The National Strategies
Primary

Teaching children to calculate mentally

Department for Education

Teaching children to calculate mentally

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Introduction

The ability to calculate in your head is an important part of mathematics. It is also an essential part of coping with society's demands and managing everyday events.

This mental calculation publication has been adapted from *Teaching mental calculation strategies – guidance for teachers at Key Stages 1 and 2*, produced in 1999 by the National Numeracy Strategy and Qualifications and Curriculum Authority (now QCDA). Its overall aim is to assist your planning by:

- listing the number facts that children are expected to recall rapidly
- setting out expectations for the types of calculations that children should be able to do mentally
- identifying the mental methods that might be taught to children to help them to calculate accurately and efficiently
- suggesting a range of suitable classroom activities and resources to help children to understand and practise calculation methods.

The four chapters of the booklet cover:

1. Progression in mental calculation skills

This describes the progression in the number facts that children should derive and recall, the calculations that they are expected to do mentally and the range of calculation strategies or methods that they can draw on.

2. Principles of teaching mental calculation

This promotes a broad interpretation of mental calculation and identifies principles that underpin teaching: for example, encouraging children to share their mental methods, to choose efficient strategies and to use informal jottings to keep track of the information they need when calculating. It also looks at the role of tests and questioning.

3. Addition and subtraction strategies

This sets out the main strategies for adding and subtracting mentally. It describes activities to support teaching of these strategies and typical problems.

4. Multiplication and division strategies

This sets out the main strategies for multiplying and dividing mentally. Again, it describes activities to support teaching of these strategies and typical problems.

1 Progression in mental calculation skills

All primary teachers need to review, consolidate and build on children's developing mental calculation skills throughout Key Stages 1 and 2. To help your planning, the progression in mental calculation with whole numbers, including the recall of number facts, and fractions, decimals and percentages, is set out for you on the next few pages.

The following tables provide further details to exemplify the expectations in the *Primary Framework for mathematics*.

Addition and subtraction

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 1 ● number pairs with a total of 10, e.g. 3 + 7, or what to add to a single-digit number to make 10, e.g. 3 + □ = 10 ● addition facts for totals to at least 5, e.g. 2 + 3, 4 + 3 ● addition doubles for all numbers to at least 10, e.g. 8 + 8	● add or subtract a pair of single-digit numbers, e.g. 4 + 5, 8 – 3 ● add or subtract a single-digit number to or from a teens number, e.g. 13 + 5, 17 – 3 ● add or subtract a single-digit to or from 10, and add a multiple of 10 to a single-digit number, e.g. 10 + 7, 7 + 30 ● add near doubles, e.g. 6 + 7	● reorder numbers when adding, e.g. put the larger number first ● count on or back in ones, twos or tens ● partition small numbers, e.g. 8 + 3 = 8 + 2 + 1 ● partition and combine tens and ones ● partition: double and adjust, e.g. 5 + 6 = 5 + 5 + 1

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 2 • addition and subtraction facts for all numbers up to at least 10, e.g. 3 + 4, 8 – 5 • number pairs with totals to 20 • all pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 + ☐ = 100 • what must be added to any two-digit number to make the next multiple of 10, e.g. 52 + ☐ = 60 • addition doubles for all numbers to 20, e.g. 17 + 17 and multiples of 10 to 50, e.g. 40 + 40	• add or subtract a pair of single-digit numbers, including crossing 10, e.g. 5 + 8, 12 – 7 • add any single-digit number to or from a multiple of 10, e.g. 60 + 5 • subtract any single-digit number from a multiple of 10, e.g. 80 – 7 • add or subtract a single-digit number to or from a two-digit number, including crossing the tens boundary, e.g. 23 + 5, 57 – 3, then 28 + 5, 52 – 7 • add or subtract a multiple of 10 to or from any two-digit number, e.g. 27 + 60, 72 – 50 • add 9, 19, 29, … or 11, 21, 31, … • add near doubles, e.g. 13 + 14, 39 + 40	• reorder numbers when adding • partition: bridge through 10 and multiples of 10 when adding and subtracting • partition and combine multiples of tens and ones • use knowledge of pairs making 10 • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add a multiple of 10 and adjust by 1 • partition: double and adjust
Year 3 • addition and subtraction facts for all numbers to 20, e.g. 9 + 8, 17 – 9, drawing on knowledge of inverse operations • sums and differences of multiples of 10, e.g. 50 + 80, 120 – 90 • pairs of two-digit numbers with a total of 100, e.g. 32 + 68, or 32 + ☐ = 100 • addition doubles for multiples of 10 to 100, e.g. 90 + 90	• add and subtract groups of small numbers, e.g. 5 – 3 + 2 • add or subtract a two-digit number to or from a multiple of 10, e.g. 50 + 38, 90 – 27 • add and subtract two-digit numbers e.g. 34 + 65, 68 – 35 • add near doubles, e.g. 18 + 16, 60 + 70	• reorder numbers when adding • identify pairs totalling 10 or multiples of 10 • partition: add tens and ones separately, then recombine • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add or subtract 10 or 20 and adjust • partition: double and adjust • partition: count on or back in minutes and hours, bridging through 60 (analogue times)

Recall: Children should be able to derive and recall:	Mental calculation skills: Working mentally, with jottings if needed, children should be able to:	Mental methods or strategies: Children should understand when to and be able to apply these strategies:
Year 4 • sums and differences of pairs of multiples of 10, 100 or 1000 • addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves • what must be added to any three-digit number to make the next multiple of 100, e.g. 521 + ☐ = 600 • pairs of fractions that total 1	• add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. 47 + 58, 91 – 35 • add or subtract a near multiple of 10, e.g. 56 + 29, 86 – 38 • add near doubles of two-digit numbers, e.g. 38 + 37 • add or subtract two-digit or three-digit multiples of 10, e.g. 120 – 40, 140 + 150, 370 – 180	• count on or back in hundreds, tens and ones • partition: add tens and ones separately, then recombine • partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7 • subtract by counting up from the smaller to the larger number • partition: add or subtract a multiple of 10 and adjust, e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2 • partition: double and adjust • use knowledge of place value and related calculations, e.g. work out 140 + 150 = 290 using 14 + 15 = 29 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)
Year 5 • sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • doubles and halves of decimals, e.g. half of 5.6, double 3.4 • what must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + ☐ = 5000 • what must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + ☐ = 8	• add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. 38 + 86, 620 – 380, 350+ 360 • add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. 235 + 198 • find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 – 4087 • add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.8	• count on or back in hundreds, tens, ones and tenths • partition: add hundreds, tens or ones separately, then recombine • subtract by counting up from the smaller to the larger number • add or subtract a multiple of 10 or 100 and adjust • partition: double and adjust • use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 6 ● addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. 650 + ☐ = 930, ☐ – 1.4 = 2.5 ● what must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. 7.26 + ☐ = 8	● add or subtract pairs of decimals with units, tenths or hundredths, e.g. 0.7 + 3.38 ● find doubles of decimals each with units and tenths, e.g. 1.6 + 1.6 ● add near doubles of decimals, e.g. 2.5 + 2.6 ● add or subtract a decimal with units and tenths, that is nearly a whole number, e.g. 4.3 + 2.9, 6.5 – 3.8	● count on or back in hundreds, tens, ones, tenths and hundredths ● use knowledge of place value and related calculations, e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43 ● use knowledge of place value and of doubles of two-digit whole numbers ● partition: double and adjust ● partition: add or subtract a whole number and adjust, e.g. 4.3 + 2.9 = 4.3 + 3 – 0.1, 6.5 – 3.8 = 6.5 – 4 + 0.2 ● partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24-hour clock)

Multiplication and division

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 1 ● doubles of all numbers to 10, e.g. double 6 ● odd and even numbers to 20	● count on from and back to zero in ones, twos, fives or tens	● use patterns of last digits, e.g. 0 and 5 when counting in fives
Year 2 ● doubles of all numbers to 20, e.g. double 13, and corresponding halves ● doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves ● multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts ● odd and even numbers to 100	● double any multiple of 5 up to 50, e.g. double 35 ● halve any multiple of 10 up to 100, e.g. halve 90 ● find half of even numbers to 40 ● find the total number of objects when they are organised into groups of 2, 5 or 10	● partition: double the tens and ones separately, then recombine ● use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two ● use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five
Year 3 ● multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables, and corresponding division facts ● doubles of multiples of 10 to 100, e.g. double 90, and corresponding halves	● double any multiple of 5 up to 100, e.g. double 35 ● halve any multiple of 10 up to 200, e.g. halve 170 ● multiply one-digit or two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 x 100 ● find unit fractions of numbers and quantities involving halves, thirds, quarters, fifths and tenths	● partition: when doubling, double the tens and ones separately, then recombine ● partition: when halving, halve the tens and ones separately, then recombine ● use knowledge that halving and doubling are inverse operations ● recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts ● recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 4 • multiplication facts to 10 × 10 and the corresponding division facts • doubles of numbers 1 to 100, e.g. double 58, and corresponding halves • doubles of multiples of 10 and 100 and corresponding halves • fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. $\frac{3}{10}$ is 0.3 and $\frac{3}{100}$ is 0.03 • factor pairs for known multiplication facts	• double any two-digit number, e.g. double 39 • double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100 • halve any even number to 200 • find unit fractions and simple non-unit fractions of numbers and quantities, e.g. $\frac{3}{8}$ of 24 • multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 325 × 10, 42 × 100, 120 ÷ 10, 600 ÷ 100, 850 ÷ 10 • multiply a multiple of 10 to 100 by a single-digit number, e.g. 40 × 3 • multiply numbers to 20 by a single-digit, e.g. 17 × 3 • identify the remainder when dividing by 2, 5 or 10 • give the factor pair associated with a multiplication fact, e.g. identify that if 2 x 3 = 6 then 6 has the factor pair 2 and 3	• partition: double or halve the tens and ones separately, then recombine • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder • use knowledge of multiplication facts and place value, e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80 • use partitioning and the distributive law to multiply, e.g. 13 × 4 = (10 + 3) × 4 = (10 × 4) + (3 × 4) = 40 + 12 = 52

Recall: **Children should be able to derive and recall:**	**Mental calculation skills:** **Working mentally, with jottings if needed, children should be able to:**	**Mental methods or strategies:** **Children should understand when to and be able to apply these strategies:**
Year 5 • squares to 10 × 10 • division facts corresponding to tables up to 10 × 10, and the related unit fractions, e.g. 7 × 9 = 63 so one-ninth of 63 is 7 and one-seventh of 63 is 9 • percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths • factor pairs to 100	• multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8 • multiply two-digit numbers by 5 or 20, e.g. 320 × 5, 14 × 20 • multiply by 25 or 50, e.g. 48 × 25, 32 × 50 • double three-digit multiples of 10 to 500, e.g. 380 × 2, and find the corresponding halves, e.g. 760 ÷ 2 • find the remainder after dividing a two-digit number by a single-digit number, e.g. 27 ÷ 4 = 6 R 3 • multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100, 74 ÷ 100 • multiply pairs of multiples of 10, e.g. 60 × 30, and a multiple of 100 by a single digit number, e.g. 900 × 8 • divide a multiple of 10 by a single-digit number (whole number answers) e.g. 80 ÷ 4, 270 ÷ 3 • find fractions of whole numbers or quantities, e.g. $\frac{2}{3}$ of 27, $\frac{4}{5}$ of 70 kg • find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80 • find factor pairs for numbers to 100, e.g. 30 has the factor pairs 1 × 30, 2 × 15, 3 × 10 and 5 × 6	• multiply or divide by 4 or 8 by repeated doubling or halving • form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10 • use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2 • use knowledge of division facts, e.g. when carrying out a division to find a remainder • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder • use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10 • use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10% • use knowledge of multiplication and division facts to find factor pairs

Recall: Children should be able to derive and recall:	Mental calculation skills: Working mentally, with jottings if needed, children should be able to:	Mental methods or strategies: Children should understand when to and be able to apply these strategies:
Year 6 • squares to 12 × 12 • squares of the corresponding multiples of 10 • prime numbers less than 100 • equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $^{35}/_{100}$	• multiply pairs of two-digit and single-digit numbers, e.g. 28 × 3 • divide a two-digit number by a single-digit number, e.g. 68 ÷ 4 • divide by 25 or 50, e.g. 480 ÷ 25, 3200 ÷ 50 • double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 • multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20 • divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300 • multiply and divide two-digit decimals such as 0.8 × 7, 4.8 ÷ 6 • find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g • simplify fractions by cancelling • scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges • identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves	• partition: use partitioning and the distributive law to divide tens and ones separately, e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23 • form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double • use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division • recognise how to scale up or down using multiplication and division, e.g. if three oranges cost 24p: one orange costs 24 ÷ 3 = 8p four oranges cost 8 × 4 = 32p • Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors

2 Principles of teaching mental calculation

This chapter promotes a broad interpretation of mental calculation and identifies principles that underpin teaching: for example, encouraging children to share their mental methods, to choose efficient strategies, and to use informal jottings to keep track of the information they need when calculating. It also looks at the role of tests and questioning.

It also draws on the National Numeracy Strategy's publication *Mathematical vocabulary* produced in 1999. The section on developing children's use of mathematical vocabulary and language and the use of different questions can be found towards the end of this chapter.

Is mental calculation the same as mental arithmetic?

For many adults, mental calculation is about doing arithmetic; it involves rapid recall of number facts – knowing your number bonds to 20 and the multiplication tables to 10 × 10.

Rapid recall of number facts is one aspect of mental calculation but there are others. This involves presenting children with calculations in which they have to work out the answer using known facts and not just recall it from a bank of number facts that are committed to memory. Children should understand and be able to use the relationship between the four operations and be able to construct equivalent calculations that help them to carry out such calculations.

Examples

A Year 2 child who knows that 9 + 9 = 18 and who can add 10 to any two-digit number can work out other results, e.g.

$9 + 8 = 9 + 9 - 1 = 17$

$9 + 18 = 9 + 8 + 10 = 17 + 10 = 27$

$19 + 9 = 10 + 9 + 9 = 10 + 18 = 28$

$19 + 19 = 10 + 9 + 10 + 9 = 20 + 18 = 38$

A Year 3 child who knows that 6 × 4 = 24 can use this to calculate 12 × 4 by doubling.

A Year 4 child who knows that 6 × 7 = 42, should immediately recognise three other facts: 7 × 6 = 42, 42 ÷ 7 = 6 and 42 ÷ 6 = 7, and apply their knowledge of place value to work out: 60 × 7, 70 × 6, 420 ÷ 7 and 420 ÷ 6.

