

Many staff at the Royal Society contributed to the production of this report.
The programme team is listed below.

Royal Society Secretariat
Catherine Boulton, Senior Policy Adviser
Peter Finegold, Head of Education Policy
Dr Helen Harth, Senior Policy Adviser (until July 2021)
Abigail Harvey, Programme Co-ordinator (until June 2022)
Alice Kwan, Programme Co-ordinator (from September 2022)
Dr Rupert Lewis, Chief Science Policy Officer
David Montagu, Senior Policy Adviser
Sam Murphy, Mathematical Futures Programme Manager (from November 2022)

Review group

This report was reviewed by a panel of experts, before being approved by the Officers of The Royal Society. The review group members were not asked to endorse the conclusions or recommendations of the report, but to act as independent referees of its technical content and presentation. Members acted in a personal and not a representative capacity and were asked to declare any potential conflicts of interest. The Royal Society gratefully acknowledges the contribution of the reviewers.

Review group
Professor Jane Clarke FMedSci FRS, Professor of Molecular Biophysics and President of Wolfson College, University of Cambridge.
Professor Alison Etheridge OBE FRS, Professor of Probability, University of Oxford and President, Academy for Mathematical Sciences.
Dame Alison Peacock DL DLitt, Chief Executive, Chartered College of Teaching.
Professor Simon Peyton Jones OBE FRS, Epic Games, and Chair of Computing at School.

Responses to the consultation in autumn 2023 were received from the following organisations:
(Individuals without affiliation to an organisation have not been listed)

Academy for the Social Sciences

Association for Citizenship Teaching

Association of Colleges

Association of Mathematics Education Teachers (AMET)

Association of Teachers of Mathematics, Functional Programming and Computer Algebra Working Group

Association of Teachers of Mathematics/ Mathematical Association, Joint Primary Group

Bishop Challoner Catholic College

Bill & Melinda Gates Foundation

British Association for Early Childhood Education (Early Education)

BCS, the Chartered Institute for IT

Cabinet Office, National Security and intelligence

Cambridge Mathematics

Cambridge University Press and Assessments

Carmel College

Casio Education

Chartered College of Teaching

Council for Subject Associations

Cygnus Education

Department for Education

Early Childhood Mathematics Group

Education Endowment Foundation (EEF)

Education Policy Institute

Institute of Analytics

Institute of Directors

Institute of Education, University College London

Institute of Physics

Institute of Mathematics and its Applications

Institution of Chemical Engineers

Institution of Mechanical Engineers

Intelligent Plant

Joint Mathematical Council

King's College London

Kingston University

London Mathematical Society

Maths4Girls

Maths Anxiety Trust

Maths Inspiration

MathsWorld UK

MEI (Mathematics in Education and Industry)

Members of the Maths to 18 expert advisory group

National Foundation for Educational Research (NFER)

National Numeracy Leadership Council

NCETM

NRICH, University of Cambridge

Nottingham Trent University

Nuffield Foundation

OCR

Office for Statistics Regulation

Ofsted

Operational Research Society

Pearson

Qinetiq

Royal Academy of Engineering

Royal Aeronautical Society

Royal Geographical Society

Royal Statistical Society

Royal Society Advisory Committee on Mathematics Education (RS ACME) expert panels

Science Education Policy Alliance (SEPA)

Seedlings Preschool, Woking

The Pensions Network

Scottish Mathematical Council

STEM Learning Ltd

Universities' Council for the Education of Teachers (UCET)

University of Birmingham

University of Derby

University of Edinburgh

University of Glasgow

University of Hertfordshire

University of Liverpool

University of Southampton

The Royal Society is a self-governing Fellowship of many of the world's most distinguished scientists drawn from all areas of science, engineering, and medicine. The Society's fundamental purpose, as it has been since its foundation in 1660, is to recognise, promote, and support excellence in science and to encourage the development and use of science for the benefit of humanity.

The Society's strategic priorities emphasise its commitment to the highest quality science, to curiosity-driven research, and to the development and use of science for the benefit of society. These priorities are:

- The Fellowship, Foreign Membership and beyond
- Influencing
- Research system and culture
- Science and society
- Corporate and governance

For further information
The Royal Society
6 – 9 Carlton House Terrace
London SW1Y 5AG

T +44 20 7451 2500
W royalsociety.org

Registered Charity No 207043

9 781782 527282

ISBN: 978-1-78252-728-2
Issued: September 2024 DES8939