

Key Stage 3

National Strategy

Mathematics

***Interacting with mathematics
in Key Stage 3***

School file for Year 9

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Guide to the Year 9 pack

Introduction

These materials for guidance and training aim to support mathematics departments in:

- planning for teaching that engages and challenges pupils;
- developing mathematical reasoning;

and so raise standards of achievement in Year 9.

The materials are part of the series *Interacting with mathematics in Key Stage 3*. This series aims to support departments in the long-term development of schemes of work by focusing in depth on selected key topics.

The materials

The materials supplement those produced in the summer term 2002: *Interacting with mathematics in Year 8*. These Year 9 materials are designed to be inserted into the school file *Interacting with mathematics in Key Stage 3*.

The Year 9 materials contain two mini-packs, one on geometrical reasoning and the other on proportional reasoning (a sequel to the Year 8 multiplicative relationships unit). As with the Year 8 materials, the mini-packs include:

- a plan for the unit of work, including:
 - introduction to the unit;
 - the teaching objectives;
 - an outline of possible activities for a sequence of lessons;
 - prompts in the form of notes to support teaching;
 - guidance on adapting the activities to meet the differing needs of pupils;
- two training sessions to help departments explore the specific issues about teaching and learning and so decide how to use the unit;
- supporting materials including video and audio.

Features of both mini-packs are:

- objectives taken from the Year 9 teaching programmes – forming the **core** of each lesson;
- mathematical activities which help pupils to conjecture, link different ideas, generalise and solve problems – providing scope for **extending** their thinking;
- class management strategies and questioning techniques which foster discussion and collaborative ways of working – **supporting** pupils who will find these objectives challenging.

How to use the materials

It is recommended that, following the training in the spring term 2003, departments choose **one** of the mini-packs as the basis for collaborative work. You will have begun to explore both packs during the initial training and started the process of deciding how to use the material.

Each mini-pack contains notes for two departmental meetings on the unit, including some video and audio materials. The notes are structured to focus the meetings effectively and make best use of time and resources. You may be used to a more informal approach to meetings, but it is important to consider the benefits of structured group study, while maintaining the established ethos of the department. The following points may help:

- Read the notes carefully in advance and consider sharing the management of the meetings with a colleague.
- Consider handing out copies of the unit plan in advance, with the expectation that everyone will read it through before the first meeting.
- Make it clear to colleagues that you want to work to certain timings in order to ensure that all issues are covered effectively in the time available – the sessions are timed at 75 minutes, but could be expanded to 90 minutes.

After working with one mini-pack you may decide to continue with the second pack at a suitable time, as well as evaluating your approach to planning other units in your scheme of work. Allow time for in-depth work on chosen units of work and do not worry if your in-school developments are slightly out of phase with the roll-out of national materials. Aim for lasting changes which will raise standards at Key Stage 3.

The audio-visual materials

The video: Multiplicative relationships

| Sequence | Meeting | Duration | Title | Description |
|----------|--------------------------|----------|-----------------------------|---|
| 1 | Proportional reasoning 1 | 16 min | Scale factors | Raj teaches a Year 9 upper middle set, showing a sample of precursor activities and the main teaching |
| 2 | For later viewing | 13 min | Precursors to scale factors | An extended sequence of precursor activities |

The audio

| Tracks | Meeting | Duration | Title | Description |
|--------|-------------------------|----------------------|--|---|
| 1–7 | For classroom use | 1½–3½ min each track | Various | Visualisation exercises for oral and mental starters |
| 8 | For classroom use | 2 min | Extract from <i>Alice's Adventures in Wonderland</i> | For discussion in plenary on reasoning |
| 9 | Geometrical reasoning 1 | 4 min | Pupils proving with build-up 7 | A group of pupils explain to another group a proof of the angle sum of a triangle |
| 10 | Geometrical reasoning 2 | 6 min | Pupils discussing visualisation 6 | Sue discusses the visualisation 'Two pairs of parallel lines' with a group of Y9 set 2 pupils |

In addition to the audio tracks, the CD-ROM includes the following text files in Word format:

- geometrical reasoning: unit plan, resource sheets and problem bank;
- proportional reasoning: unit plan and resource sheets.

Acknowledgements

We are grateful to the teachers and pupils of the schools featured in the video and audio. They are:

| School | LEA | No. on roll | % FSM* |
|---------------------------|------------------|-------------|--------|
| Almondbury High School | Kirklees | 856 | 25% |
| Manor School | Northamptonshire | 782 | 6% |
| Cottenham Village College | Cambridgeshire | 942 | 5% |

* Proportion of pupils eligible for free school meals

We gratefully acknowledge the contributions of Buckinghamshire, Cambridgeshire, Kirklees, Lincolnshire, Luton and Northamptonshire LEAs in helping to produce these materials.

Resource sheet

Build-ups: figures 1 and 2

Figure 1

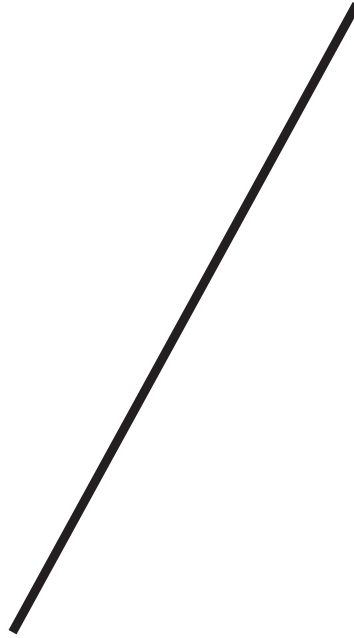
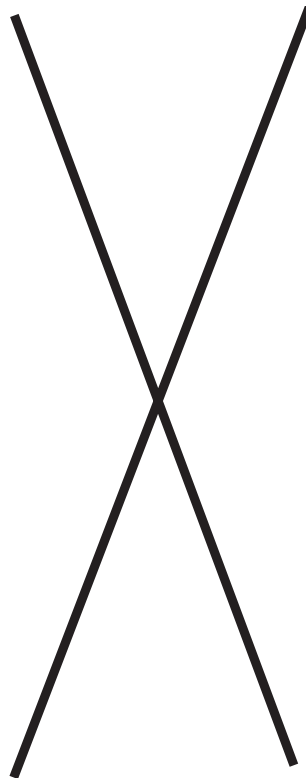