A Year 5 child who knows that 2 × 7 = 14 and 6 × 7 = 42, can work out 26 × 7 by partitioning 26 into 20 and 6 and multiplying the tens and ones separately, from which the answer of 140 + 42 = 182 is readily obtained.

A Year 6 child who knows that 42 ÷ 6 = 7 can use this to calculate 84 ÷ 6 by splitting 84 into 42 + 42 and using their knowledge of 42 ÷ 6 and doubling to reach the answer 14.

Research shows that learning key facts ‘by heart’ enables children to concentrate on the calculation which helps them to develop calculation strategies. Using and applying strategies to work out answers helps children to acquire and so remember more facts. Many children who are not able to recall key facts often treat each calculation as a new one and have to return to first principles to work out the answer again. Once they have a secure knowledge of some key facts, and by selecting problems carefully, you can help children to appreciate that from the answer to one problem, other answers can be generated.

In the National Strategies’ suite of publications: *Securing levels in mathematics* (for example *Securing level 4 in mathematics* Ref: 00065-2009BKT-EN) we have identified six key points to remember when planning and teaching mathematics:

- every day is a mental mathematics day
- hands-on learning is important
- seeing mathematics through models and images supports learning
- talking mathematics clarifies and refines thinking
- make mathematics interesting
- learning from mistakes should build children’s confidence.

Applying these to teaching mental calculation leads to the following teaching principles:

- Commit regular time to teaching mental calculation strategies.
- Provide practice time with frequent opportunities for children to use one or more facts that they already know to work out more facts.
- Introduce practical approaches and jottings with models and images children can use to carry out calculations as they secure mental strategies.
- Engage children in discussion when they explain their methods and strategies to you and their peers.

Revisiting mental work at different times in the daily mathematics lesson, or even devoting a whole lesson to it from time to time, helps children to generate confidence in themselves and a feeling that they control calculations rather than calculations controlling them. Look out too for opportunity to introduce short periods of mental calculation in other lessons or outside lessons when queuing for some activity. Regular short practice keeps the mind fresh. Mental calculation is one of those aspects of learning where – if you don’t use it you will end up losing it!

What's special about mental calculations?

Calculating mentally may involve 'seeing' objects, images or quantities that help you manage the process. But this is not the same as just picturing in your head how to do the calculation using a traditional paper and pencil method.

Examples

A group of Year 4 children is working on mental addition of two-digit numbers.

Jo explains that 36 + 35, a near double, must be 71, since it is double 35 plus 1.

Sam explains that 36 + 35 is 36 plus 30, making 66, plus 4 to make 70, plus 1 more, to make 71. He has worked this out by partitioning the second number into tens and ones, then counting on the tens, then the ones, bridging through 70.

Misha explains that 38 + 37 is 30 plus 30, or 60, plus 8 add 7, which is 15, giving a total of 75. She has partitioned both numbers into tens and ones, and added the tens first. She has also recalled a known fact that she knows by heart: 8 + 7 = 15.

A feature of mental calculation is that a type of calculation can often be worked out in several different ways. Which method is the best will depend on the numbers involved, the age of the children and the range of methods that they are confident with.

This feature differs from a standard written method: the strength of a standard method is that it is generally applicable to all cases irrespective of the numbers involved. For example, the following calculations would all be treated in the same way if the decomposition method of subtraction were used:

61 – 4   61 – 41   61 – 32   61 – 58   61 – 43

If carried out mentally, each calculation would probably be done in a different way.

We expect children to do these kinds of calculation mentally, applying strategies to reflect their confidence with and understanding of the alternative approaches. The clear advantages are that children develop a much stronger 'number sense', better understanding of place value and more confidence with numbers and the number system.

The underlying teaching principle is therefore to:

- Ensure that children can confidently add and subtract any pair of two-digit numbers mentally, using jottings to help them where necessary.

Many children manage to do this by the end of Year 4, when they can then apply themselves more successfully to written column methods for hundreds, tens and ones and larger numbers.

How do I help children to develop a range of mental strategies?

During their primary schooling, children are likely to be at different stages in terms of the number facts that they have committed to memory and the strategies available to them for figuring out other facts. This publication is intended to help you to be more aware of the range of possible strategies children might be taught and use as they progress in mathematics. This way you will be:

- in a better position to recognise the strategies children are using when they calculate mentally
- able to draw attention to and model a variety of the strategies used by the children in your class
- able to make suggestions to children that will move them on to more efficient strategies.

To help children to learn and draw on a range of mental methods, you need to raise their awareness and understanding of the range of possible strategies, develop their confidence and fluency by practising using the strategies, and help them to choose from the range the most efficient method for a given calculation.

Example

A Year 1 teacher is working with a 0 to 10 number line and getting the children to use it to count on.

She deliberately chooses examples of the sort 1 + 6, 1 + 8 and so on to provoke the children into putting the larger number first. Quite quickly one of the children suggests this, and the teacher picks up the strategy and works with the children explicitly on it. Afterwards, she explains that had no one suggested this after a couple of examples she would have pointed it out to the children. It became known as Linka's method after the child who had suggested it and soon this method was in common use by the whole class.

The underlying teaching principle here is to:

- Teach a mental strategy explicitly but in addition invite children to suggest an approach and to explain their methods of solution to the rest of the class.

This has the advantages that:

- children get used to looking out for an approach they can call their own
- children doing the explaining clarify their own thinking
- children who are listening develop their awareness of the range of possible methods
- the activity can lead to a discussion of which methods are the most efficient.

Can mental calculations involve practical equipment?

- Hands-on learning is important

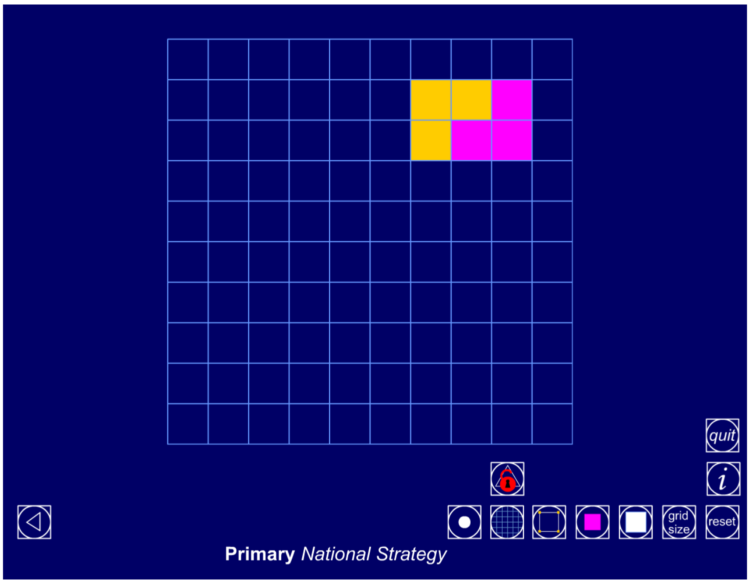
Mental calculations involve visualising, imagining and working things out in your head. In German there is a word for it (but there is no direct equivalent in English): *Gedankenexperimente*, thought experiments, which involve exploring ideas in one's imagination.

But children will not be able to visualise and 'see' how something works if they have not had any practical experiences to draw on or been shown any models and images that support the approaches taught. The underlying teaching principle here is to:

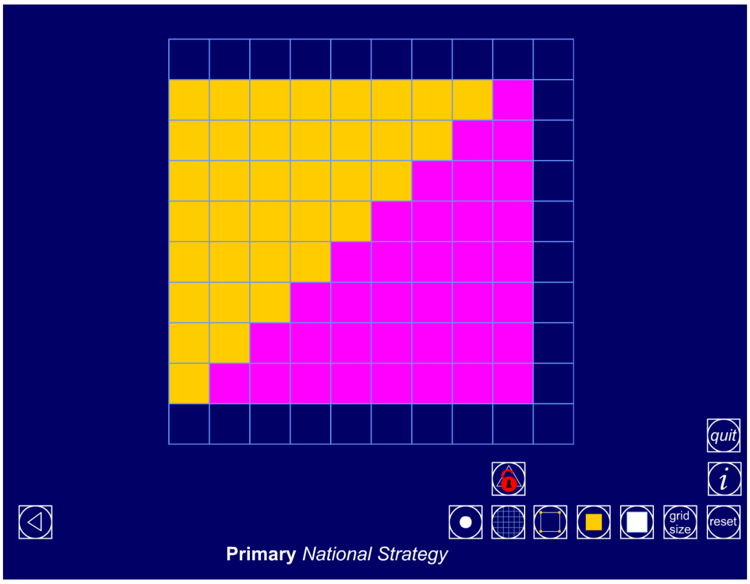
- Provide suitable equipment for children to manipulate and explore how and why a calculation strategy works, and that helps them to describe and visualise or 'see' the method working.

The equipment can include objects like counters, interlocking cubes, coins, counting sticks, bead strings, number lines, 100-squares, place-value cards, structural apparatus like base 10 blocks, diagrams of shapes divided into fractional parts, and so on. An interactive whiteboard is also a powerful tool for manipulating images.

For example, using the 'Area' Interactive Teaching Program (ITP), a Year 1 teacher worked with a group of children to develop the pattern shown and related this to the addition facts 2 + 1 = 3 and 1 + 2 = 3. With the children she increased the number of rows and columns one at a time and each time the children had to give the corresponding number facts represented.



In time the children reached the stage where they had listed all the addition facts for each number up to 9 and had understood that knowing that 3 + 6 = 9 meant that 6 + 3 was also 9. The teacher allowed the children to use the ITP in later lessons to consolidate their learning – she would give them a number for which they had to generate the addition facts and come and tell her all the addition facts for that number with their backs to the board to show they could remember them. Over time they completed the task without the board and generated speed and accuracy of recall.



Selecting when and how to use and to withdraw resources and visual images is a key part of teaching. It is having and applying the principles of fit-for-purpose pedagogy. This involves planning how best to construct a blend of teaching approaches that are selected and designed to match intended learning outcomes and children's needs, and to take account of the context and organisation of children. It is asking: 'What is the purpose I want to achieve?' and 'Is this fit for that purpose or is there a better way?'

Visualisation is important in mathematics. The ability to visualise representations, pictures or images and then adapt or change them is an important tool for example when problem solving, pattern spotting and reasoning in mathematics. Like everything else it is a skill that can be acquired through practice and this involves the use of practical equipment and tactile resources. Giving children carefully structured learning experiences with supporting discussion to describe and refine ideas and thinking will help them develop these visualisation skills.

Visualisation takes place at all stages and ages. Young children recognise pictures and representations of objects or people; they learn to describe something they have seen but cannot see at that time; they associate an image with an object, stimulus or emotion. In mathematics we need to focus and develop these skills. This usually takes place after children have some tactile experiences to draw upon. Practising visualisation helps to develop the brain's capacity to 'see' and to 'draw pictures' in the same way that we need to practise doing so on paper.

Can mental calculations involve pencil and paper?

Young children often support their early mathematical thinking through the use of 'mark making' involving drawings, writing, tally-type marks and invented and standard symbols including numerals. This 'informal' recording helps us all to represent our ideas as we go along. We do not just record the mathematics in a neat and complete way after we have worked something out. Informal ongoing recording is an extension of our thinking.

Pencil and paper can support mental calculation in various ways:

- through jottings – informal notes during the intermediate steps in a calculation

357 ÷ 17

57 − 17 = 40
340 ÷ 17 = 20

- through recording for an explanation of the method used. This is a variation on getting children to explain their methods orally. A written account can help them begin to use appropriate notation and form the basis for developing more formal written methods

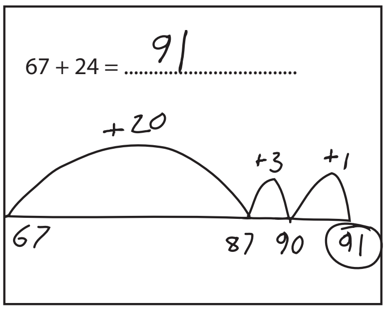
37+28 = ...

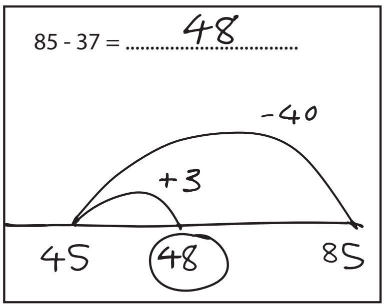
30 + 20 = 50
7 + 8 = 15
50 + 15 = 65

85 - 37 = ...48......

85 − 40 = 45
45 + 3 = 48

- through models and diagrams that support the development of mental imagery and which are visual representations of the way in which a calculation is being carried out.





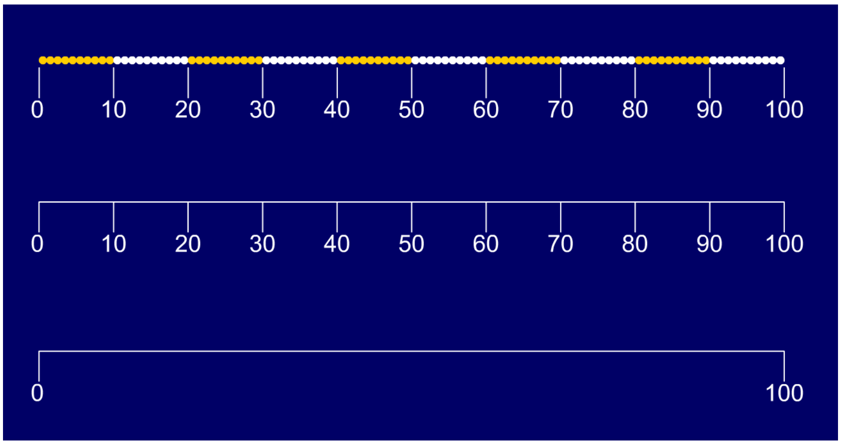
The underlying teaching principle here is to:

- Encourage children to make jottings as they work and to recognise how these can support their thinking; model this process for them and distinguish between a presentation and a jotting.

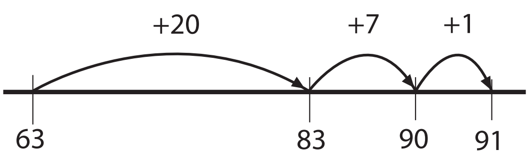
Using an empty number line

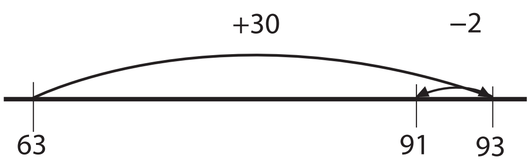
The empty number line is a powerful model for developing children's calculation strategies and developing their understanding.

