Section 2

Booster lessons

Template

Booster lessons

Oral and mental starter [Suggested timing]

Objectives
From the Framework teaching programmes (with adaptations)
Need to be specific
Vocabulary
From the Framework
Include notation
Resources
To support the activities

In booster lessons oral and mental work should focus on skills needed in the main part of the lesson and might include:

- factual recall;
- developing and explaining strategies;
- applying calculation skills;
- developing estimation skills;
- interpreting data;
- visualising shapes;
- developing mathematical vocabulary;
- developing the ability to generalise, prove and reason.

Main teaching [Timing to maintain pace]

Objectives
From the Framework teaching programmes (with adaptations)
Need to be specific
Vocabulary
From the Framework
Include notation
Resources
To support the activities

The main part of a booster lesson can be used to:

- consolidate previous work;
- develop vocabulary and the use of correct notation;
- use and apply concepts and skills;
- assess and review concepts and skills using examples from the Framework.

Use direct interactive teaching to:

- direct and tell;
- demonstrate, explain and model;
- question and discuss both facts and understanding;
- explore and investigate;
- consolidate and embed;
- reflect and evaluate;
- summarise and remind.

Consider grouping pupils – class, small groups, pairs, individual work. Make connections between topics. Rectify misconceptions. Booster lessons should not just involve practising past questions. There needs to be interaction between teacher and pupil and between pupils.

Plenary [Suggested timing]

By the end of the lesson pupils should:
…
Framework supplement of examples page …
[This gives examples of what pupils should be able to do at levels 5 and/or 6.]

The plenary can be used to:

- summarise key facts, ideas and vocabulary;
- generalise from examples generated earlier;
- assess work informally through key examples and questions, and rectify remaining misconceptions;
- make links to other and future work;
- highlight progress made towards pupil targets;
- set homework.

Lesson 1 Place value

Oral and mental starter 10 minutes

Objectives

Understand and use decimal notation and place value; multiply and divide integers and decimals by 10, 100, 1000, and explain the effect (Y7)

Read and write positive integer powers of 10 (Y8)

Vocabulary

place value, zero place holder, tenths, hundredths, thousandths, equivalent, equivalence, power, index

Resources

OHT of M1.1

Ask pupils to write the number 5.7, multiply it by 10 and record their answer.

Ask a pupil to read their answer aloud and to explain how they arrived at it.

Q What has happened to the digits? (Note that the decimal point does not move; the digits shift one place to the left.)

Repeat with numbers involving one and two places of decimals, asking pupils to multiply and divide by 100 and 1000.

Q What do 10^2 and 10^3 mean?

Q How do you write 10 000 as a power of 10? How do you write 10 as a power of 10? What about 1?

Ensure that pupils recognise that increasing powers of 10 underpin decimal notation.

Extend to division to cover 10^0 and negative powers of 10.

Choose numbers from the place value chart (**OHT M1.1**) and ask pupils to multiply and divide them by integer powers of 10 (using powers of 10).

Main teaching 40 minutes

Objectives

Extend knowledge of integer powers of 10 (Y9)

Multiply and divide integers and decimals by 0.1, 0.01 (Y8)

Vocabulary

place value, zero place holder, tenths, hundredths, thousandths, equivalent, equivalence, power, index

Resources

OHT of M1.2
Handouts of M1.1
OHT of M1.3 (for plenary)

Introduce the spider diagram (**OHT M1.2**). Ask pupils to explain the results and to consider other solutions, using the digits 4, 0 and 1 only. Invite pupils to show their responses and to talk through their reasoning. Make sure that $4 = 0.04 \div 0.01$ is considered.

Discuss what happens when a number is multiplied/divided by a number less than 1.

Q Does division always make a number smaller?
Does multiplication always make a number larger?

Repeat with 5.7 in the centre of the spider diagram and record pupils' responses. Establish that there are several different ways of recording the answers. For example:

$$0.057 \times 100 = 5.7 \qquad 0.057 \div 0.01 = 5.7$$
$$0.057 \times 10 \times 10 = 5.7 \qquad 0.057 \div 0.1 \div 0.1 = 5.7$$
$$0.057 \times 10^2 = 5.7 \qquad 0.057 \div 10^{-2} = 5.7$$

Ask pupils to work in groups to develop their own spider diagram showing equivalent calculations for other numbers (e.g. 3.2, 67.3, 0.43).

Remind them that their work should be recorded clearly.

Use place value charts (**handout M1.1**) to support pupils as appropriate.

Differentiate by using different starting numbers for different ability groups and encourage pupils to record their responses in ways in which they are confident.

Move pupils on to develop their understanding of place value and notation through the following stages:

- multiplication/division by 10, 100 and 1000
- multiplication/division by positive integer powers of 10
- multiplication/division by 0.1 and 0.01
- multiplication/division by negative integer powers of 10

Plenary 10 minutes

By the end of the lesson

pupils should understand that:

- when a number is multiplied (divided) by 10/100/1000, the digits move one/two/three places to the left (right)
- multiplying by 0.1 has the same effect as dividing by 10
- multiplying by 0.01 and dividing by 100 are equivalent
- multiplying by 10^{-3} and dividing by 10^3 are equivalent

Framework supplement of examples, page 39
Levels 5 and 6

Use the place value target board (**OHT M1.3**).

Q If I divide a number by 0.1 and then again by 0.1 the answer is 0.03. What number did I start with? How do you know?

Q Why do $3.3 \times 10 \times 10$ and $3.3 \div 0.01$ give the same answer?

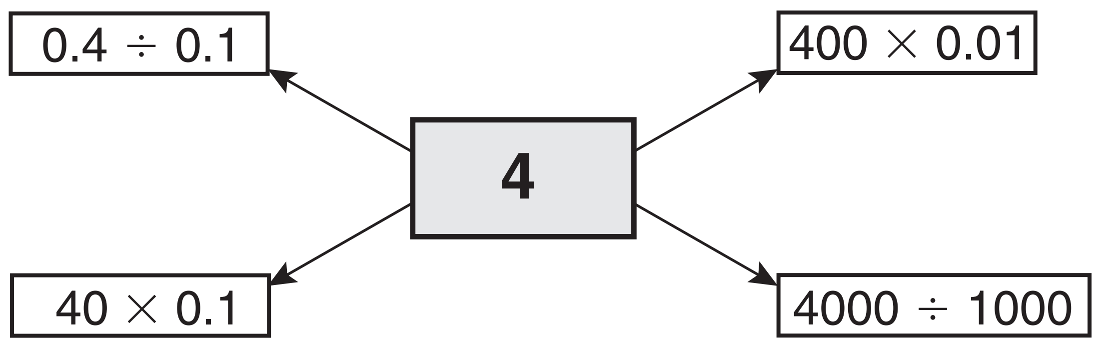
Ask similar questions to check pupils' understanding.

Place value chart

M1.1

0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000

Place value spider diagram



Place value target board

0.4	17.6	3
0.33	30	156
4000	1.56	1.76
3.3	0.0176	0.0003

Lesson 2 Fractions, decimals and percentages 1

Oral and mental starter — 15 minutes

Objectives
Use the equivalence of fractions, decimals and percentages to compare proportions (Y8)
Vocabulary
equivalent, recurring decimal
Resources
OHTs of M2.1 and M2.2

Show **OHT M2.1** and ask pupils to find the other two values – fraction, decimal or percentage – equivalent to the one you point at.

Q If you know $\frac{1}{5}$ as a decimal, how do you find $\frac{4}{5}$?

Pay particular attention to 0.4 and 0.04, 0.3 and 0.03.

Next, show **OHT M2.2** and ask pupils to calculate mentally the fractions, decimals or percentages of £240. Write '£240' in the centre of the web.

Q How did you work out …?

Q Do you find it easier to use fractions, decimals or percentages when calculating in your head?

Change the starting amount (e.g. 40 grams) and repeat the process.

Q Which calculations can still be done mentally?

Main teaching — 35 minutes

Objectives
Calculate percentages (Y7, 8, 9)
Vocabulary
unitary method
Resources
OHP calculator
Calculators for pupils
OHT of M2.3

Ask pupils to calculate 13% of 48 and then discuss in pairs how they did the calculation. Invite one or two pairs to explain their methods.

Discuss the methods used. These might include:

- 10% + 1% $\times$ 3
- Find 1%, then 13% (unitary method)
- 0.13 $\times$ 48

Explain the equivalence of 13%, $\frac{13}{100}$ and 0.13.

Use an OHP calculator to demonstrate the key sequence to calculate 13% of 48.

Give pupils a few examples to practise similar calculations on their own calculators. Include examples such as 8% of £26.50, $12\frac{1}{2}$ % of £98.

