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In Improving Scottish Education 2005-2008¹, I said that HMIE will assist national and local bodies as they consider the implications of that report for their work. I am pleased to provide this Learning Together: Mathematics guide as part of the new Learning Together series. Its purpose is to promote improvements in Scottish education through illustrating effective practice, raising current issues, and stimulating reflection and debate. This guide builds on the evidence presented in Improving Achievement in Mathematics in Primary and Secondary Schools published by HMIE in 2005.

Inspectors have worked with a wide range of practitioners and stakeholders to develop this guide. It aims to help schools and centres to improve mathematics provision as they continue to implement *Curriculum for Excellence*.

If we are to achieve the Scottish Government's purpose of sustainable economic growth, children and young people need to develop a broad range of mathematical and numeracy skills. Effective education in mathematics across the population is in the national interest. As young people compete in an ever changing world and face increasingly complex challenges, they will need to be able to apply various aspects of mathematics in order to be successful. To maximise their life chances, young people need the confidence to recognise when and how to apply their skills in a range of situations, some of which are predictable and others which are new to them. National and international studies of Scottish young people's current performance confirm some key strengths but there is a need to continue to improve achievement in mathematics. Teachers in the vast majority of Scottish schools recognise this.

'To face the challenges of the 21st century, each young person needs to have confidence in using mathematical skills, and Scotland needs both specialist mathematicians and a highly numerate population' (Building the Curriculum 1).

Curriculum for Excellence provides opportunities for teachers to reflect on how they can meet learners' needs more effectively in mathematics and improve their achievement as successful learners, confident individuals, responsible citizens and effective contributors. Learning mathematics gives children and young people access to the wider curriculum and the opportunity to pursue further studies and interests. Numeracy is not just a subset of mathematics, it is also a life skill which permeates and supports all areas of learning. The following sections of this guide explore ways in which professionals are working together to improve how children and young people learn mathematics.

¹ Improving Scottish Education: A report by HMIE on inspection and review 2005-2008. (HMIE, 2009).



In *Improving Scottish Education 2005-2008*, I wrote that: 'Scotland's future economic prosperity requires an education system within which the population as a whole will develop the kind of knowledge, skills and attributes which will equip them personally, socially and economically to thrive in the 21st century'. High-quality learning experiences in mathematics, such as those outlined in this guide, will help us to turn this aspiration into reality for all young people in Scotland.

Graham HC Donaldson HM Senior Chief Inspector



1. Introduction

This guide is intended to stimulate professional reflection, dialogue and debate about mathematics and how to improve it. It draws together themes, features and characteristics of effective improvement in mathematics and descriptions of good practice. It offers a reference point for staff and teachers² who are working together to improve mathematics education. This guide will also be useful for learners³, parents⁴ and communities as they discuss how they can contribute to improving learning in mathematics.

Staff are able to make substantial progress in improving learning and teaching in mathematics by sharing practice with each other and by involving children, young people, parents and communities as partners.

Staff in many schools⁵ have been developing approaches designed to improve learners' mathematical skills and understanding to enable them to play a meaningful role in society. They know that levels of achievement in mathematics have an impact on important future life opportunities for their learners. An increasing number of teachers are transforming the curriculum by engaging actively with colleagues to evaluate continuously the outcomes of learning to improve learners' experiences and attainment. However, in too many schools, innovation has been insufficient and the outcomes for young people have not improved. Implementing *Curriculum for Excellence* should lead to improvements in attainment and achievement, raising the bar for all children and young people. To achieve this, a stronger focus on leadership for learning in mathematics is required at all levels in local authorities, centres and schools.

We need to engage, challenge and motivate all young people to increase their levels of achievement in mathematics. National and international assessments in mathematics show that learners continue to make insufficient progress from P6 to S2. Many schools have engaged already with the experiences and outcomes in mathematics⁶ and are developing numeracy across learning. This guide aims to promote reflection about how best to develop young people's ability and inclination to apply their mathematical or numeracy skills in managing their day-to-day activities and meeting the challenges of a modern society. It also provides examples of good

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² The term 'teacher' or 'staff' in this document is used we mean early years practitioners, local authority staff, student teachers, initial teacher education staff, further education staff, adult numeracy tutors and teaching staff in schools.

³ In this guide, the terms *learners*, *children*, *young people* are used to describe all of those engaged in the learning process as appropriate to sector or service, as distinct from practitioners or professionals who provide the service.

⁴ The term 'parents' should be taken to include foster carers, residential care staff and carers who are relatives or friends.