A progression from practical bead strings, to number lines with landmark numbers such as those with intervals representing multiples of 5 or 10 marked, to empty number lines can help children to develop the image of the number line in their head. The 'Ordering numbers' ITP provides a useful way to help children to make the links between these different representations of whole number sequences.



Here are two examples of the calculation 63 + 28 = 91 represented on the empty number line.



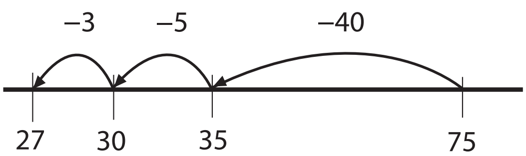


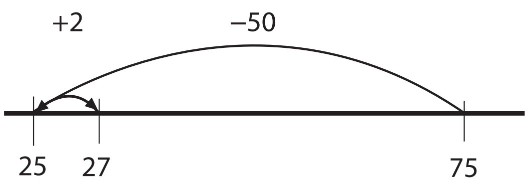
Both of these examples model common mental approaches using counting on. Each starts at the larger number (63).

In the first example, 28 is partitioned into 20 + 8. The tens are added first, then the ones are added by 'bridging through a multiple of 10', in this case 90.

In the second example, we count on more than 28, the next higher multiple of 10, which in this case is 30. We adjust, in this case by 2. This method is sometimes called the 'compensation method'. In either case, the answer of 91 appears as a position on the number line.

One model of subtraction, the inverse of addition, involves counting back. Here are two examples of 75 – 48 = 27 represented on the empty number line. In each case, we count back from the larger number, 75.

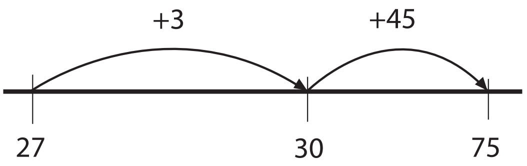




In the first example, 48 is partitioned into 40 + 8. The tens are subtracted first, then the ones, bridging through a multiple of 10 (30).

In the second example we count back too many (the next higher multiple of 10, which is 50) and then adjust, in this case by 2. In either case, the answer of 27 appears as a position on the number line.

Subtraction is also the difference between two numbers. The difference can be represented as the distance between the two numbers on the number line. This distance is usually worked out by counting up from the smaller to the larger number, bridging through a multiple of 10. In this case the answer to 75 – 27 is equivalent to 45 + 3 = 48 represented by the two jumps, not a point on the line. The counting up method is sometimes called 'shopkeeper's method' because it is a bit like a shop assistant counting out change.



We know that many children find it easier to subtract by counting up rather than by counting back.

Is speed important?

Once a mental strategy has been introduced to children, there comes a time to encourage them to speed up their responses and either use more efficient strategies or expand their repertoire. The underlying teaching principle here is:

- Encourage children to compete against themselves, aiming to better their previous performance.

A 'mental test' can help children to monitor changes in their performance over time.

The traditional mental mathematics test involves a set of unseen questions. A worthwhile alternative is to give children examples of the type of questions 10 minutes in advance, so that they can think about the most efficient way to answer the questions. The purpose of this preparation time is not to try to commit answers to memory but to sort the questions into those they 'know' the answer to, and those that they need to figure out. Pairs of children can talk about their 'figuring out' methods and after the test the whole class can spend some time discussing the strategies they used.

Collecting the questions, then giving children the test with the questions in a random order, also encourages attention to strategies. The same test can be used at a different time for children to try to beat their previous score.

Example: One-minute multiplication test

Warn children in advance that they are going to be given a mental multiplication test.

Classes or individuals can be set different challenges according to levels of attainment, so one class or child may be working on multiples of seven while another is working on multiples of three. The same incomplete test paper can be duplicated for particular groups: 10 randomly ordered 'multiplied by' questions:

$\square \times 3 = \square$

$\square \times 7 = \square$

$\square \times 4 = \square$

and so on.

Children are given the multiple which they write in the first box in each calculation. They then have one minute to do this and to fill in the answer boxes. The challenge is to beat their previous best scores.

Example: Related numbers test

Give children a multiplication fact they all know. They each record this on their paper so they can refer to it throughout the test. For example:

$6 \times 3 = 18$

The 10 questions all relate to the recorded number fact. The class agrees a set time that will be allowed for each question. The questions can vary in difficulty over the test so that all children can engage and no-one gives up in the middle of the test. For example the questions might be:

$6 \times 30 = \square$

$12 \times 3 = \square$

$6 \times 1.5 = \square$

$12 \times 6 = \square$

$12 \times 0.3 = \square$

The questions can also involve inverse operations or could follow a particular set of facts that needs consolidation. After the test the different methods and strategies can be compared and discussed in groups, with the children identifying those questions that they found most difficult to answer. At the end of the lesson a target time per question is set for the next challenge and particular question types are identified for further consolidation in preparation for the next test.

When a 'mental test' is marked, there is a chance to discuss any errors made and why they happened. Did the children guess the answer rather than work it out? Or did they make a careless slip in trying to answer within the time limit? Maybe the error indicates a genuine misunderstanding.

Once again, discussion of the methods used can help to reveal why the error was made. Help children to understand that most of us make mistakes at first when we are learning any new skill but that thinking about the mistake and how and why it was made helps us to improve that skill. The underlying teaching principle here is:

- Encourage the children to discuss their mistakes and difficulties in a positive way so that they learn from them and share the ownership of targets to help children to manage and recognise their rate of progress.

How do I make mental calculation practice fun?

Children are easily bored by repetitive practice. Yet practice is essential if children are to increase fluency and confidence with the mental strategies they are learning.

Many of the ITPs can provide fun mental practice (see **www.standards.dcsf.gov.uk/nationalstrategies** and search for 'Guide to Interactive teaching programs'). There are also many enjoyable games and puzzles that involve mental calculation.

Example

Invite a pair of children to the front of the class and give one of them a calculator.

Pose a calculation that you expect the children to be able to do mentally. The child with the calculator must use it to find the answer, even if they can mentally work more quickly. The other child works mentally and records on a small wipe board. The first child to indicate they have the answer scores a point, providing it is correct and confirmed by the rest of the class. If they are wrong they lose a point. The best score out of five questions wins the game.

You may have to be a strong referee to ensure that the 'calculator' child doesn't call out the answer without keying in the calculation! The rest of the class complete the questions on small wipe boards and when requested show the answer to you – from time to time they are given a challenge to beat the calculator.

Some of these games involve recall of facts while others involve more complex calculations. The best of the games and puzzles also involve an element of reasoning. There are increasing numbers of websites that provide free mathematics games for primary children. A carefully selected game from sites like these makes very good homework providing suitable practice in a motivating context.

It is also fairly easy to set up your own number puzzles on an interactive whiteboard using one of the available toolkits. If the context is sufficiently interesting most children are motivated to find solutions. In addition, the context may well suggest methods of finding a solution and these methods can then be discussed.

Example

Some seven-year-olds are working on this problem: three cars, three bicycles and two lorries go past the school gate. How many wheels went by?

After agreeing that there are eight wheels on each lorry, Tess and Sam quickly agree that there are 12 + 6 + 16 wheels. Tess counts on from 12 and announces 33 as her answer. Sam, after a few moments' reflection, announces that it is 34. Asked to explain his method, he replies: 'Well there's 12 and the 10 from there (pointing to the 16) makes 22, there's another six left (from the 16), so that's 28. Two from there (the 6) makes 30 and there's four left so that's 34.'

The teaching principle here is:

- Make use of the National Strategies' ITP, a variety of games and puzzles and realistic contexts around the school, to make mental calculation practice interesting and enjoyable.

How should children respond to oral questions?

The traditional method of asking a question and inviting a volley of hands to go up has several drawbacks for children who are figuring out answers. It emphasises the rapid, the known, over the derived – children who 'know' are the first to answer, while those who are collecting their thoughts are distracted by others straining to raise hands.

Ways of getting round these problems are:

- insisting that nobody puts a hand up until the signal, and silently counting to five or so before giving it
- using digit cards for all children to show their answer at the same time
- asking children to answer by writing their answers on small wipe boards.

Whatever method of response you use, bear in mind the last underlying teaching principle to:

- Allow time to discuss the various ways that children reached the answer, to point out the range of possible strategies and use of jottings; help children to recognise why one method is more efficient than another.

What are the different aspects of mental calculations?

This section has emphasised that mental calculation is more than just recalling number facts, but this is an important skill that helps children to concentrate on their calculations, the problems and the methods involved. Below are six aspects of mathematics that involve mental calculation. They are supported with questions to exemplify what might be asked of children to engage them in mental calculation activity and to stimulate discussion.

Recalling facts

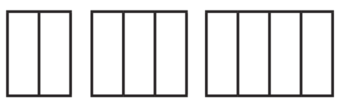
- What is 3 add 7?
- What is 6 x 9?
- How many days are there in a week?... in four weeks?
- What fraction is equivalent to 0.25?
- How many minutes in an hour, in six hours?

Applying facts

- Tell me two numbers that have a difference of 12.
- If 3 x 8 is 24, what is 6 x 0.8?
- What is 20% of £30?
- What are the factors of 42?
- What is the remainder when 31 is divided by 4?

Hypothesising or predicting

- The number 6 is 1 + 2 + 3, the number 13 is 6 + 7. Which numbers to 20 are the sum of consecutive numbers?
- Roughly, what is 51 times 47?
- How many rectangles in the next diagram? And the next?



- On a 1 to 9 key pad, does each row, column and diagonal sum to a number that is a multiple of 3?

Designing and comparing procedures

- How might we count a pile of sticks?
- How could you subtract 37 from 82?
- How could we test a number to see if it is divisible by 6?
- How could we find 20% of a quantity?
- Are these all equivalent calculations: 34 – 19; 24 – 9; 45 – 30; 33 – 20; 30 – 15?

Interpreting results

- So what does that tell us about numbers that end in 5 or 0?
- Double 15 and double again; now divide your answer by 4. What do you notice? Will this always work?
- If 6 x 7 = 42 is 60 x 0.7 = 42?
- I know 5% of a length is 2 cm. What other percentages can we work out quickly?

Applying reasoning

- The seven coins in my purse total 23p. What could they be?
- In how many different ways can four children sit at a round table?
- Why is the sum of two odd numbers always even?

What types of closed and open questions might I ask?

In the table below are examples of closed questions and open questions. Closed questions generally have just one correct answer, while open questions usually offer alternatives and may have a number of different answers, each of which is correct for a different reason. Each type of question has their use and purpose.

The use of questions is generally to:

- **Prompt** thinking and get children started. For example, we might ask what number facts involving the number 12 the children can identify. This is an example of an open question. We can close it down by limiting the examples to division facts to focus thinking about division as the purpose of the lesson may be to extend knowledge of division and its use in solving problems.
- **Probe** thinking and establish the confidence and security of the children's knowledge, skills and understanding. For example, having established that the children can work out 12 ÷ 3 = 4 we might ask whether they could now apply this to work out the answer to 24 ÷ 3. This is an example of a closed question that can help to determine how children 'see' division as we follow up with more probing questions. If children recognise, for example, that 12 ÷ 3 = 4 means that when 12 is divided into 3 groups, each group has 4 items, doubling the 12 will double the group size so the answer is 8. Our probing questions may oscillate between closed and open questions as we engage children in the discussion. They also help us to identify the reasons for any mistakes we might observe children making: they are a key part of assessment for learning in everyday teaching.
- **Promote** thinking to set a new challenge, problem or line of enquiry that the children can follow. For example, having established that children know number facts about 12 and can work out related facts we set children a problem involving how £1200 might be shared out equally between charities. We can have any number of charities but we will only use £5 notes when we distribute the money. What are the possibilities? This is an example of an open question but it could be closed down as new conditions are introduced to promote new thinking and limit the range of possible solutions.

Closed questions help to establish specific areas of knowledge, skills and understanding; they often focus on children providing explanations as to how and why something works and can be applied when identifying and developing approaches and strategies for a particular purpose.

Open questions help to generate a variety of alternative solutions and approaches that offer children a chance to respond in different ways; they often focus on children providing explanations and reasons for their choices and decisions and a comparison of which of the alternative answers are correct or why strategies are more efficient.

Closed Questions

Count these cubes.

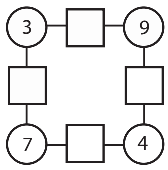
A chew costs 3p. A lolly costs 7p.
What do they cost altogether?

What is 6 – 4?

What is 2 + 6 – 3?

Is 16 an even number?

Write a number in each box so that it equals the sum of the two numbers on each side of it.



Copy and complete this addition table.

+	4	7
2		
6		

What are four threes?

What is 7 x 6?

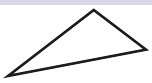
How many centimetres are there in a metre?

Continue this sequence: 1, 2, 4...

What is one-fifth add four-fifths?

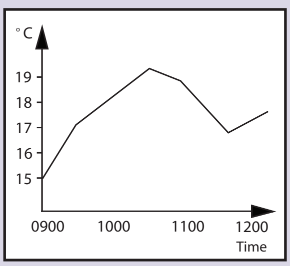
What is 10% of 300?

What is this shape called?



This graph shows room temperature on 19 May.

What was the temperature at 10.00am?



Open Questions

How could we count these cubes?

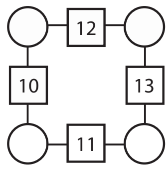
A chew and a lolly costs 10p altogether. What could each sweet cost?

Tell me two numbers with a difference of 2.

What numbers can you make with 2, 3 and 6?

How do you know whether 16 is even?

Write a number in each circle so that the number in each box equals the sum of the two numbers on each side of it. Find different ways of doing it.



Find different ways of completing this table.

	3	4
	7	

Tell me two numbers with a product of 12.

If 7 x 6 = 42, what else can you work out?

Tell me two lengths that together make 1 metre.

Find different ways of completing this sequence: 1, 2, 4...

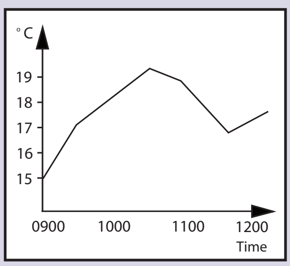
Write eight different ways of adding two numbers to make 1.

Find ways of completing: ...% of ... =30

Sketch some different triangles.

This graph shows room temperature on 19 May.

Can you explain it?



What types of questions can I ask to extend children's thinking?