Check pupils' understanding of these examples.

Ask pupils where they might see percentages in real life. Introduce **OHT M2.3**, which includes some applications of percentages, and ask pupils to solve the problems.

For level 6 use questions from the Year 9 supplement of examples, page 75.

Plenary — 10 minutes

By the end of the lesson
pupils should be able to:

- understand and calculate equivalent fractions, decimals and percentages
- use a calculator to calculate percentages of quantities

Framework supplement of examples, pages 73–75
Level 5

Pick two of the questions on **OHT M2.3** and ask pupils to share their methods and solutions.

Q What is the equivalent decimal?

Q How did you find 3.5%? Show the key sequence on the OHP calculator.

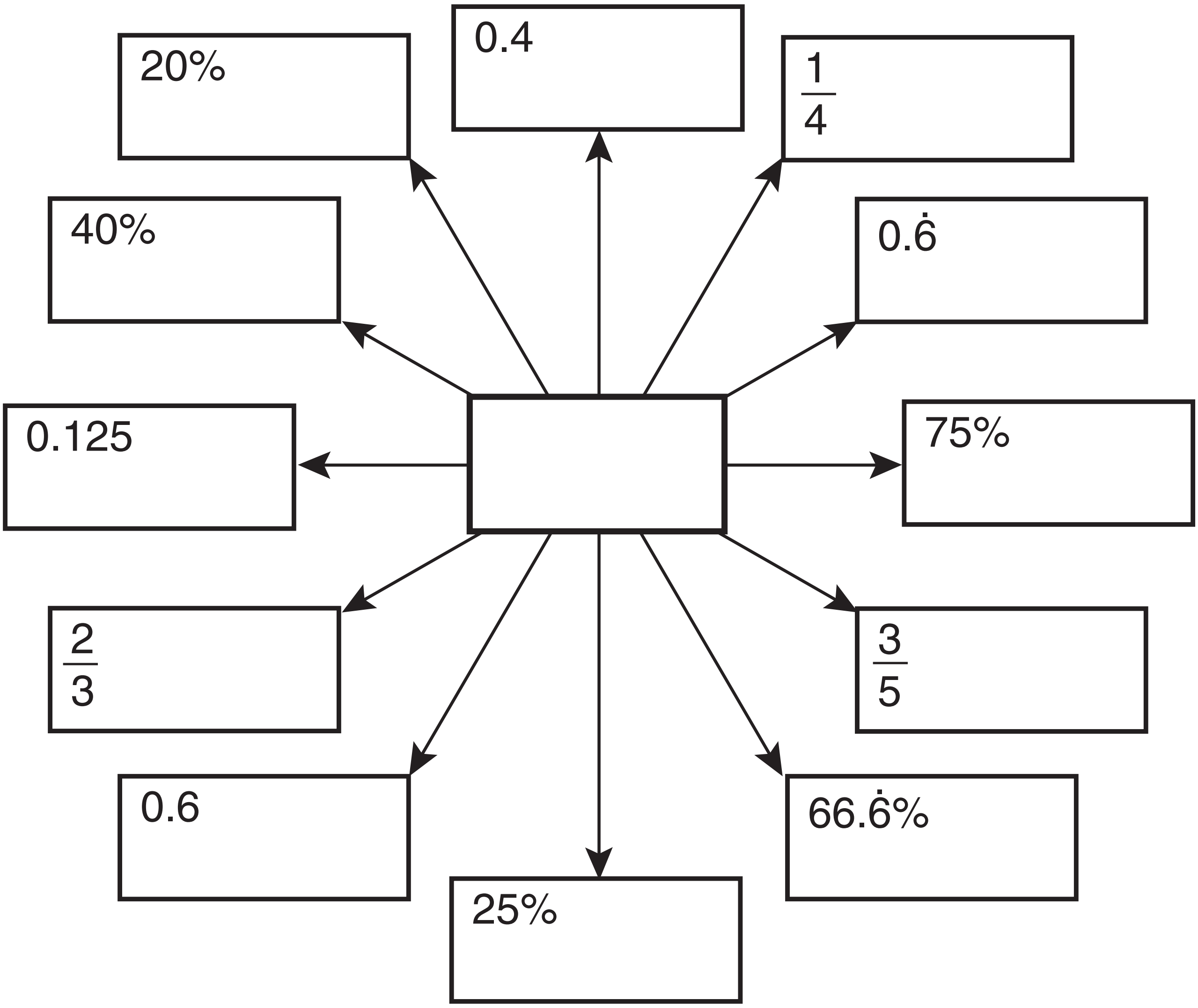
You could extend calculations to include a percentage increase, which is covered in lesson 3.

Matching fractions, decimals and percentages

M2.1

20%	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	0.125	$\frac{2}{3}$
$\frac{3}{10}$	$\frac{3}{4}$	12.5%	$\frac{1}{50}$	$\frac{1}{4}$	30%
25%	0.02	$\frac{1}{5}$	0.6	75%	2%
0.1	$66.\dot{6}\%$	0.25	$\frac{1}{8}$	$0.\dot{6}$	60%
0.4	0.2	$\frac{1}{25}$	$0.\dot{3}$	40%	0.04
4%	$\frac{3}{5}$	$33.\dot{3}\%$	10%	0.3	0.75

Fractions, decimals and percentages spider diagram



Percentage problems M2.3

1 An alloy is made from 95% copper, 3.5% tin and 1.5% zinc. How much tin is there in 1 kg of the alloy?

2 The population of Greece is 10 million. 37% of the population is aged 15 to 39. How many people is this?

3 An ice-cream label lists the contents as 17.5% fat. How much fat is there in 750 g of ice-cream?

4 A fabric is made from 83% viscose, 10% cotton and 7% nylon. How much viscose is needed to make 1.5 tonnes of the fabric?

Lesson 3 Fractions, decimals and percentages 2

Oral and mental starter 15 minutes

Objectives

Calculate percentages and find the outcome of a given percentage increase or decrease (Y8)

Vocabulary

percentage increase, percentage decrease, partition

Resources

OHT of M3.1

Using the target board on **OHT M3.1**, ask pupils to calculate a percentage increase/decrease of one of the amounts. Invite them to explain how they arrived at their answers. Discuss their methods.

Encourage pupils to use jottings, when appropriate, to record steps in their working.

Q How did you work that out?

Q How did you partition 35%?

As preparation for the main teaching, ask:

Q If you increase an amount by 15%, what percentage of the original will you then have?

Main teaching 30 minutes

Objectives

Calculate percentages and find the outcome of a given percentage increase or decrease (Y8, 9)

Vocabulary

percentage increase, percentage decrease

Resources

Objects (e.g. pencils)
Whiteboards (if available)
OHP calculator
Calculators for pupils
OHT of M3.2

Discuss examples involving whole numbers of objects, using statements such as:

Q If something increases by 100%, it doubles. What percentage do you then have?

Q How can you describe an increase by 500%?

Demonstrate this pictorially or with real objects. You need to explain that you have the original 100% *plus* the increase of 500%.

Model an increase of 10%. Demonstrate that this results in a total of 110%: 100% can be represented by 10 pencils, so 1 pencil represents 10% and the new amount, 11 pencils, is 110%.

Q How do you write 110% as a decimal?

Model a decrease of 10%. Demonstrate that this leads to 90%: 10 pencils represent 100%, so 1 pencil represents 10%; the new amount of 9 pencils represents 90%.

Q How do you write 90% as a decimal?

Repeat this with another example, such as a 20% increase/decrease.

Use a set of short questions to assess whether pupils can generalise these results.

Q If something increases by 15%, what percentage of the original amount do you then have? (Check that pupils have written 115% on their whiteboards.)
How do you write that as a decimal?

Repeat this with a decrease of 35%. Pupils write 65% to represent the final amount.

Q How do you write this as a decimal?

Extend to decreasing £450 by 17%. Recap how to calculate 83% of £450, using the OHP calculator (see lesson 2).

Q How would you increase £450 by 17%?

Pupils then need to practise similar calculations. Use the target board on **OHT M3.2**. These examples require the use of calculators.

For level 6 use questions from the Year 9 supplement of examples, page 77.

Plenary 15 minutes

By the end of the lesson

pupils should be able to:

- calculate percentages of quantities using a calculator
- calculate percentage increases and decreases

Framework supplement of examples, page 77
Levels 5 and 6

Discuss these problems:

Q I start with £250 on January 1st. This increases by 10% on February 1st. How much do I then have?
This further increases by 10% on March 1st. How much have I now?

Q I start with £250 on January 1st. This increases by 20% on March 1st. Is this the same result as before?

Discuss how a 10% increase followed by another 10% increase is not the same as a 20% increase. Go on to illustrate how a 20% increase + 20% increase is not the same as a 40% increase.