⁵ Where we use 'schools' we mean schools, early learning centres or other establishments where children and young people learn.

⁶ Curriculum for Excellence: Numeracy and Mathematics: experiences and outcomes, Scottish Government, 2009.



practice along with questions which teachers can use to reflect on how well they encourage high achievement in mathematics for all learners.

In meeting key challenges staff need to:

- invigorate teaching, and so learning, in mathematics, drawing on successful practice from learning and teaching across the curriculum;
- provide learning environments rich in number and mathematical contexts, reflecting that the school community values mathematics and recognises that mathematical skills and, importantly, understanding are key to being a successful learner:
- use more relevant and exciting contexts to engage and enthuse learners, including well-planned interdisciplinary projects which extend mathematical skills and allow their application in new contexts; and
- improve achievement and post-school outcomes.

The HMIE resources associated with *The Journey to Excellence*⁷, which include self-evaluation guidance and exemplification of effective practice, will be of particular help in taking this work forward.

⁷ Journey to Excellence HM Inspectorate of Education (www.journeytoexcellence.org.uk)



2 Good practice in learning, teaching and promoting achievement

2.1 What are the strong characteristics of effective learning?

The curriculum design principles⁸ are a key tool for planning and organising effective learning in mathematics. The appendix provides reflective questions on the use of the principles to improve practice. Inspection activities and national and international research suggest the following key themes for improving learning in mathematics. By discussing and exploring the nature of these key themes, staff will extend their skills in identifying when and how to make use of them in mathematical activities.

Pace

Pace is not about how quickly tasks and activities can be completed: learning is not a race. Rather, pace can be viewed as one aspect of challenge. Teachers need to plan mathematical tasks and activities which meet learners' needs well. Key features of well-paced lessons include:

- well-established routines;
- good planning including a clear focus on the purpose of learning activities;
- shared purpose for learning;
- a high proportion of time spent on active learning tasks with minimum interruptions;
- high levels of learner stimulation and engagement in thinking;
- a range of activities that provide the right level of challenge to match everyone's learning needs in the class; and
- opportunities for learners to demonstrate that they are confident and proficient in achieving the learning outcome(s).

Sharing and demonstrating learning

Characters are used to help P1 children understand what they are learning. *Mr What?* shares the learning outcomes with children, *Miss How?* shares the process and *Mrs Why?* encourages children to transfer skills, understand the purpose of learning and how this can be applied across the curriculum. The characters are used at the end of learning experiences to encourage children to demonstrate new learning.

⁸ Curriculum for Excellence: Building the Curriculum 3, Scottish Government 2009



To achieve an appropriate pace of learning, teachers need to take account of:

- the rate of learning, including the size of learning 'steps', taking account of the fact that in mathematics learners progress at different rates and with different levels of facility at different times, in different contexts and when learning different ideas;
- the proportion of time spent on active engagement in thinking, the quality of learning, and time focused on task; and
- the learner's resilience, patience, ability to concentrate, application (learning new things needs effort), and depth of understanding.

The right pace is influenced by a number of factors, including the ethos within the class. Children need to share with their teachers high and realistic expectations about their engagement in learning and the belief that they can succeed. They need to feel secure in class and valued for the effort they put into their learning. At all stages in their mathematical journey, children need to know what they have done well and also be clear about what they need to do as next steps for improvement. This interaction between the learner and teacher helps to shape learning and influence pace.

Sharing and planning learning in a primary school

Personal learning planning encourages children to become better mathematical learners. By reflecting on feedback from their teachers and peers, they set clear next steps for learning. This enables all children to recognise success at their own level. Teachers adjust their planning to take account of individual next steps. Children enjoy their learning more and their confidence in mathematics often increases greatly.

Challenge, enjoyment and positive attitudes

By providing challenge for learners, teachers enable them to gain a sense of achievement as they apply, consolidate and extend their existing skills in a number of ways. On too many occasions, teachers provide extension work for higher-achieving learners which does not lead to new learning. As part of learning, teachers need to promote positive attitudes to mathematics and an understanding of how it equips young people with many of the skills required for life, learning and work. National and international surveys indicate that in Scotland, young people's enjoyment of mathematics declines through primary into secondary education, compared to the positive attitudes of young people in other countries.



Effective challenge includes teachers setting tasks and activities which:

- are of increasing levels of complexity or abstraction;
- continually develop, reinforce and extend understanding;
- help learners to make connections to, and build on, their prior mathematical learning;
- require higher-order thinking including analysis, reasoning and deduction;
- depend on learners applying their mathematical and numeracy knowledge and skills to solve problems in familiar and unfamiliar contexts; and
- extend learning within a lesson to meet individual learning needs.