Ask children who are getting started on a piece of work:

How are you going to tackle this? What information do you have? What do you need to find out or do? What operation/s are you going to use? Will you do it mentally, with pencil and paper, using a number line, with a calculator…? Why? What method are you going to use? Why? What equipment will you need? What questions will you need to ask? How are you going to record what you are doing? What do you think the answer or result will be? Can you estimate or predict?

Make positive interventions to check progress while children are working, by asking:

Can you explain what you have done so far? What else is there to do? Why did you decide to use this method or do it this way? Can you think of another method that might have worked? Could there be a quicker way of doing this? What do you mean by…? What did you notice when…? Why did you decide to organise your results like that? Are you beginning to see a pattern or a rule? Do you think that this would work with other numbers? Have you thought of all the possibilities? How can you be sure?

Ask children who are stuck:

Can you describe the problem in your own words? Can you talk me through what you have done so far? What did you do last time? What is different this time? Is there something that you already know that might help? Could you try it with simpler numbers… fewer numbers… using a number line…? What about putting things in order? Would a table help, or a picture/diagram/graph? Why not make a guess and check if it works? Have you compared your work with anyone else's?

To help assess children's progress ask:

How did you get your answer? Can you describe your method/pattern/rule to us all? Can you explain why it works? What could you try next? Would it work with different numbers? What if you had started with… rather than…? What if you could only use…? Is it a reasonable answer/result? What makes you say so? How did you check it? What have you learned or found out today? If you were doing it again, what would you do differently? Having done this, when could you use this method/information/idea again? Did you use any new words today? What do they mean? What are the key points or ideas that you need to remember for the next lesson?

3 Addition and subtraction strategies

This chapter sets out the main methods for adding and subtracting mentally.

Each section starts with examples of typical problems and describes activities to support teaching of the methods. It covers these strategies:

- Counting forwards and backwards
- Reordering
- Partitioning: counting on or back
- Partitioning: bridging a multiple of 10
- Partitioning: compensating
- Partitioning: using 'near' doubles
- Partitioning: bridging through 60 to calculate a time interval

Features of addition and subtraction

Numbers can be added in any order. Take any pair of numbers, say 7 and 12, then:

$$7 + 12 = 12 + 7$$

This is the commutative law of addition. It applies to addition but not subtraction. In subtraction, order does matter. So, 5 – 3 is not the same as 3 – 5.

But a series of consecutive subtractions can be taken in any order. For example,

$$15 - 3 - 5 = 15 - 5 - 3$$

When three numbers are added together, they too can be taken in any order because of the associative law and the commutative law. In practice, two of the numbers have to be added together or associated first, and then the third number is added to the associated pair to give the result of the calculation. For example:

$$\begin{aligned} 7 + 5 + 3 &= (7 + 5) + 3 \\ &= 7 + (5 + 3) \\ &= (7 + 3) + 5 \end{aligned}$$

Because of the inverse relationship between addition and subtraction, every addition calculation can be replaced by an equivalent subtraction calculation and vice versa. For example the addition:

$5 + 7 = 12$

implies $5 = 12 - 7$

and $7 = 12 - 5$

In the same way:

$13 - 6 = 7$

implies $13 = 7 + 6$

and $6 = 13 - 7$

Any numerical equivalence can be read from left to right or from right to left, so

6 + 3 = 9 can always be rearranged as 9 = 6 + 3.

Knowing addition and subtraction facts

The National Curriculum and the *Primary Framework for mathematics* make clear that children should learn number facts and multiplication tables 'by heart'. If they cannot recall these facts rapidly and always resort to a basic counting strategy instead, they are distracted from thinking about the calculation strategy they are trying to use.

The first chapter in this booklet lists the facts that children in each year group should be able to derive and recall rapidly. These expectations are based on the objectives in the 'Knowing and using number facts' strand of the *Primary Framework for mathematics*.

Counting forwards and backwards

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and backwards. They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

Examples of expectations over Years 1 to 6

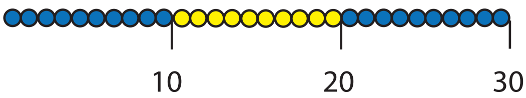
	Example calculations	Possible counting strategy
Year 1	4 + 5	count on in ones from 4 (or in ones from 5)
	8 – 3	count back in ones from 8
	10 + 7	count on in ones from 10 (or use place value)
	13 + 5	count on in ones from 13
	17 – 3	count back in ones from 17
	18 – 6	count back in twos
Year 2	23 + 5	count on in ones from 23
	57 – 3	count back in ones from 57
	60 + 5	count on in ones from 60 (or use place value)
	80 – 7	count back in ones from 80 (or use knowledge of number facts to 10 and place value)
	27 + 60	count on in tens from 27
	72 – 50	count back in tens from 72
Year 3	50 + 38	count on in tens then ones from 50
	90 – 27	count back in tens then ones from 90
	34 + 65	count on in tens then ones from 34
	87 – 23	count back in tens then ones from 87
	35 + 15	count on in steps of 5 from 35

Year 4	73 – 68	count up from 68, counting 2 to 70 then 3 to 73
	47 + 58	count on 50 from 47, then 3 to 100, then 5 to 105
	124 – 47	count back 40 from 124, then 4 to 80, then 3 to 77
	570 + 300	count on in hundreds from 570
	960 – 500	count back in hundreds from 960
Year 5	3.2 + 0.6	count on in tenths
Year 6	1.7 + 0.55	count on in tenths and hundredths

Activities

Ask children to count from zero in ones, one after the other round the class. When you clap, they must count backwards. On the next clap, they count forwards, and so on.

Extend to counting in tens, twos, fives, threes, and other multiples. Vary the starting number. Extend to negative numbers.



Ensure children have experience of using bead strings to underpin their understanding of the number line. Make a number line that goes up in tens, large enough for the whole class or a group to see.

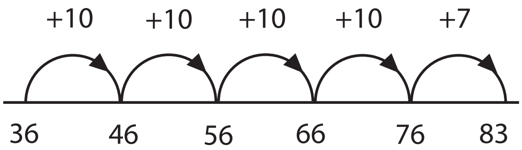
Ask an individual child to show where a number like 26 would fit on the line. Ask other children to fit some numbers close to 26, such as 23 or 28. They may find that they need to adjust the positions of the numbers until they are satisfied with them. Get them to explain what they did to the rest of the class.

This activity encourages children to imagine where the numbers 1 to 9, 11 to 19 and 21 to 29 would appear on the line, and to count on mentally before they decide where to place the number they are given. The idea can be extended to decimals, for example, with a line numbered 1, 2, 3 for positioning of numbers in tenths, or numbered 0.1, 0.2, 0.3 for positioning of numbers in hundredths.

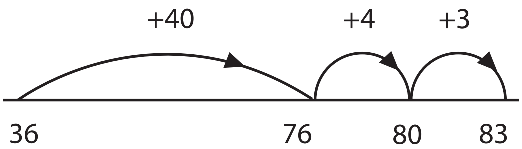
Some of the ITPs can support counting on and back, e.g.

'Counting'	'Counting on and back'
'Number grid'	'Number line'
'Bead sticks'	'Thermometer'

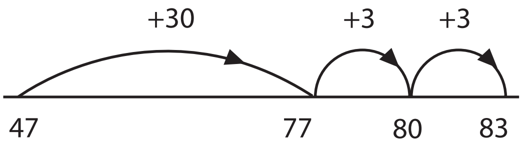
Get children to record a two-digit number on an empty number line. For example, 36 + 47 might be seen as counting on from 36 initially in steps of 10:



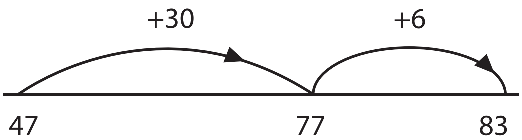
or by first counting on a step of 40 to 76, then bridging through 80 using two steps:



or by reordering the calculation and then counting on from 47, bridging through 80 using two steps:



or by counting on 30 to 77, then using knowledge of number facts to 20 and place value to reach 83 in one step:



Empty (or blank) number lines or bead strings give a useful way for children to record their working and help you to see what method they are using. Discuss the different methods that children use. Encourage them to move to more efficient methods using fewer steps.

Tell the class that you will move along an imaginary number line. You will tell them what number you are standing on and what size steps you are taking.

For example, 'I am on 15 and am taking steps of 10.' Invite them to visualise the number 15 on a number line and to tell you where you will be if you take one step forward (25). Take three more steps forward and ask: 'Where am I now?' (55). Take two steps back and ask: 'Where am I now?' (35), and so on.

Activities such as this help children to visualise counting on or back. The activity can be used for larger numbers, visualising taking steps for 1s, jumps for 10 and leaps for 100s. For example, tell them you are standing on 1570 and making leaps of 100 and ask them to visualise this, asking questions such as: 'Where am I if I make two leaps forwards?'

Reordering

Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly.

It is important for children to know when numbers can be reordered:

e.g. 2 + 5 + 8 = 8 + 2 + 5 or 15 + 8 – 5 = 15 – 5 + 8 or 23 – 9 – 3 = 23 – 3 – 9

and when they can't be reordered:

e.g. 8 – 5 ≠ 5 – 8

The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally.

Examples of expectations over Years 1 to 6

	Example calculations	Possible reordering strategy
Year 1	2 + 7	7 + 2
	5 + 13	13 + 5
	10 + 2 + 10	10 + 10 + 2
Year 2	5 + 34	34 + 5
	5 + 7 + 5	5 + 5 + 7
Year 3	23 + 54	54 + 23
	12 – 7 – 2	12 – 2 – 7
	13 + 21 + 13	13 + 13 + 21 (using double 13)
Year 4	6 + 13 + 4 + 3	6 + 4 + 13 + 3
	17 + 9 – 7	17 – 7 + 9
	28 + 75	75 + 28 (thinking of 28 as 25 + 3)
Year 5	12 + 17 + 8 + 3	12 + 8 + 17 + 3
	25 + 36 + 75	25 + 75 + 36
	58 + 47 – 38	58 – 38 + 47
	200 + 567	567 + 200
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
Year 6	3 + 8 + 7 + 6 + 2	3 + 7 + 8 + 2 + 6
	34 + 27 + 46	34 + 46 + 27
	180 + 650	650 + 180 (thinking of 180 as 150 + 30)
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
	4.7 + 5.6 – 0.7	4.7 – 0.7 + 5.6 = 4 + 5.6

Activities

Present children with groups of three then four numbers that they are to add in their head. Make sure that, in each group of numbers, there are two numbers that have a total of 10. For example:

8 + 3 + 5 + 2

Discuss their methods. See if any children chose to add 8 + 2 first and then add on the 5 + 3, or linked the 3 + 5 and added 8 + (3 + 5) + 2.

Give children similar examples and encourage them to look for pairs that add to make 10 or that make doubles before they start to add. Get them to make up similar examples for each other.

Have regular short, brisk practice sessions where children are given 10 questions such as:

2 + 7 + 8 + 5 + 4 + 3

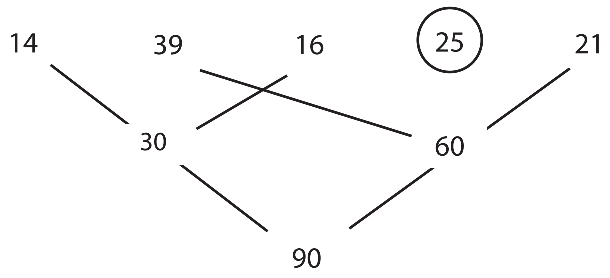
where some pairs total 10. Encourage children to time their responses, keep a personal record of their times and try to beat their personal best.

Give children the same set of questions at regular intervals and encourage them to see how rapidly they can get to the answers. This should ensure that every child sees that they have made progress.

When children can find pairs of numbers that add to make multiples of 10, they can make use of this information when adding several numbers together. For example, when they add

14 + 39 + 16 + 25 + 21

it is sensible to pair numbers:



90 + 25 =115

Children should learn that it is worth looking at all numbers that are to be added to see whether there are pairs that make convenient multiples of 10. The number tree shown in the diagram can be a helpful model for pairing numbers.

In some sequences of numbers, the re-ordering strategy is useful and can give opportunities for an investigative approach. For example:

Find quick ways of finding these answers:

$1 + 2 + 3 + 4 + 5 + 6 = ?$

$5 + 7 + 9 + 11 + 13 = ?$

$3 + 6 + 9 + 12 + 15 + 18 = ?$

$1 + 2 + 3 + 4 + \ldots + 98 + 99 = ?$

Series of numbers such as these are always easier to add by matching numbers in pairs.

In the first example, it is easier to add $1 + 6 = 7$, $2 + 5 = 7$, $3 + 4 = 7$, and then to find 7×3.

In the last example, combining $1 + 99$, $2 + 98$, and so on up to $49 + 51$, gives $(100 \times 49) + 50 = 4950$.

Use a set of number cards, making sure that there are pairs that make multiples of 10. Divide the class into groups of three and give each child a card. Ask each group to add their numbers together. Encourage them to look for pairs of numbers to link together.

List all the totals on a board or projector. Ask whose numbers give the largest total. Get children to swap their cards with someone in another group and repeat. The numbers on the cards can depend on the knowledge of the children. For example, they could be:

23	30	17	52	24	8	70	16
12	30	60	140	170	50	80	

You can organise sets of cards so that each group gets cards that match their number skills.

The activity can be extended to decimals, but in this case the aim is to make pairs that make a whole number: e.g. use 1.4, 3.2, 0.6, 0.2, 1.6, 0.8, 2.3.

Partitioning: counting on or back

It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that 326 = 300 + 20 + 6. In this way, numbers are seen as wholes, rather than as a collection of single digits in columns.

This way of partitioning numbers can be a useful strategy for adding and subtracting pairs of numbers. Both numbers can be partitioned, although it is often helpful to keep the first number as it is and to partition just the second number.

Examples of expectations over Years 2 to 6

	Example calculations	Possible partitioning and counting strategy
Year 2	30 + 47	30 + 40 + 7
	78 – 40	70 + 8 – 40 = 70 – 40 + 8
	17 + 14	10 + 7 + 10 + 4 = 10 + 10 + 7 + 4
Year 3	23 + 45	40 + 5 + 20 + 3 = 40 + 20 + 5 + 3
	68 – 32	60 + 8 – 30 – 2 = 60 – 30 + 8 – 2
Year 4	55 + 37	55 + 30 + 7 = 85 + 7
	365 – 40	300 + 60 + 5 – 40 = 300 + 60 – 40 + 5
Year 5	43 + 28 + 51	40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1
	5.6 + 3.7	5.6 + 3 + 0.7 = 8.6 + 0.7
	4.7 – 3.5	4.7 – 3 – 0.5
Year 6	540 + 280	540 + 200 + 80
	276 – 153	276 – 100 – 50 – 3

Some of the ITPs can help children to partition numbers, e.g.