Percentage target board 1

Increase	Decrease

£40	70 cm
300 g	1 kg
£12	650 m

by

10%	15%	100%	35%	12.5%

Percentage target board 2 M3.2

Increase	Decrease

£70	83 cm
350 g	1 kg
£12.50	650 m

by

11%	17%	120%	38%	16.5%

Lesson 4 Using a calculator

Oral and mental starter — 10 minutes

Objectives

Make and justify estimates and approximations of calculations (Y7, 8, 9)

Vocabulary

estimate, significant digit, units digit

Resources

OHT of M4.1 (cut up the answers beforehand)
Calculator

Introduce the grid on **OHT M4.1** (note that the answers need to be cut up beforehand and spread around the grid; keep an uncut copy of the answer grid to use as a check). Ask pupils, in pairs, to match an answer with its question on the grid by estimating the answer.

Allow a few minutes, then check results:

Q How did you estimate that?

Q Did you consider the size of the numbers?

Q Did you use the unit digits?

Continue with other examples, involving other pupils, until the sheet is complete.

Main teaching — 40 minutes

Objectives

Use a calculator efficiently and appropriately to perform complex calculations with numbers of any size; use sign change keys and function keys for powers, roots, brackets and memory (Y9)

Vocabulary

square, square root, cube, fraction, brackets, keystroke

Resources

Calculators for pupils
OHP calculator
OHT of M4.2

Clarify, using examples, the use of the brackets, memory and sign change keys on a calculator. Pupils could make up an example for a partner. Invite individuals to demonstrate good examples to the class on the OHP calculator.

Explain to pupils that when doing calculations they need to decide which is the most efficient way of tackling the problem and which keys are the most appropriate.

Ask pupils to calculate problems 1–3 on **OHT M4.2**:

1 $(23 \times 37) - (42 \times 17)$

2 $(43.6 - 17.93)^3 + \sqrt{4.68}$

3 $\dfrac{63.2 \times 9.56}{8.2 - (3.5 - 1.49)}$

Invite pupils to demonstrate the order of their keystrokes using the OHP calculator. Highlight the use of the brackets, memory and sign change keys as appropriate.

Introduce and discuss problem 4:

4 Packets of biscuits are packed in boxes which hold 144 packets. A factory makes 20 000 packets of biscuits. Can all the packets be put into completed boxes? How many completed boxes will there be? How many packets will be left over?

Check that pupils are able to find the remainder and that they realise it should be an integer value.

Model ways to find this using a calculator:

> $20\,000 \div 144 = 138.8889$
> Then subtract 138, the number of complete boxes.
> 0.8889 is a fraction of a box. How many packets is this? (128 packets)

Now set problem 5:

5 How many days, hours, minutes and seconds are there in a million seconds?

Ask pupils to discuss the question in pairs.

Q How will you get started? Think about how to deal with the remainder.

Plenary — 10 minutes

By the end of the lesson

pupils should be able to:

- use a calculator efficiently, including the use of function keys for powers and roots, brackets, and memory
- deal with remainders using a calculator

Framework supplement of examples, pages 108–109
Levels 5 and 6

Invite a pair to explain and demonstrate their solution to problem 5 using the OHP calculator.

Make sure that pupils understand how to deal with the remainders.

As an extension, you could ask pupils how they would calculate the number of days, hours, minutes and seconds there are in 10 million seconds.

Estimation jigsaw **M4.1**

2.3×5.8	$5130 \div 95$	32×3.8	$371.2 \div 5.8$
$1421 \div 29$	6.5×9.8	$2769 \div 71$	38×68
51×61	$1769 \div 29$	71×19	$44.89 \div 6.7$
$2511 \div 81$	49×64	$345.8 \div 91$	49×51

Cut up one copy of the following into individual answers.
Keep a second copy as a check.

13.34	54	121.6	64
49	63.7	39	2584
3111	61	1349	6.7
31	3136	3.8	2499

1 $(23 \times 37) - (42 \times 7)$

2 $(43.6 - 17.93)^3 + \sqrt{4.68}$

3 $\dfrac{63.2 \times 9.56}{8.2 - (3.5 - 1.49)}$

4 Packets of biscuits are packed in boxes which hold 144 packets.

A factory makes 20 000 packets of biscuits. Can all the packets be put into completed boxes?

How many completed boxes will there be?
How many packets will be left over?

5 How many days, hours, minutes and seconds are there in a million seconds?

What about 10 million seconds?

Lesson 5 Ratio and proportion

Oral and mental starter 15 minutes

Objectives

Reduce a ratio to its simplest form (Y8)

Divide a quantity into two or more parts in a given ratio (Y8)

Vocabulary

equivalent, ratio notation

Resources

OHTs of M5.1 and M5.2

Show **OHT M5.1** and ask pupils to give ratios equivalent to the one in the centre of the diagram. Then ask:

Q Which is the simplest form of the ratio? How do you know?

Discuss simplifying ratios, including those expressed in different units such as 5p to £1, 400 g to 2 kg, 50 cm to 1.5 m.

Show **OHT M5.2** and discuss the unitary method for finding the amounts.

Ask pupils to complete the other questions mentally. Discuss their solutions.

The amount in the centre can be varied to produce questions that require mental, written or calculator methods for solution.

Main teaching 30 minutes

Objectives

Divide a quantity into two or more parts in a given ratio (Y8)

Use the unitary method to solve simple word problems involving ratio (Y8)

Vocabulary

unitary method

Resources

OHT of M5.3
OHT/handouts of M5.4
(for plenary)

Introduce the first problem on **OHT M5.3**:

1 The angles in a triangle are in the ratio 9:5:4.
Find the size of each angle.

Give pupils time to discuss this in pairs and attempt a solution. Then, through structured questions, lead pupils to the solution:

Q What fact do you need to know about triangles?

Q What does 9:5:4 mean?

Q How would you set out your working so that someone else can understand it?

In the same way, discuss the second question on OHT M5.3:

2 Green paint is made by mixing 2 parts of blue paint with 5 parts of yellow.
A girl has 5 litres of blue paint and 10 litres of yellow paint. What is the maximum amount of green paint she can make?

Now show pupils the third example on OHT 5.3 (Framework supplement of examples, page 79).

3 This recipe for fruit squash is for 6 people.

300 g	chopped oranges
1500 ml	lemonade
750 ml	orange juice

How much lemonade do you need to make fruit squash for:

(a) 9 people? (b) 10 people?

Ask pupils in pairs to discuss solutions to the questions, then invite individuals to explain and demonstrate how they arrived at their answers.

Sheet M5.4 lists further practice examples. Alternatively, you could use questions from previous Key Stage 3 test papers.

Plenary 15 minutes

By the end of the lesson pupils should be able to:

- divide a quantity in a given ratio
- solve simple problems using a unitary method

Framework supplement of examples, page 79–81
Levels 5 and 6

Discuss the solution of examples from **OHT M5.4**. For example:

In a game of rugby Rob's ratio of successful kicks to unsuccessful kicks was 5:3. Dave's ratio was 3:2. Who was the more successful?

Note that although Rob has the better ratio you cannot tell who had the greater number of successful kicks.

Consider examples such as:

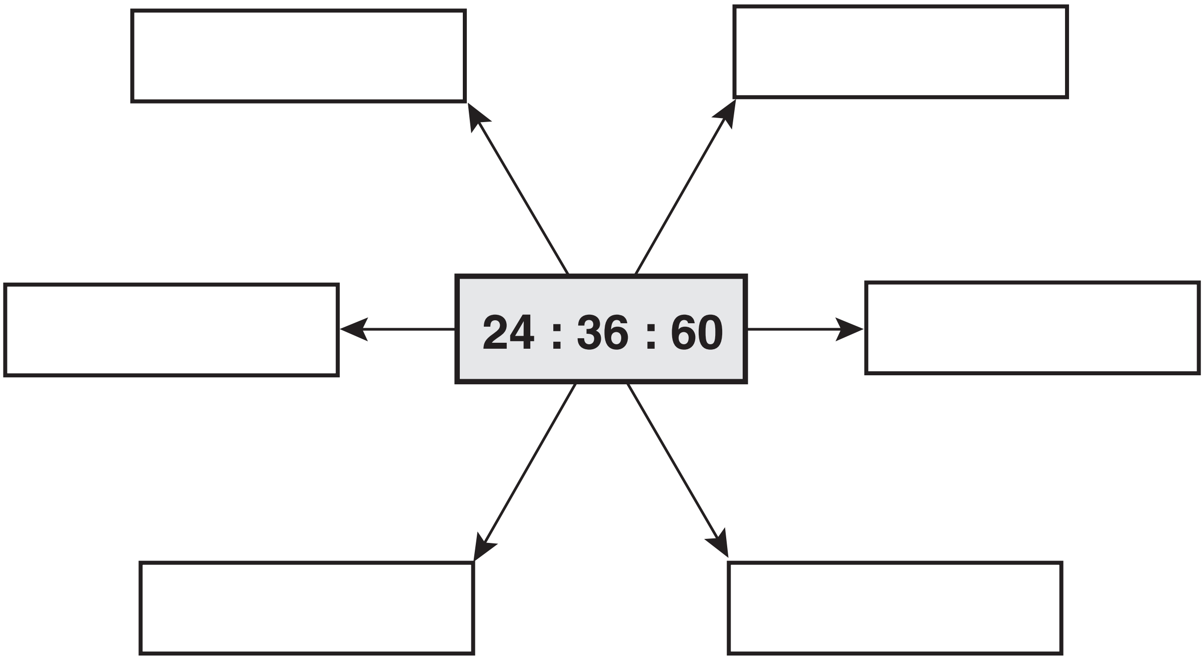
Can you split a class of 25 pupils in the ratio of 3:4?