Figure 2



Resource sheet

Build-ups: figures 3 and 4

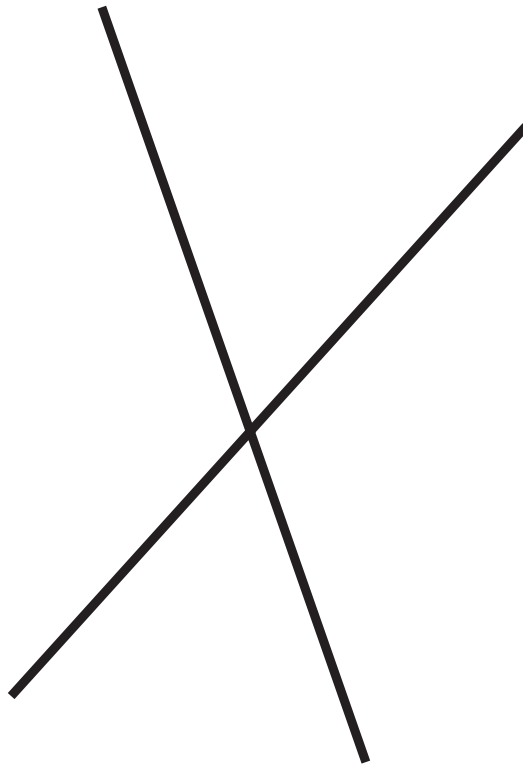


Figure 3

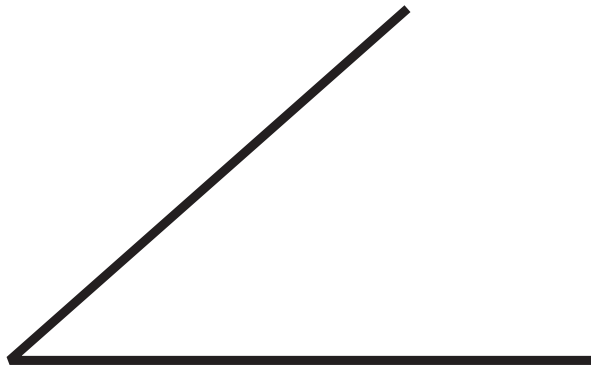


Figure 4

Resource sheet

Build-ups: figures 5 and 6

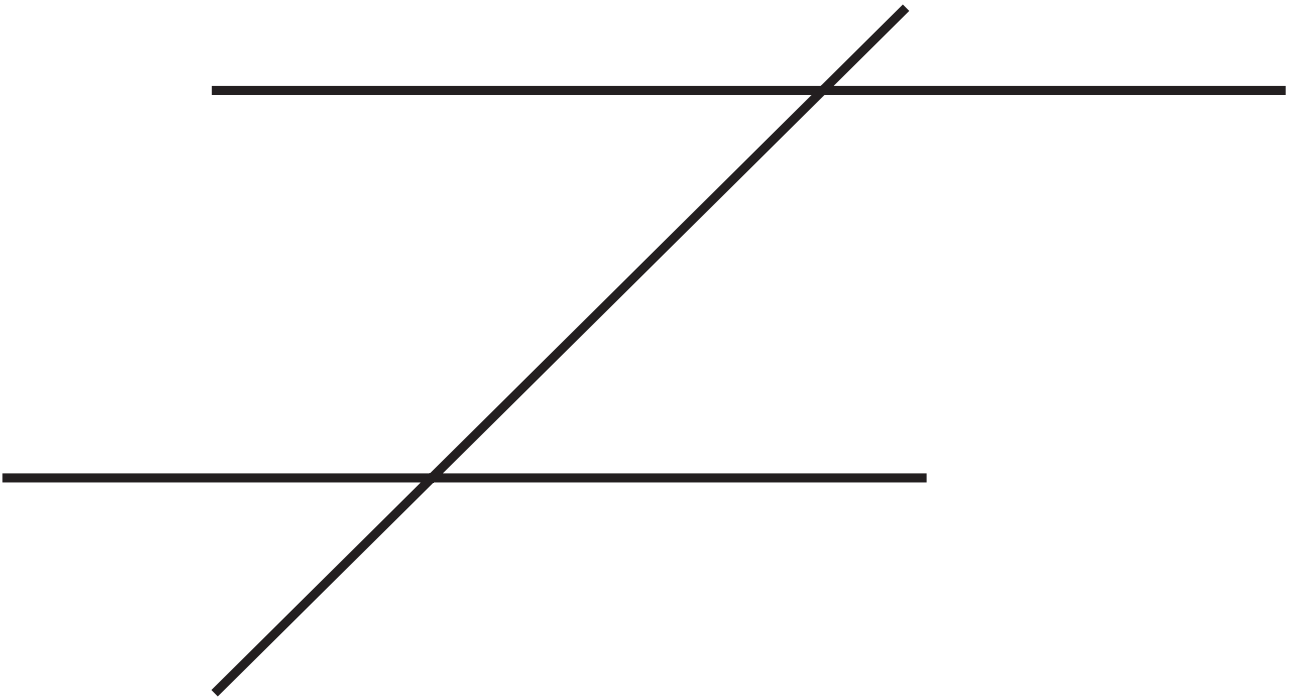


Figure 5

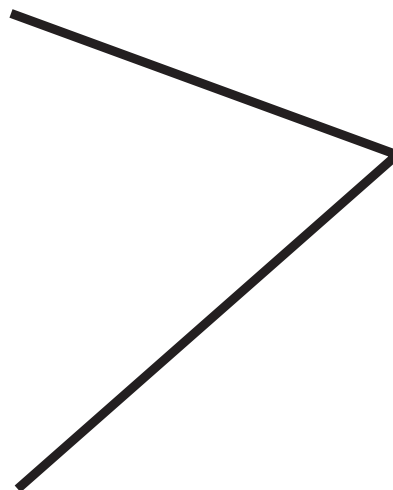


Figure 6

Resource sheet

Build-ups: figure 7

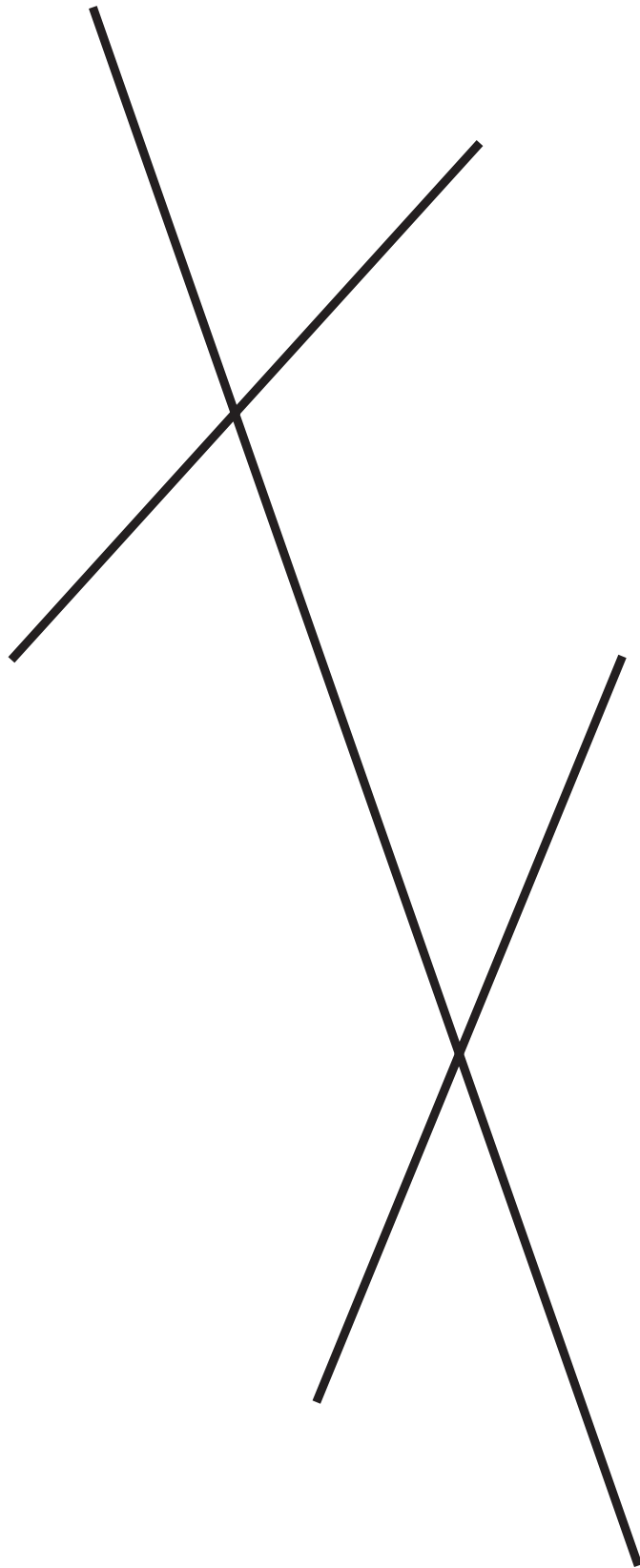


Figure 7

Resource sheet

Build-ups: figures 8 and 9

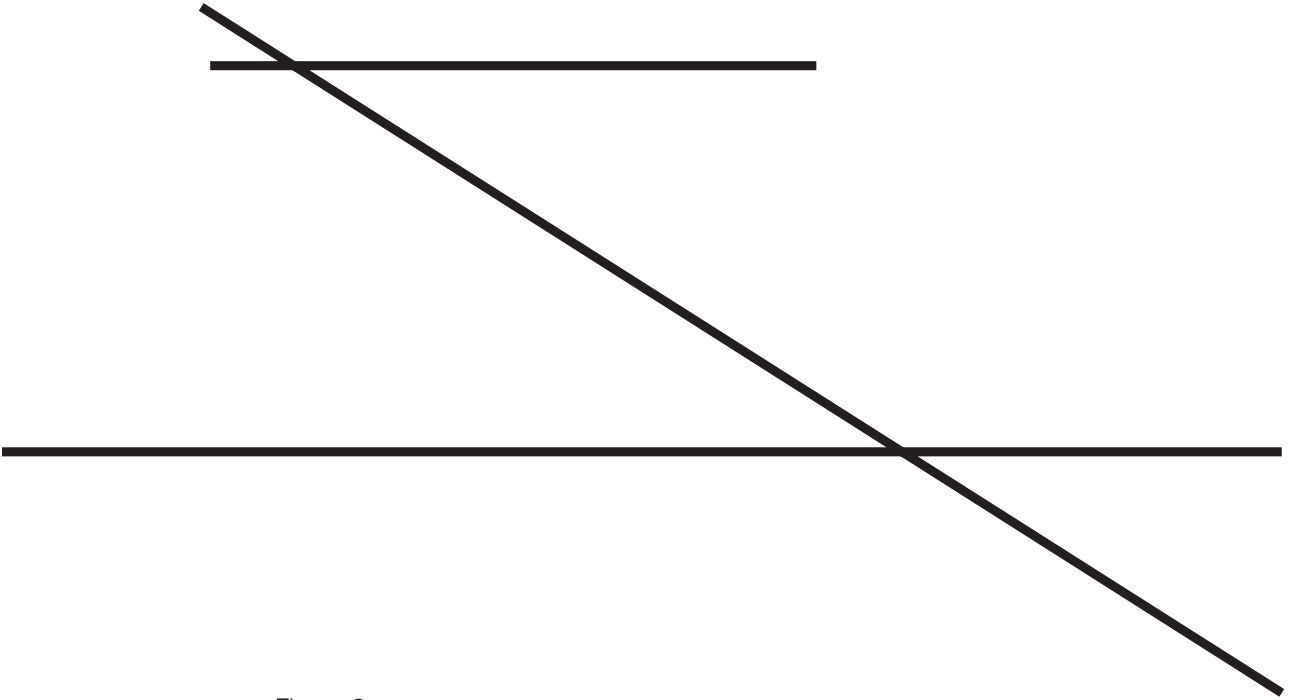


Figure 8

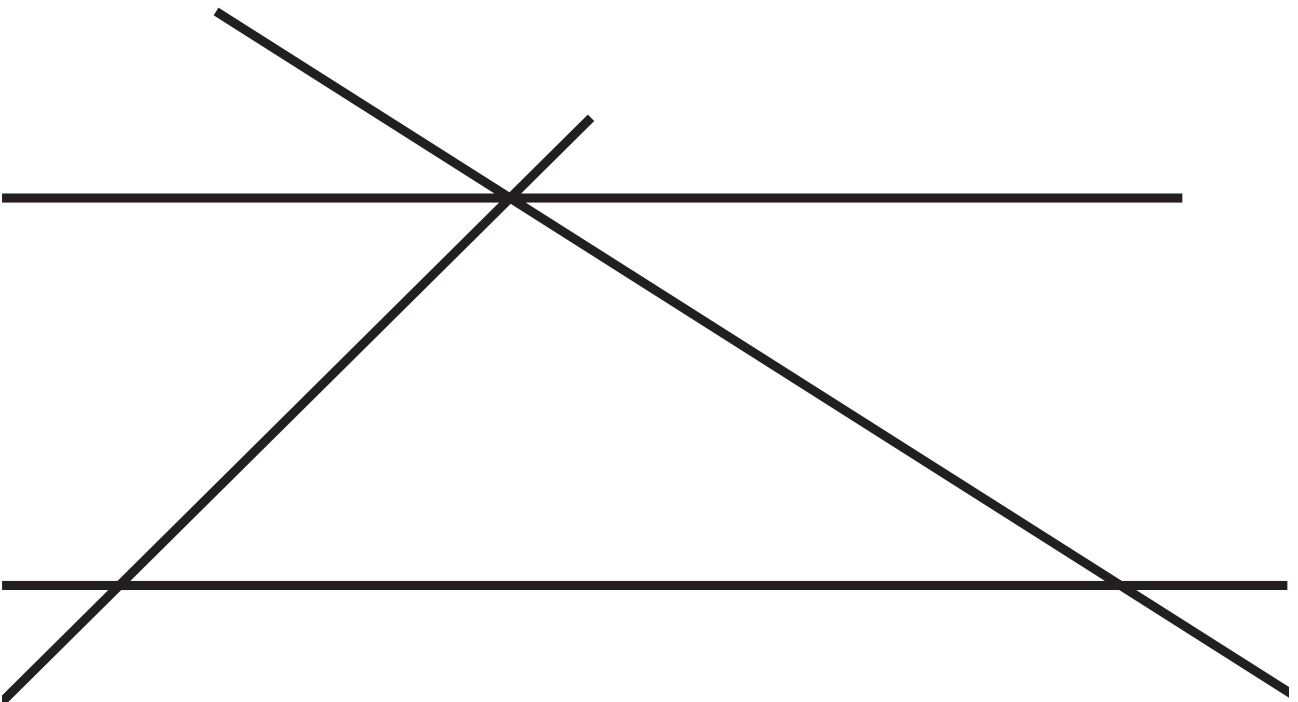


Figure 9

Resource sheet

Build-ups: figure 10



Figure 10

Resource sheet

Build-ups: figure 11



Figure 11

Resource sheet

Alice's Adventures in Wonderland

'There's more evidence to come yet, please your Majesty,' said the White Rabbit, jumping up in a great hurry: 'this paper has just been picked up.'

'What's in it?' said the Queen.

'I haven't opened it yet,' said the White Rabbit, 'but it seems to be a letter, written by the prisoner to – to somebody.'

'It must have been that,' said the King, 'unless it was written to nobody, which isn't usual, you know.'

'Who is it directed to?' said one of the jurymen.

'It isn't directed at all,' said the White Rabbit: 'in fact, there's nothing written on the *outside*.' He unfolded the paper as he spoke, and added, 'It isn't a letter, after all: it's a set of verses.'

'Are they in the prisoner's handwriting?' asked another of the jurymen.

'No, they're not,' said the White Rabbit, 'and that's the queerest thing about it.' (The jury all looked puzzled.)

'He must have imitated somebody else's hand,' said the King. (The jury all brightened up again.)

'Please your Majesty,' said the Knave, 'I didn't write it, and they can't prove that I did: there's no name signed at the end.'

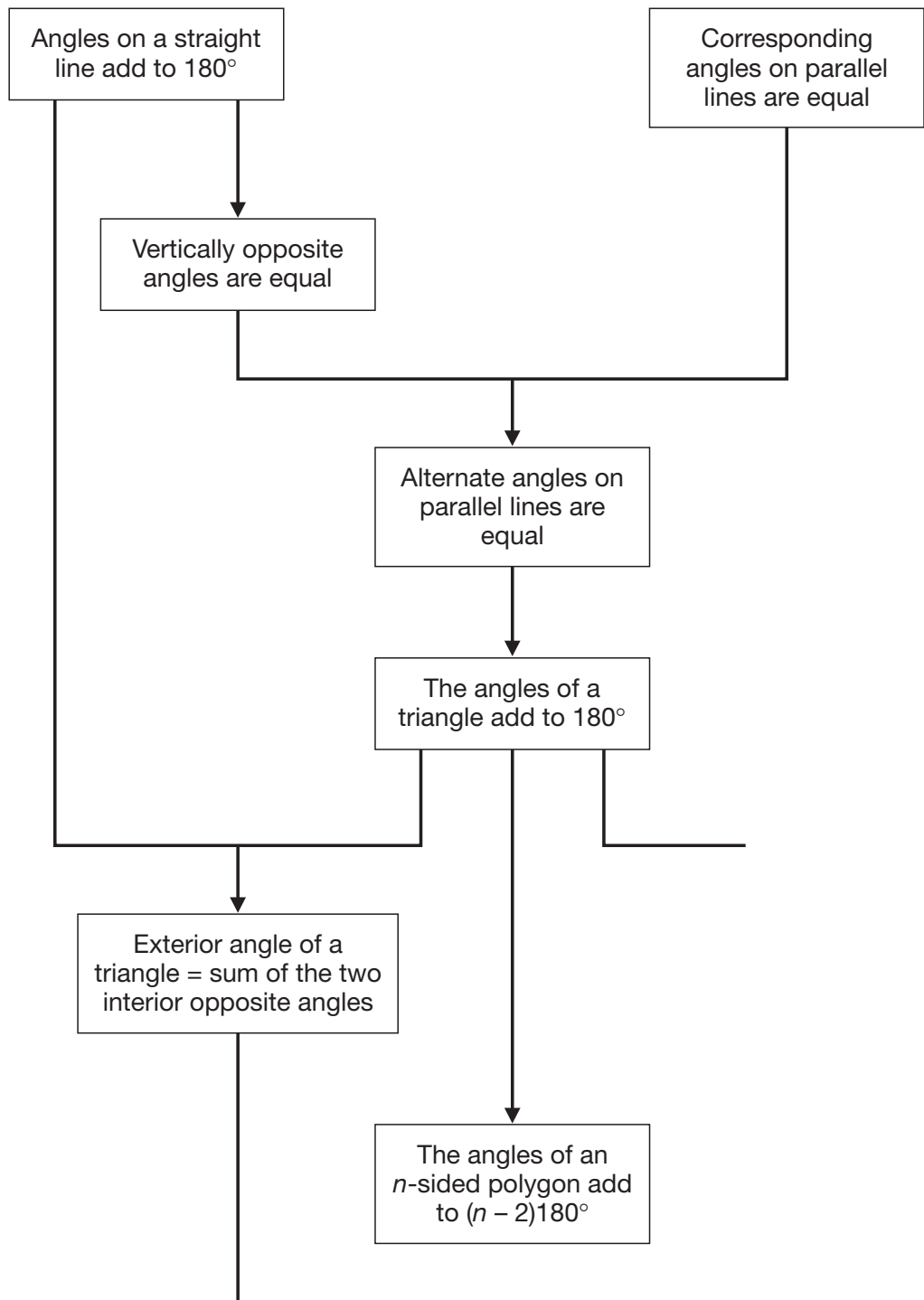
'If you didn't sign it,' said the King, 'that only makes the matter worse. You *must* have meant some mischief, or else you'd have signed your name like an honest man.'

