Teaching of Calculation in Primary Schools

A report by HMI

April 2002

HMI 461

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Background

1. Since the introduction of the National Numeracy Strategy (NNS) in 1999, Her Majesty's Inspectors (HMI) and Additional Inspectors (AIs) have carried out a series of annual visits to a nationally representative sample of primary schools to evaluate the impact of the strategy on the quality of teaching in mathematics. There were 300 schools in the sample in the first year of the implementation of the strategy, reduced to 200 in the second year.

2. In the spring and summer terms of 2001, inspectors gave particular attention in 68 schools to three important themes in primary mathematics: progression from mental to written calculation from Year 1 to Year 6; the solving of word problems; and the use of calculators in Years 5 and 6. These visits included the observation of the daily mathematics lesson; the scrutiny of pupils' work and, in 24 schools, discussions with groups of four pupils. The discussions focused on how pupils calculated using the four operations of number and on their approaches to problem-solving. This paper draws upon those focused visits, as well as on evidence from the visits to the other schools in the national sample.

Introduction

3. Schools have made significant progress in the past two years in teaching mathematics and, in particular, in teaching pupils how to calculate mentally. There is still more to do, however, and this report highlights aspects of calculation and the methods of teaching it that need further improvement.

Main Findings

- Teachers give appropriate emphasis to pupils' recall of number facts and mental methods of calculation during oral and mental sessions. However, at Key Stage 2 they often overlook the importance of linking pupils' mental strategies to the introduction of expanded and compact written methods.
- In the majority of schools, pupils are taught horizontal and expanded forms of calculation leading towards the use of a standard written form.
- Pupils make too little use of personal jottings to support and record mental strategies or explain their methods of calculation.
- Teachers rely too much on worksheets and commercial schemes, particularly in Key Stage 1, limits opportunities for pupils to develop and use their own methods of recording.
- Whilst teachers recognise the importance of estimating before calculating and teach this to pupils, pupils rarely use it in practice. As a result, they do not always have a clear idea of what constitutes a sensible answer.

- Pupils do not always make appropriate decisions about when to use written methods and when it is more sensible to work things out in their head.
- Insufficient attention is given to helping pupils acquire a range of strategies to solve word problems, particularly more complex problems where two or more steps are needed to solve them.
- Errors or misconceptions evident in pupils' recording of calculations are not always tackled in the main body of the lesson or in the plenary.
- Schools are recognising, increasingly, the importance of adopting a common approach to the recording and layout of pupils' work, but much remains to be done to put policies into practice.
- There is not enough good use of calculators, either as a teaching tool or by the pupils themselves, in the daily mathematics lesson at Key Stage 2.

Progression from Mental to Written Methods of Calculation

4. The link between the teaching of mental skills and that of written calculations, as well as the development of pupils' ability to explain orally their methods of working, is crucial in ensuring that pupils move progressively from informal to more formal methods of recording in mathematics. In order to ensure that pupils make the move smoothly, teachers need:

- a good understanding of mathematical progression and the ability to match this to the various stages and rates of pupils' understanding (the NNS *Framework* outlines the various stages clearly)
- a clear understanding of how pupils can use jottings, expanded methods and standard written forms to support, explain and record their work in mathematics
- to be clear about the difference between mental recall and mental calculation and the implications of these differences for teaching and learning.

5. Teachers' knowledge and understanding of these factors have a strong influence on the quality of teaching. In the best teaching, expanded written methods and standard written forms are used as aids to pupils' understanding and not solely as a means of recording for someone else. Effective teaching ensures that pupils' written responses derive from their knowledge of mental strategies of calculation and from their skills in being able to explain orally how they have reached their answers.

Key Stage 1

6. In **Year 1,** teachers emphasise appropriately the vital oral and mental skills which underpin pupils' written recording. In particular, they focus on counting, ordering numbers and ensuring that pupils have instant recall of number bonds to

10. Pupils know that addition is adding, making bigger and so on and that subtraction is taking away, although they do not always understand the term ‘difference’.

7. A positive feature at this stage is the wide and varied range of approaches to the teaching of counting. This includes counting forwards and backwards from different starting-points, with individual pupils and with groups. One pupil, when asked to order the numbers 10, 5 and 9, used his knowledge of counting facts to explain, ‘I can count it. Five comes before nine, then it’s ten’. Another pupil made use of his counting knowledge to count on in order to answer 13 + 6 = 19. Much of this understanding is enhanced by teachers’ systematic teaching of number facts and strategies to support pupils’ calculations. The good use of questioning encourages pupils to explain their thinking, whilst effective assessment enables teachers to help them to move forward in their learning.