Introduce questions of the type:

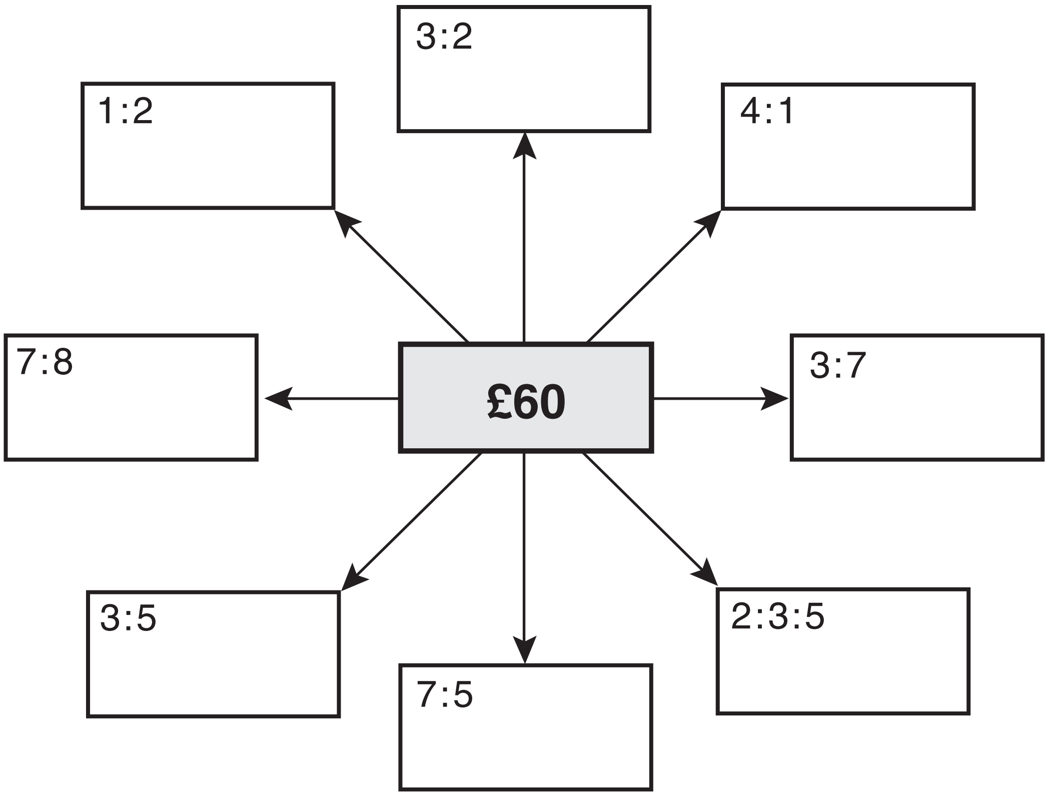
The ratio of Pat's savings to spending is 2:3. If she spent £765, how much did she save?

You could follow up with similar questions in future lessons.

Ratio spider diagram 1



Ratio spider diagram 2



Ratio problems 1 M5.3

1 The angles in a triangle are in the ratio 9:5:4. Find the size of each angle.

2 Green paint is made by mixing 2 parts of blue paint with 5 parts of yellow.

A girl has 5 litres of blue paint and 10 litres of yellow paint. What is the maximum amount of green paint she can make?

3 This recipe for fruit squash is for 6 people.

300 g	chopped oranges
1500 ml	lemonade
750 ml	orange juice

How much lemonade do you need to make fruit squash for:

(a) 9 people? (b) 10 people?

1 In a game of rugby Rob's ratio of successful to unsuccessful kicks was 5:3. Dave's ratio was 3:2. Who was the more successful?

2 The gears of a bicycle travelling along a flat road are such that for every 2 turns of the pedals the rear wheel makes 5 turns. If the pedals make 150 turns, how many turns will the rear wheel make?

When travelling up a steep hill in a different gear, would you expect the rear wheel to make more or less than 5 turns for each 2 turns of the pedals? Explain your answer.

3 The answers to a survey are shown in a pie chart. The angle representing 'Yes' is 120°, the angle for 'No' is 150°, and the angle for 'Don't know' is 90°.

If 300 people took part in the survey, how many replied 'No'?

Lesson 6 Algebraic expressions

Oral and mental starter — 10 minutes

Objectives

Understand that algebraic operations follow the same conventions and order as arithmetic operations (Y7)

Use index notation for small positive integer powers (Y8)

Vocabulary

symbol, variable, equals, brackets, evaluate, simplify, term, expression, equation

Resources

OHT of M6.1
Whiteboards (if available)

Show **OHT M6.1**. Ask pupils to discuss in pairs which expressions match. Take feedback and ask pupils to explain their reasoning.

Next, read out the following. Ask pupils to write the expressions or equations on paper or on whiteboards, using algebra.

1 A number x, plus 9, then the result multiplied by 6.
2 Add 7 to a number y, and then multiply the result by itself.
3 Think of a number t, multiply it by itself and then subtract 5.
4 Think of a number z, square it, add 1 and then double the result. The answer is 52.
5 Cube a number p, and then subtract 7. The answer is 20.

Check whether pupils:

- know how multiplication is represented in algebraic expressions;
- understand the meanings of $2n$ and n^2, $3n$ and n^3;
- understand the difference between an expression and an equation.

Main teaching — 40 minutes

Objectives

Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket (Y8)

Vocabulary

symbol, variable, equals, brackets, evaluate, simplify, term, expression, equation

Resources

OHT of M6.2
Multiple sets of M6.3, cut up and put in envelopes

Show question 1 on **OHT M6.2** and ask pairs of pupils to consider the statements, identify the errors and correct them.

Invite pupils to show their responses and to talk through their reasoning.

Q What needs to be done first?
Q What is the coefficient of d?
Q Which terms can we collect together?

Check pupils' understanding of the errors:

- the distributive law is applied incorrectly;
- there are errors in signs;
- mistakes are made in collecting terms.

At this point, it may be useful to illustrate grid multiplication with numerical examples, then extending it to:

$\times$	$4 - 2p$
$^-2$	$^-8 + 4p$

Ask pupils to simplify the expression $5(p - 2) - (4 - 2p)$ (question 2 on **OHT M6.2**). Invite them to show their response and talk through their reasoning.

Discuss how useful it is to think of $-(4 - 2p)$ as $^-1(4 - 2p)$.

Next, discuss how each of the sets of expressions in question 3 on **OHT M6.2** is equivalent.

Give each group of pupils the sets of expressions from **M6.3**, to match and identify the simplified form.

Q Which expressions do not match? Why not?

You may need to support pupils by extending grid multiplication to include products of types $(x + 2)(x + 3)$ and $(x + 2)^2$.

Plenary — 10 minutes

By the end of the lesson

pupils should be able to:

- collect like terms
- multiply a single term over a bracket

Framework supplement of examples, page 117
Levels 5 and 6

Discuss particular expressions from **M6.3**. You might focus on:

$$(a + 3)^2 - (a - 1)^2 - 3(a - 5)$$

or extend the work to expressions that involve two variables:

$$2(x + 1) + 5(y + 1)$$

If pupils are confident, an alternative plenary may be to extend the work to include factorising.

Match the expressions

Draw lines to match equivalent expressions.
Write expressions to match the two left over.