There was a general clapping of hands at this: it was the first really clever thing the King had said that day.

From *Alice's Adventures in Wonderland* by Lewis Carroll (Ch XII, Alice's evidence)

Resource sheet

Start of a flowchart of logical reasoning



***Interacting with mathematics
in Key Stage 3***

*Year 9 geometrical reasoning:
notes for departmental meetings*

Introduction

These notes can be used to guide your department through an exploration of the sample Year 9 unit on geometrical reasoning. The notes have been structured to make best use of time and resources. Your usual departmental meetings may have an informal meeting style, but it is important to consider the benefits of structured group study promoted in these notes, while maintaining the ethos of the department. The following programme of action may help:

- Read the notes carefully in advance and consider sharing the management of the meeting with a colleague.
- Consider handing out copies of the unit plan in advance, with the expectation that everyone will read it before the first meeting.
- Make it clear to colleagues that you will want to work to certain timings in order to ensure that all issues are covered effectively in the time available – the sessions are timed at 75 minutes, but could be expanded to 90 minutes.

Each section of the notes is structured as follows. Key points and features are outlined in boxes. These are followed by suggested activities. The activities are designed to stimulate discussion and exploration of the mini-pack on geometrical reasoning. Activities are followed by points you may wish to make in summary.

Year 9 geometrical reasoning: meeting 1

Objectives

- To indicate some current issues in teaching geometry
- To try a visualisation and reconstruction exercise
- To outline a Year 9 unit on geometrical reasoning
- To consider how to help pupils work towards proof in geometry

Resources

- Equipment: CD player, overhead projector, flipchart or board
- Year 9 CD-ROM
- Two acetate copies of figure 1 and one acetate copy of figure 2 (see resource sheets 'Build-ups')
- For each teacher, unless otherwise indicated:
 - *Year 9 geometrical reasoning: mini-pack* (you may wish staff to have chance to read this prior to the meeting)
 - *Framework for teaching mathematics: Years 7, 8 and 9* (could be shared)
 - Figures 1, 5 and 8 printed or photocopied on to good-quality tracing paper
 - two or three small pieces of blank tracing paper

Session outline

75 minutes

Introduction

| | | |
|---|------|-----------|
| Introducing the Year 9 unit and setting out the agenda for the meetings | Talk | 5 minutes |
|---|------|-----------|

Visualisation: an oral and mental starter

| | | |
|---|-------------------------|------------|
| Trying a visualisation exercise from a collection suggested as oral and mental starters | Activity and discussion | 10 minutes |
|---|-------------------------|------------|

Exploring the geometrical reasoning unit plan

| | | |
|---|-----------------------|------------|
| Outlining the unit and where it fits into the Year 9 plan | Talk and reading time | 10 minutes |
|---|-----------------------|------------|

Towards proof

| | | |
|-----------------------------------|-------------------------|------------|
| Exploring the build-up activities | Activity and discussion | 45 minutes |
|-----------------------------------|-------------------------|------------|

Preparing for the second meeting

| | | |
|--|-----------------------|-----------|
| Outlining plans for the second meeting | Reading time and talk | 5 minutes |
|--|-----------------------|-----------|

Key points

The mathematics strand of the Key Stage 3 Strategy has produced two mini-packs to support mathematics departments in their planning for Year 9. The packs, similar to the Year 8 mini-packs, are part of the series *Interacting with mathematics in Key Stage 3*. The focus is on collaborative planning of a unit of work designed to engage and challenge pupils across a wide range of attainment.

You will be aware that in recent years there has been a shift towards strengthening the place of geometry in the mathematics curriculum. This is reflected in:

- the 2000 revision of the National Curriculum for Key Stages 3 and 4;
- a joint report by the Royal Society and Joint Mathematical Council, *Teaching and learning geometry 11–19*, published in 2001 (Royal Society, www.royalsoc.ac.uk).

This unit aims to support departments in developing the teaching and learning of geometrical reasoning, particularly where this is a new focus.

This first meeting provides an opportunity to:

- try a visualisation and reconstruction exercise as an oral and mental starter;
- explore a six-hour (two-week) sample unit on geometrical reasoning described in two phases;
- consider ways of helping pupils to reason deductively in geometry and begin to understand the nature of proof.

The second meeting will provide an opportunity to:

- take a further look at visualisations and consider their value as oral and mental starters;
- examine ways of solving geometrical problems with a focus on developing clear explanations and reasons;
- discuss how the unit will be used.

Start by using the notes above to explain the background to the materials to your department, particularly the renewed attention being given to the place of geometry in the mathematics curriculum. Then outline what will be covered in the two departmental meetings.

Visualisation: an oral and mental starter**10 minutes**

Say that you would like to begin with an oral and mental starter – a visualisation exercise. Ask teachers to shut their eyes. Play **audio track 1**, ‘Two intersecting lines’ (about 2½ minutes).

Ask teachers to describe what they saw, working through the sequence of images and taking views from different people. Encourage them to stay with verbal descriptions and avoid drawing pictures. Discuss why differences of perception may have occurred.

Ask everyone to note the reference to visualisations in the oral and mental starters for this unit (**page 8** of the mini-pack). Allow time for everyone to scan the ‘Prompts for oral and mental starters in phase 1’ on **pages 10–14** of the mini-pack, noting the introduction and seven visualisations, scripted and recorded on the audio CD.

Encourage everyone to try at least one visualisation exercise with a class to inform further discussion. In particular, it would help to be familiar with visualisation 6, ‘Two pairs of parallel lines’, on **track 6** as you will have the opportunity to listen to a recording of pupils’ responses to this example at the next meeting. If you are new to such exercises, follow the guidance notes, deciding whether to use the CD or to read the prepared script.

Key points

- The geometrical reasoning unit is about two weeks of work assuming 6 or 7 lessons of around 1 hour.
- The unit aims to draw together pupils' geometrical knowledge into a logical and reasoned framework.
- Oral and mental starters include some visualisation exercises. These aim to develop pupils' insights into geometrical relationships and their ability to describe what they see.
- Phase 1 of the unit uses 'build-ups' with acetate overlays and tracings to develop pupils' ability to reason logically and prove elementary results in geometry.
- Phase 2 concentrates on solving problems, using the facts developed in phase 1 as 'givens', and setting out written explanations in logical order.
- Activities within a phase are not prescribed lesson by lesson. A possible sequence is given for teachers to adapt and modify as appropriate for their classes.

Allow time for everyone to read **pages 4–5** of the geometrical reasoning mini-pack (introduction and objectives). Use the notes above to highlight the key features of the unit.

Referring to the Framework guide (section 1 of the Framework) ask everyone to read the section on 'Geometrical reasoning' on **page 16** and the key features of shape, space and measures in Key Stage 3 on **page 17**.

Next ask teachers to turn to **page 50** of the Framework guide, the example planning chart for Year 9. Use this chart along with the details in the introduction to the unit on **page 4** in the mini-pack beginning, 'In planning the unit several decisions ...'. Together, these give a quick overview of how the unit fits into the sample plan for Year 9 and the adjustments made to other units.

In summary

The emphasis of the unit is on establishing familiar geometrical properties in a logical hierarchy and developing the ability to reason with them (orally and in writing).

The detail of rebalancing the units in Year 9 can be considered after the two departmental meetings, when the emphasis on different objectives has become clear.

Key points

The main activities in phase 1 of the unit focus on 'build-ups', using acetate overlays and tracing paper.

- The 'build-up' process is designed to engage pupils with the notion of developing a coherent sequence of geometrical facts.
- Given facts (represented on acetates) are used to 'build up' new geometrical properties.
- All the build-up activities use **conventions** in the form of notation and labelling and all offer the chance to rehearse both oral and written forms of an argument.
- Some build-ups establish **definitions** and some prove **properties** from statements of given facts established previously.

In order to engage fully with the unit pupils must appreciate:

- the nature of the 'game', that is starting from certain assumptions (or 'givens'), from which other results are deduced;
- that they will be asked to justify results which are obvious to them or which they regard as well-established truths;
- the importance of setting out the steps of an argument in logical order, with clear reasons at each stage.

Allow about **20 minutes** to complete this section down to the first summary point.

Ask everyone to scan the plan for phase 1 of the unit on **page 8** of the mini-pack. Say that you will return to the plan later, after you have clarified some of the detail.

Explain the purpose of build-ups and how they deal with conventions, definitions and derived properties.

Demonstrate build-up 1, 'To prove that vertically opposite angles are equal', using the three acetate sheets and following the notes in the 'Prompts for main activities in phase 1' (**page 18** of the mini-pack). Involve another member of the department in recording notes on a flipchart or board, as outlined in the third column of the prompts.

Briefly discuss what such activities could offer in developing pupils' understanding of geometrical deductive argument. (You may find it helpful to refer to the points above.)

Allow time for everyone first to scan the guidance for build-up 1 on **page 18** of the mini-pack, and second to read the first three pages of the 'Prompts for main activities in phase 1' starting on **page 15**.

In summary

The novelty of the build-ups helps to engage pupils in the steps of the deductive process by clearly identifying the component properties of each final image.

Key points

The recording which you are about to play captures the discussion between two groups of three pupils. The first group has just taken part in a teacher-led discussion of build-up 7, the proof of the interior angle sum of a triangle using these given facts:

- Pairs of alternate angles on parallel lines are equal.
- Pairs of corresponding angles on parallel lines are equal.
- Angles on a straight line add to 180° .

The second group of pupils has been asked to question as many aspects of the proof as possible – in fact to role-play doubters!

Allow about **25 minutes** for this section.

Use the notes above to introduce the recording and describe the background to the pupil discussion.

Distribute the tracing-paper copies of figures 1, 5 and 8 and encourage colleagues as they listen to the recording of the pupils' discussion to:

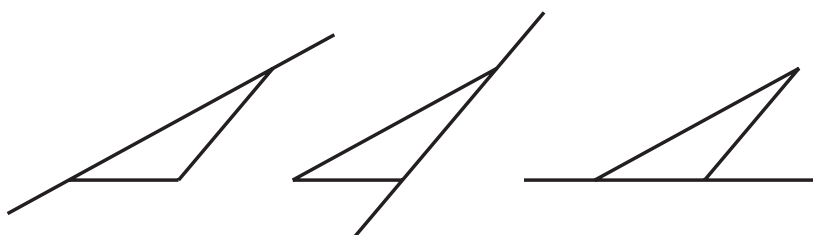
- reconstruct the conventions used by the pupils to label the diagram;
- identify areas of vocabulary which could be more precise;
- be prepared to record a written version of the stages of their proof.

Play **audio track 9**, 'Pupils proving with build-up 7' (approximately 4 minutes). Allow a minute or two afterwards for everyone to write and discuss their explanation.

Discuss the value of both demonstration using acetate and the 'hands-on' approach using tracing paper.

Allow time for everyone to read the suggested sequencing of the main part of the lessons in phase 1 (**page 8** in the mini-pack).

Try the second activity described in the section 'Proving the angle sum of a triangle' (note that blank tracing paper is needed). This poses the question: 'Can the build-ups be created for one given triangle, beginning from any side?', using the argument outlined in the recording (build-up 7).



Note the purpose of this activity, which is to illustrate the generality of the argument, and the suggested plenary to draw out the distinction between demonstration and proof.

Conclude with a discussion as to how you could manage the sequence of lessons in phase 1 with different classes, starting with the build-ups and leading to the work on angles in a polygon.

In summary

Build-ups allow pupils to reconstruct the process of proof, developing their ability to reason, set out logical explanations (orally and in writing) and understand the nature of deductive proof.

Key points

The following features of the unit plan are to be discussed at the second meeting:

- visualisation exercises tried since the first meeting;
- activities in phase 2 of the unit, devoted to solving geometrical problems, with a focus on clear written explanations and reasons.

Allow a minute or two for everyone to examine phase 2 of the unit plan on **page 9** of the mini-pack, explaining what will be discussed at the next meeting.

Explain that you will also discuss how to implement the unit. In the meantime, people may be able to find time to examine the unit in greater detail.

Remind everyone that they have agreed to try one or more of the visualisation exercises before the next meeting.

Year 9 geometrical reasoning: meeting 2

Objectives

- To revisit the use of visualisation exercises
- To explore approaches to solving geometrical problems and how pupils can develop their ability to reason and set out logical arguments in writing
- To decide on practical steps needed to implement the unit
- To discuss any general implications for the work of the department

Resources

- Equipment: CD player
- Year 9 CD-ROM
- For each teacher, unless otherwise indicated:
 - *Year 9 geometrical reasoning: mini-pack*
 - *Framework for teaching mathematics: Years 7, 8 and 9*

Session outline

75 minutes

Introduction

| | | |
|--|------|-----------|
| Recapping key points and outlining the session | Talk | 5 minutes |
|--|------|-----------|

Visualisations revisited

| | | |
|---|----------------------|------------|
| Discussing examples tried with classes and listening to a recorded discussion | Audio and discussion | 15 minutes |
|---|----------------------|------------|

Solving geometrical problems

| | | |
|--|-------------------------|------------|
| Solving geometrical problems with a focus on developing clear written explanations and reasons | Activity and discussion | 35 minutes |
|--|-------------------------|------------|

Planning the way forward

| | | |
|--|------------|------------|
| Considering how to use the prepared unit and develop the key ideas | Discussion | 15 minutes |
|--|------------|------------|

Conclusion

| | | |
|--|------------|-----------|
| Considering some general issues arising from the meetings on geometrical reasoning | Discussion | 5 minutes |
|--|------------|-----------|

Key points

The key points from the last meeting are:

- Geometrical reasoning now has a more prominent place in the curriculum at Key Stage 3. The sample unit aims to help teachers engage more pupils in this strand of mathematics.
- A distinctive approach is adopted: using visualisation exercises to manipulate images; using 'build-ups' to develop proofs.
- Pupils are helped to appreciate arguments starting from given facts to derive new facts. The emphasis is more on reasoning than on presenting new content.

