ACME Draft GCSE subject criteria for mathematics

Version : 10 June 2008

Contents

Introduction

GCSE criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in mathematics. They provide the framework within which an awarding body creates the details of the specification.

Specifications must also meet the regulators' general requirements, including the common and GCSE criteria as defined in *The statutory regulation of external qualifications (*QCA/04/1293).

Subject criteria are intended to:

- Help ensure consistent and comparable standards in the same subject across the awarding bodies
- Ensure that the rigour of GCSE is maintained
- Ensure that specifications build on the knowledge, understanding and skills established by the national curricula for England, Northern Ireland and Wales and facilitate progression to higher level qualifications in mathematics
- Help higher education institutions, employers and other stakeholders such as learners and parents/guardians know what has been studied and assessed.

Any GCSE specification that contains significant elements of mathematics must be consistent with the relevant parts of these subject criteria.

Aims and learning outcomes

GCSE specifications in mathematics should help learners to be able to use and apply mathematics in their everyday lives, employment and in their study of other subjects, both contemporaneous with their study and in the future. They should enable learners to be creative and develop confidence in their own abilities to communicate mathematically and to apply mathematics to solving real-life problems. They should also help learners to recognise the importance of mathematics both in their own actual and potential lives and to society more generally. They should encourage educational establishments to provide a broad, coherent, satisfying and worthwhile course of study. Specifications should prepare learners to make informed decisions about further learning opportunities and career choices.

GCSE specifications in mathematics must enable learners to:

- develop mathematical functionality
- develop knowledge, skills and understanding of useful mathematical methods and concepts
- select and apply mathematical techniques and methods in everyday and real world situations, including financial and unfamiliar contexts
- acquire strategies for problem solving and modeling in real world contexts
- reason mathematically in realistic contexts, making deductions and inferences and drawing conclusions
- adopt a critical approach towards solving realistic problems and the limitations and scope of modeling
- interpret and communicate mathematical information in a variety of forms appropriate to the information and context
- use technology appropriately

Subject content

The content of GCSE specifications in mathematics must reflect the learning outcomes.

GCSE specifications in mathematics must be consistent with the national curriculum key stage 4 programmes of study requirements in the orders for England and Wales and the statutory requirements for key stage 4 in Northern Ireland.

GCSE specifications in mathematics must enable learners to develop the ability to apply the knowledge, skills and understanding specified below. Content **in bold** is higher tier only.

- Real numbers, their properties and their different representations **including upper and lower bounds**
- Rules of arithmetic applied to calculations and manipulations with rational numbers, including standard index form **and manipulation of non-integral powers including surds**

- Proportional reasoning, direct and inverse proportion, proportional change and exponential growth
- Linear, quadratic and other expressions in one unknown
- Graphs of polynomial, exponential and trigonometric functions and their properties
- Graphs of simple loci
- **Algebraic methods involving quadratic equations, expressions and inequalities**
- **Transformations of functions**
- Properties and mensuration of 2D and 3D shapes
- Understanding the trigonometrical ratios **and using trigonometry to solve triangles, including the sine and cosine rules, and in mensuration**
- Circle theorems
- Properties and combinations of transformations
- 3D coordinate systems
- Vectors in two dimensions
- Conversions between measures and compound measures
- The data handling cycle
- Presentation and analysis of large sets of grouped and ungrouped data, including **cumulative frequency diagrams and histograms with unequal class intervals**
- Lines of best fit and their interpretation
- Measures of central tendency and spread
- Experimental and theoretical probabilities of single and combined events

Assessment objectives

All specifications must require candidates to demonstrate their ability to:

Assessment objectives		% weighting
AO1	Recall, use and communicate their knowledge of mathematics	35-45
AO2	Demonstrate their understanding by selecting and applying mathematical methods in a range of realistic contexts using appropriate terms	30-40
AO3	Interpret and analyse problems in realistic contexts and generate strategies to solve them	25-35

5. Scheme of assessment

GCSE specifications in mathematics must allocate a weighting of 100% to external assessment.

Question papers in mathematics must be targeted at either Foundation or Higher tier.

Each scheme of assessment must allocate a minimum weighting of 20%, to assessment without a calculator.