$n + n$	$n \times n$
$n \div 3$	$4n + 8$
$2n + 3$	$3 \times n$
n^2	$3 + n + n$
$4(n + 2)$	$2n$
$5n$	$n \div 5$
$3n$	$\frac{n}{3}$

1 What is wrong with this?

$$2(3d + 7) - 2(d - 4) = 6d + 7 - 2d - 8$$
$$= 4d - 15$$
$$= {}^{-}11d$$

2 Simplify $5(p - 2) - (4 - 2p)$

3 Each set of expressions below is equivalent. Can you explain why?

(a) $3(b + 5) - (b + 3)$

$2b + 12$

$2(b + 6)$

(b) $3(b + 5) - (b - 3)$

$2b + 18$

$2(b + 9)$

Expression sort

Cut up these cards and place them into an envelope, one set for each group of pupils.

$2(3a - 2)$	$4a^2 + 2(3a - 2) - (2a)^2$	$6a - 4$	$2 + 5a - 6 + a$
$3(a - 1)$	$(a - 5) + 2(a + 1)$	$3a - 3$	$7a - 3 - 4a$
$2a + 18$	$3(a + 5) - (a - 3)$	$2(a + 9)$	$20 + 5a - 2 - 3a$
$2(a + 6)$	$3(a + 5) - (a + 3)$	$2a + 12$	$10 - 5a + 2 + 7a$
$2(a - 6)$	$4(1 - a) + 2(3a - 8)$	$2a - 12$	$9 - 5a - 21 + 7a$
$5(a - 1)$	$(a + 3)^2 - (a - 1)^2 - 3(a - 5)$		$5a - 5$
$a - a^2$	$4a + 3 - 2a^2 - 3a + 2a^2 - 2$		$a(1 - a)$

Lesson 7 Sequences

Oral and mental starter — 10 minutes

Objectives

Generate and describe sequences (Y7, 8)

Vocabulary

sequence, term, position, term-to-term rule, position-to-term rule

Ask pupils to continue sequences such as:

0.7, 0.4, 0.1, … 1005, 1003, 1001, …

1, $^{-}2$, 4, $^{-}8$, … 1, 0.5, 0.25, 0.125, …

Make sure the level of difficulty matches examples in the Framework (pages 144–145).

Q What is the rule to get the next term?

Explain that you have been using a term-to-term rule to describe these sequences.

Extend the activity: give pupils the first three terms and ask for the 4th, 5th, 10th terms.

Make sure that pupils realise that there are pitfalls in continuing sequences. For example, ask them to describe these sequences:

1, 2, 4, then 8, 16, … 1, 2, 4, then 7, 11, …

Now give pupils the first three terms of a sequence. Ask them to suggest what the 4th and subsequent terms might be. Point out that the same few given terms can lead to several different sequences.

Spend a few minutes considering how the sequence 1, 4, 9, 16, … can be described using term-to-term and position-to-term rules.

Main teaching — 40 minutes

Objectives

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence (Y8, 9)

Vocabulary

sequence, term, *n*th term, general term, position-to-term rule, expression, justify, symbol, pattern, generate

Resources

OHT of M7.1
OHT of M7.2 (for plenary)

Show the first set of shapes on **OHT M7.1** and ask a pupil to draw the next shape in the pattern.

Q How many rectangles will there be in pattern 7? How did you work it out?

Bring out through discussion that they have used the term-to-term rule (as in the starter).

Q How many rectangles are there in pattern 20?

Ask pupils to discuss this in pairs and record their values.

Organise the values in a table.

Pattern	1	2	3	…	7	…	20
No. of rectangles	5	8					

Ask pupils to explain how they got the 20th term.

Counteract the misconception that you multiply the 2nd term by 10 to obtain the 20th term by offering a counter-example from the table, e.g. the 1st and 3rd terms.

Ask pairs of pupils to work out how many rectangles there are in pattern 50, and to explain their method.

Model how to write the correct general term in words and symbols, linking the explanation to the diagrams. Make sure that links are made between the numerical and spatial patterns. For example (referring to the lower diagram on **OHT M7.1**):

Think of the original two rectangles (on the left-hand end). Each time three rectangles are added to form the next pattern. So:

Number of rectangles = 2 + pattern number × 3 (this is a position-to-term rule).

Further examples are available in the Framework supplement of examples, pages 155–157, and on previous Key Stage 3 test papers.

By the end of the lesson

pupils should be able to:

- understand sequences
- calculate terms in a sequence
- express a general term in words and symbols
- explain how the elements of a general term relate to the original sequence

Framework supplement of examples, pages 155–157
Levels 5 and 6

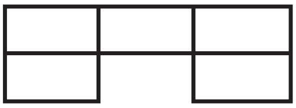
Plenary — 10 minutes

Show **OHT M7.2**, developed from a 1996 test question. Recap key language by asking pupils to make statements about this new sequence. For example:

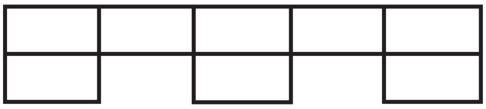
The 3rd term is …
The term-to-term rule is …
The position-to-term rule is …

Tile patterns

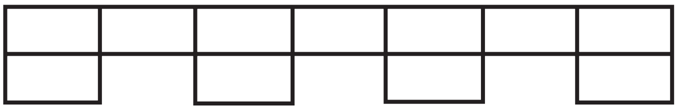
M7.1



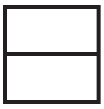
pattern 1

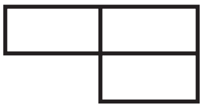


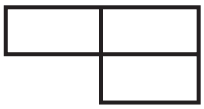
pattern 2



pattern 3







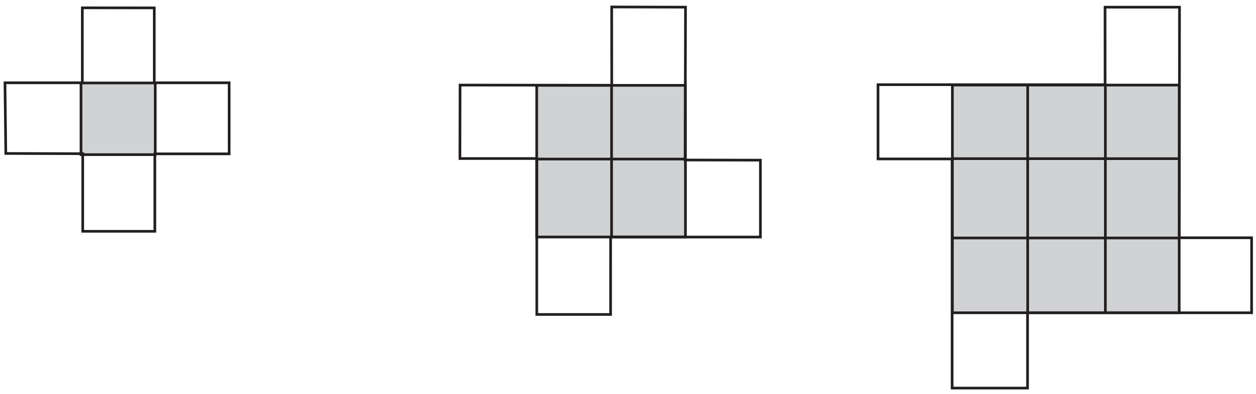
Start with
2 rectangles

Add 3 rectangles
each time for next
pattern

Windmills

M7.2

This series of patterns is made with grey and white tiles.



pattern 1 pattern 2 pattern 3

(a) How many grey tiles and white tiles will there be in pattern 8?

(b) How many grey tiles and white tiles will there be in pattern 16?

(c) How many white tiles will there be in pattern n?

(d) How many grey tiles will there be in pattern n?

(e) How many tiles altogether will there be in pattern n?

Lesson 8 Lines and angles

Oral and mental starter — 10 minutes

Objectives

Use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes (Y7)

Vocabulary

parallel, perpendicular, transversal, intersecting, vertically opposite, supplementary, complementary, alternate, corresponding, interior, exterior, equidistant, convention, definition

Resources

Whiteboards (if available)

Ask pupils to listen to the following instructions, and sketch and label the diagram:

Draw an isosceles triangle ABC.
Mark the equal angles and sides.
Mark and shade angle ABC.
Extend line AB to point D, and mark the exterior angle of the triangle.

Discuss different orientations of the triangle.

Now ask pupils to do the same for these instructions:

Draw a pair of parallel lines with an intersecting transversal.
Label the parallel lines.
Label a pair of corresponding angles with the letter *c*.
Label a pair of alternate angles with the letter *a*.
Label a pair of vertically opposite angles with the letter *v*.

Discuss pupils' answers and the relationships between the angles: Which are equal? Do any add up to 90° (complementary)? … add up to 180° (supplementary)?

