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INVESTOR IN PEOPLE

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CUSTOMER SERVICE EXCELLENCE

Commentary on

Post-Primary Mathematics Teaching

In January 2007, the Education and Training Inspectorate published Better Mathematics[1], a report on the mathematics provision in Northern Ireland post-primary schools. The findings in the Report were disseminated to department leaders at a series of seminars arranged through the auspices of the five Education and Library Boards. The following commentary on the Teaching section of the Report builds on the illustrations used during the seminars by providing further comments and suggestions about the identified aspects of mathematics teaching.

The most effective lessons were characterised by many of the following strengths. The teachers:

- share the intended learning with the pupils at the start of the lesson;

1.1 The sharing of the intended learning at the start of lessons is an important aspect of Assessment for Learning[2] and many year 8 pupils will have seen WALT and WILF[3] statements in primary school. However, the pupils' knowledge of the learning objective doesn't prevent the teacher acting contingently in response to the pupils' needs or to unforeseen learning opportunities. (See §10.2)

- recap and link the work to previous learning, or set the work in an appropriate real-world context;

2.1 Linking the current work to future or past work can provide a chance for the pupils to gain a holistic view of mathematics. (See §9.1)

2.2 A real-world context can capture the interest of pupils, making the work more relevant and providing a purpose for their learning. The degree to which a question can be a real problem, or can use real data, clearly depends on the maturity and ability of the pupils, although even a contrived scenario may help

1 www.etini.gov.uk/better_mathematics-2.pdf

2 Assessment for Learning: Beyond the Black Box, The Assessment Reform Group, ISBN: 01223 36963 1; and Resource Pack for Assessment for Learning in Mathematics, The Mathematical Association, ISBN: 090658859 6

3 'We are learning to...' and 'What I'm looking for?'

provide an answer to the question that often faces teachers, 'Why are we doing this?'

2.3 For those pupils participating in the Vocational Enhancement Programme (VEP), there can be many instances during which the mathematical skills taught in class can be applied in relevant and worthwhile contexts. The reverse process through which the teachers use the pupils' VEP experiences to make their learning in class more purposeful and engaging is good practice. This greater liaison between classroom, workshop and workplace can also inform teachers about important differences in custom and practice. For example, in the workplace, millimetres are used in preference to centimetres even when measuring objects in the order of, say, 1m 20cm in length, i.e. 1200mm would be recorded.

2.4 The history of mathematics provides examples that can capture the pupils' interest; for example, the main ideas of school trigonometry were developed to help in the drawing of maps and charts in the 15th Century, and the metre was derived from the circumference of the Earth in the late 18th Century. Pupils can also be surprised to learn that we use 360 degrees because of the ancient Babylonians and that most, if not all, of school geometry predates the birth of Christ.[4]

2.5 A discussion about careers using mathematics can also be helpful in providing a context for the work.[5] More generally, the important role of mathematics in many careers is poorly promoted. The experiences of past pupils are a source of information that is rarely used and may be an effective way of addressing the poor attitude to mathematics that is present in some young people.

- provide clear exposition involving, where appropriate, multiple explanations, with board-work modelling what the pupils should do;

3.1 Understanding in mathematics has been classified as relational or instrumental, where relational understanding is based on connections and relationships and instrumental understanding is based on a series of disconnected rules.[6] Explaining in two or more ways may provide the extra background required for the pupils to reach a relational understanding - one that is more permanent and can form the basis for further development.

3.2 Explanations that promote relational understanding often introduce thinking-models that allow the pupils to construct their own 'conceptual structures'.[8] For example, the 'empty number line' is a thinking-model that can be used for many mental mathematics strategies and for explaining rounding-up and

4 There are some useful websites available, for example, www-groups.dcs.st-and.ac.uk/~history/Indexes/HistoryTopics.html

5 Useful information can be found at www.mathscareers.org.uk and www.plus.maths.org/interview.html

6 See Tanner and Jones below for an interpretation of Skemp's ideas (The Psychology of Learning Mathematics, Skemp, ISBN: 014021310 4) rounding-down. A dissected rectangle is a useful thinking-model for the four terms of the expansion of $(x+a)(x+b)$, and may help pupils to understand why the common misconception, $(x + a)^2 = x^2 + a^2$, is incorrect. (See §8.1)