In this meeting you will:

- revisit the use of visualisation exercises;
- explore approaches to solving geometrical problems and how pupils can develop their ability to reason and set out logical arguments in writing;
- consider how you will use the unit.

Use the notes above to remind everyone of the key points from the last meeting and set out the plan for this session.

Visualisations revisited

15 minutes

Remind everyone that, as indicated in the first meeting, you would like to discuss visualisation 6, 'Two pairs of parallel lines'. Play **audio track 6** (2 minutes).

Ask teachers who have tried this with a class since the last meeting to say briefly how the follow-up discussion went.

Explain that you will now play a recording of part of a discussion conducted by Sue with her Year 9 set 2 class, who have not yet met this unit on geometrical reasoning. Note that Sue's approach is not to interject or correct pupils but to encourage discussion to develop, so that language and thought processes are revealed. Play **audio track 10**, 'Pupils discussing visualisation 6' (6 minutes).

After listening to the discussion, talk as a group about what you heard. Interesting points to discuss are:

- how an open discussion can usefully reveal areas for further work, such as the need to develop language or resolve misconceptions;
- the value of working dynamically with mental images and sharing perceptions with others, in order to develop geometrical reasoning.

In summary

- Visualisation exercises can help to develop perceptions of geometrical relationships and the language needed to describe them.
- Pupils usually enjoy the activity and, with practice, greatly improve their skills.
- There would be value in using some of these exercises in lessons prior to starting the unit on geometrical reasoning.

Key points

'Prompts for main activities in phase 2' in the mini-pack provides guidance on teaching pupils to solve problems. This includes suggestions of how to:

- structure the stages of solving a geometrical problem;
- provide strategies to support the development of written argument.

The problem bank in the mini-pack:

- contains questions of various types, including alternative wording of particular questions;
- is not intended to be copied directly as a worksheet;
- is available on the CD as a text file, from which to cut and paste.

Ask everyone to look again at phase 2 of the unit plan on **page 9** of the mini-pack to remind themselves that this phase is concerned with solving geometrical problems. Note the following points relating to the main activities of solving problems:

- the stages in the problem-solving process;
- the list of seven given facts on which pupils can draw (developed in phase 1);
- the suggestions for paired working.

Allow everyone time to read the solving problems section of the 'Prompts for main activities in phase 2' on **page 25** of the mini-pack, paying particular attention to the exemplification of written explanations. Following this, ask them to scan the problem bank.

Now allow about **10 minutes** for everyone in pairs to choose two problems to solve, discuss and write up explanations.

Allow a further **10 minutes** for the whole group to share one or two written explanations. Using the prompts as guidance, agree on a consistent approach.

Discuss:

- how the problems could be presented to your classes;
- what expectations you will have of different groups of pupils, particularly in terms of written explanations.

Finally, look at the 'Prompts for final plenary in phase 2', particularly the flowchart of logical reasoning on **page 30**.

In summary

The aim is for pupils to become aware of a hierarchy of facts, built on the foundations of more basic facts or 'givens', and to begin to make logical connections for themselves.

This hierarchy is developed in phase 1 and applied in phase 2.

Key points

The rest of this second meeting is devoted to considering how to implement this unit. There are various possibilities:

- Use the unit without adaptation.
- Use the unit but adapt the prompts or problem bank drawing on your own ideas and resources.
- Use the structure of the unit but incorporate some effective ideas for teaching activities from the department.
- Use the structure of an existing departmental unit but incorporate some ideas from this unit.

As well as considerations of structure and content, there may be implications for teaching:

- How do the approaches explored in these two meetings compare to our current teaching?
- In what ways might we adapt our teaching?
- Is this different for different groups of pupils?
- What are the implications for the department in implementing and supporting these changes?

Discuss with the department **how you will move forward**, including any adaptations to the unit and development of teaching style.

Finally, agree details of any **practical tasks** that need to be completed:

- concerning the **planning**, for example, problems to be prepared, sheets to be duplicated, notes to be adapted;
- concerning the **teaching**, for example, paired teaching, lesson observation;
- concerning the way the unit will be **evaluated** and **reviewed**.

What needs to be done, who will do it and by what date?

If your department has mathematics consultant support available, how might the support be used?

Conclusion

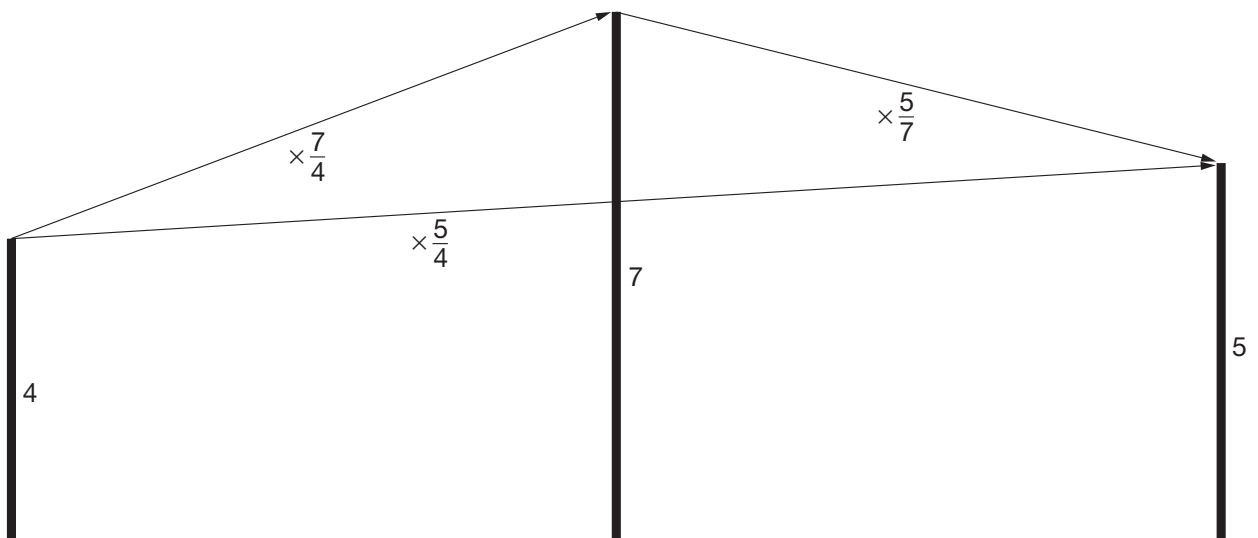
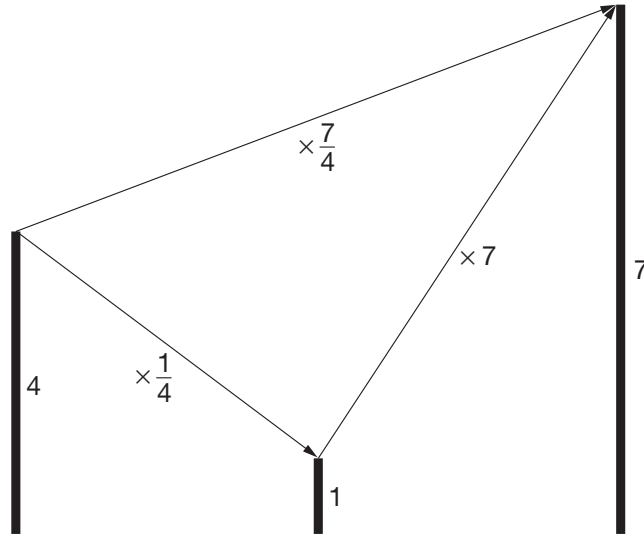
5 minutes

Allow a few minutes to pick up general points from your meetings. In particular:

- How has the link between visual images, layers of a built-up diagram and structuring of oral and written argument helped to clarify the process of geometrical reasoning?
- How might the suggested approaches to teaching geometrical reasoning be extended into Key Stage 4 and filtered down into Year 8?

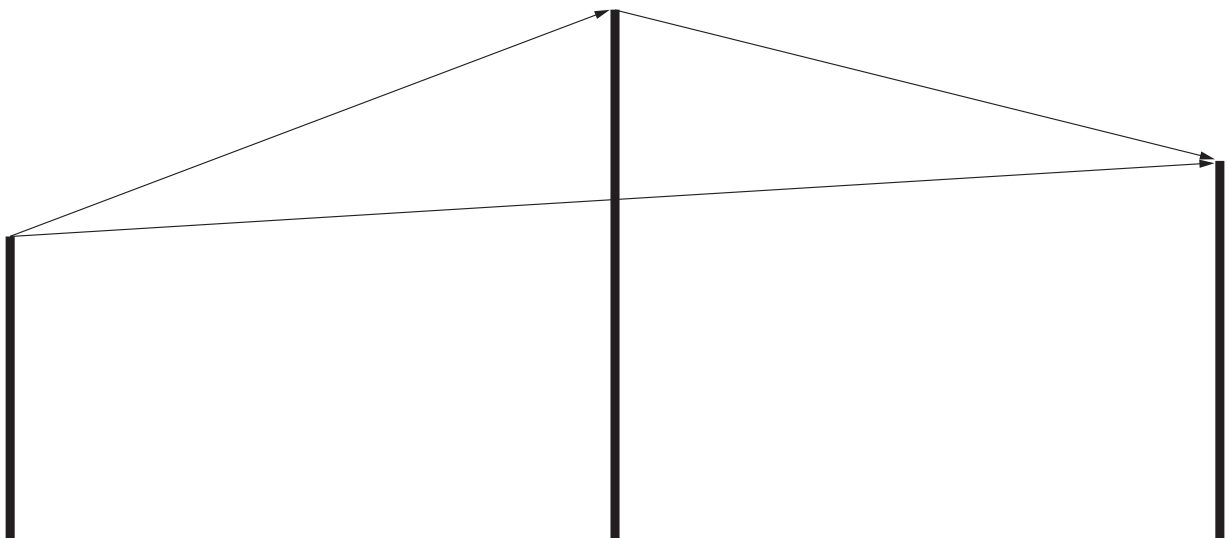
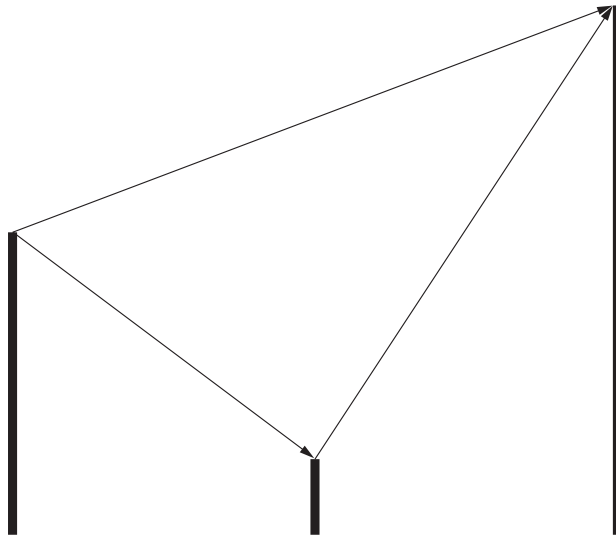
Resource sheet