Main teaching — 40 minutes

Objectives

Solve geometrical problems using side and angle properties and explaining reasoning with diagrams and text (Y8)

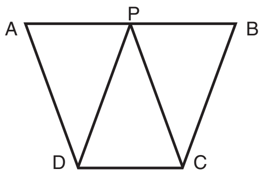
Vocabulary

parallel, perpendicular, transversal, vertically opposite, supplementary, complementary, alternate, corresponding, interior, exterior, equidistant, prove, proof, convention, definition, derived property

Resources

OHT of M8.1
OHT of M8.2 (for plenary)
Whiteboards (if available)
Mixed set of questions drawn from the Framework

Tell pupils that they are going to use their knowledge of lines and angles to solve geometrical problems. Model how to solve the problem on **OHT M8.1**.



ABCD is an isosceles trapezium with AB parallel to DC.
P is the midpoint of AB, and AP = CD, AD = DP.
∠DAP = 75°. Calculate the sizes of the other angles.

Ask pupils to draw and label the diagram and discuss the question in pairs.

Q What information can you add to the diagram?

Label AB and CD as parallel.
Label AP = PB = CD.
Label BC = AD (isosceles trapezium).
Label ∠DAP = ∠PBC = 75° (isosceles trapezium).

Q Which angles can you calculate first? Justify your answers.

Discuss pupils' solutions and model the use of correct language and geometrical reasoning.

∠DAP = ∠APD = 75° (triangle APD is isosceles)
∠ADC = 180° – 75° (interior angles, AB and DC are parallel)
∠ADP = 180° – 75° – 75° (angle sum of triangle is 180°) and so on

Ask pupils to solve a selection of mixed problems involving lines and angles. Use the Framework supplement of examples, pages 16, 17 and 183, and examples from previous Key Stage 3 test papers.

Check pupils' knowledge of interior and exterior angle properties of polygons.

Plenary — 10 minutes

By the end of the lesson pupils should:

- know and use the angle properties of parallel and lines, and of triangles and polygons
- be able to justify and explain reasoning with diagrams and text

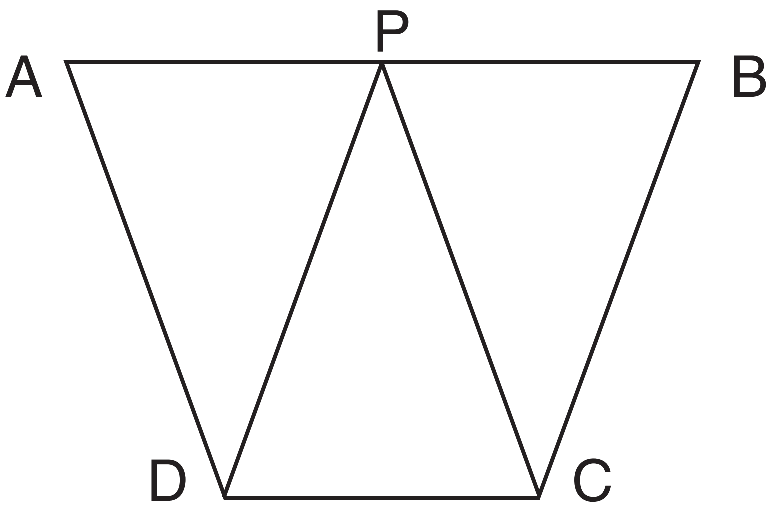
Framework supplement of examples, pages 16, 17, 183
Levels 5 and 6

Show **OHT M8.2**, developed from a 1999 test question. Ask pupils to discuss in pairs how to solve the problem and to explain their reasoning.

$k = 180 - 70$ (interior angles)
$3(m + 70) = 360$ (angles at a point total 360°)

Demonstrate how pupils should show their working concisely.

Isosceles trapezium



ABCD is an isosceles trapezium
with AB parallel to DC.

P is the midpoint of AB, and
AP = CD, AD = DP.

$\angle DAP = 75°$.

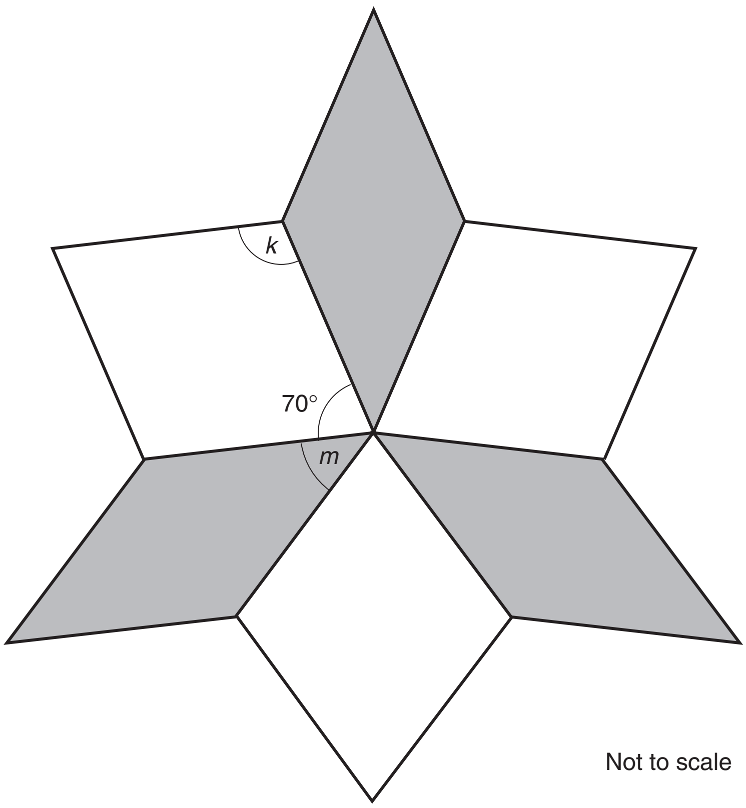
Calculate the sizes of the other angles.

Star

The shape below has three identical white tiles and three identical grey tiles.

The sides of each tile are the same length.
Opposite sides of each tile are parallel.

One of the angles is 70°.



(a) Calculate the size of angle *k*.
Give a reason for your answer.

(b) Calculate the size of angle *m*.
Give a reason for your answer.

Lesson 9 Area

Oral and mental starter 15 minutes

Objectives

Use formulae for the area of a triangle, parallelogram and trapezium (Y8)

Vocabulary

triangle, parallelogram, trapezium, compound shapes, units of measurement

Resources

OHT/handouts of M9.1

Working in pairs, pupils match the areas and perimeters to the shapes on **M9.1**.

Ask pupils to explain their answers. Check that pupils know:

- the difference between area and perimeter;
- how to calculate area and perimeter;
- formulae for the area of a triangle, rectangle, parallelogram and trapezium.

Pupils need to be secure in finding these areas and perimeters before moving on to problem solving.

Main teaching 35 minutes

Objectives

Use formulae for the area of a triangle, parallelogram and trapezium; calculate areas of compound shapes made from rectangles and triangles (Y8)

Solve increasingly demanding problems (Y9)

Vocabulary

triangle, parallelogram, trapezium, compound shapes, units of measurement

Resources

OHT of M9.2
OHT of M9.3 (for plenary)
Mixed set of word problems

Say that pupils are going to use their knowledge of the formulae for area and perimeter to solve problems.

Model how to solve the following problem (**OHT M9.2**):

An organisation has a pentagonal logo that is made from a rectangle and an isosceles triangle. The dimensions are as follows:

Length of rectangle	10 cm
Width of rectangle	3 cm
Height of logo	15 cm
Side of isosceles triangle	13 cm

What is the area of the logo? How much braid is needed to go around the outside?

Ask the pupils to analyse the problem. They will need to sketch the logo.

Q What information do you have? Label the sketch.

Q What information do you need to find out?

Q How will you calculate the area of the logo? (Pupils will need to calculate the height of the triangle: 12 cm.)

Q How do you work out the length of the braid that goes around the outside?

Q Do your calculations make sense? Check your solutions.

After modelling the solution of another problem from the Framework, ask pupils to solve a selection of mixed word problems using area and perimeter. Use examples from the Framework supplement, pages 18–19 and 234–237, and from previous Key Stage 3 test papers. For level 6 include questions on circles – see also booster lesson 10.

Plenary 10 minutes

By the end of the lesson

pupils should be able to:

- use formulae for the area of a triangle, parallelogram and trapezium
- calculate the areas of compound shapes
- solve problems

Framework supplement of examples, pages 234–237
Level 5

Show **OHT M9.3**, developed from a 1998 Key Stage 3 test question, and ask pairs of pupils to discuss solutions to part (a).

Q What method can you use to solve this? Can you do it in another way? Which is most efficient?

Discuss a variety of methods:

One triangle has area $\frac{1}{2}(5 \times 4)$; hexagon has area of 6 triangles.