3.3 Mathematics is a hierarchical subject and sometimes additional explanations can lay the foundation for future work, as well as developing a deeper, more permanent understanding. For example, it is common for percentage increase to be taught in year 8 as a multi-step process; for example, 'increase the cost by 15%' is taught as three steps: find 10%; find 5%; add them on to the cost. Moreover, it is frequently taught to all the pupils in only that way. However, when the single-step multiplicative process is also explained, that is, multiply by 1.15, the foundation for the reverse percentage problem is laid down - a type of problem that pupils find difficult if they face it later in their mathematics careers. It also reinforces decimal place value.

3.4 The assumption that lower-attaining pupils are confused by more than one explanation has been questioned.[7] It has been found that all pupils, including the lower-attaining ones, respond positively to being required to think mathematically within a range of teaching approaches that build on previous learning and promote continuity in the learning.

3.5 Beginning teachers sometimes have difficulty explaining ideas because they haven't built up their repertoire of explanations. Even some more experienced teachers use only one explanation because they believe that the way they prefer to do a question is the best way for their pupils. In the best practice, departments share ideas to broaden the range of explanations at the teachers' disposal and, when appropriate, reach an agreed consistency in order to promote a continuity and progression in pupils' learning. The role of a scheme of work as a depository of the professional expertise within a department is important in this context.

3.6 Experienced teachers are aware of the difficulties that pupils have with mathematical language and, at times, draw the pupils' attention to the use of specific words and phrases in their expositions. In particular: some words are specialised and only occur during mathematics work, e.g. *hypotenuse* and *isosceles*, and yet can still be ambiguous, 'Is the *circumference* the boundary of a circle or the length of the boundary?'; some words have a mathematical meaning in addition to their everyday one, e.g. *product, face, acute* and *improper*; and, even some simple words have a different interpretation, e.g. the use of *or* in P(A *or* B) means A or B or both, and the use of *any* in 'the sum of *any* two odd numbers is an even number' makes this a generalisation and not just a specific statement, say, 5 + 7 = 12.

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7 *Deep Progress in Mathematics: The Improving Attainment in Mathematics Project, Watson et al, University of Oxford; and Raising Achievement in Secondary Mathematics, Watson, ISBN: 033521860 1*

3.7 Often at the start of a topic, teachers provide formal notes that pupils are required to copy into a notebook – probably without thinking about what they are writing. An alternative is to ask the pupils, after completing the topic, to do the most difficult question they had previously answered correctly and then to annotate it with their own reminders about how they had done it.

3.8 In Japan, teachers generally introduce a new skill by posing, and solving collectively with the pupils, a 'complex thought-provoking problem'.[8] In this country, in contrast, teachers use simple examples that can often be solved by pupils using ad hoc methods or even guessing.[9] Moreover, the examples are often explained with little interaction with the class - the pupils being passive except for copying down the model solution from the board.

3.9 Better Mathematics also reported on the use of information and communication technology. In particular, it was found that mathematics software, such as graph plotting and dynamic geometry packages, is underused in providing the explanation and inter-connections that help develop relational understanding in the pupils.

- use a variety of activities, including ICT and practical equipment, which entails the pupils working individually, in pairs or in groups;

4.1 Group work in mathematics is generally only associated with formal investigational projects. Working collaboratively in pairs or in groups is less common during ordinary lessons and often, when it happens, teachers intervene too readily preventing the pupils from developing their understanding more deeply.

4.2 In many classes, doing mathematics is a silent, solitary exercise, even though it is generally agreed that we often learn, and certainly consolidate our understanding, by articulating what we know and what we are thinking. Beginning teachers often remark, 'I only really understood … after trying to teach it.'

4.3 When pupils are working at an exercise for a long time, teachers will often give them a break by showing the next more difficult worked example – perhaps when the better pupils are approaching the questions in the exercise similar to the worked example. Asking the pupils to solve collaboratively one of the later questions is an alternative strategy, and probably a better one. With the right prompting from the teacher, it may lead to a class discussion through which all will think mathematically and learn. (See §10.2) The use of mini-whiteboards in this collaborative problem-solving can allow the pupils to amend easily their collective solution, and also allow the teacher to scan the pupils' work from a distance.