8. Whilst pupils record accurately using conventional signs and symbols, the use of recording as an aid to calculation is a weakness, especially with larger numbers. Worksheets which provide no space for pupils’ personal jottings or expanded methods of recording, or which simply require them to write answers in the boxes provided, are unhelpful. As a result, pupils have too few opportunities to jot down or illustrate their number work in a variety of ways. Teachers see commercial schemes as support for providing for progression but, in some instances, the worksheets limit pupils’ opportunities to record their mathematical thinking in a manner which make sense for them and which might support their oral explanations later.

9. In **Year 2**, teachers continue to emphasise pupils’ oral and mental skills. As a result, most pupils are able to count to 100 and know addition and subtraction facts to 10 and beyond. They understand the commutative law for addition and this enables them, for example, to rearrange numbers to help them. For example, when adding 9 + 7 + 1, they can rearrange the order of the numbers to 9 + 1 + 7 to make 10 + 7. By contrast, fewer pupils are clear about the inverse relationship between addition and subtraction and are not, for example, able to see that if 12 + 6 = 18, then 18 - 6 = 12. Most pupils are able to carry out multiplication as repeated addition in a formulaic way, but they do not readily see the link between, for example, 2 + 2 + 2 + 2 + 2 = 10 and 2 x 5 = 10.

10. As in Year 1, pupils do not have enough opportunities to practise recording. Few see the use of jottings to record their thoughts as useful aids to calculation. Errors and misconceptions evident in pupils’ recording are not always addressed, either in the main body of the lesson or the plenary. For example, in response to the question, ‘What is double 3?’ a pupil recorded 2 x 2 x 2 = 6, but this misconception was not challenged by the teacher. In one or two lessons seen, pupils were encouraged to write their explanations as a way of supporting their mathematical thinking. For example, ‘I made the plasticine weigh 10 cubes. When I changed the shape of the plasticine, the weight stayed the same’.

11. The rapid recall of number facts receives a high priority, not only for its own sake and to support quick mental calculations, but also to enable pupils to derive new facts from what they already know. For example, in order to answer the question 4 + 5, one pupil used his knowledge of doubling and explained, ‘I know five and five equals ten, so I take one away and that gives me nine’. The use of probing questions encourages pupils to reflect on their responses and the good use of

individual whiteboards in some schools is effective in supporting pupils' recording of number facts. The use of jottings to help pupils with their mental calculations is not taught frequently enough.

12. The demonstration by teachers of ways of recording a calculation is a positive feature as well as, in some lessons, the linking of horizontal and vertical layout via partitioning. However, too few schools have an agreed approach to pupils' recording and layout of their work. When this is established successfully, it supports pupils in organising their work in mathematics and avoids careless errors, for instance in place value, which are caused by poor recording.

13. Teachers rely too much on worksheets or commercial schemes continues in Year 2. Many of these simply require an answer (often in a box) or some colouring-in. No space is given for pupils' jottings or for them to show how they worked out the answer. Linked to this is an emphasis on recording at the expense of the explanation of methods. Pupils rarely use informal methods of recording or jottings to support and explain mental addition and subtraction of numbers to 100.

Key Stage 2

14. In **Year 3**, the mental and oral skills needed for all types of calculation are generally taught well. As a result, pupils are able to read, write and order numbers. They are confident in aspects such as partitioning, counting and recalling and using number facts, but are unsure about the inverse relationship between multiplication and division. Pupils are less confident when applying their oral and mental skills to solve word problems and continue to be reluctant to use jottings to help with their calculations.

15. The use of individual whiteboards to record pupils' responses, and teachers' demonstration of recording and written methods, continue to be positive features in Year 3. Teachers use pupils' work effectively as a focus for learning through questioning with the whole class. Another positive feature is the extent to which many teachers make links across the different areas of number, such as between measurement and number. For instance, the numbers in a clock-face were used in one lesson in the calculation and recording of number bonds to twenty. There were also examples of teachers demonstrating a bridge between recording in horizontal and vertical layouts. For example, pupils were presented with 68 + 79 in a horizontal layout and shown how to use partitioning to do the calculation using the vertical layout below:

68 + 79
60 + 70 = 130
8 + 9 = 17
147

16. By the start of **Year 4** in the majority of schools, pupils are introduced to a standard, vertical method of calculation for addition. They progress readily to column addition using their knowledge of partitioning by adding the tens and units separately. Subsequently, they tackle harder questions involving 'carrying' and larger numbers using these familiar methods as a natural, and mostly successful,

progression. They use estimation prior to calculation only infrequently, however. This is a mental operation which pupils understand in theory, but rarely use as a matter of course as part of the process of calculation.