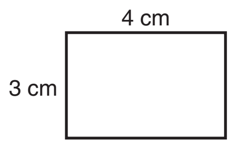
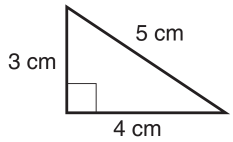
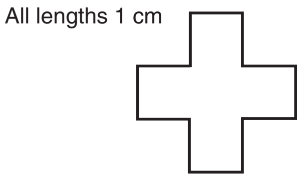
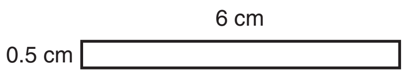
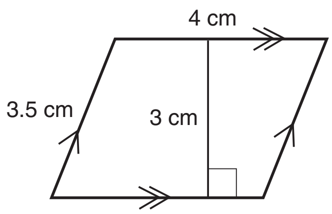
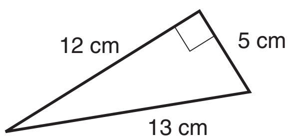
Trapezium has area $\frac{1}{2}(5 + 10) \times 4$; hexagon is area of two trapezia.

Small triangle has area $\frac{1}{2}(2.5 \times 4)$; hexagon has area of 80 minus four small triangles.

The other parts of this question revise percentages and volume.

Area and perimeter match

Match the areas and perimeters to the shapes.
For the area and perimeter left over, draw a rectangle that fits those dimensions.

Area	Shape	Perimeter
6 cm^2		12 cm
12 cm^2		12 cm
24 cm^2		15 cm
12 cm^2		14 cm
30 cm^2		20 cm
3 cm^2		13 cm
5 cm^2		30 cm

Company logo

An organisation has a pentagonal logo that is made from a rectangle and an isosceles triangle.

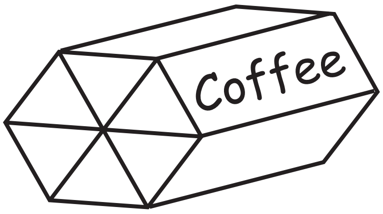
The dimensions are as follows:

Length of rectangle	10 cm
Width of rectangle	3 cm
Height of logo	15 cm
Side of isosceles triangle	13 cm

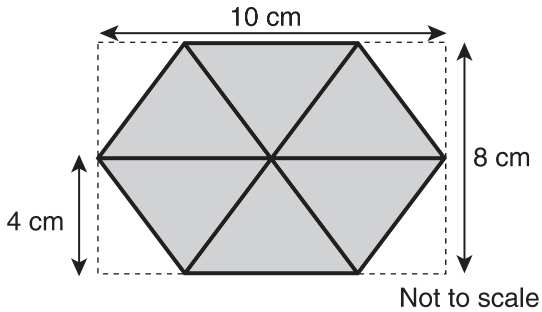
What is the area of the logo?

How much braid is needed to go around the outside?

A box for coffee is in the shape of a hexagonal prism.

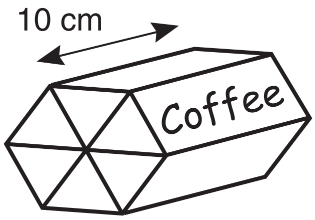


One end of the box is shown below.
Each of the six triangles in the hexagon has the same dimensions.



(a) Calculate the total area of the hexagon.

(b) The box is 10 cm long.



After packing, the coffee fills 80% of the box.
How many grams of coffee are in the box? (The mass of 1 cm^3 of coffee is 0.5 grams.)

(c) A 227 g packet of the same coffee costs £2.19. How much per 100 g of coffee is this?

Lesson 10 Circles

Oral and mental starter 15 minutes

Objectives

Know the definition of a circle and the names of its parts (Y9)

Vocabulary

centre, radius, diameter, circumference, arc, chord, semicircle

Resources

Compasses
Paper circles
Whiteboards (if available)

Ask pupils to use compasses to draw a circle and then draw and label the centre, a radius, a diameter and a chord.

Q What is the longest chord you can draw?

Q What is the name of the shape bounded by an arc and a diameter?

Q What is the relationship between the radius and the diameter? Can you write the relationship in another way?

Q Do you know any other relationships?

Demonstrate by measuring C and d that $C/d \approx 3$ and link this to previous work on ratio.

Either using paper circles or pupils' accurately drawn circles, ask pupils to work in pairs and find:

- the radius of the circle;
- the diameter of the circle;
- the circumference of the circle;
- the area of the circle.

Discuss pupils' methods; compare those involving measurement with any using calculations.

Main teaching 30 minutes

Objectives

Know and use the formulae for the circumference and area of a circle (Y9)

Solve increasingly demanding problems (Y9)

Vocabulary

centre, radius, diameter, circumference, π

Resources

Calculators
OHP calculator
OHT of M10.1
OHT of M10.2 (for plenary)
Mixed set of word problems

Remind pupils that the circumference of a circle is given by the formula $C = \pi d$ and ask for the equivalent formula using the radius.

Explain that π is a number that cannot be written down exactly. It is called an irrational number and has an approximate value of 3.142.

Introduce problem 1 on **OHT M10.1**. Ask pairs of pupils to calculate the answer and invite one pair to explain their method. Demonstrate the calculator key sequence using an OHP calculator.

Remind pupils that the area of a circle is given by the formula $A = \pi r^2$.

Introduce and model how to solve problem 2 on the OHT. Invite pupils to suggest methods. Set the solution out clearly, showing the working. Demonstrate the calculator key sequence.

Now say that pupils are going to use their knowledge of the formulae for the circumference and area of circles to solve problems. Select problems from the Framework supplement of examples, pages 19 and 235–237, and examples from previous Key Stage 3 test papers.

Plenary 15 minutes

By the end of the lesson

pupils should be able to:

- use the formulae $C = \pi d$ and $A = \pi r^2$
- solve problems involving circles

Framework supplement of examples, pages 19, 235–237
Level 6

Introduce the questions on **OHT M10.2**, adapted from a 1998 Key Stage 3 test paper.

Ask pupils to discuss how they would solve these questions.

Q What are you trying to find out?

Q What information about circles do you need?

Q What calculation do you need to do?

Q How will you do the calculation?

Q What degree of accuracy is reasonable?

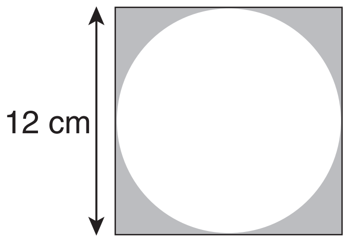
Establish what working needs to be shown.

Use an OHP calculator to demonstrate the calculations.

Circle problems

1 The London Eye has a diameter of 135 metres. How far do you travel in one revolution of the Eye?

2 A circle is drawn inside a 12 cm square so that it touches the sides. Calculate the shaded area.

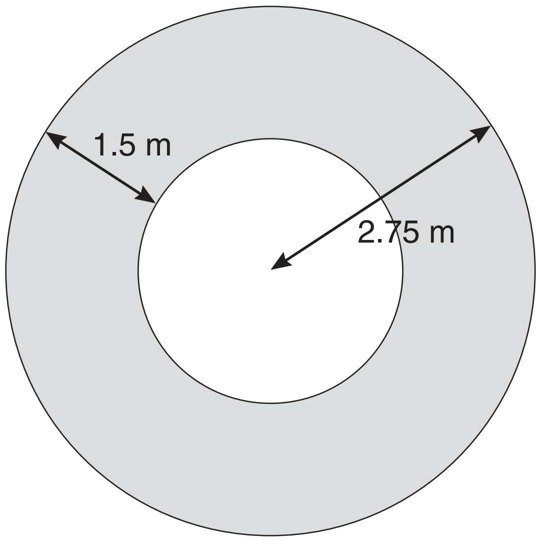


The Round Table of King Arthur M10.2

At Winchester there is a large table known as the Round Table of King Arthur. The diameter of the table is 5.5 metres.

(a) A book claims that 50 people sat around the table. Assume each person needs 45 cm around the circumference of the table. Is it possible for 50 people to sit around the table?

(b) Assume people sitting around the table could reach only 1.5 metres. Calculate the area of the table that could be reached.



Lesson 11 Probability

Oral and mental starter — 15 minutes

Objectives

Use the vocabulary of probability (Y8, 9)

Vocabulary

outcome, sample space, equally likely, mutually exclusive, relative frequency, fair, likelihood

Give each group of four pupils a term from this list:

outcome
sample space
equally likely
mutually exclusive
relative frequency
fair
likelihood

Say that each group has three minutes to discuss and agree on their understanding of the term and how it might be used.

Take feedback from each group and clarify pupils' understanding of the terminology. Illustrate with examples.