Making connections

'It's really good when young people make connections to work they have done before and someone says, 'This is just the same as'. It makes them more likely to have an 'I can do this' approach to their work'.

In mathematics as in other areas, the right environment for learning is a key component for successful learning. To foster a challenging learning environment where learners are engaged and enjoying learning, effective teachers:

- work to develop shared, high expectations of effort and success: teachers and learners together promote a culture of ambition;
- ensure that learners feel valued and supported enough to take some risks in learning. Children and young people need to feel confident about asking for support from their teacher and peers when they are unsure and feel that they need help; and
- develop an ethos of self-challenge for learners.

Successful teachers involve children and young people actively in their learning, for example by engaging them in thinking and making learning activities enjoyable. Most children in pre-school are making good progress in early number skills during their play. They show skills and enthusiasm in imaginative activities, including learning out-of-doors where appropriate opportunities are taken to embed mathematics within real life contexts. Their broader mathematical skills are less developed in activities in science and technology. In primary schools, there is increasing use of well-paced interactive learning activities in mathematics, particularly in aspects of numeracy and mental calculation. When implementing the



experiences and outcomes, we need to build on this good practice and maintain a strong focus on mental strategies and skills.

Confidence in learning

Children begin to develop independence in measuring, using arbitrary units, by working together to make dough. These skills are then transferred to enable children to measure in other contexts, for example, at water play.

Enthusiasm for learning

'In our department, we regularly include a variety of lesson starters and active and interactive activities. Our young people's enjoyment can be seen from their enthusiasm for learning and their requests for more advanced challenges and games or shorter times to complete mental and problem solving tasks.'



Independent and Collaborative Learning

Independent learning puts learners at the centre of their own learning. It depends upon children and young people being self-motivated and confident enough to make informed decisions. This can include choices about resources, pace and level of challenge and evaluating their own progress. As part of working independently, they are not solely reliant on their teacher but are accountable for their learning. Being able to work independently – either individually or in groups – helps learners to pursue mathematical interests and solve problems, as well as develop more generic skills including leadership, responsibility, resilience and team-working that are essential for lifelong learning. Collaborative learning challenges individuals to think independently and engage in discussion, debate and activity to achieve specific outcomes. In some schools, children and young people are encouraged to initiate their own learning and contribute to that of others by undertaking independent tasks and presenting their findings to the class.



Independent learning in pre-school

In pre-school, children respond well to opportunities to initiate their own learning and develop their interests. Through problem solving and experimentation, children enthusiastically explore mathematical concepts together. Effective intervention from adults supports children in taking their learning confidently into other contexts.

Independent learning in a real life context

To coincide with work experience, young people at S4 were given a task to trawl employment vacancies online, find a job they were interested in and see if they could 'survive' a month on their wages. This involved them investigating questions about tax and costing necessary expenses including car insurance, rent, council tax, utility bills and food.

Working together

"They learn so much more by working together because they see how others work and they all chip in with their ideas. And when they forget, one of their group can remind them so forgetting is no big deal."

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Collaborative interdisciplinary learning

Children in P7 used their mathematics and technology knowledge successfully to build robots to take part in the Scottish Lego League Robotics competition organised in partnership with the University of Edinburgh Informatics Department. Children were judged on teamwork skills as well as their technological skills and spoke confidently about the mathematical processes and thinking behind the design of their robot.



Homework provides rich opportunities for children and young people to demonstrate, extend and explore learning through a variety of exciting and enjoyable activities. Homework is one piece of the teaching- learning picture and in best practice is connected to what happens in the classroom. Quality homework tasks allow learners to practise or process information, introduce them to material that will be discussed in the future, or provide feedback to teachers so they may check for understanding. As well as reinforcing concepts, effective homework:

- has a clear purpose and demands active learner engagement;
- provides opportunities for parents and young people to talk about learning in mathematics and see real-life connections and applications; and
- develops higher-order skills such as analysing and researching.

A family homework challenge

Children in P7 researched the mathematics within a range of board games. Their homework challenge was to work with their family to design and create a new board game. They chose the theme, set the rules, and invented and designed all of the resources required to play the game. They made use of their mathematical knowledge of the properties of two-dimensional shapes and three-dimensional objects, scale, nets, patterns and sequences, and problem solving. They played each others' games, evaluated them based on agreed success criteria and presented their findings to the class.