'Place value'

'Bead sticks'

Visit: **www.standards.dcsf.gov.uk/nationalstrategies** and search for 'Interactive teaching programs'.

Activities

Use a dice marked: 1, 1, 10, 10, 100, 100 for the game 'Target 500' with a group of children. Each player may roll it as many times as they wish, adding the score from each roll and aiming at the target of 500. They must not 'overshoot'. If they do, they go bust!

For example, a sequence of rolls may be:

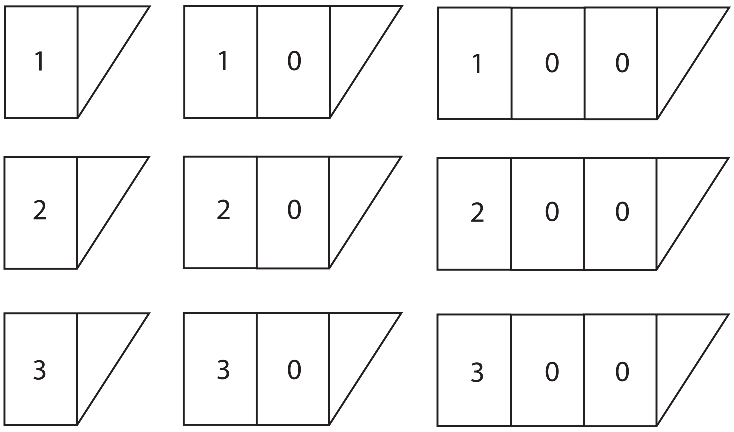
10, 10, 1, 100, 1, 100, 100, 1, 1, 10, 100.

At this point, with a total of 434, a player might decide not to risk another roll (in case 100 is rolled) and stop, or to hope for another 10.

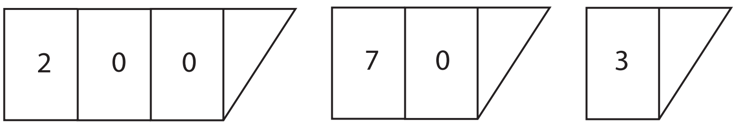
The winner is the player who gets nearest to 500.

This game practises building up numbers by mental addition using ones, tens and hundreds.

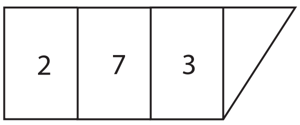
Use place value cards 1 to 9, 10 to 90 and 100 to 900, for example:



Ask children to use the cards to make a two-digit or a three-digit number by selecting the cards and placing them on top of each other. For example, to make 273, the cards



can be placed over each other to make:



This could be used as a class activity, in which case you could ask individual children to select the appropriate card and to put it in the correct place.

Alternatively, it could be used with individuals as a diagnostic task, to check whether they understand place value in this context.

With a group of three to five players, play a game using the place value cards 1 to 9, 10 to 90 and 100 to 900. You also need a set of two-digit and three-digit number cards to use as target numbers, e.g.

153	307	682	914	530	890
745	201	26	79	468	96

Put the target numbers in a pile and turn them over, one by one, to set a target. Deal the 27 place value cards between the players.

Player A inspects their cards to see if they have any part of the target number. If so, they put it on the table.

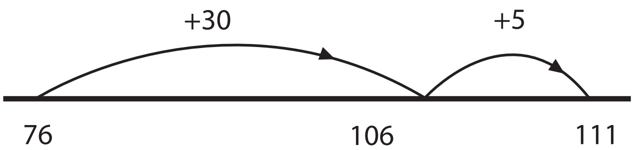
Play continues anti-clockwise. Player B checks to see whether they have another part of the number, followed by players C, D, and so on. Whoever completes the target number keeps it.

The winner is the player who wins the most target numbers.

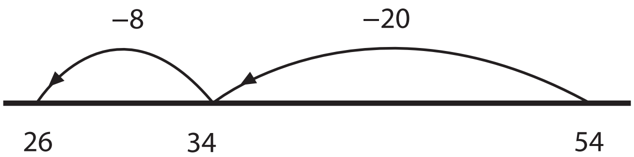
Games like this give motivating practice in partitioning numbers into hundreds, tens and ones.

Use the empty number line to add or subtract two-digit numbers by partitioning the second number and counting on or back in tens then ones, e.g.

76 + 35:



54 – 28:

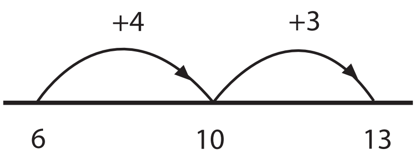


Empty number lines are a useful way to record how children use multiples of 10 or 100 to add or subtract. They give a means for discussing different methods and encourage the use of more efficient methods.

Partitioning: bridging through multiples of 10

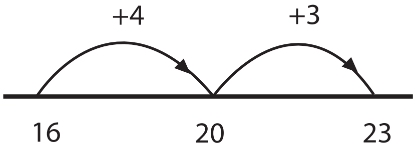
An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40.

In mental addition or subtraction, it is often useful to count on or back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example, 6 + 7 is worked out in two jumps, first to 10, then to 13. The answer is the last point marked on the line, 13.



Subtraction, the inverse of addition, can be worked out by counting back from the larger number. But it can also be represented as the difference or 'distance' between two numbers. The distance is often found by counting up from the smaller to the larger number, again bridging through multiples of 10 or 100. This method of complementary addition is sometimes called 'shopkeeper's method' because it is like a shop assistant counting out change. So the change from £1 for a purchase of 37p is found by counting coins into the hand: '37p and 3p is 40p, and 10p makes 50p, and 50p makes £1'.

The empty number line can give an image for this method. The calculation 23 – 16 can be built up as an addition:



'16 and 4 is 20, and 3 is 23, so add 4 + 3 for the answer.' In this case the answer of 7 is not a point on the line but is the total distance between the two numbers 16 and 23.

A similar method can be applied to decimals, but here, instead of building up to a multiple of 10, bridging is through the next whole number. So 2.8 + 1.6 is 2.8 + 0.2 + 1.4 = 3 + 1.4.

Examples of expectations over Years 2 to 6

	Example calculations	Possible bridging strategy
Year 2	5 + 8 or 12 – 7	5 + 5 + 3 or 12 – 2 – 5
	65 + 7 or 43 – 6	65 + 5 + 2 or 43 – 3 – 3
	24 – 19	19 + 1 + 4
Year 3	49 + 32	49 + 1 + 31
	90 – 27	27 + 3 + 60
Year 4	57 + 34 or 92 – 25	57 + 3 + 31 or 92 – 2 – 20 – 3
	84 – 35	35 + 5 + 40 + 4
Year 5	607 – 288	288 + 12 + 300 + 7
	6070 – 4987	4987 + 13 + 1000 + 70
Year 6	1.4 + 1.7 or 5.6 – 3.7	1.4 + 0.6 + 1.1 or 5.6 – 0.6 – 3 – 0.1
	0.8 + 0.35	0.8 + 0.2 + 0.15
	8.3 – 2.8	2.8 + 0.2 + 5.3 or 8.3 – 2.3 – 0.5

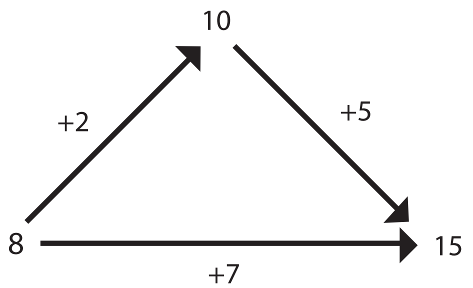
Activities

Show the class a single-digit number and ask a child to find its complement to 10. Repeat this many times, encouraging children to respond as quickly as they can. Then offer two-digit numbers and ask for complements to 100. Extend to decimals. What must be added to 0.78 to make 1?

Activities such as these give practice so that children can acquire rapid recall of complements to 10 or 100 or 1. Using practical resources such as multilink cubes or hundred squares helps children to visualise complements.

Give the class two single-digit numbers to add. Starting with the first number (the larger), ask what part of the second number needs to be added to make 10, then how much more of it remains to be added on. Show this on a diagram like this:

Example 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15



Diagrams like this are a useful model for recording how the starting number is built up to 10 and what remains to be added. Children can be given blank diagrams and asked to use them to add sets of numbers less than 10.

Give examples based on money, asking children to show what coins they would use to build up to the next convenient amount. For example:

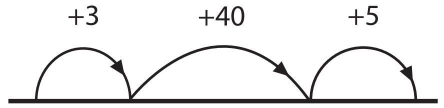
- A packet of crisps costs 27p.
  How much change do you get from 50p?
  27p and 3p is 30p. Another 20p makes it up to 50p. The change is 3p + 20p = 23p.
- A paperback book costs £4.87.
  How much change do you get from £10?

This is the natural way to find a difference when using money. Coins provide a record of how the change was given. This method can usefully be applied to other instances of subtraction, such as 'The cake went into the oven at 4.35pm. It came out at 5.15pm. How long did it take to cook?'

Use empty number lines for addition and subtraction, using multiples of 10 as landmark numbers. For example:

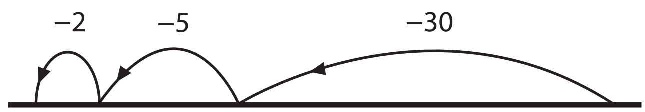
85 – 37

- by counting up from 37 to 85 (the shopkeeper's method):



'37 and 3 makes 40, and 40 make 80, and 5 makes 85. So add 3 + 40 + 5 to get the answer.' The answer is 48, the distance between 37 and 85 on the number line.

- by partitioning 37 and counting back 30 then 7 from 85



85 take away 30 is 55, take away 5 is 50, take away 2 is 48. This time the answer is the last point marked on the line, 48.

In the first example, the next multiple of 10 is a first 'landmark' from the starting number, so 37 is built up to 40. Subsequent landmarks might be other multiples of 10 (80 in the first case). In the second example, the first landmark is 55, then the next is 50. Encouraging children to use number lines in this way provides a mental image that can assist with mental calculations.

Write a set of decimals on the board, such as:

3.6, 1.7, 2.4, 6.5, 2.3, 1.1, 1.5, 1.8, 2.2, 3.9

Ask children to find pairs that make a whole number.

Extend the activity to finding pairs of numbers that make a whole number of tenths, e.g.

0.07, 0.06, 0.03, 0.05, 0.04, 0.05, 0.09, 0.01

The first activity with decimals is to build up to whole numbers, so 3.6 is added to 2.4 to make 6. In the case of hundredths, pairs of numbers such as 0.06 and 0.04 can be added to make 0.1.

Partitioning: compensating

This strategy is useful for adding and subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added or subtracted is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1; subtracting 18 is carried out by subtracting 20, then adding 2.

A similar strategy works for adding or subtracting decimals that are close to whole numbers. For example:

1.4 + 2.9 = 1.4 + 3 – 0.1 or 2.45 – 1.9 = 2.45 – 2 + 0.1.

Examples of expectations over Years 2 to 6

	Example calculations	Possible compensation strategy
Year 2	34 + 9 34 + 19 34 + 29 and so on	34 + 10 – 1 34 + 20 – 1 34 + 30 – 1 and so on
	34 + 11 34 + 21 34 + 31 and so on	34 + 10 + 1 34 + 20 + 1 34 + 30 + 1 and so on
	70 – 9	70 – 10 + 1
Year 3	53 + 12	53 + 10 + 2
	53 – 12	53 – 10 – 2
	53 + 18	53 + 20 – 2
	84 – 18	84 – 20 + 2
Year 4	38 + 68	38 + 70 – 2
	95 – 78	95 – 80 + 2
	58 + 32	58 + 30 + 2
	64 – 32	64 – 30 – 2
Year 5	138 + 69	138 + 70 – 1
	405 – 399	405 – 400 + 1
Year 6	2½ + 1¾	2½ + 2 – ¼
	5.7 + 3.9	5.7 + 4.0 – 0.1
	6.8 – 4.9	6.8 – 5.0 + 0.1

Activities

Prepare two sets of cards for a subtraction game. Set A has numbers from 12 to 27. Set B contains only 9 and 11 so that the game involves subtracting 9 and 11. Shuffle the cards and place them face down.

Each child needs a playing board.

15	3	9	16
5	18	4	17
13	7	12	8
6	11	14	10
18	1	2	17

Children take turns to choose a number from set A and then one from set B. They subtract the number from set B from the one from set A and mark the answer on their board. The first person to get three numbers in a row on their board wins.

Discuss the strategy used to subtract. Encourage the use of compensating.

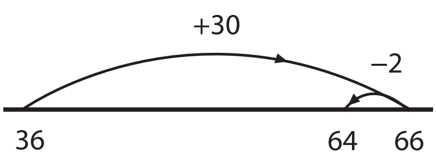
By changing the numbers in set A, negative numbers can also be used.

Use a number square for adding tens and numbers close to 10. To find 36 + 28, first find 36 + 30 by going down three rows, then compensate by going back along that row two places:

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

Children need to know that they can use the table to add 10 to any number by moving down to the number below it. For example, 36 + 10 = 46, which is just below 36, and 36 + 20 is 56, to be found two rows below. Subtracting 10 is modelled by moving to numbers in the row above. Children can use this strategy for adding or subtracting numbers that are close to a multiple of 10 by finding the correct row and then moving to the right or the left.

Use a number line to support mental calculations such as 36 + 28 by counting on 30 and compensating by counting back by 2:



Ask children to visualise a number line to show:

45 + 29, 27 + 39, …

The number line is a means of showing how the process of counting on and then back works. It is a useful way of getting children to visualise similar examples when they work entirely mentally.

Practise examples of subtracting multiples of 10 or 100 such as 264 – 50, 857 – 300.

Then ask children to subtract numbers such as 49 or 299, and so on. Encourage them to use rounding and compensating, so:

264 – 49 = 264 – 50 + 1 and

857 – 299 = 857 – 300 + 1.

Slowly move on to numbers that are further away from a multiple of 10 or 100, such as 7 or 92.

This activity could be a class activity. Individual children round the class could give the answer. Encourage explanation of strategies.

Prepare a set of cards to work with a group of children, for example:

3.9	2.9	5.1	4.1	5.8	3.2

Children take turns to choose a single-digit number, to turn over one of the prepared cards and then add the two numbers. Get children to tell others their addition strategy. Encourage them to see, for example:

7 + 4.1 as 7 + 4 + 0.1 and

7 + 3.9 as 7 + 4 – 0.1.