8 *Becoming a Successful Teacher of Mathematics*, Tanner and Jones, ISBN: 041523069 1

9 *Improving Learning in Mathematics: Challenges and Strategies*, Swan, The Standards Unit, DfES

4.4 One of the most accepted analysis of how children develop their understanding in mathematics is Sharma's six levels of knowing: Intuitive; Concrete; Representation (pictorial); Abstract; Applications; and, Communications.[10] Generally at post-primary level, there are some pupils who need to revisit their experiences at the Concrete stage even though they will have investigated the concepts using practical equipment at primary school; for example, the use of scissors and paper to make nets of simple 3-D shapes. Just as important, there will be new ideas for which the pupils will benefit from being able to explore in a concrete way. Examples include: the use of Roamer™ to explore the exterior angles of regular polygons; the use of Multilink™ cubes to explore the ratios of surface areas and volumes of similar 3-D shapes; and, the use of dice, coins, etc, to explore the inherent variability of data arising from probability experiments. Given this practical foundation, the pupils may have more success in reaching the higher levels of understanding. It has already been noted (§4.2) that pupils being able to communicate their understanding - Sharma's highest level - is an important objective of mathematics teaching.

4.5 Mathematical investigations that have a practical first stage may engage the pupils more fully than those that progress into the Abstract stage too quickly. For example: 'how many nets of a cube are there?' may be more readily accessible if the pupils first have a chance to explore practically, 'how many pentominoes are there, and which are nets of an open-ended cubical box?'

- provide opportunities for the pupils to problem-solve;

5.1 Problem-solving can be considered as being at the heart of mathematics and was one of the activities listed in paragraph 243 of the Cockcroft Report.[11]

'Mathematics teaching at all levels should include opportunities for:

- exposition by the teacher;
- discussion between the teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations; and
- investigational work.'

5.2 In this context, it is perhaps helpful to broaden the idea of a problem to any question that is beyond what the pupil currently finds routine or, in other words,

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10 www.berkshiremathematics.com/four.asp
11 Mathematics Counts, Cockcroft, ISBN: 011270522 7

any question that is a challenge. (See §4.3) For some pupils, it may well be the first question expressed in words or a question with a real-world context. Problems are not the same as puzzles: they do not always need lateral thinking before they can be solved.

5.3 Some teachers promote problem-solving by having a relevant problem on the board as the pupils arrive. In the best practice, the pupils start the problem before the lesson begins and develop the important 'have a go' attitude to problem-solving. In addition, for those who arrive on time, the effective length of the lesson is extended. The time wasted at the start of lessons is one of the factors that pupils consider when asked, 'what makes a good teacher?'

- integrate, when appropriate, the use of effective mental mathematics strategies;

6.1 Opportunities for the pupils to explain their mental mathematics methods and to share these with the rest of the class, or for the teacher to showcase alternative methods, can be exploited when they arise during ordinary class time.

- use skilful questioning, challenging the pupils' understanding and requiring them to draw conclusions and justify their thinking;

7.1 When undirected questions are asked, teachers often find the same 5 or 6 hands are raised; how many of the other pupils have opted out of the thinking? Indeed, it has been said that many pupils prefer to be labelled 'lazy' than 'stupid'! The principles of Assessment for Learning[2] make us ask, 'What is learnt by always asking those with their hands raised?' As a result, some teachers have started using a 'no hands' policy in their classes to encourage all the pupils to think.

7.2 Questions in mathematics lessons have been classified as either 'funnelling' or the more effective 'focusing':

- funnelling questions are asked in order that the pupils arrive at the answer that is in the teacher's head. They often start with 'What is...' or 'How many…' and while the teacher is doing the thinking, the pupils often end up just guessing!

- focusing questions encourage the pupils to continue to think. They direct the pupils and often start with 'Why…', 'When…' or 'Explain…'. Focusing questions provide the scaffolding that helps the pupils to construct the solution. They will also provide the opportunities for the pupils to articulate their thinking. (See §12.1)

7.3 Questions that elicit a chorus response have few, if any, benefits, especially if the pupils are responding to the teacher sounding out the first syllable of the one-word answer.