6. Grade descriptions

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

Grade	
A	Candidates recall and use mathematical content knowledge when completing tasks and solving realistic problems. They are able to combine and use different representations and symbols consistently and effectively in a range of realistic contexts Candidates apply a range of mathematical knowledge and skills in a range of realistic contexts including those which are unfamiliar and complex, utilising mathematical language and symbols when presenting a consistent, reasoned and convincing argument. They can explain why they follow particular lines of enquiry, choose a form of representation and why they reject others. In realistic contexts, they critically examine other people's mathematics and forms of representation and ways of communicating and are able to state the limitations of an approach and comment on the accuracy of results. They use this information to inform their conclusions and are aware of the needs of different audiences. Candidates make and test hypotheses and conjectures in unfamiliar but realistic settings and contexts. They interpret and analyse realistic problems, identifying appropriate knowledge, methods and skills that they might bring to bear on an unfamiliar context. They can independently plan a route through a realistic problem breaking it down, specialising and generalising as necessary. Candidates are prepared to approach a realistic problem in more than one way, returning to the early stages of such a problem if a particular approach does not appear to be working and trying something new. Candidates tackle realistic problems involving multiple variables, identifying some of the dimensions of variation and varying them systematically. They

	record and combine outcomes from the variation in the process of solving the problem. They can maintain a rigorous argument, reason inductively and deductively in real-life contexts, making inferences and drawing conclusions. They produce simple chains of reasoning in realistic contexts and can identify errors in someone else's reasoning.
C	Candidates recall and use mathematical content knowledge when completing tasks and solving realistic problems. Candidates review and refine their own work based on evaluation of their findings and input from others. They can follow someone else's logical argument and suggest improvements to other's work. Candidates justify generalisations in realistic contexts utilising algebraic, graphical or other forms of representation appropriately. They look for improvements to their work, justify their choices, and explain the effects of any changes. Candidates communicate in different ways for different audiences Candidates work confidently in realistic contexts which are unfamiliar, including those situated in financial contexts. They can make connections between their mathematics and its application and relevance to the wider community. In both these cases, candidates plan a logical sequence of work that will test their ideas. They break tasks set in realistic contexts down into manageable steps or components appropriately. They are able to identify relevant information, select appropriate representations and apply appropriate methods and knowledge. They are able to move from one representation to another, either of their own or someone else's in order to make sense of a realistic situation. When solving realistic problems, candidates deal with contexts involving more than one dimension of variation and apply methods which involve changing each one of those variables systematically. In a given realistic context, candidates can make

	up questions of their own. They specify hypotheses, and identify and collect evidence that could support or refute them. They make and test conjectures in realistic contexts using empirical data or logical argument. In such contexts, they are able to state the constraints that might apply to a generalisation and identify exceptions or special cases. Candidates make deductions and inferences in realistic contexts, look for invariance and combine related outcomes in order to draw conclusions. They understand the difference in realistic contexts between a logical argument and conclusions based on experimental evidence. .
F	In realistic contexts, candidates recall and use appropriate levels of mathematical content knowledge and skills effectively, and select appropriate representations. When trying out ideas of their own and the ideas of others in such contexts, candidates can comment on their effectiveness and efficiency. They check the accuracy and the reasonableness of their results, adapting their methods in the light of experience. They can communicate their findings in a range of forms, appropriate to a given audience. Candidates work confidently on problems situated in familiar real-life contexts and settings or which develop familiar ideas including those situated in financial contexts. They compare their methods and representations with those of others and evaluate them in the context of their own knowledge and experiences and the original context. They are able to recognise equivalent or similar methods, representations, and approaches and comment on them. Candidates can identify, use and develop their own, or other peoples', strategies for working mathematically in a range of practical and realistic settings. In these settings, they can apply key process skills to problems involving different aspects of content knowledge and which require more than one step. Candidates are able to use experimental data to look for patterns

	and relationships, reasoning inductively. They can state a generalisation arising from a series of simple real-life cases, identify counter-examples and indicate a level of confidence in their findings. Candidates set and test simple hypotheses and conjectures based on evidence collected or presented.

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Specifications must also meet the regulators' general requirements, including the common and GCSE criteria as defined in *The statutory regulation of external qualifications (*QCA/04/1293).

Subject criteria are intended to:

- Help ensure consistent and comparable standards in the same subject across the awarding bodies
- Ensure that the rigour of GCSE is maintained
- Ensure that specifications build on the knowledge, understanding and skills established by the national curricula for England, Northern Ireland and Wales and facilitate progression to higher level qualifications in mathematics
- Help higher education institutions, employers and other stakeholders such as learners and parents/guardians know what has been studied and assessed.

Any GCSE specification that contains significant elements of mathematical thinking must be consistent with the relevant parts of these subject criteria.

Aims and learning outcomes

GCSE specifications in mathematical thinking should enable students to become interested, intrigued and inspired by the power and patterns of mathematics and to develop confidence in their knowledge and ability to solve purely mathematical problems which are challenging at their level. They should develop learners ability to think and act mathematically, to become more mathematically creative and competent, to understand and appreciate the abstract relationships which form part of the interconnected structures of mathematics and to experience its aesthetic appeal.

