Interacting with mathematics in Key Stage 3

Year 8 multiplicative relationships: notes for departmental meetings

Introduction

These notes can be used to guide your department through an exploration of the sample unit on multiplicative relationships. They have been structured to make best use of time and resources. Your usual departmental meetings may have an informal meeting style, but it is important to consider the benefits of structured group study promoted in these notes, whilst maintaining the ethos of the department. The following programme of action may help:

- Read the notes carefully in advance and consider sharing the management of the meetings with a colleague.
- Consider handing out copies of the unit plan in advance, with the expectation that everyone will read it before the first meeting.
- Make it clear to colleagues that you want to work to certain timings in order to ensure that all issues are covered effectively in the time available – the sessions are timed at 75 minutes, but could be expanded to 90 minutes.

After working with one mini-pack you may decide to continue with the second pack at a suitable time, as well as evaluating your approach to planning other units in your scheme of work. The mini-packs have been produced in a ringbinder to allow you to incorporate notes of your own and the additional mini-packs that will be produced during 2002/3.

Year 8 multiplicative relationships: meeting 1

Objectives

- To examine how the sample unit fits into the plan for the year and how it is organised to address the objectives
- To explore the mathematics of multiplicative relationships: the relationship between multiplication and division, ratio and proportion
- To discuss the teaching of the mathematical content of the first phase of the unit

Resources

For each teacher, unless indicated otherwise:

- *Year 8 multiplicative relationships: mini-pack* (shared if necessary)
- *Framework for teaching mathematics: Years 7, 8 and 9*
- Handout MR 1.1, 'Exploring the mathematics of multiplicative relationships'
- Pupil resource sheets, 'Parallel number lines: examples' and 'Parallel number lines'
- Calculator

Session outline

Session outline		75 minutes
Oral and mental starter A warm-up activity on multiplicative relationships	Discussion	5 minutes
Outlining the Year 8 unit on multiplicative relationships Introducing the sessions and examining the organisation and focus of the unit	Talk and discussion	15 minutes
Exploring the mathematics Exploring the mathematics of multiplicative relationships between numbers	Mathematical activity and discussion	35 minutes
Issues related to teaching the mathematics Discussing teaching issues raised by the suggested approach	Reading and discussion	15 minutes
Preparation for second meeting Outlining phase 2 and phase 3 and plans for the second meeting	Talk	5 minutes

Oral and mental starter — 5 minutes

Say that you will explain the purpose of the meeting in a few minutes, but you would like to start with a short mathematical discussion.

Distribute copies of *Year 8 multiplicative relationships: mini-pack.* Ask everyone to find 'Prompts for oral and mental starters' on page 10 of the mini-pack. Work through each of the three given examples, either saying the sequences aloud together, or taking turns around the group. Generate about ten terms for each sequence:

One multiplied by a quarter is a quarter,
two multiplied by a quarter is a half,
three multiplied by a quarter is three quarters,
four multiplied by a quarter is one,
five multiplied by a quarter is one and a quarter, . . .

A quarter of one is a quarter,
a quarter of two is a half,
a quarter of three is three quarters,
a quarter of four is one,
a quarter of five is one and a quarter, . . .

One quarter of six is one and a half,
two quarters of six is three,
three quarters of six is four and a half,
four quarters of six is six,
five quarters of six is seven and a half, . . .

Discuss briefly how counting in fraction sequences, developed over several lessons, could help to reinforce and develop pupils' understanding of fractions and fraction calculations. Make these points:

- Multiples of a fraction (the first example) and fractions of numbers (the second and third examples) can be grasped as distinct ideas.
- Awareness of commutativity (e.g. $3 \times {}^{1}/_{4} = {}^{1}/_{4} \times 3$) can be developed.
- Language and notation can be clarified: the way we say fractions, conversion to mixed numbers, use of the word 'of' and equivalence to $\times$, etc.
- The use of visual images as suggested in the prompts for oral and mental starters can reinforce these different meanings.

Repeating sequences aloud is common in primary school classrooms, less so in secondary. Consider strategies you could use to engage Year 8 pupils if this has ceased to be a normal activity for them.

Outlining the Year 8 unit on multiplicative relationships

15 minutes

Explain that the mathematics strand of the Key Stage 3 National Strategy is producing materials to support mathematics departments in their planning for Year 8. The focus is on collaborative planning of a unit of work designed to engage and challenge pupils across a wide range of attainment.