Extend this by choosing a number with one decimal place as the starting number. So, for example, 2.4 + 3.9 = 2.4 + 4 – 0.1.

Partitioning: using 'near' doubles

If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that 6 + 6 = 12, they can be encouraged to use this to help them find 7 + 6, rather than use a counting on strategy or bridging through 10.

Examples of expectations over Years 1 to 6

	Example calculations	Possible compensation strategy
Year 1	6 + 7	is double 6 and add 1 or double 7 and subtract 1
Year 2	13 + 14	is double 13 and add 1 or double 14 and subtract 1
	39 + 40	is double 40 and subtract 1
Year 3	18 + 16	is double 18 and subtract 2 or double 16 and add 2
	60 + 70	is double 60 and add 10 or double 70 and subtract 10
Year 4	76 + 75	is double 76 and subtract 1 or double 75 and add 1
Year 5	160 + 170	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
Year 6	2.5 + 2.6	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

Activities

Choose a 'double fact' and display it on the board, for example:

8 + 8 = 16

Invite someone to give the total. Then ask for suggestions of addition facts that children can make by changing one of the numbers, for example:

8 + 9 = 17 7 + 8 = 15

Extend the activity by giving children double facts that they might not know, such as:

17 + 17 = 34 28 + 28 = 56 136 + 136 = 272

Then ask children to say how they could work out:

17 + 18 or 16 + 17 or 27 + 28 or 136 + 137.

Invite children to give their own double fact and ask other children to suggest some addition facts that they can generate from it. You can also extend this activity to decimals.

Working with a whole class, ask one child to choose a number smaller than 10. Then ask them, in turns, to double the number, then double the result and so on. Get them to write the numbers on the board or on an overhead projector, for example:

3 6 12 24 48 96 …

Challenge them to see how far they can go with this doubling sequence.

Ask them to produce another sequence by starting with a number, then doubling it and adding 1 each time. So starting again with 3, say, they would get:

3 7 15 31 63 …

Variations would be to use the rules 'double, then subtract 1' or 'double, then add 2'.

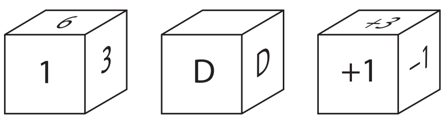
Being proficient at doubling is essential if children are to find 'near doubles'. Asking children to produce the next term in a sequence ensures that they must all take note of answers given by other children.

Play 'Think of a number'. Use a rule that involves doubling and adding or subtracting a small number, for example:

'I'm thinking of a number.
I doubled it and added 3.
My answer is 43.
What was my number?'

'Think of a number' activities require children to 'undo' a process by using inverse operations. This activity gives practice in both halving and doubling. Invite children to invent similar examples themselves.

This game for a group of children needs three dice. One is numbered 1 to 6; a second has four faces marked with a D for 'double' and two blank faces; the third is marked +1, +1, +1, +2, –1, –1.



Children take turns to throw the three dice and record the outcome. They then decide what number to make. For example, if they throw 3, D and + 1 they could: double the 3 then add 1 to make 7, or they could add 1 to the 3 to make 4 and then double 4 to make 8.

What is the smallest possible total?
What is the largest possible total?
What totals are possible with these three dice? Which totals can be made in the most ways?

Games such as this motivate children to practise the strategy and consider questions such as: 'What totals are possible?' Encourage children to reflect on the processes rather than to find just one answer.

Get children to practise adding consecutive numbers such as 45 and 46. Then give children statements such as: 'I add two consecutive numbers and the total is 63.' Ask them: 'What numbers did I add?'

Knowing doubles of numbers is useful for finding the sum of consecutive numbers. The reverse process is more demanding.

Partitioning: bridging through 60 to calculate a time interval

Time is a universal non-metric measure.

A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60.

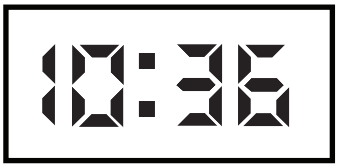
So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minutes.

Examples of expectations over Years 3 to 6

	Examples of mental questions
Year 3	It is 10.30am. How many minutes to 10.45am?
	It is 3.45pm. How many minutes to 4.15pm?
Year 4	I get up 40 minutes after 6.30am. What time is that?
	What is the time 50 minutes before 1.10pm?
	It is 4.25pm. How many minutes to 5.05pm?
Year 5	What time will it be 26 minutes after 3.30am?
	What was the time 33 minutes before 2.15pm?
	It is 4.18pm. How many minutes to 5.00pm? 5.26pm?
Year 6	It is 08.35. How many minutes is it to 09.15?
	It is 11.45. How many hours and minutes is it to 15.20?
	A train leaves London for Leeds at 22.33. The journey takes 2 hours 47 minutes. What time does the train arrive?

Activities

Have a digital clock in the classroom.

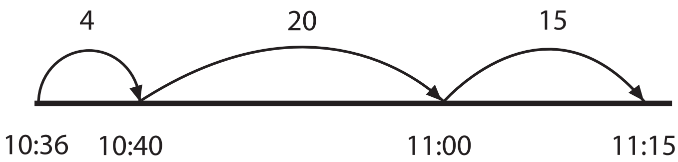


Get the class to look at it at various times of the day and ask: 'How many minutes is it to the next hour (or next o'clock)?'

Encourage children to count on from 36 to 40, then to 50, then to 60, to give a total of 24 minutes.

Then ask questions such as: 'How long will it be to 11.15?' Get them to count on to 11.00 and then add on the extra 15 minutes.

The calculation can be modelled on a number line labelled in hours and minutes.



Some children may think that minutes on digital clocks behave like ordinary numbers, so that they might count on 59, 60, 61 and so on, not realising that at 60 the numbers revert to zero as the hour is reached. It helps if you draw attention to what happens to the clock soon after, say, 9.58 and stress the difference between this and other digital meters such as electricity meters or the meters that give the distance travelled by a bicycle or car.

Give a group of children statements such as:

'Jane leaves home at 8.35 am. She arrives at school at 9.10am. How long is her journey?'

Discuss the children's methods of finding the answer, writing each on the board. Some may say:

'8.35, 8.40, 8.50, 9.00, 9.10', counting 5 and 10 and 10 and 10 to give the total time. Others may say '8.35 and 25 minutes takes us to 9.00, so add on another 10 minutes.'

Children need to remember that, for minutes, they need to count up to 60 before getting to the next hour. Some children might be tempted to say 8.35, 8.40, 8.50, 8.60, and so on, expecting to go on until they get to 100. Referring to a clock face should help them to see why this is incorrect.

Use local bus or train timetables.

This is part of a train timetable from Bristol to London:

Bristol	**London**
08.30	10.05
10.13	11.42
12.30	13.58
15.19	16.48

Ask questions such as 'How long does the 8.30 train take to get to London?' Encourage children to count up to 9:00 and then to add on the extra 45 minutes.

Ask: 'Which train takes the shortest time?' 'Which takes the longest?'

Suggest that children build the starting times up to the next hour, and then add on the remaining minutes.

Plan a journey using information from a timetable. For example, coaches arrive and leave Blackpool at these times:

	Arrive	**Leave**
Coach A	08.00	14.30
Coach B	09.30	15.45
Coach C	10.15	16.00
Coach D	11.45	17.30

Ask questions such as: 'Which coach gives you the most time at Blackpool?' 'Which gives you the least time?'

Discuss the strategies that children use to find the times.

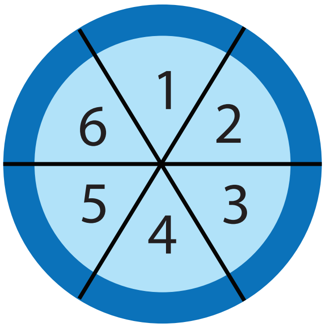
For Coach C, for example, some might bridge 10.15 up to 11.00 and then find the number of remaining hours; others might bridge from 10.15 through 12.15 to 15.15, counting in hours and then add on the remaining 45 minutes.

Children need to remember that they need to count the hours 10, 11, 12, and then start again with 1, 2, and so on.

Again, an empty number line can be used to model the calculations.

Some more addition and subtraction activities

Three darts land on this board.



Darts in the outside ring score double that number.

More than one dart can land in an area.

Find different ways of scoring 26.

How many different ways can you find?

Games and puzzles are a chance to practise mental calculation strategies in an interesting way.

You need a set of digit cards from 1 to 7.

1 2 3 4 5 6 7

Arrange your cards with + signs between them.

Use each card once.

How close can you get to a total of 100?

Here is an example.

5 2 + 1 3 + 4 6 + 7 = 118

Can you get closer to 100?

Two of the several different ways of making 100 are 12 + 35 + 46 + 7 = 100 and 15 + 47 + 36 + 2 = 100.

This is a magic square. The numbers in any row, column or the two diagonals have the same total.

23	10	17	4	11
6	18	5	12	24
19	1	13	25	7
2	16	21	8	20
15	22	9	16	3

Unfortunately, there is something wrong.

One of the numbers is incorrect.

Which number is it? What should it be?

The number 16 towards the bottom left corner should in fact be 14.

Put these eight cards in three groups.

1 2 3 4 5 6 7 8

There must be at least one card in each group.

In each group, the sum of the numbers of the cards must be the same.

Find three different ways to do it.

The eight cards have a total of 36, so the total in each group must be 12. The three ways are:

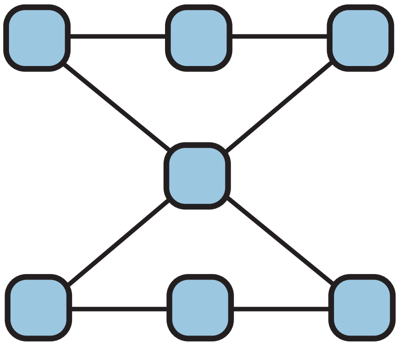
8, 4	7, 5	6, 3, 2, 1
8, 3, 1	7, 5	6, 4, 2
8, 4	7, 3, 2	6, 5, 1

You need seven number cards like these.

1 2 3 4 5 6 7

Arrange the cards on this grid.

Each line of three numbers must add up to 12.



Can you find two other ways to do it?

Put 4 in the centre and the rest fall into place.

In each question, use two + signs and two – signs.

Put them in the boxes to make the calculations correct.

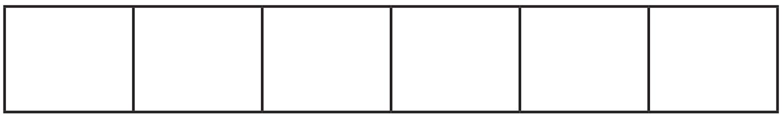
a 5 ☐ 4 ☐ 3 ☐ 2 ☐ 1 = 5

b 2 ☐ 2 ☐ 3 ☐ 4 ☐ 5 = 0

Puzzles like this can have more than one solution.

This is a game for two players with three dice.

Each of you should draw a grid like this.



Take turns to roll all three dice. Write the total score on your own grid. When the grids are full, keep rolling. This time, if the total score is on either player's grid, cross it out. The winner is the first player to get all their numbers crossed out.

4 Multiplication and division strategies

This chapter sets out the main methods for multiplying and dividing mentally. Each section starts with examples of typical problems and describes activities to support teaching of the methods. It covers strategies that involve:

- Knowing multiplication and division facts to 10×10
- Doubling and halving
- Multiplying and dividing by multiples of 10
- Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers
- Finding fractions, decimals and percentages.

Features of multiplication and division

Three different ways of thinking about multiplication are:

- as repeated addition, for example $3 + 3 + 3 + 3$
- as an array, for example four rows of three objects
- as a scaling factor, for example, making a line 3 cm long four times as long.

The use of the multiplication sign can cause difficulties. Strictly, 3×4 means four threes or $3 + 3 + 3 + 3$. Read correctly, it means 3 multiplied by 4. However, colloquially it is read as '3 times 4', which is $4 + 4 + 4$ or three fours. Fortunately, multiplication is commutative: 3×4 is equal to 4×3, so the outcome is the same. It is also a good idea to encourage children to think of any product either way round, as 3×4 or as 4×3, as this reduces the facts that they need to remember by half.

A useful link between multiplication and addition allows children to work out new facts from facts that they already know. For example, the child who can work out the answer to 8×6 (six eights) by recalling 8×5 (five eights) and then adding 8 will, through regular use of this strategy, become more familiar with the fact that 8×6 is 48.

Another feature of multiplication occurs in an expression such as $(4 + 5) \times 3$, which involves both multiplication and addition. The distributive law of multiplication over addition means that:

$(4 + 5) \times 3 = (4 \times 3) + (5 \times 3)$

This feature can be very useful in mental calculations.

The importance of learning multiplication and division facts

The National Curriculum and the *Primary Framework for mathematics* make clear that children should learn number facts and multiplication tables 'by heart'. If they cannot recall these facts rapidly and always resort to a basic counting strategy instead, they are distracted from thinking about the calculation strategy they are trying to use.

Division and multiplication are inverse operations. Every multiplication calculation can be replaced by equivalent division calculations and vice versa. This link means that if children learn multiplication facts so that they can recall them almost instantly they should also be able to recall quickly the corresponding division facts.

Because good knowledge of multiplication facts underpins all other multiplication and division calculations, written and mental, it is important that children commit the multiplication tables to 10 × 10 to memory and derive corresponding division facts by the end of Year 4, building up speed and accuracy through Years 5 and 6. They can also develop their skills at multiplying and dividing a range of whole and decimal numbers during Years 5 and 6.

The first chapter, *Progression in mental calculation skills*, lists the facts that children in each year group should be able to derive and recall rapidly. These expectations are based on the objectives in the 'Knowing and using number facts' strand of the *Primary Framework for mathematics*.

Multiplication and division facts to 10 × 10

Fluent recall of multiplication and division facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions. It is crucial that the practice involves as wide a variety of activities, situations, questions and language as possible and that it leads to deriving and recognising number properties, such as doubles and halves, odd and even numbers, multiples, factors and primes.