Specifications should encourage appreciation of the nature and development of mathematics in relation to significant mathematical problems experienced within different historic and geographical cultures and the people who helped to solve them. They should encourage educational establishments to provide a broad, coherent, satisfying and worthwhile course of study. Specifications should prepare learners to make informed decisions about further learning opportunities and career choices.

GCSE specifications in mathematical thinking must enable learners to:

- develop knowledge, skills and understanding of mathematical concepts, relationships and methods
- select and apply mathematical ideas and methods creatively in solving problems within mathematics
- acquire strategies for mathematical problem solving
- reason mathematically, make deductions and inferences and draw conclusions
- interpret and communicate mathematical information in a variety of forms appropriate to the information and context
- adopt a critical approach towards solving mathematical problems
- use technology appropriately

Subject content

The content of GCSE specifications in mathematical thinking must reflect the learning outcomes.

GCSE specifications in mathematical thinking must be consistent with the national curriculum key stage 4 programmes of study requirements in the orders for England and Wales and the statutory requirements for key stage 4 in Northern Ireland.

GCSE specifications in mathematical thinking must enable learners to develop the knowledge, skills and understanding specified below, and to use them in solving purely mathematical problems.

Content **in bold** is higher tier only.

- Deductive reasoning including following, critiquing or **developing** simple definitions and proofs, in number, algebra, geometry and probability
- Real numbers and the subsets of real numbers, their properties and relationships, and their different representations
- Properties and relationships of arithmetical operations on these subsets of numbers, including **manipulation of non-integral powers and surds**
- Proportional reasoning

- Linear expressions and equations in one unknown; polynomial, exponential and trigonometric functions and their graphs and properties**. Algebraic methods involving quadratic equations, expressions and inequalities; transformations of functions**

- Geometrical concepts, definitions and reasoning, including

  - 2D and 3D shapes and their properties

  - circle theorems **including the alternate segment theorem**

  - **trigonometric relationships and properties**

  - properties and combinations of transformations

  - 3D coordinate systems

  - vector geometry in two dimensions

- Solving probability problems using single and combined events, **including conditional probability**

- Presentation and analysis of large sets of grouped and ungrouped data, including **cumulative frequency diagrams and histograms with unequal class intervals**

- Lines of best fit and their interpretation

- Measures of central tendency and spread

Assessment objectives

All specifications must require candidates to demonstrate their ability to:

Assessment objectives		**% weighting**
AO1	Recall, use and communicate their knowledge of mathematics	25-35

AO2	Demonstrate their understanding by selecting and applying mathematical methods in a range of mathematical contexts using appropriate terms	25-35
AO3	Interpret and analyse mathematical problems and generate strategies to solve them	40-50

5. Scheme of assessment

GCSE specifications in mathematics must allocate a weighting of 100% to external assessment; all assessment must take place in controlled conditions.

Question papers in mathematics must be targeted at either Foundation or Higher tier.

7. Grade descriptions

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

Grade	
A	Candidates recall and use mathematical knowledge when completing mathematical tasks and solving mathematical problems. They are able to combine and use different representations and symbols consistently and effectively and use them to communicate ideas. Candidates apply a range of mathematical knowledge and skills in a range of mathematical situations including those which are

	unfamiliar and complex, utilising mathematical language and symbols when presenting a consistent, reasoned and convincing argument. They can explain why they follow particular lines of enquiry, choose a form of representation and why they reject others. They critically examine other people's mathematics and forms of representation and ways of communicating and are able to state the limitations of an approach and comment on the accuracy of results. They use this information to inform their conclusions and are aware of the needs of different audiences. Candidates make and test hypotheses and conjectures in unfamiliar mathematical settings. They interpret and analyse problems, identifying appropriate knowledge, methods and skills that they might bring to bear on an unfamiliar context. They can independently plan a route through a problem breaking it down, specialising and generalising as necessary. Candidates are prepared to approach a problem in more than one way, returning to the early stages of a problem if a particular approach does not appear to be working and trying something new. Candidates tackle mathematical problems involving multiple variables, identifying some of the dimensions of variation and varying them systematically. They record and combine outcomes from the variation in the process of solving the problem. They can maintain a rigorous argument, reason inductively and deductively, making inferences and drawing conclusions. They produce simple proofs and can identify errors in someone else's reasoning.
C	Candidates recall and use mathematical content knowledge when completing mathematical tasks and solving problems. Candidates review and refine their own work based on evaluation of their findings and input from others. They can follow someone else's logical argument and suggest improvements to other's work. Candidates justify generalisations utilising algebraic, graphical or other forms of representation appropriately. They look for improvements to their work, justify their choices, and explain the effects of any changes. Candidates communicate in different ways