17. During Year 4, pupils draw upon a sound understanding of partitioning to add or subtract two 2-digit numbers. For example, in calculating 64 - 32, pupils are able to subtract 30 from 60 and, separately, take 2 from 4 to arrive at the correct answer. Where this mental approach proves too difficult, pupils occasionally make good use of an empty number line for personal jottings to show the separate steps of the calculation. However, too few pupils use an empty number line to support such calculations or to tackle more challenging numbers.

18. Some pupils in Year 4 continue to record calculations horizontally as they move towards establishing vertical methods. Occasionally, mental methods are overlooked, even when they may be the most appropriate strategy for use. For instance, when asked to calculate 82 - 43, a Year 4 pupil chose a written method, but arrived at the incorrect answer of 41. When challenged, he was able to work out the correct answer mentally. Although pupils are taught an expanded form of addition in Year 3, this is not the case for subtraction in Year 4: there is little evidence of the expanded form for subtraction through decomposition being used.

19. Many **Year 5** pupils are able to use decimal notation correctly. They can represent mixed numbers such as three and three tenths or six and nine tenths as improper fractions and express these as decimal fractions. The application of pupils' knowledge of place value where mixed numbers are concerned, and their accuracy in setting out operations involving decimal numbers, however, are less secure. For example, pupils are able to identify the values of the digits in a decimal number such as 0.75, but often become confused about the position of the decimal point when asked to calculate 0.75 x 6.

20. Pupils tend to use vertical methods of addition and subtraction, even when horizontal recording would enable them to use partitioning and their recall of simpler number facts to work things out more quickly in their heads. One child, who recorded her calculation of 125 + 205 horizontally, said she had visualised it vertically in order to reach the answer; when encouraged, she could work it out in her head, using her knowledge of partitioning. As in Year 4, an empty number line for a personal jotting is used only infrequently.

21. In one school, pupils drew upon their established mental methods successfully when calculating 25 x 17:

One child used the knowledge that:		4 x 25 = 100;
	then,	16 lots of 25 make 400; and
	finally,	adding another 25 gave the answer of 425.

22. Where pupils' mental methods are less secure, some are unable to use the expanded calculations properly. Similarly, the errors in the use of partitioning to support short (TU x U) and long (TU x TU) multiplications are due to an insecure knowledge of number facts.

23. In **Year 6**, pupils use horizontal methods with decimal calculations before extending the written format to column addition and subtraction of decimals. Most pupils use this standard written method reliably, although some make the common mistake of failing to keep decimal points and digits of similar place value in line. Pupils tackle short multiplication of numbers involving decimals with mixed success, the common error being the failure to 'carry' in a calculation, 3.5 x 5 for example. In one lesson, a Year 6 pupil made good use of prior knowledge and mental methods – knowledge of 3.5 x 10 then halving – to reach the correct answer. Such a choice of strategy, however, is rare.

24. Work on the short division of numbers involving decimals is limited although, where it happens, pupils know how to calculate to one decimal place and recognise recurring decimals. The use of decimal notation becomes more firmly established in Year 6. The ordering of mixed sets of numbers and the consolidation of pupils' knowledge of place value are taught well. For instance, when questioned, pupils could record two and three-place decimals confidently and place them in the correct order, working from the smallest or the largest.

25. By Year 6, pupils use standard vertical methods for all number operations. When these are developed from other informal written methods such as empty number lines or grid methods, they generally understand such methods well and are able to follow a process carefully when it is explained by the teacher. However, pupils still tend to rely too much on written methods even when they are able to reach a correct answer in their heads.

Problem-solving

26. Pupils in Years 1 to 3 usually solve one-step problems mentally. They identify correctly which operation to use, draw upon their knowledge of number bonds and multiplication facts and are able to explain their reasoning clearly. Pupils who record the processes they have gone through invariably use the horizontal format. For example, in Year 3, in response to the question: *A spider has 8 legs. How many legs do 5 spiders have?* pupils who recorded their thinking wrote 8 x 5 = 40.

27. Pupils find word problems involving two or more steps more difficult. Most pupils are unable to identify the key information or questions involved. Few pupils approach such problems systematically or attempt to record their calculations through personal jottings. By contrast, Year 3 pupils used successfully a variety of methods to solve the problem: *A set of felt-tips costs £3. Marie saves 20p a week. How many weeks will she take to save up for the felt-tips?* One pupil calculated that it would take 5 weeks to save £1 (5 x 20p), thus 15 weeks to save £3 (5 weeks x 3). Another wrote: 20, 40, 60, 80, 100 = £1, 300 = £3, 5 x 3 = 15 weeks. While they approach the problem logically and record the information systematically, pupils often omit to note the units of measurement involved. A good example of a useful personal jotting, but one which omitted the units, was when one boy wrote: '5 is one pound and 5 x 3 = 15'.