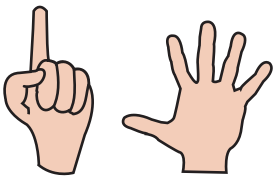
Expectations over Years 1 to 6

	Expectations
Year 1	Count on from and back to zero in ones, twos, fives and tens
	Recognise odd and even numbers to 20
	Recall the doubles of all numbers to 10
Year 2	Derive and recall doubles of all numbers to 20, and doubles of multiples of 10 to 50, and corresponding halves
	Derive and recall multiplication facts for the 2, 5 and 10 times-tables and corresponding division facts
	Recognise odd and even numbers to 100
	Recognise multiples of 2, 5 and 10
Year 3	Derive and recall doubles of multiples of 10 to 100 and corresponding halves
	Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6 and 10 up to the tenth multiple
Year 4	Identify doubles of two-digit numbers and corresponding halves
	Derive doubles of multiples of 10 and 100 and corresponding halves
	Derive and recall multiplication facts up to 10 × 10 and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9 and 10 up to the tenth multiple
Year 5	Recall squares of numbers to 10 × 10
	Use multiplication facts to derive products of pairs of multiples of 10 and 100 and corresponding division facts

Year 6	Recall squares of numbers to 12 × 12 and derive corresponding squares of multiples of 10
	Use place value and multiplication facts to derive related multiplication and division facts involving decimals (e.g. 0.8 × 7, 4.8 ÷ 6)
	Identify factor pairs of two-digit numbers
	Identify prime numbers less than 100

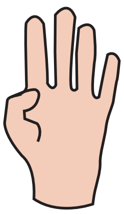
Activities

Children who are able to count in twos, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:



As children touch each of the six fingers in turn, they say '2, 4, 6, 8, 10, 12' to get the answer 12.

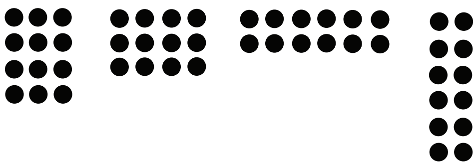
For 5 × 4 (four fives), hold up four fingers:



This time they say '5, 10, 15, 20' to get 20.

Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

Discuss ways of grouping dots in a rectangular array (single-line arrays are not allowed in this game), e.g. 12 dots can be represented as:



Play this game. Player A takes a handful of counters, counts them and tells player B how many there are. Player B then says how the counters can be arranged in a rectangular array and proceeds to make it (remember, single line arrays are not allowed). If both players agree the array is correct, player B gets a point.

Both players record the two multiplication facts that the array represents. For example, a 5 by 3 array is recorded as 15 = 3 × 5 and 15 = 5 × 3, or 3 × 5 = 15 and 5 × 3 = 15.

After the game, discuss numbers that can only be made into a single row or single column array, i.e. the primes.

Arranging counters in a rectangular array is a helpful introduction to understanding about factors. If a number can be arranged in a rectangle (excluding a straight line) then it can be factorised. Numbers that can be arranged only as a straight line are primes.

Play Fours, a game for two players using two dice.

Each player draws a 3 by 3 grid. Take turns to roll two dice. Each spot is worth 4. Write your score on your grid. Carry on until each grid is full of numbers.

Now take turns to roll the dice again. If the score is the same as a number on either player's grid, you can cross out that number.

The winner is the first to cross out all their numbers.

Adapt for different multiplication tables.

Make cards showing a multiplication on one side and the answer on the other. Children put the cards out in front of them with either all the multiplications or all the results showing.

1 x 7	7 x 9	4 x 7	9 x 7
5 x 7	6 x 7	2 x 7	6 x 7
7 x 6	3 x 7	10 x 7	7 x 5

A player touches a card, says what is on the other side and then turns it over. If not correct, the card is turned back over. Another card must then be tried. Play continues until all the cards are turned over.

Keep a record of the time taken to complete the activity and try to improve on it next time.

Write a multiplication fact in the middle of the board and ask children: 'Now that we know this fact, what other facts do we also know?' Invite children to the board to explain and record their ideas.

6 ÷ 2 = 3
4 x 3 = 12
6 ÷ 3 = 2
20 x 3 = 60
1/2 (4 x 3) = 6 — 2 x 3 = 6 — 200 x 3 = 600
2 x 1 1/2 = 3
40 x 3 = 120
12 x 3 = 10 x 3 + 2 x 3 =30 + 6 = 36

Children who are explaining to others clarify their own thoughts and others are introduced to different methods.

More activities …

Give children a multiplication square.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Ask them to highlight the facts that they already know (e.g. multiplication by 1, 2, 5 and 10).

Then identify facts that they can work out easily, e.g. the 4 times table is twice the 2 times table, the 8 times table is twice the 4 times table, the 9 times table is one multiple less than the 10 times table (e.g. 6 × 9 is 6 less than 6 × 10, and so on).

Now identify the 'tricky' facts that they need to work on, for example 7 × 6, 8 × 9.

Children could keep their own personal square and colour in facts as they are learned.

Make loop cards of questions and answers, e.g.

3 x 5	6 x 7	4 x 4	9 x 8	8 x 5
40	15	42	16	72

Distribute the cards around the class. Ask one child to read out the multiplication at the top of their card. The child who has the correct answer reads it out and then reads out the question at the top of that card. This continues until all cards are used.

The game works best if the cards form a loop where the question on the last card is linked to the answer on the first, as in the five cards above.

A benefit of this activity is that every child has to work out all the answers to see if they have the one asked for. So every child gets plenty of practice.

Children can develop new facts by partitioning one of the numbers. So, for example,

$6 \times 7 = 6 \times (5 + 2) = 6 \times 5 + 6 \times 2$

or, in words, 'seven sixes are five sixes plus two sixes'. Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.

Another strategy is to use factors, so 7×6 is seen as $7 \times 3 \times 2$.

These strategies are useful when new tables are being developed.

Distribute a number of cards with a multiplication fact with one number missing, such as:

? x 4 = 24   5 x ? = 35

Children need to place their cards on the space that gives the missing number on a sorting tray like this:

2	3	4	5
6	7	8	9
10	11	12	13

Children should time themselves to see how long it takes them to sort all the cards correctly. Repeat several times over a month to see if they improve

Ask children to work in pairs on a given multiplication table, e.g. multiples of 4. The first child claps out a number between 1 and 10. The other child has to count the number of claps, for example 6, and then say the corresponding multiple. They both record the multiplication fact, and then swap roles.

… and more …

Show children this grid and explain that it contains the numbers in the 2, 5 and 10 times-tables. Some numbers appear in more than one table.

4	**6**	**8**
10	**12**	**14**
15	**16**	**18**
20	**25**	**30**
35	**40**	**45**
50	**60**	**70**
80	**90**	**100**

Point to a number and invite children to give a related multiplication fact. Encourage a quick response.

Use this chart for the 3, 4, 6, 7, 8 and 9 times tables.

9	**12**	**16**	**18**
21	**24**	**27**	**28**
32	**36**	**42**	**48**
49	**54**	**56**	**63**
64	**72**	**81**	

This activity requires children to recognise the product of two numbers and to say what those numbers are. The activity links to work on factors.

Many of the ITPs can support children's learning of multiplication and division facts, in particular:

'Multiplication array'	'Multiplication board'
'Multiplication facts'	'Multiplication grid'
'Multiplication tables'	'Number dials'

The quickest way to find these is to visit: **www.standards.dcsf.gov.uk/nationalstrategies** and search for 'Interactive teaching programs'.

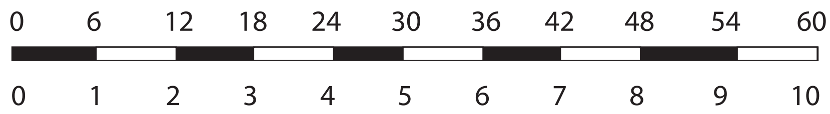
Chant division tables, using 'divided by':

0 ÷ 6 = 0

6 ÷ 6 = 1

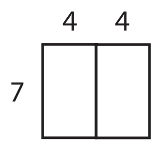
12 ÷ 6 = 2,

and so on. This can be supported with a counting stick. The stick helps to establish the relationship between the increasing steps and the corresponding quotients.

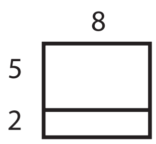


Using a vertical counting stick makes a more direct correspondence to a recorded division table.

Use rectangles to model multiplication and division facts that can be worked out from known facts, e.g. 7 x 8 is 7 x 4 and another 7 x 4:



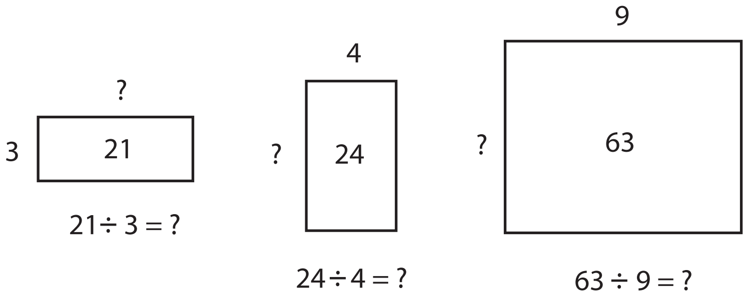
7 x 8 is 5 x 8 and 2 x 8:



Use the same method for bigger numbers, for example, 23 × 4 is 20 × 4 and 3 × 4.

These area diagrams are useful later when pupils study multiplication in algebra. They show how the distributive law works with multiplication.

Use rectangles to practise division:



This area model helps to show that multiplication and division are inverse operations.

Doubling and halving

The ability to double numbers is useful for multiplication.

Historically, multiplication was carried out by a process of doubling and adding. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations.

Sometimes it can be helpful to halve one of the numbers in a multiplication calculation and double the other.

Expectations over Years 1 to 6

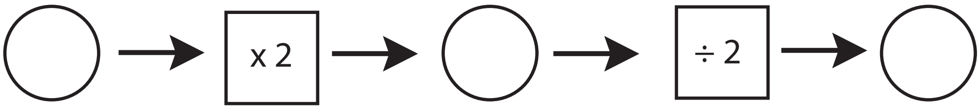
	Expectations with examples
Year 1	Double all numbers to 10, e.g. double 9
Year 2	Double all numbers to 20 and find the corresponding halves, e.g. double 7, half of 14
	Double multiples of 10 to 50, e.g. double 40, and find the corresponding halves
	Double multiples of 5 to 50 and find the corresponding halves, e.g. double 35, half of 70
Year 3	Double multiples of 10 to 100, e.g. double 90, and corresponding halves Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170
Year 4	Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94
	Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680
Year 5	Form equivalent calculations and use doubling and halving, e.g. • multiply by 4 by doubling twice, e.g. $16 \times 4 = 32 \times 2 = 64$ • multiply by 8 by doubling three times, e.g. $12 \times 8 = 24 \times 4 = 48 \times 2 = 96$ • divide by 4 by halving twice, e.g. $104 \div 4 = 52 \div 2 = 26$ • divide by 8 by halving three times, e.g. $104 \div 8 = 52 \div 4 = 26 \div 2 = 13$ • multiply by 5 by multiplying by 10 then halving, e.g. $18 \times 5 = 180 \div 2 = 90$ • multiply by 20 by doubling then multiplying by 10, e.g. $53 \times 20 = 106 \times 10 = 1060$
	Multiply by 50 by multiplying by 100 and halving
	Multiply by 25 by multiplying by 100 and halving twice
Year 6	Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 Form equivalent calculations and use doubling and halving, e.g. • divide by 25 by dividing by 100 then multiplying by 4 e.g. $460 \div 25 = 4.6 \times 4 = 18.4$ • divide by 50 by dividing by 100 then doubling e.g. $270 \div 50 = 2.7 \times 2 = 5.4$

Activities

Play 'Doubles dominoes' with a group of children: this needs a set of dominoes in which, for example, 7 × 2, 2 × 7, 7 + 7 and 14 can be matched.

Watch to see which facts the children can recall quickly.

Use 'doubling' and 'halving' function machines.



Ask one child to choose a number and another to choose whether to use the 'doubling' or the 'halving' machine. Then ask a third child to say how the number is transformed by the machine.

With this activity, children can give each other lots of quick practice.

Ask children to halve a two-digit number such as 56. Discuss the ways in which they might work it out. Show children that, unless they 'know', it may be better to partition it as 50 + 6 and to work out half of 50 and half of 6, then add these together.

Ask a child to suggest an even two-digit number and challenge other children to find a way of halving it.

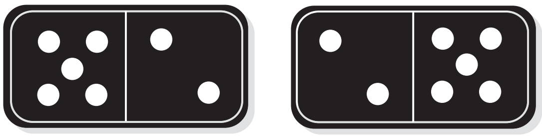
Some children may be able to halve an odd number (47, say), by saying that it is 23 and a half.

If children are familiar with the halves of multiples of 10 beyond 100, they can partition three-digit numbers in order to halve them. So they could halve 364, for example:
half of (300 + 60 + 4) = 150 + 30 + 2.

Start with a small number, e.g. 2, 3, 5 or 7. Start doubling it by going round the class. How far can you go?

This is a chance to try doubling larger numbers. Ask children to explain how they worked out their double.

Use matching dominoes from two sets to make 'doubles' patterns like this counting the dots (14).



What other dominoes can be placed in this way to make a total of 14 dots? 18 dots? 20 dots? …

This activity provides opportunity for lots of short, quick practice.

Investigate doubling and halving number chains.

Ask someone to choose a number. Say that the rule is: 'If the number is even, halve it; if it is odd, add 1 and halve it.'

Go round the class generating the chain. Write all the numbers in the chain on the board, e.g.

14 → 7 → 4 → 2 → 1

Ask for a new starting number. Continue as before.

Number chains can be quite intriguing as it is usually not possible to guess what will happen. As more and more starting numbers are chosen, the chains can build up to a complex pattern. For example, the starting number 8 joins the chain above at 4; the starting number 13 joins the chain at 7. A starting number of 23, for example, goes to 12, then 6, then 3, then joins the chain at 2.

When finding 20% of an amount, say £5.40, discuss how it is easier to first find 10% and then double. 10% of £5.40 is 54p, so 20% of £5.40 is £1.08.

Ask children how they would find 5% of £5.40. Then get them to work out 15% of £5.40.

Encourage children to use a range of methods for working out other percentages. For example, they might find 15% of £15.40 by finding 10% then halving that to find 5% and adding the two together. Or, having found 5%, they might multiply that result by 3. They could work out 17.5% by finding 10%, 5% and 2.5% and adding all three together.

Multiplying and dividing by multiples of 10

Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.