Key aspects of the curriculum have been identified where, if pupils do not grasp the underlying ideas in Year 8, it will be more difficult to raise standards of attainment by the end of the key stage. The first two aspects chosen are handling data, particularly interpretation and inference, and multiplicative relationships, particularly understanding of ratio and proportion.

Explain that this is the first of two departmental meetings to discuss a unit on multiplicative relationships in Year 8. These meetings aim to:

- explore a sample teaching unit on multiplicative relationships;
- explore the mathematics of multiplicative relationships and how it may be approached in the classroom;
- consider strengths and weaknesses in pupils' ability to solve problems involving multiplication and division, ratio and proportion;
- examine teaching styles and discuss how the teaching of problem-solving skills can be improved;
- consider how to use these materials – whether to adopt as a unit or as a source of ideas for adapting existing units.

Now allow a few minutes for everyone to read the introduction to the mini-pack and to note the objectives listed on page 5. Draw attention to the points made after the sentence beginning 'This unit has been structured into three phases . . .'

Point out these features:

- Phase 1 explores multiplicative relationships within the field of pure number.
- Phase 2 briefly considers practical examples of variables that are in proportion.
- Phase 3 develops strategies for solving problems involving multiplication and division, ratio and proportion.
- Phase 3 draws on ideas from phase 1, but not in a rigid or formulaic way – there is recognition that informal methods are often appropriate and that pupils will progress gradually in their understanding of the underlying ideas.
- In phase 1 the oral and mental starters address pupils' understanding of fractions and fraction operators. In phase 3, the oral and mental starters work towards automating key aspects of calculating with proportions.
- The unit represents nine hours of lesson time.
- The objectives of the unit are referenced by letter in the unit plan. Ways of providing differentiation are listed.

Ask everyone to scan **pages 12–13** of the Guide to the Framework, noting the distinctive features of number in Key Stage 3.

Now ask everyone to turn to **page 49** of the Guide to the Framework. This gives an example planning chart for Year 8. Look at this chart alongside the bulleted points on page 4 of the introduction to the sample unit. Together these should give you an overview of how the unit fits into the overall plan for Year 8 and the adjustments made to the content and sequence of other units in the Year 8 plan.

If you are making detailed use of the example planning charts you may wish to look at the adjustments in more depth at some other time. It is important at this point simply to get to grips with the content of this unit.

Exploring the mathematics of multiplicative relationships — 35 minutes

Explain that most of this meeting will be spent considering phase 1 of the unit. Allow a minute or so for everyone to scan the outline of phase 1 on page 4 of the mini-pack.

Distribute **handout MR 1.1.** Everyone will need a calculator.

Explain that the idea is to work first on the mathematics of multiplicative relationships between numbers, then to consider issues for teaching. The reason for considering the mathematics first is to reach a common understanding of the content. In particular, how powerful mathematical ideas can be developed within a coherent sequence of work to challenge and take pupils' thinking forward.

Spend about **30 minutes** working together through the three sections of the handout. Discuss the mathematical points raised, leaving issues for teaching until later.

Conclude this part of the meeting by saying that the level of mathematical discussion will have been higher than can be expected when teaching the unit. It is useful and interesting to discuss and debate these points as a department. It is only through getting to grips with the underlying issues involved in proportional reasoning that we will find more effective ways of explaining and illustrating these aspects in our teaching.

Issues related to teaching the mathematics — 15 minutes

Ask everyone to turn to 'Prompts for main activities in phase 1' on pages 16–18 of the mini-pack, and to have a copy of phase 1 of the unit plan alongside for reference. Work through the each section of the prompts. Allow about **10 minutes** in total.

Read each section, then discuss the teaching points raised. When considering how to adapt the approach for different teaching groups, emphasise the importance of meeting the challenge to integrate central mathematical ideas for as many pupils as possible. These crucial concepts can and should be accessed at an intellectual level with most pupils.

Say that evading the difficulties at this stage can result in pupils tackling questions in a procedural way without understanding the underlying ideas. We know, to our cost, that pupils rarely remember mere procedures beyond the teaching units within which they are rehearsed.

Finally, spend a few minutes looking at the unit plan for phase 1, identifying how the different elements of oral and mental starters, main activities and plenaries are linked.