28. By Year 6, many pupils know and use a systematic approach to solving word problems and teachers often provide pupils with good strategies for tackling them. For example, Year 6 pupils were asked how to: *find the number of marbles in only*

three bags when 960 are divided equally into 16 bags. They recognised the task as a two-step problem and were able to identify the steps to find an answer. One school used a problem frame with pupils to help them to analyse what type of word problem it might be and how to solve it. The teacher highlighted the key words and numbers with the pupils and then considered how the problem and the solution could be written mathematically.

29. Although systematic approaches to problem-solving are taught, too little attention is given to encouraging pupils to predict a sensible answer, using personal jottings and drawing upon mental strategies as good starting-points. Estimation is not a sufficiently established feature of pupils' approaches to problem-solving, nor is the use of checking to decide if the answer arrived at is a sensible one. As a result, pupils rely too much on the mechanical process of written calculation. In some cases, they are confident enough to work out word problems in their heads, but few use an appropriate combination of mental skills, personal jottings and standard written methods.

30. Pupils' anxiety to solve the problem and 'get it right' often leads them to use immediately what they feel is the security of a standard written method rather than a range of strategies related to the nature of the task. For example, in a group of four Year 5 pupils (2 boys and 2 girls) only one was able to calculate accurately the length of a train journey from 11.50 to 15.45. Two of the pupils made inappropriate use of a vertical format:

$$\begin{array}{r} 15.45 \\ \underline{-11.50} \end{array}$$

The one pupil in the group who answered correctly used his knowledge of number lines and time to count on from 11.50 to 12.00; he then recognised that three hours and 45 minutes remained to which he could add the ten minutes from 11.50 to 12.00.

Points for action: from mental to written calculation and problem-solving

31. To improve the quality of teaching of calculation and pupils' standards of attainment, schools need to:

- give more emphasis in the teaching of mathematics to the use of jottings as an aid to calculation, particularly when pupils are using commercial materials
- clarify the links between repeated addition and multiplication, especially for younger pupils, and between repeated subtraction and division
- give more attention, at Key Stage 1, to pupils' understanding of the inverse relationship between addition and subtraction; and at Key Stage 2, to pupils' understanding of multiplication and division and the inverse relationship between them

- give more emphasis to practising the recall of division facts
- ensure that, when pupils' errors and misconceptions are identified, time is taken to remedy them, particularly in the main teaching activity or during the plenary
- give attention to pupils' accuracy in the vertical layout of calculations involving decimal numbers
- give more attention towards the end of Key Stage 2 to the short division of numbers involving decimals
- help pupils to develop strategies for solving word problems through the use of a combination of methods that include mental strategies, personal jottings, estimation and checking
- strengthen the teaching of place value and the accuracy of setting out operations where calculations involving decimal fractions are concerned
- ensure that mathematics policies include guidance, for both key stages, on when to move from mental calculation to using informal jottings and, finally, to using formal written methods.

Use of Calculators by Pupils in Years 5 and 6

32. A mental method of calculation should always be pupils' first strategy; they need to learn that a calculator does not replace this. They need to be able to weigh up the relative merits of mental, written and calculator methods and to apply them appropriately, depending on the task set. If they choose to use a calculator, they should be able to draw upon established skills, such as rounding numbers, to check the reasonableness of the answer.

33. The National Numeracy Strategy points out that calculators should not normally be used in Key Stage 1 at the point in pupils' learning of mathematics when recall and mental calculation need to receive strong emphasis:

> *. . . the calculator's main role in mathematics lessons is not as a calculating tool, since children are still developing the mental calculation skills and written methods that they will need throughout their lives.*

However, at Key Stage 1, calculators can be used as an effective teaching tool rather than for calculation and, at Key Stage 2, as a way of teaching pupils about numbers and the number system.

34. Most of the work with calculators occurs in Years 5 and 6. By the end of Key Stage 2, pupils should have learned to enter numbers (money, measurements or fractions), carry out multiple-stage calculations (including the use of the memory function), interpret the display and appreciate what sort of work requires the use of a calculator.

35. Despite its potential value, the use of the calculator is not a regular feature in the teaching of the daily mathematics lesson. Although a significant number of

teachers refer to its use, it is not given enough emphasis. Furthermore, when it is used, teachers give too little attention to encouraging a range of options, such as using mental methods and personal jottings alongside the use of calculators.