Expectations over Years 2 to 6

	Expectations with examples
Year 2	Recall multiplication and division facts for the 10 times table, e.g. 7 × 10, 60 ÷ 10
Year 3	Multiply one-digit and two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 × 100
	Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence
Year 4	Multiply numbers to 1000 by 10 and then 100, e.g. 325 × 10, 42 × 100
	Divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 120 ÷ 10, 600 ÷ 100, 850 ÷ 10
	Multiply a multiple of 10 to 100 by a single-digit number, e.g. 60 × 3, 50 × 7
	Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. 599 pence to £5.99, 2.5m to 250cm
Year 5	Multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100
	Divide a multiple of 10 by a single-digit number (whole number answers), e.g. 80 ÷ 4, 270 ÷ 3
	Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g. 60 × 30, 900 × 8
	Multiply by 25 or 50, e.g. 48 × 25, 32 × 50 using equivalent calculations, e.g. 48 x 100 ÷ 4, 32 x 100 ÷ 2
	Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm
Year 6	Multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20
	Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300
	Divide by 25 or 50
	Convert between units of measurement using decimals to two places, e.g. change 2.75 l to 2750 ml, or vice versa

Activities

Use the ITP 'Moving digits'. Visit: **www.standards.dcsf.gov.uk/nationalstrategies** and search for 'Interactive teaching programs'.

This program gives a demonstration of the way that the digits move to the left or the right when they are multiplied or divided by 10, 100 or 1000.

Use function machines that multiply by 10.

Enter a number → x 2 → x 10 = ?

Enter the same number → x 10 → x 2 = ?

What do you notice?

Try some divisions.

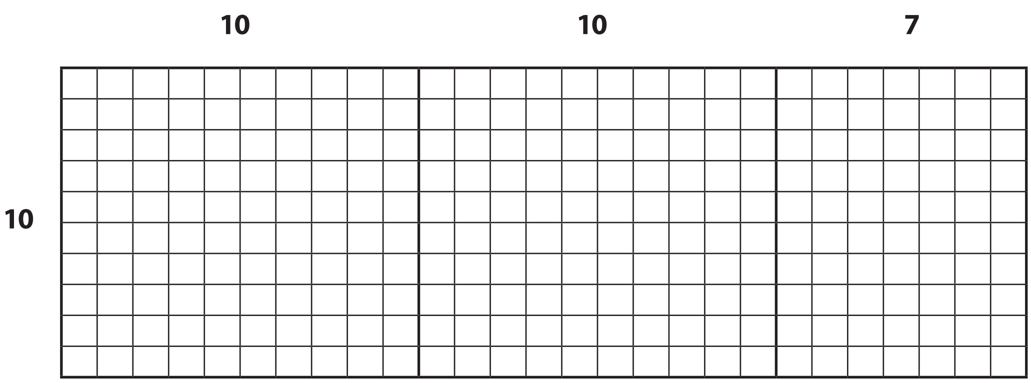
40 → ÷ 4 → ÷10 = ?

40 → ÷10 → ÷ 4 = ?

Try other starting numbers, such as 60, 20, 80, …

The function 'machine' is a useful way to focus on particular operations, in this case multiplication and then division. In the first part, children will notice that the order of multiplication does not matter – the effect of multiplying by 10 and then by 2 is the same as multiplying by 2 and then by 10. The machines can also work backwards, again illustrating that multiplication and division are inverse operations.

Use a rectangular array, e.g. to show 27 × 10.



The area model is useful, especially to show how a number is partitioned into tens and units. It provides an image for children to visualise to aid mental calculation.

Use a multiplication chart.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
100	**200**	**300**	**400**	**500**	**600**	**700**	**800**	**900**
1000	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**

Explain that the numbers on each row are found by multiplying the number above them by 10. So:

8 × 10 is 80, 40 × 10 is 400, and 500 × 10 is 5000.

If you skip a row, the numbers are multiplied by 100, so:

2 × 100 is 200, 70 × 100 is 7000.

Use the chart for dividing:

50 ÷ 10 = 5, 600 ÷ 10 = 60, and 4000 ÷ 100 = 40.

Extend the chart to show decimals by inserting decimals 0.1 to 0.9 above the numbers 1 to 9.

This chart is very helpful for showing multiplication by powers of 10. Going down a row has the effect of multiplying by 10, while going down two rows produces a multiplication by 100. Similarly, it demonstrates nicely that multiplication and division are inverse operations. Going up a row represents division by 10, and two rows division by 100.

Use a multiplication grid. Ask children to find the missing numbers.

x	2		7
	40		
10		50	

Children are probably familiar with a conventional tables square. It can be more interesting to be given smaller grids in a random order.

Here children must recognise that the first vertical column is part of the 2 times table; the number in the cell below the 40 must be found by 2 × 10 = 20. They can deduce that the middle column must be × 5 in order to get the 50.

Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers

Once children are familiar with some multiplication facts, they can extend their skills.

- One strategy is to partition one of the numbers and use the distributive law of multiplication over addition. So, for example, $6 \times 7 = 6 \times (5 + 2) = 6 \times 5 + 6 \times 2$ or, in words, 'seven sixes are five sixes plus two sixes'. Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.
- Another strategy is to make use of factors, so 7×6 is seen as $7 \times 3 \times 2$.

Once children understand the effect of multiplying and dividing by 10, they can start to extend their multiplication and division skills to larger numbers.

- A product such as 26×3 can be worked out by partitioning 26 into $20 + 6$, multiplying each part by 3, then recombining.
- One strategy for multiplication by 2, 4, 8, 16, 32, … is to use doubling, so that 9×8 is seen as $9 \times 2 \times 2 \times 2$. A strategy for dividing by the same numbers is to use halving.
- A strategy for multiplying by 50 is to multiply by 100, then halve, and for multiplying by 25 is to multiply by 100 then divide by 4.

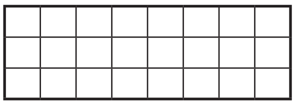
Since each of these strategies involves at least two steps, most children will find it helpful to make jottings of the intermediate steps in their calculations.

Expectations over Years 4 to 6

	Expectations with examples
Year 4	Find one quarter by halving one half
	Multiply numbers to 20 by a single-digit number, e.g. 17×3
Year 5	Multiply and divide two-digit numbers by 4 or 8, e.g. 26×4, $96 \div 8$
	Multiply two-digit numbers by 5 or 20, e.g. 32×5, 14×20
	Multiply by 25 or 50, e.g. 48×25, 32×50
Year 6	Multiply a two-digit and a single-digit number, e.g. 28×7
	Divide a two-digit number by a single-digit number e.g. $68 \div 4$
	Divide by 25 or 50, e.g. 480×25, 3200×50
	Find new facts from given facts, e.g. • given that three oranges cost 24p, find the cost of four oranges

Activities

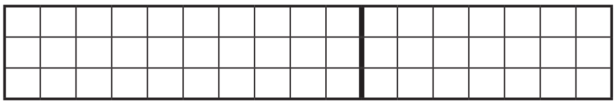
Use an area model for simple multiplication facts. For example, illustrate 8 × 3 as:



How many … rows? columns? small squares?

Encourage children to visualise other products in a similar way.

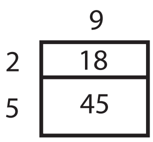
Extend this model to larger numbers, such as 17 × 3: split the 17 into 10 + 7 and use 10 × 3 + 7 × 3.



How many … rows? columns? small squares?

> The rectangles give a good visual model for multiplication: the areas can be found by repeated addition (in the case of the first example, 8 + 8 + 8), but children should then commit 3 × 8 to memory and know that it gives the same answer as 8 × 3.

Use multiplication facts that children know in order to work out others. For example, knowing 9 × 2 and 9 × 5, work out 9 × 7.



> Area models like this discourage the use of repeated addition. The focus is on the separate multiplication facts. The diagram acts as a reminder of the known facts, which can be entered in the rectangles, and the way that they are added in order to find the answer.

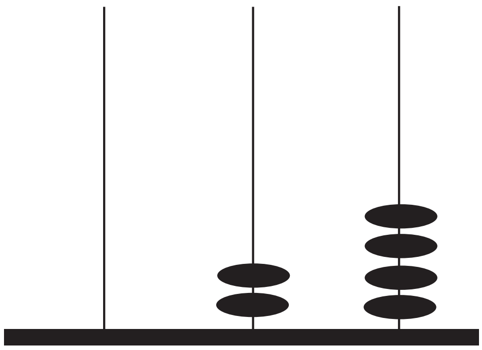
Discuss the special cases of multiplying by 25 and 50, which are easily done by multiplying by 100 and dividing by 4 or 2 respectively.

> The use of factors often makes a multiplication easier to carry out.

Use base 10 material to model multiplication. Ask a child to put out rods to represent, say, 24.

Then ask for two more groups of 24 to be added, making three groups altogether. Make sure the child knows that as soon as there are 10 'ones' they are exchanged for a '10' or a 'long'. Record the result as 24 × 3.

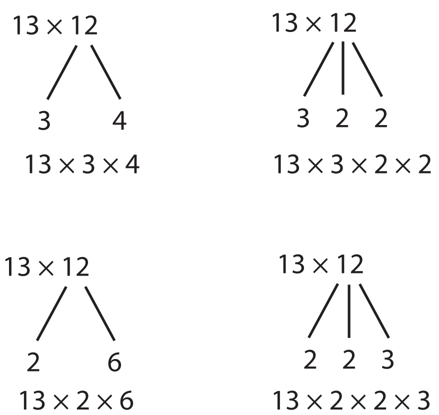
Repeat using a spike abacus. This time, as soon as there are 10 beads on a spike, they are removed and replaced by one bead on the spike to the left.



Base 10 material also lends itself to the demonstration of multiplying by 10 as each time a 'one' becomes a '10' and a '10' becomes a '100'. With the spike abacus, each bead moves to the spike on the left when multiplying by 10.

Base 10 materials and spike abacuses are more effective if digit cards are used alongside them to show the digits being represented.

Use factors. For example, work out 13 × 12 by factorising 12 as 3 × 4 or 6 × 2:



Discuss which are easier to use.

Diagrams like these can help children to keep track of the separate products.

Fractions, decimals and percentages

Children need an understanding of how fractions, decimals and percentages relate to each other. For example, if they know that $\frac{1}{2}$, 0.5 and 50% are all ways of representing the same part of a whole, then they can see that the calculations:

half of 40

$\frac{1}{2} \times 40$

$40 \times \frac{1}{2}$

40×0.5

0.5×40

50% of 40

are different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

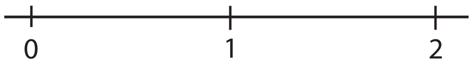
There are strong links between this section and the earlier section 'Multiplying and dividing by multiples of 10'.

Expectations over Years 2 to 6

	Expectations
Year 2	Find half of any even number to 40 or multiple of 10 to 100, e.g. halve 80
Year 3	Find half of any multiple of 10 up to 200, e.g. halve 170
	Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ of numbers in the 2, 3, 4, 5 and 10 times tables
Year 4	Find half of any even number to 200
	Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g. $\frac{3}{8}$ of 24
	Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and $\frac{3}{10}$, and 0.03 and $\frac{3}{100}$
Year 5	Recall percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths
	Find fractions of whole numbers or quantities, e.g. $\frac{2}{3}$ of 27, $\frac{4}{5}$ of 70 kg
	Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80
Year 6	Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$
	Find half of decimals with units and tenths, e.g. half of 3.2
	Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g

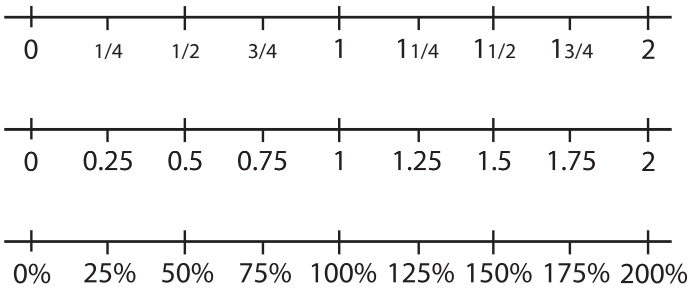
Activities

Draw a number line on the board, marking on it the points 0, 1 and 2:



Invite children to show where the fractions $\frac{1}{4}$, $\frac{1}{2}$, $1\frac{1}{4}$, $1\frac{1}{2}$ and $1\frac{3}{4}$ fit on the line. Ask what other fractions between 0 and 2 they could add to the line.

When they are familiar with fractions, draw a new line under the first one and ask for the decimals 0.5, 1.5, 0.25, 1.25, 1.75, 0.75 to be placed on this line. Repeat with a line for percentages from 0% to 200%.



Discuss the equivalence of, for example, $\frac{1}{4}$, 0.25 and 25%. Choose any number and ask children to call out the equivalents on the other two lines.

Number lines are useful for showing fractions as points on the number line between the whole numbers. This helps to move children from the idea of fractions as parts of shapes. The first part of the activity requires them to think about the relative sizes of fractions. Separate number lines with, for example, halves, quarters and eighths placed one under the other, help to establish the idea of equivalent fractions. Gradually the lines can be built up to include more fractions, up to, say, twelfths or twentieths. As in the example above, they can also be used to demonstrate the equivalence between fractions, decimals and percentages.

Use the ITP 'Fractions'. Visit: **www.standards.dcsf.gov.uk/nationalstrategies** and search for 'Interactive teaching programs'.

This program is useful for showing equivalent fractions, decimals and percentages.

Write a sum of money on the board, e.g. £24.

Ask children, in turn, to tell you what half of £24 is, then $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{12}$.

Then give fractions such as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{5}{12}$. Ask how they could calculate these fractions of £24.

To answer questions like these, children will use the basic strategy of using what they already know to work out related facts. In this case they will need to know how to find unit fractions (with a numerator of 1) of an amount and use this to find other fractions. For example, knowing that $\frac{1}{3}$ of £24 is £8 they can say $\frac{2}{3}$ is twice as much, or £16. Similarly, knowing that $\frac{1}{8}$ of £24 is £3, then $\frac{3}{8}$ is three times as much.

Put a percentage example on the board, say 25% of £60.

Discuss different ways of interpreting the question, such as $^{25}/_{100}$ of £60, or $^{1}/_{4}$ of £60.

Children might calculate $^{1}/_{4}$ of £60 by saying that $^{1}/_{2}$ of £60 is £30, and $^{1}/_{2}$ of £30 is £15.

Alternatively, they might calculate 10% of £60, which is £6, so 5% of £60 is £3, and 20% of £60 is £12, so 25% of £60 is £12 + £3, or £15.

Ask children to find 17.5% of £60. Since they know that 5% of £60 is £3, they can work out that 2.5% of £60 is £1.50. Then they can find the total by adding 10% + 5% + 2.5%.

Invite children to suggest other examples.

Aim to give actual examples that children have seen in local shops or newspapers, to give a more realistic and motivating context.

It is important that children understand the basic fact that % means 'out of a hundred' or 'per hundred', rather than to learn any rules about working with percentages. Examples of percentage calculations that can be done mentally can usually be worked out using this basic knowledge.

Acknowledgements

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