Preparation for second meeting 5 minutes

Allow a few minutes for everyone to read phases 2 and 3 of the unit plan.

Point out these features, which will be discussed in greater detail at the second meeting:

- Phase 2 links the concept of proportion to practical contexts, many of which involve constant rates.
- The oral and mental starters in phase 3 work towards automating key processes such as expressing, comparing and finding proportions.
- The main activities in phase 3 are designed to enhance the traditional approach of working individually through exercises, by providing activities requiring a greater level of discussion around methods of solution.
- Specific problem-solving strategies are to be taught, drawing on the ideas of phase 1, but building flexibly on a range of methods, including informal mental approaches.

Draw the meeting to a close by telling everyone that in the next meeting you will use a video to discuss some of the issues pupils face in solving problems involving multiplication, division, ratio and proportion. You will then consider effective problem-solving strategies and how they can be taught, and discuss practical steps towards implementing the unit and any general issues that affect the work of the department.

In the meantime, some people may be able to give more time to examining the unit in more detail, or preparing any examples that you might want to present to pupils in phase 1.

Exploring the mathematics of multiplicative relationships

This handout is for group study by mathematics staff, to explore the mathematics underpinning phase 1 of the unit. There is a separate set of prompts to support teaching with different groups of pupils. This will be explored later in the meeting.

Where appropriate, use a calculator.

Scaling numbers

Consider the question:

How can you get from 4 to 5 using only multiplication and division?

Discuss these possible approaches:

Multiplying by 5 and then dividing by 4	$\times 5 \div 4$
Dividing by 4 and then multiplying by 5	$\div 4 \times 5$
Multiplying by $^5/_4$ or $1^1/_4$ (as a single number)	$\times {}^5/_4$

Pay attention to the mathematical language used and any images drawn or described. How could you use the pupil resource sheets of parallel number lines to provide representative images?

What about the inverse problem of scaling 5 to 4? How does each of the approaches invert?

Try scaling between other pairs of numbers – include one non-integer pair. How would you generalise the result?

Returning to the first example, consider how $^5/_4$ and $^4/_5$ could be written as decimals ($5 \div 4$ and $4 \div 5$) or as percentages (hundredths):

		multiplicative inverse	inverse as divisor
Multiplying by $^5/_4$ or $1^1/_4$	$\times {}^5/_4$	$\times {}^4/_5$	$\div {}^5/_4$
Multiplying by 1.25	$\times$ 1.25	$\times$ 0.8	$\div$ 1.25
Finding 125% of 4	$\times$ 125%	$\times$ 80%	$\div$ 125%

Briefly discuss how the inverse can be expressed as a divisor – as an alternative to the multiplicative inverse, particularly when the operator is a single decimal: $\div$ 1.25.

Discuss the merits of:

- thinking of $\times {}^5/_4$ as a single operation (scale factor), which is easy to generalise, and the usefulness of the concept of a multiplicative inverse ($\times {}^4/_5$);
- being able to move between equivalent fraction, decimal and percentage forms of a number;
- sometimes thinking of the inverse operation as division, $\div$ 1.25 ($\div {}^5/_4$), as an alternative to the multiplicative inverse, $\times$ 0.8 ($\times {}^4/_5$).

Ratio and proportion

a	*b*
7	10.5
0.8	1.2
$1\frac{1}{2}$	$2\frac{1}{4}$
24	36
1.4	2.1

$a \xrightarrow{\times ?} b$

For each pair of entries in this table, find the multiplier that scales *a* to *b*. Confirm that these are all equal, so that these sets of numbers are in proportion. The multiplier or scale factor is the ratio *b* : *a* or *b*/*a*.

Repeat the process, this time to find the multiplier that scales *b* to *a*:

$b \xrightarrow{\times ?} a$

Confirm that this multiplier too is the same for each pair of entries. It is the inverse scale factor, which is the ratio *a* : *b* or *a*/*b*.

For the following examples, identify the sets that are in proportion:

Example 1		Example 2		Example 3		Example 4	
a	***b***	***a***	***b***	***a***	***b***	***a***	***b***
5	11.25	8	26.25	7	49	12	3.6
0.3	0.675	2	8.25	2	4	2.5	0.75
7	15.75	3	11.25	3.5	12.25	7	2.1
12	27	40	122.25	20	400	60	18
24	54	5.5	18.75	8	64	9	2.7

Choose *one* of the proportions and record the:

- scale factor as a ratio of two numbers (not necessarily integers or in lowest terms), as a fraction, as a decimal and as a percentage;
- inverse scale factor as a fraction multiplier, as a decimal multiplier, as a percentage multiplier.