Active learning

Active learning engages children and young people, extending their thinking and developing their mathematical skills. Well-judged contexts and issues motivate learners and encourage them to find things out for themselves. These range from activities based on imaginative play for younger children to those which require older learners to use their mathematical skills in response to current real-life issues.

Problem solving is embedded within the experiences and outcomes in *Curriculum for Excellence* and is no longer a separate strand of mathematics. A problem solving approach is at the heart of effective learning and teaching of mathematics. Through active learning we challenge young people, for example by providing non-routine tasks and activities of increasing levels of complexity which require them to analyse and make decisions, explain their thinking and synthesise aspects of their existing skills. In best practice, we challenge them to challenge themselves.

Inspection evidence indicates that many primary schools need to build on active learning at the early stages. Across all sectors, teachers do not always provide sufficient challenge for higher-achieving children and young people. Expectations of children's and young people's capacity to learn need to be high enough in order to extend their knowledge and understanding in challenging experiences rather than by



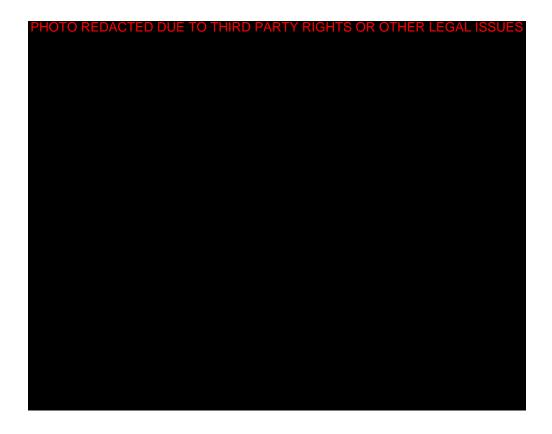
spoon feeding' knowledge with little reference to other aspects of learning or relevant contexts.

Active learning at the early years

In a nursery class, children worked together to order numbers. In order to ensure appropriate challenge for individual children, they were given further opportunities to transfer this skill to other areas of their play, for example, in playing games by ordering numbers successfully on a simple washing line.

When children and young people are increasingly active in their learning, they:

- think deeply about mathematical ideas and concepts and construct their own understanding about them; and
- use their existing skills and knowledge in different contexts, test out their ideas and conjectures, and solve problems.





Mathematics through active learning

As a result of learning experiences and outcomes in mathematics being more relevant to real-life learning contexts, children from P5 to P7 are carrying out the majority of their mathematics lessons through activities which make links across the curriculum where appropriate. The main aim of teaching mathematics in this way is to ensure that children enjoy exploring, investigating and applying mathematical concepts to understand and solve problems. There is an emphasis on collaborative learning to encourage children to reason logically and creatively through discussion. When children learn about the properties of three-dimensional objects, they link it to their topic work, where they learn about churches on the Isle of Lewis. Children work in groups to plan and design a three-dimensional church. During this design and make process, they use their knowledge to build nets, measure accurately and identify faces, edges, vertices and angles. This type of task results in children being actively engaged and their understanding being enhanced. Groups present their findings to their peers, including delivering a presentation. In their evaluations, children commented on how much this approach to learning has impacted on their learning and enjoyment of the topic.

Confidence and security

Curriculum for Excellence emphasises the enrichment of learners' experiences and on the development of higher-order thinking skills. This identifies clearly the importance for learners to be secure in their mathematical knowledge and understanding. They need to be able to recognise the mathematics in a range of problems and contexts, and have the confidence to apply the requisite skills. Underpinning this is recognition of the importance of consolidation of their learning in a range of appropriate contexts. National and international studies indicate mathematics learning often takes place within a narrow range of contexts. When young people's understanding lacks depth, they are unable to apply their learning with confidence, understanding and accuracy to everyday, practical, real-life situations. The studies highlight learners' ability to complete accurately mathematical exemplars from textbooks and worksheets. These studies emphasise learners often lack the deeper understanding of mathematical concepts gained from rich experiences explored through relevant contexts. Often, learners experience a high proportion of repetitive problems of low procedural complexity and an absence of deductive reasoning.

The following features are key to establishing greater confidence in mathematics.

- Learners need to understand what they are supposed to be learning and related success criteria, and have the opportunity to reflect on how successfully they have learned.
- Tasks and activities need to be well paced and engage and challenge learners.