Scaling line segments: examples



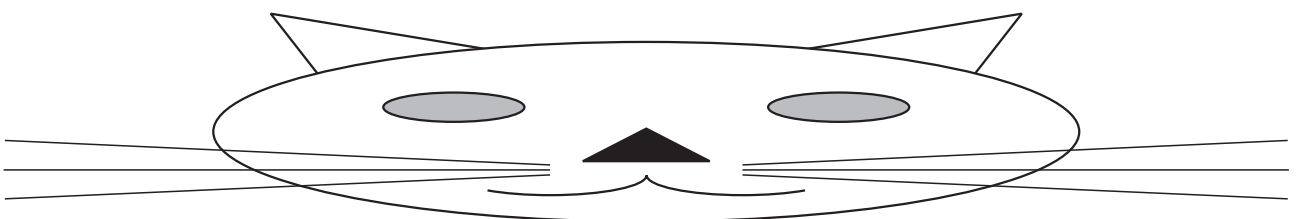
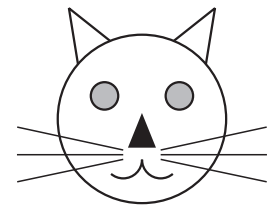
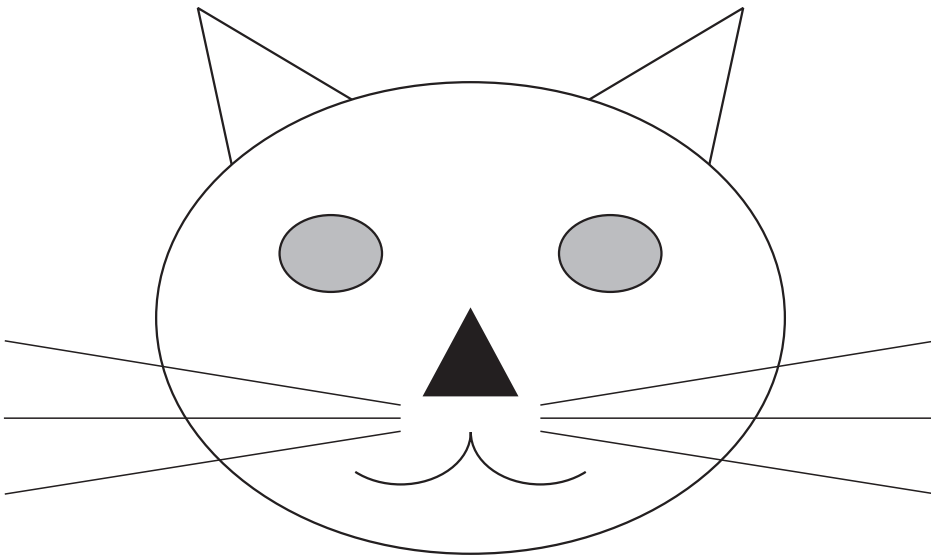
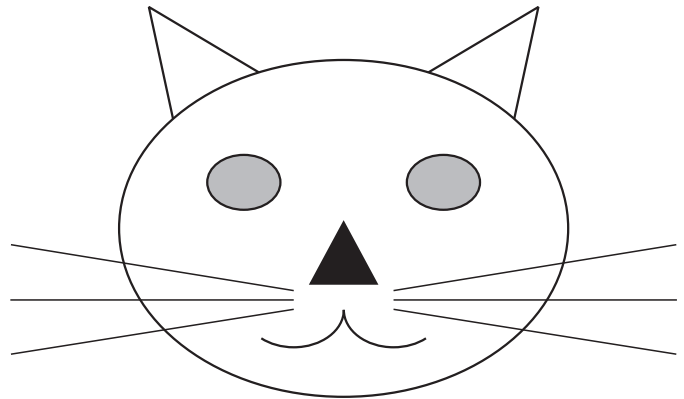
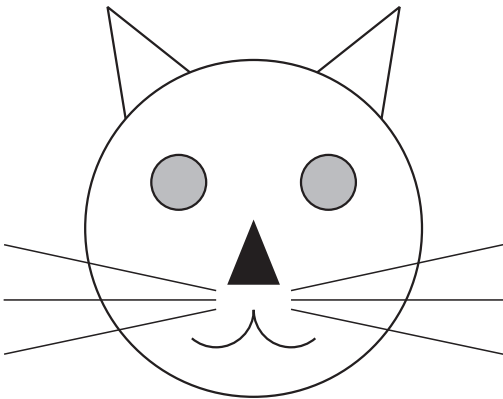
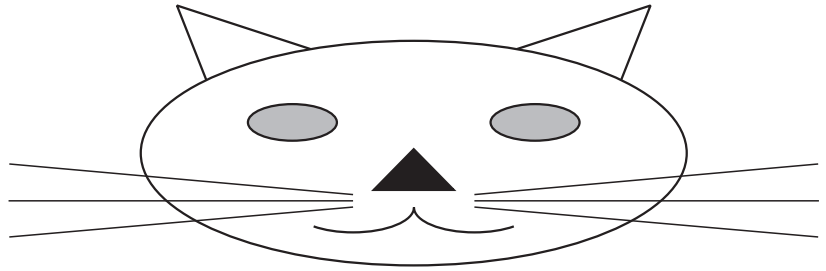
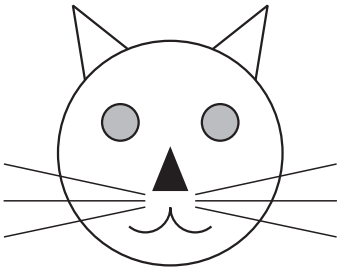
Resource sheet

Scaling line segments



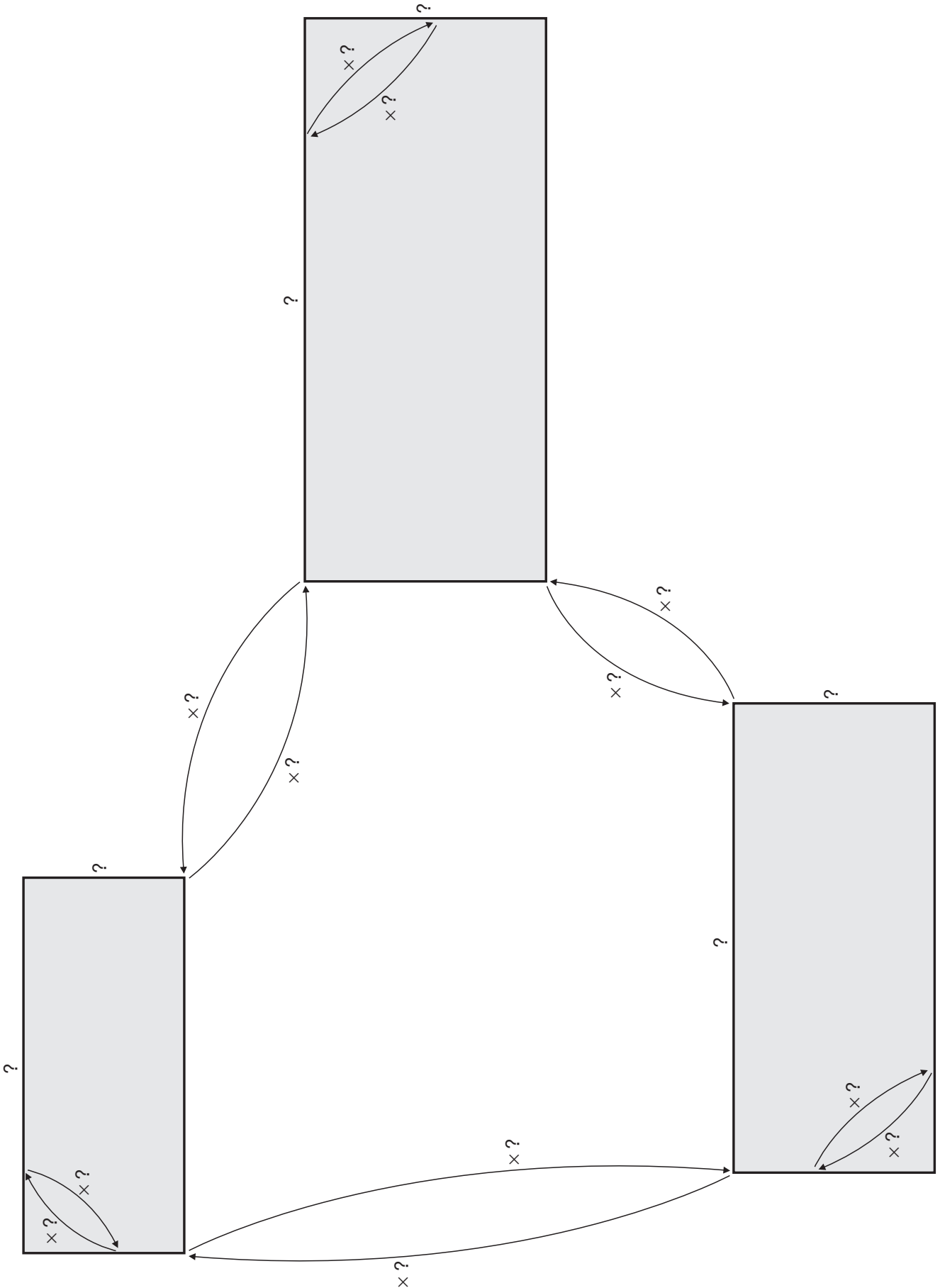
Resource sheet

Cat faces



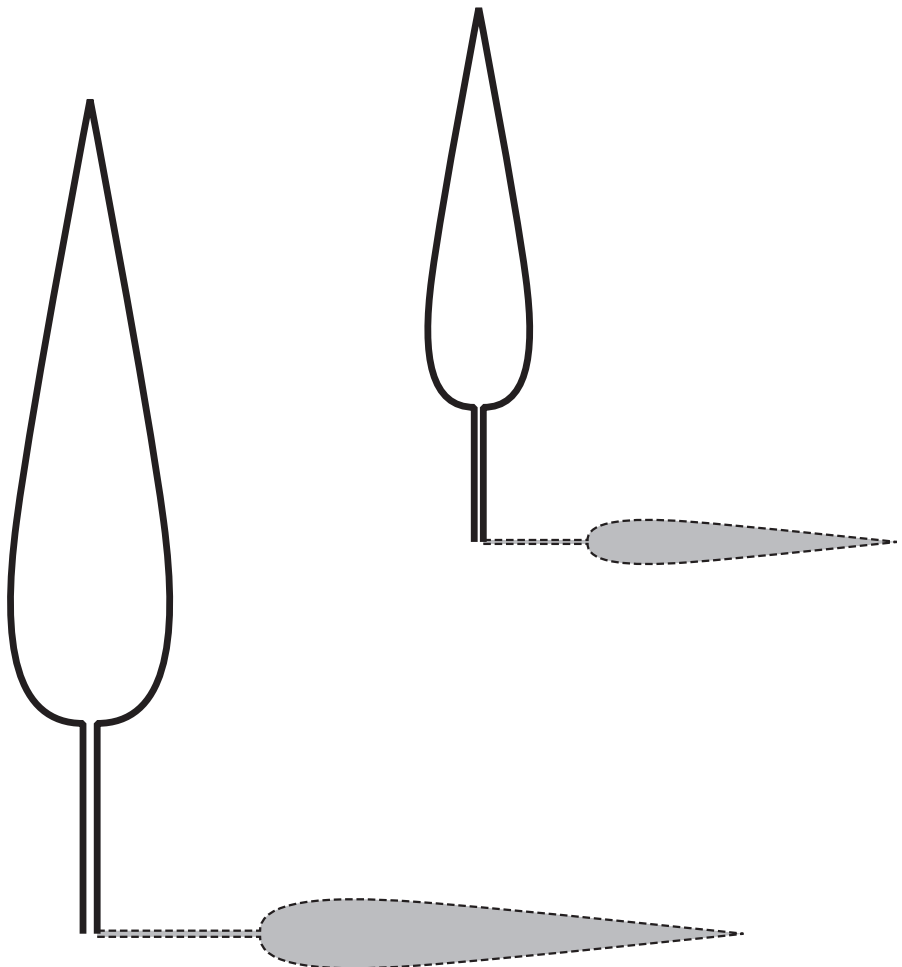
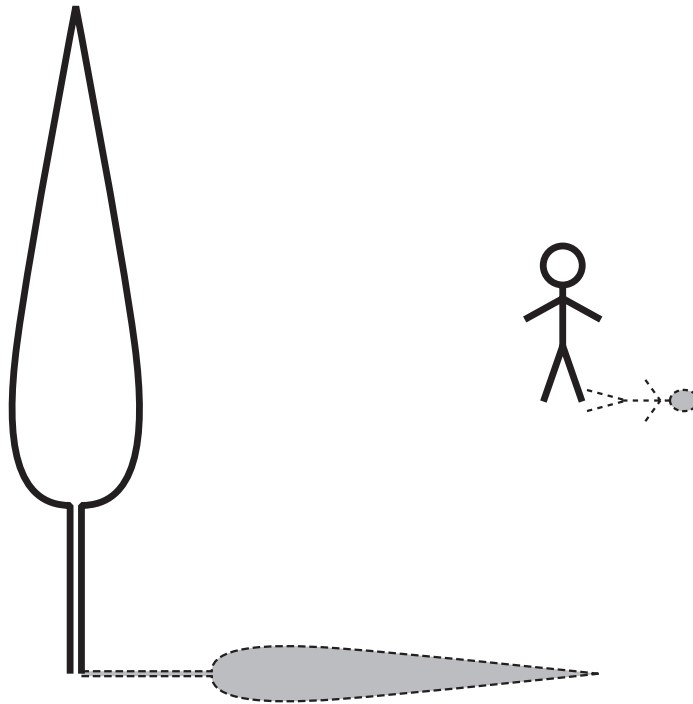
Resource sheet