Then remind yourself of the value of the original scale factor as a decimal multiplier and use this to find the decimal divisor that would give you an alternative way of calculating the inverse.

		scale factor	inverse scale factor
	ratio		
inverse as divisor	fraction	×	×
÷	decimal	×	×
	percentage	×	×

Discuss the merits of:

- establishing the concept of proportionality by examining sets of ‘difficult’ numbers in unordered sequences;
- attending to the symmetry of the relationship by considering the scale factor in each direction.

Using ratio and proportion

The following sets of numbers are in proportion. Find the missing entry in each set by calculating the appropriate scale factor as a multiplier:

Example 1	
a	***b***
9	13
13.5	19.5
20	28.89
15	*x*

Example 2	
a	***b***
7	2
38.5	11
y	10
56	16

Example 3				
a	28	10	8	64
b	77	27.5	*z*	176

Example 4				
a	12	*t*	19.2	144
b	5	17	8	60

Consider now a set of just two pairs of numbers in proportion. Find the ratios (scale factors) between the columns and within the columns and check for equivalences:

$$\begin{tikzcd} 4 \arrow[r, "\times 5/4"] \arrow[d, "\times ?"'] & 5 \arrow[d, "\times ?"] \\ 6.8 \arrow[r, "\times ?"'] & 8.5 \end{tikzcd} \qquad \begin{tikzcd} 4 & 5 \arrow[l, "\times ?"'] \\ 6.8 \arrow[u, "\times ?"] & 8.5 \arrow[l, "\times ?"] \arrow[u, "\times ?"'] \end{tikzcd}$$

Before moving on, reflect briefly on the fact that proportions involve equality of ratios both ‘between’ and ‘within’ sets of numbers.

A practical illustration would be a photographic enlargement.

The ratio of lengths *between* the original and the enlargement is equal to the ratio of widths *between* the original and the enlargement;

The ratio of length to width *within* the original is equal to the ratio of length to width *within* the enlargement.

A typical proportion problem distils to two pairs of numbers, where one number is unknown – for example:

x	5	4	y	4	5	4	5
6.8	8.5	6.8	8.5	z	8.5	6.8	t

(*Note:* In a different representational form, the first of these examples might be presented as $x : 5 = 6.8 : 8.5$ or $x : 6.8 = 5 : 8.5$.)

Write *two* equations for calculating each of x, y, z and t (e.g. x can be calculated by scaling 5 or by scaling 6.8).

Discuss the merits of:

- considering the number you are scaling and thinking ‘Will the answer be bigger or smaller?’;
- looking for relationships that are easy to deal with by a mental method. For example, these problems can be solved by doubling and trebling respectively:

6	7.65	18	6
12	x	y	23

Year 8 multiplicative relationships: meeting 2

Objectives

- To consider pupils' responses to test questions on ratio and proportion and the implications for teaching and learning
- To discuss how to teach the techniques needed to solve problems involving multiplication and division, ratio and proportion
- To decide on practical steps needed to implement the unit, including adaptations to the materials to meet the needs of different groups of pupils
- To discuss any general implications for the work of the department

Resources

- Equipment: video player
- Video sequence 3, 'Test questions on proportion'
- For each teacher, unless otherwise indicated:
  - *Year 8 multiplicative relationships: mini-pack* (shared if necessary)
  - *Framework for teaching mathematics: Years 7, 8, and 9*
  - Handout MR 2.1, 'Reflecting on pupils' responses to Key Stage 3 test questions' (one between two or three teachers)
  - Handout MR 2.2, 'Pupils' written responses to Key Stage 3 test questions' (one between two or three teachers)
  - Current scheme of work for Year 8, including notes for ratio and proportion units

Session outline — 75 minutes

Discussing pupils' responses to test questions Considering some questions on ratio and proportion from the 2001 Key Stage 3 test papers	Video and discussion	25 minutes
Teaching problem solving Discussing techniques to use when teaching pupils to solve problems involving multiplicative relationships, ratio and proportion	Mathematical activity and discussion	25 minutes
Planning the way forward Considering the use of the prepared unit and the ideas it contains	Discussion	20 minutes
Conclusion Considering some general issues arising from the meetings on multiplicative relationships	Discussion	5 minutes