- Breadth through practise and application needs to be established across a range of contexts, including familiar and unfamiliar situations.
- Connections need to be made through making conjectures and exploring ideas individually and with others.
- Depth needs to be increased through higher-order learning activities, including discussing, explaining and making sense of important mathematical concepts.

'Getting advice and help from a fellow student breaks down barriers to learning, and often has the added bonus of giving the helper a real boost to his or her confidence'

2.2 What are the strong characteristics of effective teaching?

Teachers need to continue to develop their professional content knowledge in mathematics, have a strong grasp of skills progression and development, and of how young people learn mathematics.

Opening up learning in mathematics can lead to more confident teaching. In turn, this can instil confidence in learners to contribute to the learning of the group and class. In the 2008 Scottish Survey of Achievement (SSA)⁹, primary teachers confirmed a lack of confidence in teaching ratio and algebraic processes. It is widely agreed that fractions form an important part of the mathematics curriculum, underpinning the development of proportional reasoning and algebraic thinking. *Curriculum for Excellence* experiences and outcomes place a greater emphasis on the development of key numeracy skills and algebraic thinking, requiring teachers to be more secure in their teaching approaches. Those skilled in using a range of effective teaching techniques, and with a greater depth of subject knowledge, foster young mathematicians who have a disposition to recognise embedded mathematical ideas and to apply them. They demand success for all, expecting nothing less than passion for mathematics, openness to new thinking and ideas, curiosity and determination to maximise the potential of every single learner.



⁹ 2008 Scottish Survey of Achievement (SSA) – Mathematics and Core Skills, Scottish Government, 2009.

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Teacher to teacher coaching

The 'lead teacher' programme provides subject specialist support for teachers in each school. Teacher to teacher coaching provides the opportunity to work alongside an expert teacher. The lead teachers share good practice, model effective teaching and coach, mentor and provide CPD for colleagues. Lead teachers require a well established subject knowledge base to provide this level of support.

Over a number of years, many teachers have recognised the need to increase children and young people's ability to use mathematics to solve problems as part of day-to-day life. Most provide examples of the use of mathematics where the context lends itself to such illustration, for example about money calculations or extracting information from tables and charts. However, not all teachers identify appropriate and relevant contexts consistently for the mathematics being taught. Sometimes, schools have taken steps to promote aspects of numeracy which provide children and young people with many of the life skills required as responsible citizens of Scotland in the 21st century. Relatively few make enough use of relevant contexts and meaningful purposes from other curriculum areas to motivate learners. Teachers often rely too heavily on commercially produced schemes. The need now is for the best practice in achieving high standards in mathematics and making mathematics relevant to learners to become more widespread and consistent across all schools and classes. In order to achieve this, we need more subject-specific professional development to build further skills and confidence in teaching mathematics.

As schools implement *Curriculum for Excellence*, teachers are beginning to organise and connect learning in mathematics with other curriculum areas through interdisciplinary learning. In the most effective examples, the learning ensures appropriate challenge and progression in skills within enriched, stimulating contexts.

Connecting learning

Children in P6 developed and produced a leaflet on a local tourist attraction for other children. They used their mathematics knowledge to examine and compare quotes for publication and learned valuable skills across a small number of curriculum areas. They contributed to the promotion of their local community.

In S1/S2, young people's learning in mathematics, geography and physical education is connected through an orienteering project. They draw maps to scale, identify quickest routes, discuss rotation, estimate distance, calculate time intervals and use bearings to navigate an orienteering course.

Teaching approaches which are presently contributing particularly well to successful learning in mathematics include the following.



- Consistently sharing and discussing the purpose of learning with children and young people, including long-term and short-term learning goals. In particular, every mathematics lesson can be justified on the basis that it has real-life applications, and is used in other subjects, extends personal interests or is a necessary skill for future learning in mathematics.
- Teachers and learners asking open and searching questions to probe understanding, develop higher-order thinking skills and highlight misconceptions. In successful lessons, teachers use young people's answers skilfully to illustrate the type of thinking which leads to the successful solving of mathematical problems. They use 'wrong' answers to promote learning, praising 'smart mistakes' and encouraging learners to explain the thinking which led to their answer.
- Setting high expectations of what children can achieve in mathematics.
- Maximising child-to-child interaction in order for them to develop thinking, explain, explore and demonstrate their learning, as opposed to passively listening to the teacher for extended periods.
- Well-planned opportunities for children and young people to learn through investigative, active approaches.
- Making explicit reference to key mathematical concepts when connecting learning across curriculum areas.
- Making good use of information and communications technology (ICT) to promote understanding and to enliven teaching and learning and, as part of homework tasks, to engage and motivate learners. In some schools, young people send in their homework online and receive individual feedback on it from their teacher. In a few schools, young people use technology within their mathematics projects and this aspect is accredited within their National Qualification certificated ICT course.