Main teaching — 35 minutes

Objectives

Estimate probabilities from experimental data (Y9)

Find and record all possible mutually exclusive outcomes for single events and two successive events in a systematic way (Y8)

Compare experimental and theoretical probabilities in different contexts (Y8)

Vocabulary

event, experimental probability, sample space, theoretical probability, mutually exclusive

Resources

Handout of M11.1
Scissors

Introduce the pairs game on **handout M11.1**. Discuss how an outcome is a pair of animals and not a single animal, and that a 'success' is obtaining two identical animals ('snap').

Ask pupils to work in pairs and carry out the experiment to estimate the probability of a snap.

Q How are you going to record your results?

Q How many times are you going to repeat the experiment in order to get a meaningful result?

Allow time for pupils to complete their experiment and to estimate the probability. Compare their estimates. Discuss any issues that arise.

Next introduce a theoretical approach to the problem.

Q What are the possible outcomes in the pairs game?

Q How can you be sure that you have found them all?

Q How can you list them systematically?

Encourage pupils to use a systematic method of recording the outcomes, for example:

bb bc bd   cb cc cd   db dc dd

or

	b	c	d
b	bb	bc	bd
c	cb	cc	cd
d	db	dc	dd

Q Using the list of outcomes, what is the theoretical probability of winning?

Use additional problems from the Framework supplement of examples, Year 9, page 281.

Plenary — 10 minutes

By the end of the lesson

pupils should be able to:

- systematically record all the outcomes of an experiment
- understand that increasing the number of times an experiment is repeated generally leads to better estimates of probability
- understand the links between experimental and theoretical probabilities

Framework supplement of examples, pages 281–285
Levels 5 and 6

Discuss the results of the pairs game experiment.

Q How does your experimental probability compare with the theoretical one? Are they the same? If not, why are they different?

Q How could you improve your estimate?

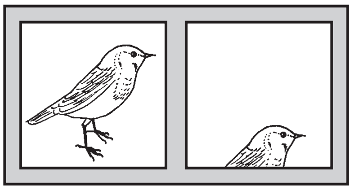
Ask other questions based on the outcomes listed.

Q What is the probability of getting two different animals?

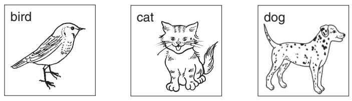
Q What is the probability of getting a bird and a cat?

Q Have you simplified your answer?

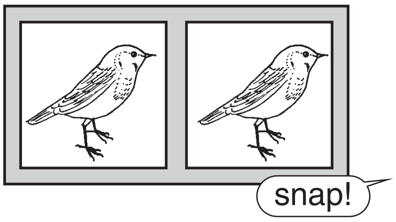
Pairs game M11.1



A child’s game has two windows.



In each window, one of three different animals – a bird, cat or dog – is equally likely to appear.



When both windows show the same animal, the child shouts ‘snap’ (this counts as a ‘success’).

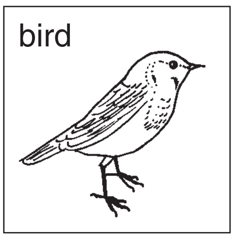
Estimate the probability of getting a ‘snap’, like this.

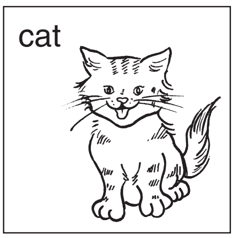
- Cut out the three animal cards, place them face down and shuffle them.
- Pick a card. This represents the animal that appears in window 1.
- Replace the card, face down. Shuffle the cards again.
- Pick a card. This represents the animal that appears in window 2.

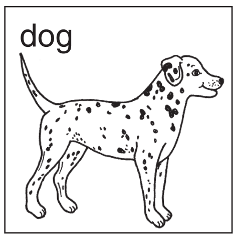
Decide how to record this result.

Decide how many times you are going to repeat this process.

Use your results to estimate the probability of getting two animals the same.







Lesson 12 Solving word problems

Oral and mental starter 15 minutes

Objectives

Identify the necessary information to solve a problem (Y8)

Vocabulary

sum, difference, total, altogether

Resources

Sheet M12.1

Use **sheet M12.1**, which contains questions adapted from Key Stage 3 mental test papers. Read each question aloud, one at a time. Ask pupils to explain the calculation that they would do.

Q Which key words help you decide which operation(s) to use?

Q Could you solve the problem another way? Which is the most efficient method?

Highlight those questions that require two steps to reach a solution and encourage pupils to jot down the intermediate step.

Main teaching 35 minutes

Objectives

Solve more complex problems by breaking them into smaller steps or tasks (Y8, 9)

Enter numbers into a calculator and interpret the display in different contexts (Y8)

Carry out more difficult calculations effectively and efficiently (Y8)

Vocabulary

depends on context of questions

Resources

OHTs of M12.2–M12.4

Calculators

Mixed set of test questions (for plenary)

Introduce the problem on **OHT M12.2**.

Give pupils two minutes to discuss in pairs how to solve the problem, then invite one or two pairs to explain their method on the board or OHT.

Model a process of solving word problems and recording the solution clearly (**OHT M12.3**).

> ***Stage 1: Make sense of the problem***
> *Read the problem. Underline key information. Decide what you need to find out. Is any information not needed?*

If necessary, support pupils with writing frames or ask them to write down a statement: 'I need to find out …'

> ***Stage 2: Calculate the answer***
> *Decide what you need to calculate. Should the calculation be done mentally, using a written method or using a calculator? Show the working, if appropriate.*

Discuss when it is appropriate to use a calculator to do the calculations.

Q What working do you need to show?

Ensure that pupils know that when using a mental or calculator method the working is just a record of the calculations that have been done. With written calculations the working is the calculating process itself.

> ***Stage 3: Check the answer***
> *Write down a solution to the problem. Look back at the original problem and make sure the answer makes sense.*

It might help some pupils to look back at stage 1 ('Decide what you need to find out') and write a sentence that answers this.

Now ask pupils to complete the questions on **OHT M12.4**.

For further examples refer to the Framework supplement of examples, pages 2–25.

Plenary 10 minutes

By the end of the lesson pupils should:

- know the three stages for solving word problems;
- be able to read a problem and identify key information;
- be able to carry out calculations using an appropriate method;
- know what to record.

Framework supplement of examples, pages 2–25

Levels 5 and 6

Ask a pupil to share their solution of one of the problems on **OHT M12.4**. Discuss the solution, highlighting the necessary working.

Choose appropriate Key Stage 3 test questions, for example 1995 tins (non-calculator), 1996 dive rating (calculator), 2000 museum (non-calculator), and ask pupils to read the question and discuss how they would solve it.

Pupils could complete the solutions for homework or as part of a future lesson.

1 A room is 4.2 metres long. How many centimetres is that?

2 What is the sum of 2.4, 3.6, 3.6 and 4.4?

3 Ann bought a car for £3000. She sold it for a quarter of this price. How much did she lose on the sale?

4 Bruce weighs 82 kg and Wayne weighs 9 kg less. How much do they weigh together?

5 A CD costs £9.99. How much will 15 cost?

6 37% of a class are boys. What percentage are girls?

7 240 kilometres are equivalent to 150 miles. How many kilometres are equivalent to 50 miles?

8 A square has a perimeter of 36 centimetres. What is its area?

9 A DVD player costs £200. Its price is reduced by 15%. What is its new price?

10 What is the difference between £9.65 and £7.89?

The cost of hiring a van is a basic charge of £43.75 per day plus 24p per mile.

Winston hires a van for two days.

The mileage shows 23 412 at the start of the first day.
It shows 23 641 at the start of the second day.
When Winston returns the van at the end of the second day the mileage is 23 812.

How much will the van hire cost?

How to solve problems M12.3

Stage 1 Make sense of the problem

- Read the problem.
- Underline key information.
- Decide what you need to find out.
- Is any information not needed?

Stage 2 Calculate the answer

- Decide what you need to calculate.
- Should the calculation be done
  - mentally?
  - using a written method?
  - using a calculator?
- Show the working, if appropriate.

Stage 3 Check the answer

- Write down a solution to the problem.
- Look back at the original problem and make sure the answer makes sense.

1 A supermarket sells biscuits in three different sized packets:
- 15 biscuits for 65p
- 24 biscuits for 88p
- 36 biscuits for £1.33

Which packet is the best value for money?

2 Six friends went to a restaurant for lunch. The total cost for the set menu was £40.50.

How much would the set menu cost for eight people?

3 Annie has some small cubes. The edge of each cube is 1.5 cm long.

She makes a larger cube out of the small cubes. The volume of the larger cube is 216 cm^3.

How many small cubes does Annie use?