7.4 Mini-whiteboards have the potential to change radically the nature of questioning in mathematics lessons: teachers can say, 'Show me…' instead of asking, 'What is…', or even, 'Why...'.

7.5 Teachers often do not allow sufficient thinking time for their pupils to understand the question, never mind to solve it. One approach used is to let a pupil count to, say, 20 before the teacher chooses someone to answer. Alternatively, the pupil who counts can choose the pupil who will answer. In addition, the time before the teacher confirms whether the answer is correct, or not, is probably as important: pupils will think again about whether they are correct.

7.6 Making mathematical statements, and prompting the pupils to state whether they are 'always true', 'sometimes [or not necessarily] true' or 'never true', has been found to be sometimes preferable to asking, even, focusing questions. For example: 'The diagonals of a rectangle meet at right angles', 'All prime numbers are odd' and 'Only one diagonal in a kite is bisected'. An effective follow-up question to the last statement can be, 'How could the statement be changed to make it always true?' thus emphasising the importance of precise language in mathematics. [9]

- highlight common misconceptions and exploit these in a sensitive way;

8.1 A misconception is often the over-generalisation of a rule; examples include: 23.70 is ten times greater than 23.7 because a zero has been added; parallelograms have bi-lateral symmetry because they can be divided into two congruent halves by a line parallel to one of their sides; and, when a number is squared, the answer is bigger. It is now accepted that when misconceptions occur, it is best to show incorrect and correct examples and let the pupils experience the 'cognitive conflict'. This has been found preferable to the teacher showing an earlier, or even another, correct use of the rule in the hope that the pupils will appreciate that its use has limitations.[12] [9]

8.2 There are, of course, errors other than misconceptions and how teachers react to these is as important. Sometimes teachers over-emphasise an error as the pupil's mistake and use 'silly' or 'careless' to soften the criticism. Whether pupils appreciate the intended softening is doubtful. A preferable approach is to be positive and constructive, for example, 'If you do …. you'll get it right the next time.'

- relate the ongoing work to other parts of the course to encourage the pupils to make interconnections and think of mathematics holistically;

9.1 It has been shown that generally teachers who think of, and teach, mathematics holistically are more effective teachers.[13]

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12 Recent Research in Mathematics Education 5-16, Askew and Wiliam, ISBN: 011350049 1

13 Making Connections: Effective Teachers of Numeracy, Askew, BEAM RES02

- engage the pupils fully by ensuring that the lesson had appropriate pace, challenge and progression;

10.1 As noted earlier in §3.4, all pupils, including the lower-attaining ones, gain by being required to think mathematically and by being challenged.[7] Generally, they do not improve their understanding by doing repetitive questions that do not increase in difficulty. In fact, low-challenge exercises can often result in the pupils' low motivation and poor behaviour.

10.2 Effective whole-class teaching is now associated with the development and promotion of a 'community of inquiry'[14] in which teachers orchestrate the ideas being brought to the discussion from fully engaged and challenged pupils. On these occasions, the teachers work contingently, responding positively to the pupils' specialising, generalising and justifying – these arising through the teachers' skilful prompting. ['The teacher as the manager of pupil-talk' is an important aspect of the approach practised in the Cognitive Acceleration in Mathematics Education (CAME) Project[15], which some Belfast schools have been piloting.]

10.3 The pace of a lesson is governed as much by the quality of the discourse of inquiry as by the completion of the planned agenda of activities.

- teach step-by-step algorithms only when necessary; and

11.1 Step-by-step algorithms or routines often result in instrumental understanding[6] and will be readily forgotten by the pupils. When routines are taught, they should be accompanied preferably by explanations that promote understanding and provide thinking-models for the pupils. (See §3.2)

- encourage the pupils to think and talk about how they learn and what they have learnt, often through appropriate plenary sessions at the end of lessons.

12.1 Metacognition is the ability to assess and monitor one's own thinking, and it is developed by pupils when they reflect on their work and articulate their thinking. Interestingly, it has been found that the ability to problem-solve and the level of metacognition are closely associated. [8]

12.2 Metacognition can be promoted in less direct ways: the formal phrases in solutions, for example, 'Let x be the …', 'Substituting x into equation 1' or '$(\ldots)(\ldots) = 0$ implies either $(\ldots) = 0$ or $(\ldots) = 0$', are often prompts that encourage pupils to re-think what they have done when checking their work.