- Responsive planning which takes account of children's and young people's
 progress, interests and preferences. As a result of assessing learning on a
 continual basis, plans are developed to meet the needs of groups and individuals.
- Lessons make effective use of plenary sessions to encourage children and young people to reflect on learning, collaboratively identify next steps and transfer learning.



Effective questioning

Staff use effective questioning, listen to responses and interact with young people, to help them identify areas of strengths and weaknesses. They then discuss what would help. For example, would mnemonics help young people remember their trig ratios? Would they like a peer to explain the work in their own way? Would a mind map help with retaining information? Young people are given the responsibility for deciding what they would like to try and reassess the situation in the near future.

Teachers, with a strong grasp of the mathematics curriculum, are best placed to support children and young people across the spectrum of ability to achieve success. High-quality teaching resources and CPD are important to provide them with the support to organise learning to best meet the needs of learners.

2.3 What are the strong characteristics of achievement?

Mathematics education provides a wide range of opportunities for learning and personal achievement. It contributes effectively to the development of the skills and attributes underpinning the four capacities of *Curriculum for Excellence*. The table overleaf highlights the characteristics that successful learners, confident individuals, responsible citizens and effective contributors develop through mathematics.



Successful Learners

- Are motivated and enthusiastic about learning mathematics because they understand why the topic is being taught.
- Think flexibly about how to apply their skills and enjoy having to puzzle out an answer.
- Use technology effectively to save time in routine calculation and demonstrate understanding by sketching graphs.
- Understand how their mathematical knowledge can be used and apply appropriate strategies to solve problems in a range of contexts and across the curriculum.
- Achieve high standards.

Confident Individuals

- Use a range of mathematical and numeracy skills in school subjects and everyday life.
- Are independent, mathematical thinkers who can discuss and explain their reasoning.
- Use their mathematical and numeracy skills to provide evidence for informed decisions.
- Collaborate effectively to solve problems.

Responsible Citizens

- Interpret numerical information to draw conclusions based on evidence and analysis and not on opinion or prejudice.
- Interpret tables and graphs to assess trends and can use their conclusions to take and justify decisions.
- Evaluate data to gain an accurate view of a situation and make informed choices.
- Recognise the importance and role of mathematics within society.

Effective Contributors

- Have an enterprising 'can do' attitude.
- Work productively in teams to solve problems and reach decisions.
- Work as self-reliant individuals.
- Are motivated and enjoy being challenged and solving problems.
- Apply critical thinking skills in different contexts.
- Are not afraid to take risks and can solve problems.

Pathways of progression

Teachers know the importance of smooth progression as learners develop their mathematical skills. Taking full account of, and building on, mathematical concepts and previously developed skills are essential conditions for successful learning and confident application of mathematics. This is particularly important as children move from stage to stage and at key transition periods. Planning for effective progression requires recognition that learning in mathematics is not linear and that close attention



is paid to relevant factors such as pace, challenge, collaboration and independence in learning.

Progress in learning mathematics relates directly to the range of skills and attributes embedded in the mathematics and numeracy experiences and outcomes. What does 'progress' look like to learners, their parents and teachers? Some of the features of progress are:

- extension and development in learners' mathematical knowledge, understanding and skills;
- improvement in children's propensity to use their learning and apply their developing skills;
- becoming better at learning how to learn mathematics;
- being more self-reliant and independent as learners;
- making their own connections across their mathematical learning and from different contexts; and
- engaging in higher-order thinking skills and asking probing questions to develop deeper understanding of mathematical concepts.

A number of key features are necessary for continuous and continuing progress.

These include:

- mathematically knowledgeable teachers at all stages, who know and understand
 in detail and from a more advanced perspective, the concepts, content and its
 connections to other important mathematics, both prior to and beyond the level
 they are assigned to teach;
- teachers identifying accurately the learning needs of children and young people, and using this information to plan tasks and activities which support and challenge them, providing both depth and breadth of learning;
- teachers and learners talking together about what they need to do to improve and planning next steps. Learners need to be clear what success looks like and they may find it helpful to agree appropriately challenging targets with their teacher. This is linked to them feeling valued as learners and sharing high, realistic expectations; and
- learners who believe that 'success breeds success' and who respond positively to a 'can do' ethos and well-judged praise.