	for different audiences Candidates work confidently in mathematical contexts which are unfamiliar. They have a broad appreciation of mathematical structures. Candidates plan a logical sequence of work that will test their ideas. They break tasks down into manageable steps or components appropriately. They are able to identify relevant information, select appropriate representations and use appropriate methods and knowledge. They are able to move from one representation to another, either of their own or someone else's in order to make sense of a situation. When solving mathematical problems, candidates deal with more than one dimension of variation and apply methods which involve changing each one of those variables systematically. In a given mathematical setting, candidates can make up questions of their own. They specify hypotheses, and generate results that could support or refute them. They make and test conjectures using specific cases or logical argument. They are able to state the constraints that might apply to a generalisation and identify exceptions or special cases. Candidates make deductions and inferences. They look for invariance and combine related outcomes in order to draw conclusions. They understand the difference between a general logical argument and conclusions based on consideration of particular cases.
F	Candidates recall and use appropriate levels of knowledge and skills effectively in solving mathematical problems and select appropriate representations. When trying out ideas of their own, and the ideas of others, in such settings candidates can comment on their effectiveness and efficiency. They check the accuracy and the reasonableness of their results, adapting their methods in the light of experience. They can communicate their findings in a range of forms, appropriate to a given audience. Candidates work confidently on problems situated in familiar but

	abstract settings or which develop familiar ideas. They compare their methods and representations with those of others and evaluate them in the context of their own knowledge and experiences and the setting. They are able to recognise equivalent or similar methods, representations, and approaches and comment on them. Candidates can identify, use and develop their own, or other peoples', strategies for working mathematically in a range of meaningful mathematical settings. They can apply key process skills to problems involving different aspects of content knowledge and which require more than one step. Candidates are able to use particular cases to look for patterns and relationships, reasoning inductively. They can state a generalisation arising from a series of simple cases, identify counter-examples and indicate a level of confidence in their findings. Candidates set and test simple hypotheses and conjectures based on results that they have generated or which are presented to them.

Jim Knight MP
Minister of State for Schools and Learners
Department for Children, Schools and Families
Sanctuary Buildings
Great Smith Street
London
SW1P 3BT

Office of the Qualifications
and Examinations Regulator
Spring Place
Coventry Business Park
Herald Avenue
Coventry CV5 6UB

Telephone 0300 303 3344
Textphone 0300 303 3345
info@ofqual.gov.uk
www.ofqual.gov.uk

13 June 2008

Dear Jim

GCSE Maths

In the light of recent discussions with QCA and ACME about the possibility of developing two GCSE qualifications in mathematics, I thought it might help your consideration to set out in writing the principles which Ofqual brings to this question.

We would need to make sure that the integrity and quality of the qualifications on offer were retained and that they could continue to command the confidence of learners, higher education and employers. If there were to be two GCSE qualifications in place of one, it would be important to ensure that the first had not been reduced in quality or demand to make room for the second. This would be particularly important if more candidates were to sit the first qualification than the second, and if the first qualification were to be used as the main indicator of the mathematical ability of youngsters. It would also be important for learners to know which of the qualifications paved the way for progression to A-level maths.

As you know, our regulatory requirements stipulate that additional GCSE specifications in the same subject must be significantly different in terms of the knowledge or skills assessed. This is not pedantry – it reflects the importance of ensuring that each qualification is valued and fully supports an appropriate course of study. It is also important to avoid a proliferation of qualifications that may not add educational value.

Ofqual retains an open mind about whether two sufficiently distinctive GCSEs in maths could be developed. If specifications were submitted to us, we would apply the principles set out in this letter. We would welcome public and professional debate about some of the questions that have arisen from the work done in recent months by QCA and ACME, including the feasibility of distinguishing the testing of skills or procedures in an “applied” setting from the testing of the same skills or procedures in "purer", conceptual, tasks; and the scope for distinguishing more clearly the subject content of the two specifications.

Ofqual will stand by to finalise new criteria for GCSE maths in the light of the outcome of the forthcoming consultation by QCA. We recognise that subject experts have worked very hard over the past weeks and months to try to advance this issue further. As a new organisation, if Ofqual can help move the debate forward, then we shall be pleased to do so.

I am copying this letter to Adrian Smith and Ken Boston.

Yours sincerely

Kathleen Tattersall
Chair, Ofqual

Direct line 02476 716742