36. Teachers demonstrate and explain the different functions of the calculator keys well. This is reflected in most pupils' competent use of the keys for the four basic operations. Teachers extend pupils' knowledge and understanding, for instance, by teaching about recurring decimals in division in Year 6.

> *The teacher made good use of paired work by setting Year 6 pupils calculator tasks such as dividing 2,000 by 60. The concept of a recurring decimal was discussed and was followed up effectively in the plenary to reinforce the connection between recurring decimals and fractions.*

37. Good attention is given to the role of the calculator when dealing with fractions, decimals and percentages. In one Year 6 class, the teacher gave a good demonstration of how to use the percentage key to calculate 20% of 137. Pupils were then encouraged to apply this simple skill in problem-solving. Other instances included exploring the equivalences of fractions, decimals and percentages, for example ¾ = 0.75 = 75%, and the use of this knowledge in the context of problem-solving to compare the relative values of given amounts expressed in different units.

38. Teachers often emphasise the importance of the correct interpretation of the numbers displayed.

> *In a Year 6 class, the teacher asked what 16.4 meant in the calculator's display in the context of a money problem. Having established that it meant £16.40, the teacher extended the pupils' knowledge through skilful questioning, for example by asking how 4p would be represented on the display. The pupils' knowledge of decimals and place value was reinforced.*

Where such effective questioning does not take place, calculator work can become a mechanical exercise with insufficient emphasis on pupils' understanding of the underlying processes.

39. While teaching relating to decimals generally receives good attention, other important aspects of calculator use in Years 5 and 6 are not taught effectively. Little emphasis is given, for example, in Year 6, to using the memory function, although this was done effectively in one mixed Year 5/6 class:

> *The teacher gave clear, step-by-step explanations and instructions for the use and application of the memory function. The pupils were asked to carry out two- and three-stage calculations, firstly by using simple numbers, for example, (3 x 8) + (4 x 7), then larger numbers in money problems. This involved selecting correct key sequences for more than one operation and using the memory function to store answers to calculations within brackets.*

40. Increasingly, teachers are encouraging pupils to estimate the size of an answer and its reasonableness without relying solely upon the calculator. For example, one teacher required a Year 6 class to estimate and consider what would be a reasonable answer to the question: 'Are 1,000 minutes more or less than a day? How do you know?" There is also growing evidence of teachers encouraging pupils to check their answers when using a calculator. The most frequently used strategy

is the application of the inverse operation. For example, 3,756 - 937 = 2,819 is checked by the calculation 2,819 + 937. The use of rounding, however, in order to estimate an answer to a calculator problem is not well established.

41. Although many of the oral and mental sessions in Years 5 and 6 do not involve calculators, most of them form a useful link with the main teaching activity through the reinforcement of important strategies which pupils might use later with calculators. Examples include doubling, rounding to the nearest ten, hundred or thousand and counting forwards and backwards to include negative numbers.

42. The use of the calculator occurs predominantly during the main part of the lesson where the quality of teaching is mostly good. The use of an overhead projector (OHP) calculator is often a positive feature. It enables teachers to demonstrate the use of a calculator to the whole class, to generate discussion about strategies and to allow pupils to practise them. Effective use of the OHP calculator also enables teachers to emphasise the importance of the correct interpretation of the display. The follow-up usually involves pupils working individually or in pairs. In the best examples, teachers intervene by drawing the class back together to discuss the work before moving on to the next stage. These 'mini-reviews', which often focus on errors and misconceptions identified by the teacher, help to maintain a good pace.

43. In most lessons that use calculators in the main teaching activity, the learning objectives are revisited appropriately in the plenary. However, in better plenaries, teachers go much further than this by identifying pupils' errors and misconceptions and correcting them through discussion and explanation. Other features of good plenaries include the use of pupils' work to illustrate and explain teaching points, and the use of direct questions about how the calculator helped with the calculations needed to solve problems. Occasionally, the plenary is also used to reinforce or extend pupils' ability to use the calculator. For example, in a Year 6 class, following a revision of the main features of the lesson, the pupils were set an additional problem to enable them to practise the use of the percentage key on their calculators in a different context.

Points for action: the use of calculators

44. To improve the quality of teaching and pupils' standards of attainment, schools need to:

- teach pupils to judge when it is sensible to use mental, written and calculator methods, to choose the appropriate method or combination of methods, and to apply these accordingly
- encourage pupils' use of mental strategies, such as rounding, to check an answer when using a calculator
- continue to look for ways to make use of the calculator as an effective teaching and learning tool.