Discussing pupils' responses to test questions 25 minutes

Remind everyone of key points from the last meeting:

- Pupils often encounter difficulties with questions involving multiplicative relationships. In the last meeting you explored the mathematics underlying phase 1 of the unit.
- You discussed ways of explaining, modelling or illustrating the ideas in this unit, in an effort to find a common approach.
- The sample unit addresses the concept of multiplicative relations in three phases. Pupils will explore the pure number aspect of multiplicative relationships, then consider relationships within and between proportional sets (in practical contexts involving constant rates). In phase 3 of the unit pupils use the skills developed in phases 1 and 2 to solve problems.

Outline the plans for this meeting. As a department you will:

- reflect on pupils' methods of solving problems involving multiplication, division, ratio and proportion;
- consider the teaching approaches suggested in the unit and compare them to current practice;
- consider which aspects of the unit you wish to incorporate into your teaching.

Now explain that you are going to spend **20 minutes** examining the teaching and learning issues arising from pupils' responses to problems involving multiplication, division, ratio and proportion in the 2001 national test at Key Stage 3. You will use video clips from discussions by pupils who achieved either level 6 or level 7 in the test. Although this is only a small sample of responses, the issues raised are likely to apply to pupils in many schools, including pupils whose attainment is lower than those on the video.

Distribute **handouts MR 2.1** and **MR 2.2.** Handout MR 2.1 provides the focus for each of the video clips.

For each test question you will need to guide everyone through the following stages:

- becoming familiar with the question;
- considering pupils' written responses;
- watching the video of the pupils explaining their solutions and discussing the implications for teaching.

Point out that the three video clips are very short. If necessary you can view a clip twice to help you to address the questions raised.

Teaching problem solving 25 minutes

Say that as well as what you teach, an important consideration is how you teach. A key question is:

> How can we engage more pupils in discussion of the specific strategies that are effective in solving problems?

Explain that this meeting will focus on the main activities in phase 3. These are designed to enhance the traditional approach of working individually through exercises, by providing activities that require discussion around methods of solution.

Specific problem-solving strategies are taught in phase 3, drawing on the skills of phase 1, but building flexibly on a range of methods, including informal mental approaches.

Discuss the following approaches to the main teaching part of the lesson in phase 3:

- Choose one problem: discuss alternative strategies for solving the problem; change the numbers in the problem (e.g. make them more difficult) and consider how the methods can be adapted; ask different or supplementary questions from the same context. (See pages 19–20 of 'Prompts for main activities in phase 3' in the mini-pack.)
- Choose a small set of problems: concentrate on extracting and organising the data (e.g. putting it in tabular form) before deciding on possible methods of solution rather than working the problems through to an answer. (See pages 21–23 in the mini-pack.)
- Ask pupils to make up similar problems for a partner to solve.
- Give part solutions and ask pupils to continue and complete the solution or give a complete solution and ask pupils to evaluate the efficiency of the strategy chosen and to identify any errors.

Now assess the usefulness of the suggestions included in 'Prompts for main activities in phase 3' by working through parts of it. If your department is large enough you could split into two groups, one following option A and the other option B. In smaller departments simply choose one of the options and work together. Allow **15 minutes** for this activity.

Option A

The first part of 'Prompts for main activities in phase 3' shows how one problem can be approached using different strategies.

- Work though the suggestions. Make any additional notes that will be helpful as further teaching prompts.
- Follow this up by working on another question, from the problem bank, the Framework or a suitable text, and prepare a similar set of prompts showing alternative strategies.
- Think about how a main activity based on this approach could allow independent work (individual or in pairs) as well as whole-class discussion.

Option B

The second part of 'Prompts for main activities in phase 3' shows how a small set of problems can be used to illustrate the translation of word problems into simpler numerical forms. This then leads to a discussion of possible methods of solution, although the problems are not carried through to the final stage of solution.

- Work though the suggestions. Make any additional notes that will be helpful as further teaching prompts.
- Follow this up by collecting together another small set of questions, from the problem bank, the Framework or a suitable text, and prepare a similar set of prompts showing the translated forms.
- Think about how a main activity based on this approach could allow independent work (individual or in pairs) as well as whole-class discussion.

Allow **5 minutes** for the two groups to exchange feedback on the activity.