Talking together to improve learning

Upper primary school children undertook an audit of *Assessment is for Learning* and found that tools and strategies were not always used to identify and help them understand what to do to improve in mathematics. Most of the marks in their jotters were ticks and crosses. The pupil council worked with senior managers to introduce new ways of responding to learning in mathematics to help children to think and reflect.

Place of assessment

Teachers use assessment for a number of reasons, including monitoring pace and challenge and evaluating learners' progress. *How* teachers assess aspects of mathematical learning depends for the most part on *Why? What?* and *When?*

Teachers need to know what progress learners are making and which areas require challenge and support, as well as whether they are 'getting there' or not. Teachers are aware of Assessment is for Learning approaches. To be fully effective and help learners to learn mathematics, teachers need to have a deep understanding of a range of strategies and embed them in their practice. The principles of Curriculum for Excellence mean that progress is defined in terms of breadth and depth of achievement at the level for each stage. To ensure that children become confident and secure in their mathematical learning, teachers now need to think more about planning opportunities for learners to demonstrate what they can do, in both familiar and unfamiliar contexts. Effective assessment practices inform next steps in learning as well as providing reassurance - at national, local, school or classroom level – that learners have achieved the levels of skills, attributes and knowledge they will need if they are going to make their way in an increasingly competitive world. Assessment evidence is more than written 'tests' and includes individual portfolios. photographs, oral presentations, artefacts and constructions, written pieces of work as part of problem solving activities, movies and peer reviews.



Personal profiles in the early years

In a children's centre, staff work effectively together to observe children engaged in mathematical activity. They then meet regularly to discuss and analyse the progress of individual children before agreeing on what children should learn next. Effective teaching strategies ensure that numeracy is embedded across the playroom. For example, children were exploring the conservation of number through using shapes they had made at the dough table. Staff engage in regular professional dialogue with teachers from local primary schools to share teaching approaches and assessment information. Teachers had the opportunity to observe nursery staff at work with the children and discuss the progress of individuals.



Numeracy folios

A secondary department of mathematics works closely with their local college where young people complete a range of 'skills for life, skills for work' courses. Individual numeracy folios record achievements of young people in developing and applying key numeracy skills through contextualised learning experience. For example, young people studying construction recorded evidence of applying skills in measuring, estimating and calculating. This included photographs, course work and assignments.



What are the features of effective practice in improving mathematics?

Learning improves when staff at all levels take greater ownership of improvement through self-evaluation, involving learners and their parents. For self-evaluation to give an accurate, rounded view, it must triangulate evidence from people's views, direct observation of learning and teaching, and quantitative data, as illustrated in the following diagram from *How Good is our School.*



The following sections of this guide take each of the three sources of evidence in turn, exploring the ways in which teachers are working to improve learning in mathematics.

Reflective questions: the Appendix contains reflective questions for professionals exploring further the issues covered in this section.

3.1 Using people's views

All stakeholders have important views about mathematics. Individually, each perspective is valuable and can contribute to the successful change that is needed to improve the outcomes for children and young people in mathematics. Taken together, people's views can influence the changes that will ensure that all children and young people will have a confidence in using mathematical skills in life, learning and work. Teachers also need to take due cognisance of important national and international views of children's and young people's progress in mathematics to bring about effective improvement. For example, by using valuable information on young people's performance in national examinations provided by the Scottish Qualification Authority's principal assessors.

What do learners think?

Children, young people and adults use mathematical skills in their everyday life. Principles and practice in mathematics as part of *Curriculum for Excellence* discusses the engagement and fascination of learners in logical reasoning, analysis, problem-solving skills, creativity and the ability to think in abstract ways. Therefore, all children and young people have first hand experience of mathematical learning and teaching. As they move through school, they will have an increasing view of



how they acquire the disposition and ability to recognise mathematical ideas and apply them appropriately.

Children improving learning experiences

Children were asked about ways the school could improve learning in mathematics. Most children felt that they wanted more time to talk about what they were learning and spend less time completing exercises from textbooks: 'when you know you understand, there isn't any point doing the whole page' (P7 child). 'We want more time talking and listening to each other rather than watching the teacher at the board' (P6 child). The school improvement plan took this into account and focused on ways of increasing the use of talking, listening and collaborative group work within mathematics to improve learners' experiences.