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14 *Teaching for Learning Mathematics, Sutherland, ISBN 033521390 1*

15 *www.kcl.ac.uk/schools/sspp/education/research/projects/came.html*

Frequently, less effective mathematics lessons were characterised by the following:

- the pupils were shown one or two worked examples on the board, which sometimes were the ones provided in the textbook;

13.1 The pupils' level of independence can be promoted through their use of the textbook worked examples; for example, as a part of their homework they can be asked to prepare to teach one of the worked examples to the rest of the class. This will not only help them to develop their metacognition, but also be a useful recapping strategy at the start of the next lesson.

- the pupils began an exercise of questions from the textbook, which were often routine, repetitive and insufficiently challenging;

14.1 Ideally, exercises of questions should be well graduated, increasing in challenge at an appropriate rate. There are, of course, pupils who work more slowly than others or may require more support. A frequent strategy used is to let the pupils do only the even-numbered questions in class: the homework is then all, or a selection, of the odd-numbered ones. Through this approach, all the pupils may reach the more challenging questions in class and the homework will be better suited for consolidation. Of course, at other times, it is good to set a challenging homework because at home pupils have the luxury of time to come to terms with the problems and to learn to try different strategies.

- the teacher gave individual support which consisted of his or her completion of the question for the pupil;

15.1 The pupils should still construct the solution by themselves although with the help of scaffolding provided by the teacher. (See §7.2)

- the lessons were not drawn to an appropriate conclusion; and

16.1 Too often the school bell dictates the end of the lesson resulting in there being insufficient time for the pupils to reflect on their learning and for the teacher to put the learning in context, either by re-emphasising the purpose of the lesson or by promoting a holistic view of mathematics. (See §9.1 and §12.1)

- the teacher gave homework without due regard to the quantity and difficulty of the work entailed for each individual, for example, pupils being asked to 'finish the exercise'.

17.1 The pupils who work more slowly may consider doing more mathematics as a form of sanction thus causing a poor attitude to mathematics. Moreover, some of the pupils who have difficulties are being asked to do the challenging questions without the teacher being at hand to support them.

In these lessons, teachers generally taught, without sufficient explanation, step-by-step algorithms which the pupils were required to memorise. For many pupils, being good at mathematics is perceived as being able to memorise and apply accurately well-practised methods.

18.1 ‘An over-reliance on your memory brings with it the danger of forgetting.’ Although able to remember routines in the short-term, many pupils forget them within a few weeks.

Summary

19.1 '[Mathematics] Teaching is more effective when it:

- builds on the knowledge learners already have;
- exposes and discusses common misconceptions;
- uses higher-order questions;
- uses co-operative small group work;
- encourages reasoning rather than 'answer getting';
- uses rich, collaborative tasks;
- creates connections between topics; and
- uses technology.'[16]

19.2 This description matches the conclusion in a review of the Northern Ireland Numeracy Strategy (NINS) commissioned by the NINS Steering Group in 2006.

"The consensus [of recent research] is that successful teaching of mathematics is based on an eclectic mix of approaches that comprises the following elements:

- engaging in higher-order questioning;
- emphasising mental calculations in a wide range of contexts;
- developing strategic thinking by setting more challenging tasks;
- encouraging pupils to explain and discuss their ideas rather than to resort to memorize routines; and
- encouraging pupils to engage in collaborative problem-solving.

This type of 'discourse of inquiry' is also a feature of the Revised [NI] Curriculum which places renewed emphasis on 'investigating and problem-solving', 'challenging and engaging', 'enquiry based', 'on-going reflection' in the description of pupils' learning experiences."[17]

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16 Maths4Life, Swan, DfES, www.maths4life.org

17 'Review of the NINS: Final Report, Nov 2006', Pirrie et al, The SCRE Centre, The University of Glasgow.

Copies of this report are available on the DE website: **www.deni.gov.uk** or may be obtained from the Inspection Services Branch, Department of Education, Rathgael House, 43 Balloo Road, Bangor, Co Down BT19 7PR.