Planning the way forward **20 minutes**

Introduce a discussion with the department as to how you will move forward, including any adaptations to the unit and development of teaching style.

Note the various possibilities for adaptation:

- Use the unit without adaptation.
- Use the unit but adapt the prompt sheets, drawing on your own ideas and resources.
- Use the structure of the unit but incorporate effective ideas for teaching activities from the department.
- Use the structure of an existing departmental unit but incorporate some ideas from this unit.

In addition to considerations of structure and content, there may be implications for teaching. Questions to consider are:

- How do the approaches explored in these two meetings compare to our current teaching?
- In what ways might we adapt our teaching?
- Is this different for different groups of pupils?
- What are the implications for the department in implementing and supporting these changes?

Say that the department may wish to consider trialing the materials or aspects of teaching style with specific classes before making them available to the whole department. In many schools a key group of pupils are those being targeted to achieve level 5 in the Key Stage 3 tests in Year 9. This group should certainly be included in any trials.

Finally, agree details of any practical tasks that need to be completed:

- concerning the **planning** – for example, examples to be prepared, sheets to be duplicated, notes to be adapted;
- concerning the **teaching** – for example, paired teaching, lesson observation;
- concerning the way the unit will be **evaluated** and **reviewed**.

Decide what needs to be done, who will do it and by what date.

If your department has support from a mathematics consultant, you will also want to consider how he or she will contribute to the development.

Conclusion **5 minutes**

Allow a few minutes to pick up general points from your meetings. In particular:

- Has exploring the mathematical approach to multiplicative relationships suggested a model of how to work collaboratively as a department to review teaching styles?
- Has video sequence 3 (viewed in the second meeting) stimulated ideas about how pupils’ outcomes may be evidence for future reviews and auditing?

Reflecting on pupils' responses to Key Stage 3 test questions

2001 Tier 5–7, Paper 2 question 11a (Protein in yoghurt)

The label on yoghurt A shows this information.

How many grams of protein does 100 g of yoghurt provide?

Show your working.

Yoghurt A **125 g**	
Each 125 g provides	
Energy	430 kJ
Protein	4.5 g
Carbohydrate	11.1 g
Fat	4.5 g

Answer the question thinking about the likely approaches by pupils.

Consider the written responses of a small sample of Year 9 pupils (handout MR 2.2).

Watch the first video clip (about 1 minute), in which Catherine, Lee and Linsey explain their answers.

In discussing Catherine's explanation you may wish to consider:

- What is the meaning of the value resulting from her calculation?
- Would it have helped her to articulate this as a rate '… per …'?
- How would a sense of size (estimate of the answer) have helped her?

Lee has a good initial strategy of finding the protein in 25 g of yoghurt but makes an error in his calculation.

Would extracting the data and rewriting prior to calculation help to support the stages of solution and provide a checking strategy? For example:

yoghurt	protein
125 g	4.5 g
25 g	?
100 g	?

Notice how the interaction between Lee and Catherine helps both pupils to see an efficient method of solution.

Linsey articulates the steps of her calculation very clearly. Notice, in particular, that she can identify what rate she has calculated at each stage.

2001 Tier 5–7, Paper 2 question 11b (Carbohydrate in yoghurt)

The label on yoghurt B shows different information.

A boy eats the same amount of yoghurt A and yoghurt B.

Which yoghurt provides him with more carbohydrate?

Show your working.

Yoghurt B **150 g**	
Each 150 g provides	
Energy	339 kJ
Protein	6.6 g
Carbohydrate	13.1 g
Fat	0.2 g

Answer the question thinking about the likely approaches by pupils.

Consider the written responses of a small sample of Year 9 pupils (handout MR 2.2).

Watch the second video clip (about 1 minute), in which Catherine and Linsey reflect on their solutions.

In discussing Catherine's explanation you may wish to consider:

- What is the meaning of the values resulting from her calculation?
- How do these values inform her choice of yoghurt as having 'more carbohydrate in the same amount of yoghurt'?
- How could she improve upon the language she uses to better explain the stages of her calculation?

Linsey again has a clear overview of the steps in her calculation although not all of this is recorded.

For pupils who require a little more support, what simple form of recording might scaffold their thinking in questions such as this?

2001 Tier 5–7, Paper 2 question 16 (Pay)

The table shows the average weekly earnings for men and women in 1956 and 1998.