Photographic enlargements



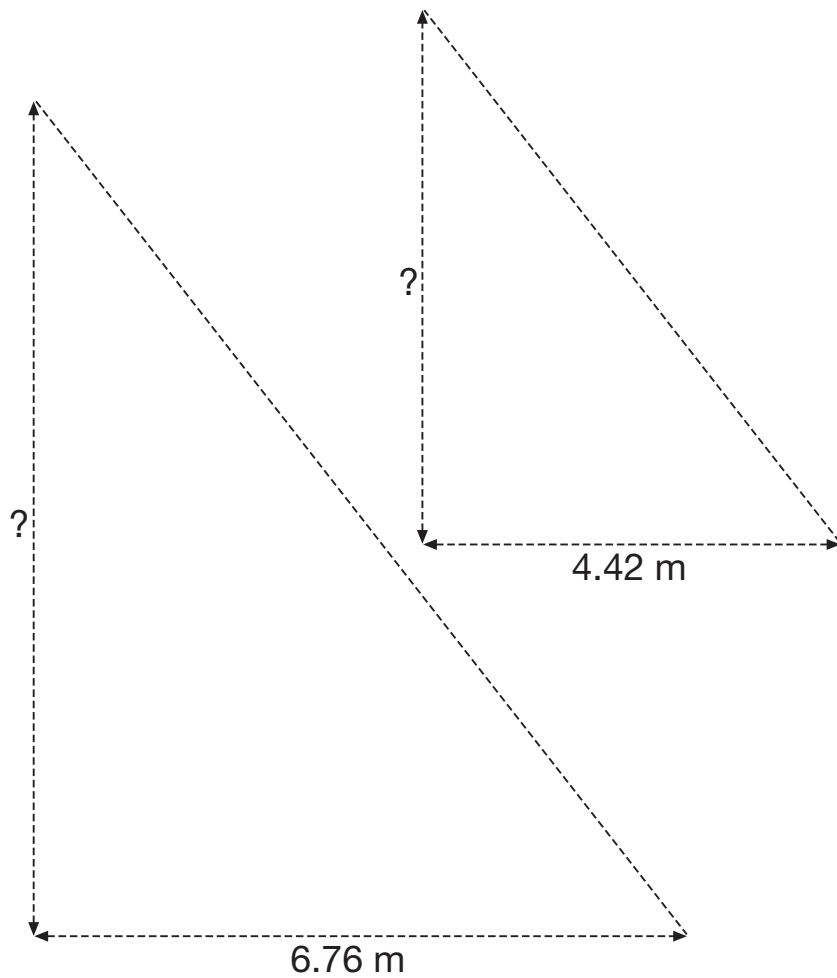
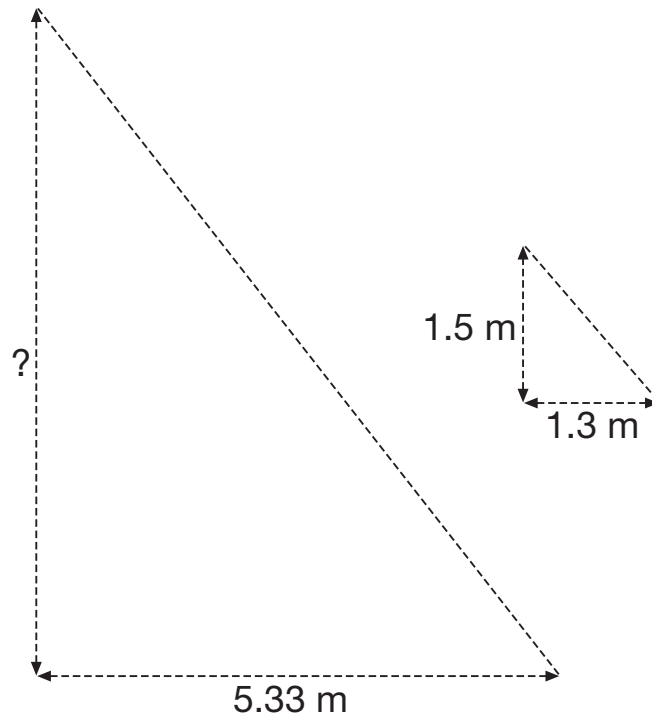
Resource sheet

Shadows 1



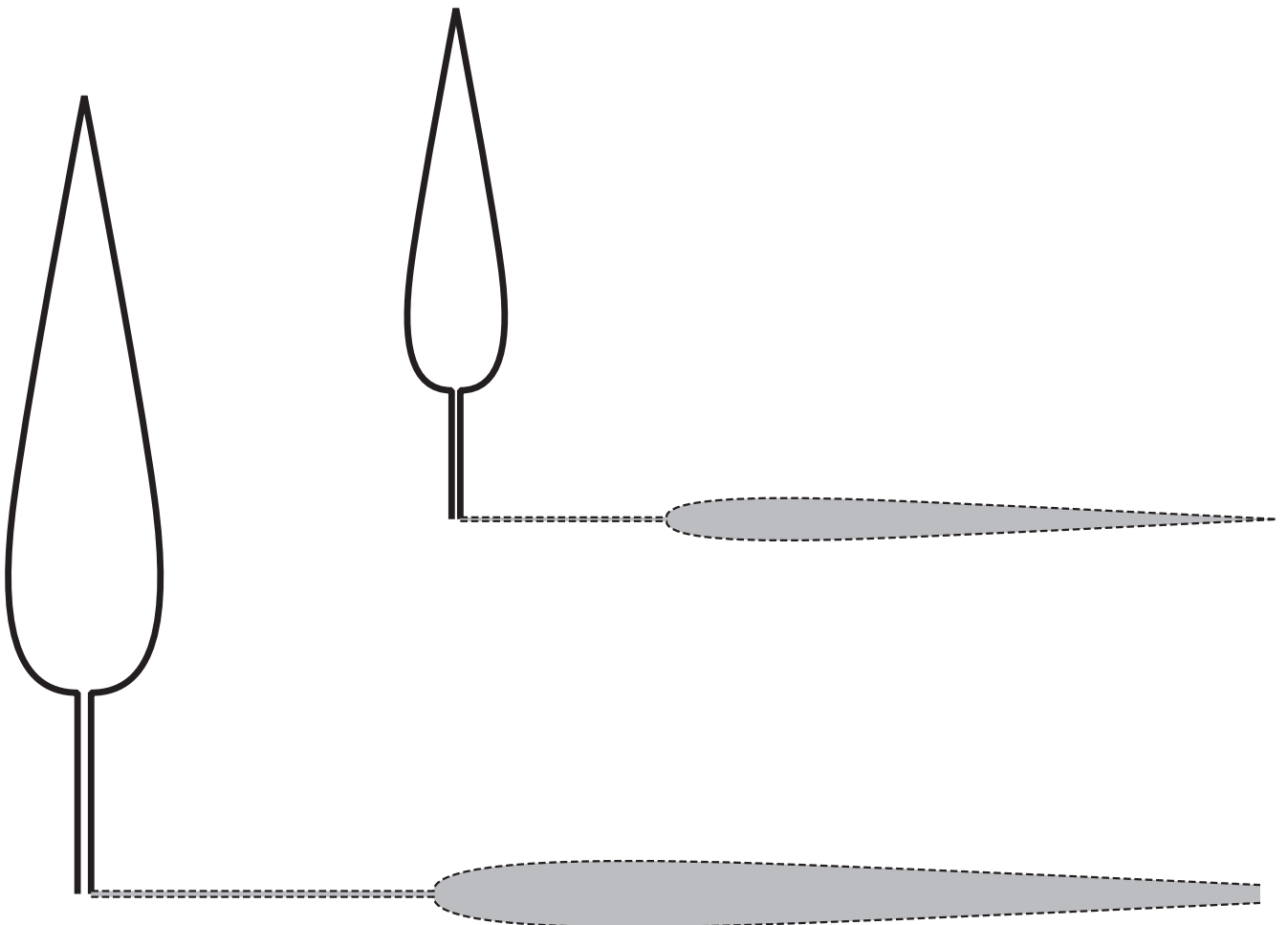
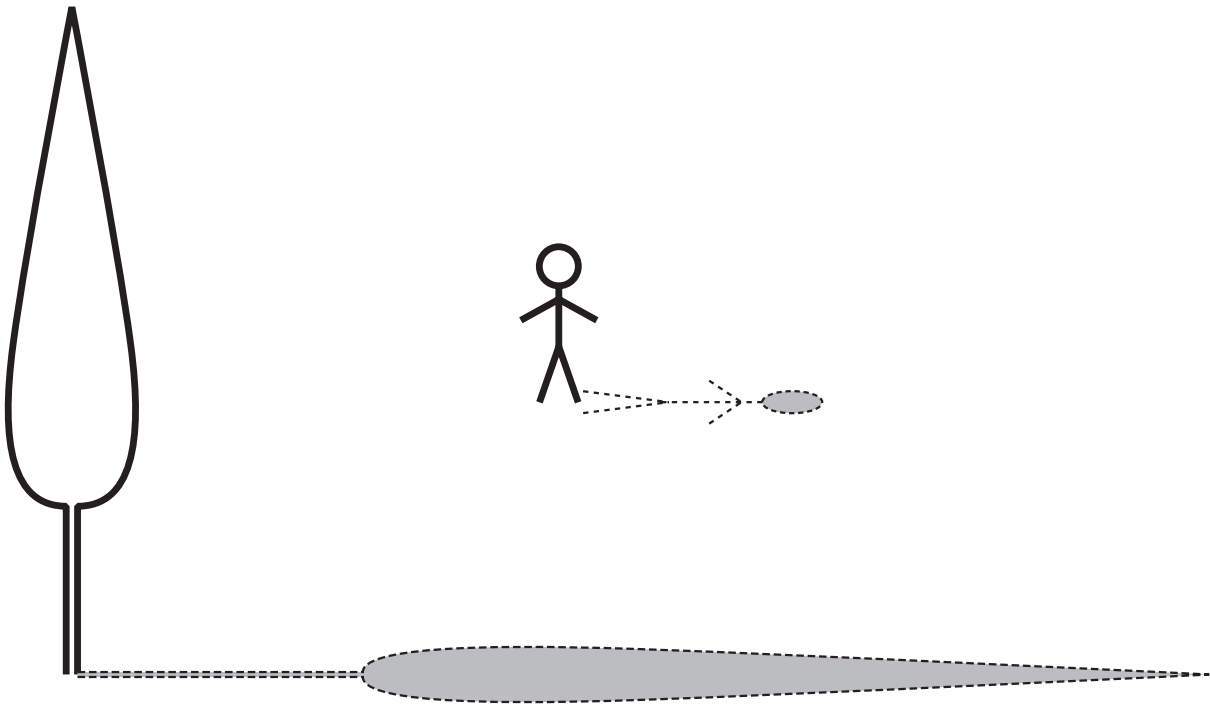
Resource sheet