	1956	1998
Men	£11.89	£420.30
Women	£6.16	£303.70

(a) For 1956, calculate the average weekly earnings for women as a percentage of the average weekly earnings for men.

Show your working and give your answer to 1 decimal place.

(b) For 1998, show that the average weekly earnings for women were a greater proportion of the average weekly earnings for men than they were in 1956.

Answer the question thinking about the likely approaches by pupils.

Consider the written response of Catherine (handout MR 2.2).

Watch the third video clip (about 1 minute), in which Catherine explains her answer.

Catherine's approach to the final part of the question is perhaps the opposite to what would be expected. (She calculates by dividing the men's wage by the women's wage for each year and then chooses the lower of these two values).

- What supplementary questions could we ask Catherine to be sure that her strategy is sound?
- How could we help Catherine to explain this solution to other pupils?
- Why do you suppose Catherine took this approach rather than finding the women's average wage as a proportion of the men's?

Briefly discuss the correctness of these explanations and the quality of expression.

Pupils' written responses to Key Stage 3 test questions

2001 Tier 5–7, Paper 2 question 11a: Catherine's response

(a) The label on yoghurt A shows this information.

How many grams of **protein** does **100 g** of yoghurt provide?

Show your working.

Yoghurt A	125 g
Each 125 g provides	
Energy	430 kJ
Protein	4.5 g
Carbohydrate	11.1 g
Fat	4.5 g

125 ÷ 4.5 = 27.77 8

27 g

2001 Tier 5–7, Paper 2 question 11a: Lee's response

(a) The label on yoghurt A shows this information.

How many grams of **protein** does **100 g** of yoghurt provide?

Show your working.

Yoghurt A	125 g
Each 125 g provides	
Energy	430 kJ
Protein	4.5 g
Carbohydrate	11.1 g
Fat	4.5 g

0.18

25 | 4.5

0.18 × 4 = 0.72

4.5 − 0.72 = 3.78

3.78 g

2001 Tier 5–7, Paper 2 question 11a: Linsey's response

(a) The label on yoghurt A shows this information.

Yoghurt A	125 g
Each 125 g provides	
Energy	430 kJ
Protein	4.5 g
Carbohydrate	11.1 g
Fat	4.5 g

How many grams of **protein** does **100 g** of yoghurt provide?

Show your working.

4·5g ÷ 125g = 0·036

0·036 × 100 = 3·6g

3·6 g

2001 Tier 5–7, Paper 2 question 11b: Catherine's response

(b) The label on yoghurt B shows different information.

A boy eats the same amount of yoghurt A and yoghurt B.

Which yoghurt provides him with more **carbohydrate**?

Show your working.

Yoghurt B	**150 g**
Each 150 g provides	
Energy	339 kJ
Protein	6.6 g
Carbohydrate	13.1 g
Fat	0.2 g

150 ÷ 13·1 = 11·45
125 ÷ 11·1 = 11·26

Yoghurt B

2001 Tier 5–7, Paper 2 question 11b: Linsey's response

(b) The label on yoghurt B shows different information.

A boy eats the same amount of yoghurt A and yoghurt B.

Which yoghurt provides him with more **carbohydrate**?

Show your working.

Yoghurt B	**150 g**
Each 150 g provides	
Energy	339 kJ
Protein	6.6 g
Carbohydrate	13.1 g
Fat	0.2 g

13·1 g ÷ 150 = 0·087 × 125 =
10·9g

yoghurt A

2001 Tier 5–7, Paper 2 question 16: Catherine's response

The table shows the average weekly earnings for men and women in 1956 and 1998.

	1956	1998
Men	£11.89	£420.30
Women	£6.16	£303.70

(a) For **1956**, calculate the average weekly earnings for women as a percentage of the average weekly earnings for men.

Show your working and give your answer to 1 decimal place.

$6 \cdot 16 \div 11 \cdot 89 = 0 \cdot 5$~~18~~

~~42·~~
~~8%0~~
41·8 %

(b) For **1998**, show that the average weekly earnings for women were a **greater proportion** of the average weekly earnings for men than they were in 1956.

~~303·70 ÷ 11·89 = 25·542~~

$420 \cdot 30 \div 303 \cdot 70 = 1 \cdot 3839$

$11 \cdot 89 \div 6 \cdot 16 = 1 \cdot 930$