Shadows 1 overlay



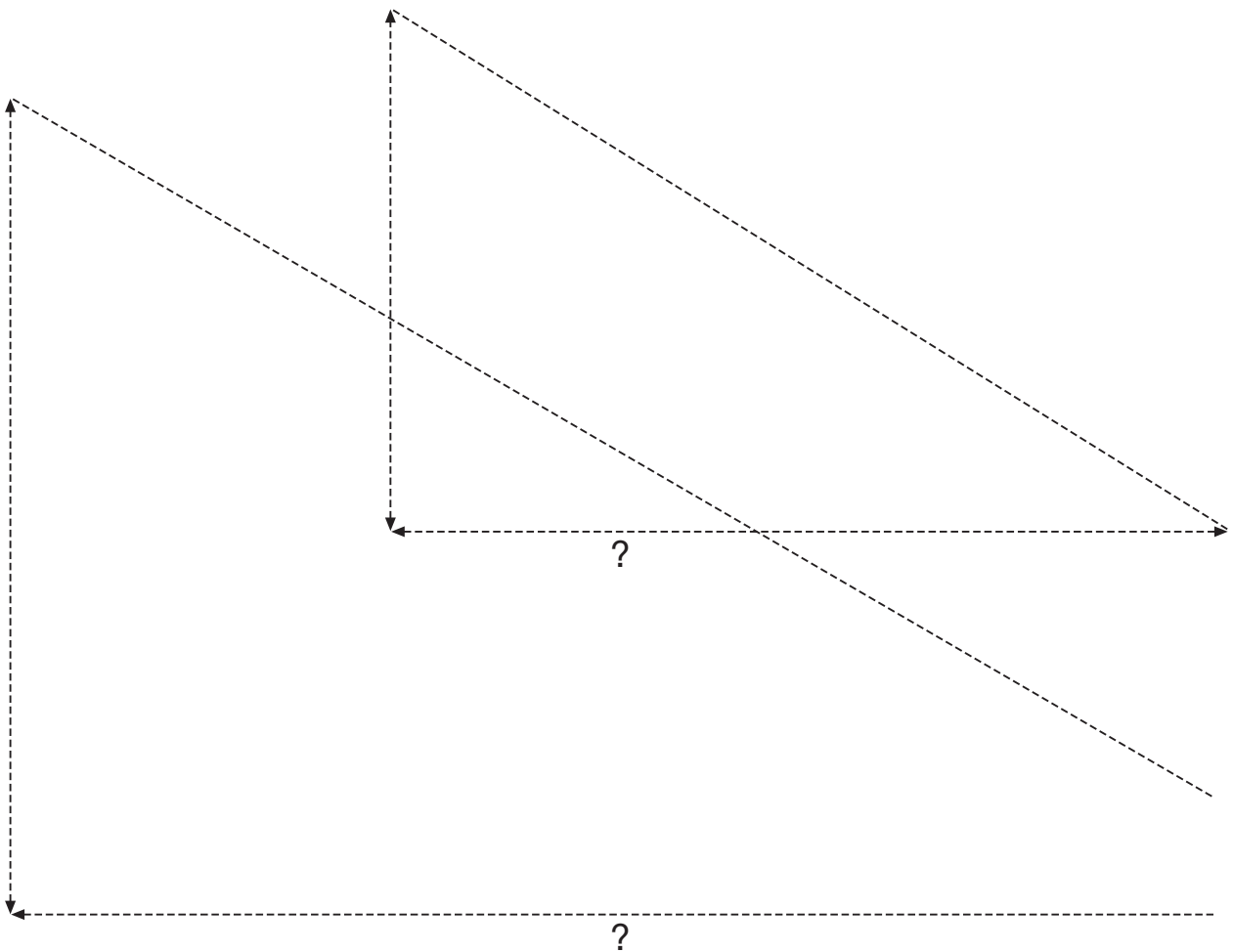
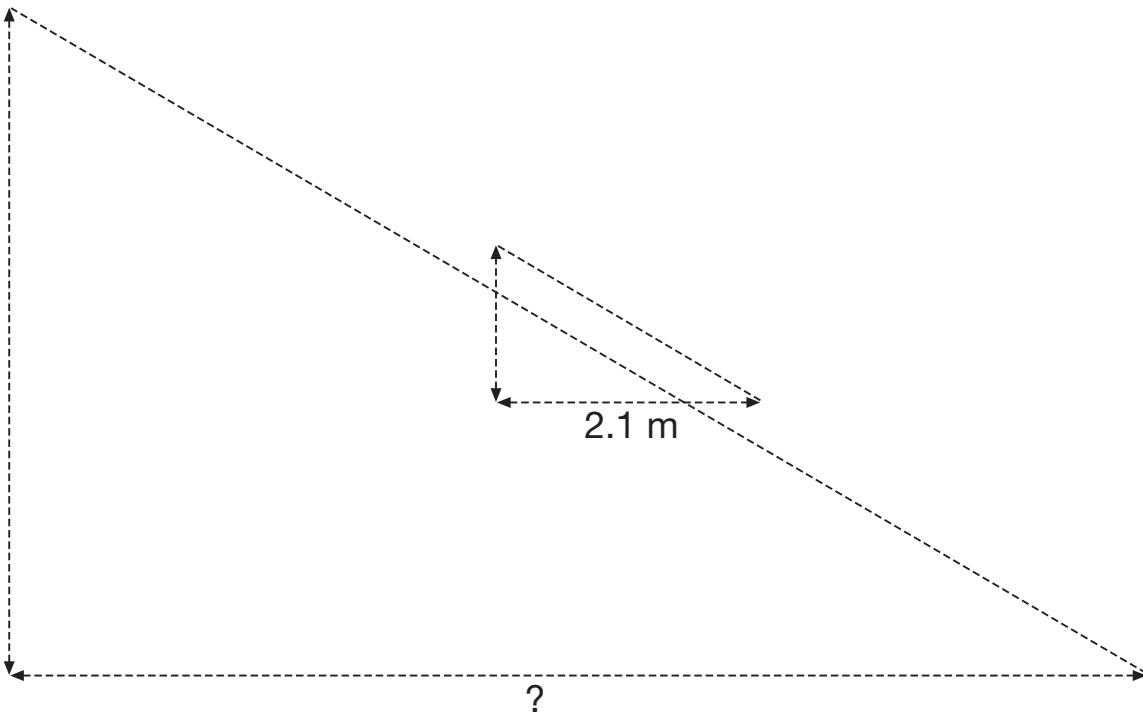
Resource sheet

Shadows 2



Resource sheet

Shadows 2 overlay

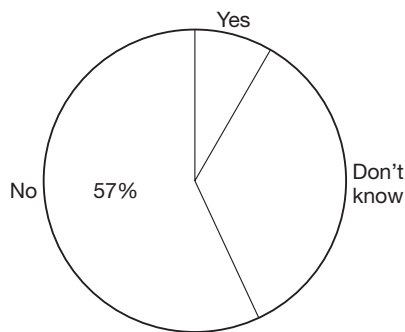


Resource sheet

Four problems: making the links

Six-term survey

This pie chart shows the results of a survey about changing the length and number of school terms. 158 responses were received.

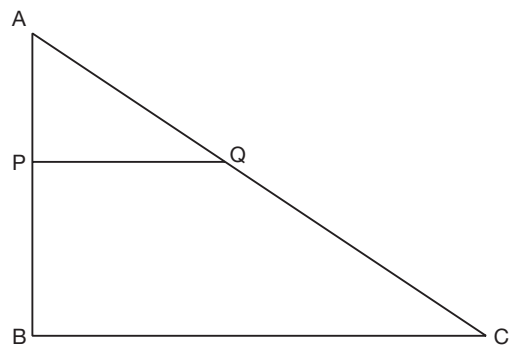


How many responses were 'No'?

55 responses were 'Don't know'. Calculate the percentage of 'Don't know' and 'Yes' responses.

Similar triangles

The triangles ABC and APQ are similar.



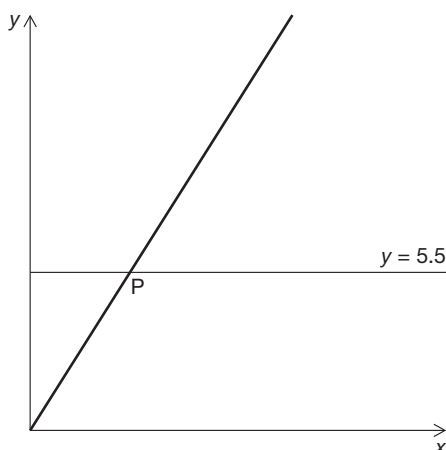
AP = 3.48 m, PQ = 5 m,
BC = 7.9 m

What is the scale factor of enlargement?

What is the length of AB?

Straight line

These points lie on the straight line below: (0, 0), (5, 7.9), (10, 15.8)



P is the point of intersection of this straight line with the line $y = 5.5$

What are the coordinates of P?

Currency

- Use $\text{£}1 = 1.58$ euros to work out how much 57p is in euros.
- Use 69.7 Indian rupees = $\text{£}1$ to work out how much 120 rupees is in pounds.
- Use $\text{£}1 = 1.58$ euros and $\text{£}1 = 69.7$ rupees to work out how much 1 euro is in rupees.

***Interacting with mathematics
in Key Stage 3***

*Year 9 proportional reasoning:
notes for departmental meetings*

Introduction

These notes can be used to guide your department through an exploration of the sample Year 9 unit on proportional reasoning. They have been structured to make best use of time and resources. Your usual departmental meetings may have an informal meeting style, but it is important to consider the benefits of structured group study promoted in these notes, while maintaining the ethos of the department. The following programme of action may help:

- Read the notes carefully in advance and consider sharing the management of the meetings with a colleague.
- Consider handing out copies of the unit plan in advance, with the expectation that everyone will read it before the first meeting.
- Make it clear to colleagues that you will want to work to certain timings in order to ensure that all issues are covered effectively in the time available – the sessions are timed at 75 minutes, but could be expanded to 90 minutes.

Each section of the notes is structured as follows. Key points and features are outlined in boxes. These are followed by suggested activities. The activities are designed to stimulate discussion and exploration of the mini-pack on proportional reasoning. Activities are followed by points you may wish to make in summary.

It is presumed that you have already incorporated elements of the Year 8 multiplicative relationships unit into your scheme of work and are preparing to teach the Year 9 proportional reasoning unit as a sequel. Any new colleagues will need to familiarise themselves with the unit on multiplicative relationships, and any adaptations you made to it, in advance of the meeting. Note also that revision of the Year 8 unit for future use is a separate discussion, for which there will not be time here.

Year 9 proportional reasoning: meeting 1

Objectives

- To review the effectiveness of the Year 8 unit on multiplicative relationships
- To outline the Year 9 unit on proportional reasoning
- To discuss teaching of the main activities in phases 1 and 2 of the unit

Resources

- Equipment: video player
- Video sequence 1, 'Multiplicative relationships: scale factors'
- Resource sheet 'Photographic enlargements' (OHT or poster size)
- For each teacher, unless otherwise indicated:
 - *Year 9 proportional reasoning: mini-pack* (you may wish staff to have a chance to read this prior to the meeting)
 - *Year 8 multiplicative relationships: mini-pack* (could be shared)
 - *Framework for teaching mathematics: Years 7, 8 and 9* (could be shared)
 - Plain paper for folding: A4 and trimmed A3 (i.e. non-metric)
 - Ruler, calculator

Session outline

75 minutes

Introduction and review of Year 8 work

Explaining the purpose of the two departmental meetings and reviewing the Year 8 multiplicative relationships unit

Talk and discussion

10 minutes

Exploring the Year 9 proportional reasoning unit plan

Outlining the unit and where it fits into the Year 9 plan

Reading time and talk

10 minutes

Scaling and proportional sets

Discussing main activities in phase 1 of the unit

Video and discussion

30 minutes

Enlargement and similarity

Discussing main activities in phase 2 of the unit

Reading and discussion

25 minutes

Key points

The Key Stage 3 mathematics strategy has produced two mini-packs to support mathematics departments in their planning for Year 9. The packs, similar to the Year 8 mini-packs, are part of the series *Interacting with mathematics in Key Stage 3*. The focus is on collaborative planning of a unit of work, designed to engage and challenge pupils across a wide range of attainment.

This proportional reasoning unit is a sequel to the Year 8 unit on multiplicative relationships.

The first meeting provides an opportunity to:

- review the Year 8 unit on multiplicative relationships;
- explore a nine-hour (three-week) sample unit on proportional reasoning described in three phases;
- discuss activities and teaching approaches in the main part of lessons in phases 1 and 2 of the unit.

The second meeting will provide an opportunity to:

- discuss the purpose of oral and mental starters in this unit;
- discuss how to teach strategies for solving problems involving ratio, proportion and percentages (phase 3 of the unit);
- discuss how the unit will be used.

Use the notes above to explain the background to the materials and outline what will be covered in the two departmental meetings.

Ask for *brief* comments from colleagues on the extent to which different classes progressed with the main ideas of the Year 8 multiplicative relationships unit:

- scaling numbers, including fractional scale factors and inverses;
- understanding proportion and tables of proportions;
- recognising problems involving multiplicative and proportional relationships and developing strategies for solving them.

In summary

- Different classes will have responded differently.
- The content was pitched to be challenging for many pupils.
- The Year 9 unit revisits the topic and keeps the level of challenge high.

Exploring the Year 9 proportional reasoning unit plan 10 minutes

Key points

- The proportional reasoning unit is about three weeks of work assuming nine lessons of around 1 hour.
- The unit is organised in a similar way to the Year 8 units and links closely with the content of the multiplicative relationships unit.
- Phase 1 (three lessons) revisits the idea of scaling numbers and the tabulation and solution of proportion problems within the field of pure number.
- Phase 2 (three lessons) offers images for proportion and scaling by making links with enlargement and similarity in geometry.
- Phase 3 (three lessons) revises, consolidates and extends strategies for solving problems, including examples involving rates, change of units and percentage changes.
- The oral and mental starters work towards automating key aspects of calculating with proportions.
- Activities within a phase are not prescribed lesson by lesson. A possible sequence is given for teachers to manage as appropriate to their class.

Allow time for everyone to read **pages 4–5** of the Year 9 proportional reasoning mini-pack (introduction and objectives). Use the notes above to highlight the key features of the unit.

Referring to the Framework guide (section 1 of the Framework) ask everyone to read the section on 'Proportional reasoning' on **pages 12 and 13**, paying particular attention to the four bullet points in the middle of page 13.

Next ask teachers to turn to **page 50** of the Framework guide, the example planning chart for Year 9. Use this chart along with the details in the introduction to the unit on **page 4** of the mini-pack, beginning: 'In planning the unit several decisions ...'. Together, these give a quick overview of how the unit fits into the sample plan for Year 9 and the adjustments made to other units.

In summary

As with the Year 8 materials, the challenge is to address this aspect of mathematics systematically. Pupils need to think multiplicatively and acquire a range of strategies for solving proportion problems.

The detail of rebalancing units in Year 9 can be considered after the two departmental meetings, when the emphasis on different objectives has become clear.

Key points

The main activities in phase 1 of the unit revisit and build on those in the Year 8 multiplicative relationships unit:

- the idea of scaling numbers is extended to consider repeated scalings;
- tables of numbers in proportion are revisited, extending to three columns, where extracts have to be made and alternative methods for finding unknowns are considered.

When studying phase 1 of the Year 8 unit, you will have worked together on the mathematics involved. This time, you will view and discuss a video which shows Year 9 pupils getting to grips with the basic idea of scaling between two numbers. They have not experienced the Year 8 unit on multiplicative relationships. After the videoed lesson, the pupils moved on to the content of the Year 9 proportional reasoning unit. The teacher is Raj and the class is an upper middle set, most of them aiming for level 6 in the Key Stage 3 tests.

Allow a minute or so for everyone to scan the outline of the main activities in phase 1 of the unit on **page 8** of the mini-pack.

Use the notes above to explain the context of the video sequence. Ask everyone to consider how Raj's approach to teaching scale factors compares with their own experience of teaching the Year 8 unit. Show video sequence 1, 'Multiplicative relationships: scale factors', pointing out that it is about **16 minutes** long.

After viewing the video, discuss the following questions:

- How do the precursors enable pupils to engage with the main teaching about scale factors?
- What strategies does Raj use to emphasise the importance of mathematical language?

Follow the discussion by asking everyone to spend a minute or two working through the 'Prompts for main activities in phase 1' on **pages 17–20** in the mini-pack, giving particular attention to the section on proportional sets.

In summary

In order for pupils fully to understand proportionality it is necessary to draw attention to interconnected mathematical ideas. This can be done at various levels:

- within a carefully structured lesson;
- within a unit of work (e.g. the Year 9 unit links across mathematical strands);
- in general, by using teaching strategies which bridge between units, emphasising mathematical discussion and clear use of language.

There is a second (13 minute) sequence on the video, showing an extended set of precursor ideas. Viewed at a later date, this could help with a more detailed analysis of mathematical interconnections.

Key points

As well as addressing part of the shape and space curriculum, enlargement and similarity are included because:

- links between different aspects of mathematics can be made explicitly;
- this provides visual images to extend pupils' understanding and help them grasp more clearly the fundamental concept of proportion.

There are three main activities in phase 2, all with detailed prompts for teaching:

- 'Folding paper' – teachers who attended the course 'Planning and teaching mathematics' will have discussed this activity and worked through part of the task; they could be asked to take the department through the examples;
- 'Cat faces' will not be examined here, but note that this activity can be used to draw out that there are both 'within' and 'between' relationships in any proportion situation;
- 'Photographic enlargements'.

There are also prompts for the final plenary in phase 2:

- 'Shadows'. This is a development of ideas which teachers may recall from video used in the Key Stage 3 conference on the National Numeracy Strategy in the summer of 2000. The lesson was focused on ratio and proportion and was taught by Walt. Schools have this video in the purple box of course materials. Participants on the course 'Planning and teaching mathematics' have also seen this video. The activity is a precursor to trigonometry.

Allow a minute or so for everyone to scan the main activities for phase 2 on **pages 8–9** of the plan of the unit. Use the notes above to clarify why this section is included.

Refer to 'Prompts for main activities in phase 2' on **pages 21–24** in the mini-pack. Allow about **10 minutes** to discuss the first activity, 'Folding paper'.

- Very briefly discuss folding a square sheet (or try the alternative visualisation exercise).
- Work in pairs on 'Folding a rectangular sheet', giving some pairs two sheets of A4 and other pairs two sheets of trimmed A3 (i.e. non-metric).
- Compare data for both shapes of paper and discuss the merits of considering non-metric as well as metric paper.

Spend about **10 minutes** on the third main activity, 'Photographic enlargements':

- Display an OHT or poster-size copy of the resource sheet, 'Photographic enlargements'; point out that there are 18 missing values – dimensions, scale factors and internal ratios.
- Discuss what values might be given, sufficient for all others to be calculated.
- Discuss other possible sets of given values. What is the minimum number required?
- Consider whether it would capture pupils' interest if actual values were hidden under flaps.

Allow everyone a minute or two to scan the 'Prompts for main activities in phase 2' and the 'Shadows' plenary on **page 28**.

In summary

The activities could be trialled with classes in other year groups. With familiarity, it should be clearer how to sequence lessons within this phase of the unit.

Sequencing lessons and making links between phases of the unit will be discussed at the next meeting.

Year 9 proportional reasoning: meeting 2

Objectives

- To consider the purpose of oral and mental starters in this unit
- To discuss how to teach the techniques required to solve proportion problems
- To decide on practical steps needed to implement the unit
- To discuss any general implications for the work of the department

Resources

- For each teacher, unless otherwise indicated:
 - *Year 9 proportional reasoning: mini-pack*
 - *Year 8 multiplicative relationships: mini-pack* (could be shared)
 - *Framework for teaching mathematics: Years 7, 8 and 9* (could be shared)
 - Resource sheet 'Four problems: making the links'
 - Calculators
 - Small whiteboards (optional)

Session outline

75 minutes

Introduction

Summarising the key points from the first meeting and clarifying the purpose of the second meeting

Talk

5 minutes

Exploring the oral and mental starters

Considering the role of oral and mental starters in this unit and trying some of the activities

Mathematical activity and discussion

20 minutes

Teaching problem solving

Discussing techniques to use when teaching pupils to solve proportion problems in phase 3

Mathematical activity and discussion

30 minutes

Planning the way forward

Considering how to use the unit and develop the key ideas

Discussion

15 minutes

Conclusion

Considering some general issues arising from the meetings on proportional reasoning

Discussion

5 minutes

Key points

The key points from the last meeting are:

- The Year 9 unit on proportional reasoning revises, consolidates and extends the ideas encountered in the Year 8 multiplicative relationships unit.
- Phase 1 develops repeated scaling and re-examines proportional sets of numbers.
- It is important to use teaching strategies that make connections between mathematical ideas and develop pupils' understanding.
- Links with enlargement and similarity enrich pupils' stock of images and help them to appreciate interconnections within mathematics.

In this meeting you will:

- consider the role of oral and mental starters in the unit;
- discuss how to teach techniques for solving proportion problems;
- consider how you will use the unit.

Use the notes above to remind everyone of key points from the last meeting and set out the plan for this meeting.

Key points

The main features of oral and mental starters in this unit are:

- The suggestions are not lesson-by-lesson prescriptions, but are intended to be used selectively by teachers, as judged to be appropriate for a particular class.
- Many activities rehearse skills or ideas which are precursors to later work. For example, 'Equivalent lists' incorporates ideas relating to percentage changes.
- Some processes need to be automated, so that pupils can concentrate on those aspects of solving the problem that will require greater thought.
- Developing fluency requires pace, working quickly through a series of examples, interspersed with short discussion of methods and links between ideas. The main part of the lesson is a better place for any extended discussion needed to secure understanding.

Engage everyone for **2 or 3 minutes** in the activity 'Equivalent lists', as described in the 'Prompts for oral and mental starters', on **page 12** of the mini-pack. Model the activity by keeping up a lively pace, and allowing time for several quick responses before discussing strategies. Spend another **couple of minutes** discussing the value of this as an oral and mental starter.

Ask everyone to turn to the unit plan on **pages 8–9** of the mini-pack and allow **a minute** or so for them to scan the oral and mental starters for each phase, emphasising the principles set out above.

Return to the 'Prompts for oral and mental starters' on **pages 10–16** of the mini-pack. Suggest that teachers spend about **5 minutes** in pairs, talking through the suggestions for phases 1 and 2 of the unit.

Note that the suggestions for phase 3 are identical to those for the Year 8 unit, with improved layout. Allow **5 minutes** in pairs to talk these through, sharing experiences of using the ideas with Year 8 pupils and how they might be developed further in Year 9.

In summary

If the oral and mental starter is extended, the balance of the lesson can be maintained by planning a briefer plenary, and vice versa. In some instances, this is indicated on the unit plan.

Key points

Pupils often have difficulties when solving problems involving proportionality – the video of pupils discussing test questions (published with the Year 8 multiplicative relationships unit) illustrates this point. One of the aims of the Year 8 unit was to enhance the traditional approach of working individually through exercises by including more discussion of problem-solving strategies.

The Year 9 materials develop these approaches further and the notes on pages 19–22 of the *Year 8 multiplicative relationships: mini-pack* are still appropriate. For example:

- Choose *one* problem: discuss alternative strategies for solving the problem; change the numbers in the problem (e.g. make them more difficult) and consider how the methods can be adapted; ask different or supplementary questions from the same context.
- Choose *a small set* of problems: concentrate on the translation and deciding on possible methods of solution, rather than working the problems through to an answer.

The Year 9 pack includes additional suggestions for:

- problems involving rates and change of units;
- problems involving percentages;
- using and applying the structure of proportional relationships in different contexts.

A bank of suitable problems, from all strands of mathematics, needs to be collected for this phase of the unit. Suggested sources include:

- the problem bank on pages 23–28 of the *Year 8 multiplicative relationships: mini-pack*;
- further questions selected from past Key Stage 3 test papers;
- Framework supplement of examples, pages 3, 5, 21, 25, 75–81, 137, 167, 217, 229, 233, 269 (also available as on the website www.standards.dfes.gov.uk/keystage3);
- textbooks.

Ask everyone to turn to the outline of the main activities for phase 3 on **page 9** of the unit plan in the Year 9 proportional reasoning mini-pack. Note that the suggestions are almost identical to those for Year 8, but with reference to additional ideas in the notes.

Ask everyone to spend a minute or two scanning **pages 19–22** of the Year 8 multiplicative relationships mini-pack, as a reminder of the approaches to problem solving developed in that unit. Note that, with familiarity, these activities become even more productive as pupils mature.

Returning to the Year 9 mini-pack, find the ‘Prompts for main activities in phase 3’ on **pages 25–27**. Spend about **10 minutes** working on the first two suggested activities: ‘Rates and change of units’ and ‘Percentage changes’. For each:

- Individually, read the two examples.
- Make up a problem like one of the given examples. **Either**
 - (a) give the problem to a colleague to make a table of proportions; **or**
 - (b) give the table and ask them to devise a possible question.

Now spend about **10 minutes** on the third activity: 'Links between different strands of mathematics'.

- From the resource sheet 'Four problems: making the links' construct tables of data for each problem. (Share the work to save time – possible tabulations are given on **page 14** of these notes.)
- Discuss any issues that arise in formulating the tables, e.g. what headings to use.
- Compare the tables, discuss the relationships between the data they reveal and identify what mathematical links there are between four apparently different contexts.

Remind everyone of possible sources of problems. Allow a couple of minutes for pairs to scan the page references to the Framework supplement of examples (see the key points box).

In summary

Tabulating data as suggested can help all pupils to make links, and will directly support those pupils who are struggling with the solution.

Asking pupils to make up examples, or giving them a set of data and asking them to devise a question to match it, is a valuable activity.

As teachers, we naturally make links. Pupils need help in establishing these links. These activities offer some suggestions.

Planning the way forward

15 minutes

Key points

When deciding how to implement the Year 8 multiplicative relationships unit, you considered various possibilities. Since this Year 9 unit is a sequel, the following points will need to be considered:

- how the Year 8 unit was used and the response of different groups of pupils;
- how this unit compares with current teaching of proportional reasoning in the Year 9 scheme of work;
- how to move forward with the unit you have been examining.

Discuss with the department how you will move forward from here.

Agree details of any practical tasks that need to be completed.

Conclusion

5 minutes

Allow a few minutes to pick up general points from your meetings. In particular:

- Are there aspects of Raj's teaching which you would consider adopting as effective strategies? How could you work together as a department on this?
- Has the structure of this unit and the Year 8 unit suggested a model for tracking through a mathematical topic and considering issues of progression across two or more years?
- Has the content of this unit suggested ways in which further links could be developed between strands of the yearly teaching programmes?

Four problems: making the links

Six-term survey

| | | | | |
|--------|-----|----|------------|-------|
| | Yes | No | Don't know | Total |
| % | ? | 57 | ? | 100 |
| Number | ? | ? | 55 | 158 |

Or

| | | |
|--------|-----|------------|
| Number | % | Total |
| 158 | 100 | Yes |
| ? | ? | No |
| ? | 57 | Don't know |
| 55 | ? | |

Similar triangles

| | |
|--------------|--------------|
| Triangle APQ | Triangle ABC |
| AP = 3.48 | AB = ? |
| PQ = 5 | BC = 7.9 |

Or

| | | |
|--------------|------|------|
| | Side | Base |
| Triangle APQ | 3.48 | 5 |
| Triangle ABC | ? | 7.9 |

Straight line

| | |
|-----|------|
| x | y |
| 0 | 0 |
| 5 | 7.9 |
| 10 | 15.8 |
| ? | 5.5 |

Or

| | | | | |
|-----|---|-----|------|-----|
| x | 0 | 5 | 10 | ? |
| y | 0 | 7.9 | 15.8 | 5.5 |

Currency

| | | |
|------|-------|--------|
| £ | Euros | Rupees |
| 1 | 1.58 | |
| 0.57 | ? | |
| 1 | | 69.7 |
| ? | | 120 |
| | 1.58 | 69.7 |
| | 1 | ? |

Interacting with mathematics in Year 9

Key Stage 3
National Strategy

Evaluation form

For completion by teachers

LEA: _____ Tutor: _____

What were the most successful aspects of the course?

What changes would you suggest if the course were repeated?

Please grade each session for usefulness and clarity.

| Session | Grade* | | | | Comments |
|-------------------------------------|----------------|---|---|------|----------|
| | Very useful | ← | → | Poor | |
| 1 Geometrical reasoning 1 | 1 | 2 | 3 | 4 | |
| 2 Geometrical reasoning 2 | 1 | 2 | 3 | 4 | |
| 3 Proportional reasoning 1 | 1 | 2 | 3 | 4 | |
| 4 Proportional reasoning 2 | 1 | 2 | 3 | 4 | |
| Overall grade for the course | 1 | 2 | 3 | 4 | |

* Grade: 1 = very useful; 2 = useful; 3 = limited use; 4 = poor

School: _____ Post held: _____

Please return this form to your tutor before leaving.