Teaching staff use many different approaches to obtain the views of children, young people and parents. They work alongside learners and make focused observations of their responses within learning experiences. Teachers also gather views as part of their on-going self-evaluation practice. The findings of activities such as these can be used to help improve learning and teaching in mathematics as part of self-evaluation. They can also contribute to supporting learners in developing the confidence they need to recognise how they can apply their mathematical skills in different contexts.

Asking children about learning

Staff took account of the views of children attending the after school mathematics club. Each week, children were encouraged to describe ways to improve the activities and how they were delivered. Subsequent sessions were adapted and improved based on children's suggestions and ideas. For example, they wanted to use more technology. Staff then identified a range of web-based games which supported the development of skills in mental calculation in an interesting and motivating way.

Taking greater account of learners' views, together with important information about them as learners, can help ensure effective teaching and learning in mathematics. As a result, levels of challenge and enjoyment in mathematics can increase.



Young people informing improvement planning

Staff place a lot of emphasis on young people's evaluations. After every assessment, young people complete an evaluation to identify their strengths and next steps. These evaluations are shared with parents. Young people are encouraged to give their views on a wide range of issues covering learning and teaching, assessment, courses and resources. These comments inform staff of current issues and concerns for young people, and also influence improvement planning for the sessions ahead.

What do parents think?

Engaging with parents and gathering their views are important steps in encouraging all parents to take an active role in their child's learning in mathematics. Parents need to have access to information about their child's progress and what they will learn next. If a strong partnership exists, then parents will help children learn as well as gain confidence. All parents can engage in their child's learning through a variety of home link resources which promote and extend children's learning.

Engaging parents in learning

To help parents improve their children's learning as they move from nursery to P1, a support group was set up in partnership with an adult literacy tutor from the community. Parents attend a five-week course in the local health centre. The course helps them develop an understanding of how children learn mathematics and how to support this at home. The local primary school work closely with the group to share approaches and resources used in lessons. As well as improving support for children as they move into P1, many parents go on to study for nationally accredited numeracy awards.

'I'm more confident than I used to be and I can help my kids with some of their homework and get a qualification too'.



The *Skills for Scotland* strategy highlights the need to help parents with numeracy to enhance their ability to support their children's learning. Pre-school centres encourage parental involvement in their child's early learning through activities such as teaching songs, nursery rhymes, painting and playing with numbers. In effective practice observed within the primary sector, there is on-going dialogue between staff and parents about supporting learning and improving provision. Not all parents are confident about being involved in their child's learning of mathematics, particularly as it becomes more complex. Pre-school centres and schools who recognise the important role that parents play in their child's mathematics development provide them with specific guidance on how to do this. They help parents to support learning both at home and school through workshops, on-line materials, leaflets and discussion forums.



Engaging parents in home study

Staff set out to engage parents more in supporting their child with Higher mathematics. The 'Yes you can do Higher mathematics' initiative includes a workshop to share course content and approaches to learning. Parents receive a pack of activities to work through with their child at home. These include quizzes, games and sets of questions to reinforce key concepts. The partnership continues as young people progress. Parents use the school website to download support materials and resources to help their children. Parents views are sought to refine and improve the support provided by the department and the range and quality of resources.

What do staff think?

Through self-evaluation, teachers have a detailed knowledge about the quality of their learning and teaching. Often, identified members of staff have a key role in developing and improving learning and teaching in mathematics. It is the responsibility of all to improve learning and teaching in mathematics and, in doing so, draw on good practice from colleagues and other areas of the curriculum.

When implementing *Curriculum for Excellence*, teachers need to consider how teaching and learning in mathematics requires to change in order to ensure the best outcomes for all children and young people. They need to reflect on how they will develop their own practice to respond to necessary changes. In doing so, they will require opportunities to engage in professional discussion. This will help teachers identify what practice can be built upon and crucially, what needs to change.

Creating a culture of improvement through self-evaluation

The mathematics department is committed to improvement through self-evaluation and has effective arrangements for reviewing the quality of its work. These include tracking young people's progress and monitoring of homework and coursework. Teachers use examination results to target improvements to courses and programmes. They continually seek the views of colleagues and young people on the effectiveness of learning and teaching. They support and challenge each other through learning visits to each others classes. This helps the department to identify its strengths, share good practice and agree areas for improvement. Staff discuss and debate educational improvements and the impact on improving outcomes for young people. They reflect on their practice continually to plan for on-going improvements to learning.



What does the community think?

Improving Scottish Education 2005-2008 reported on the need to build partnerships with individuals, establishments and services to support and ease progression for learners. The views and support of partners such as employers, industry, Scotland's colleges and voluntary groups are essential when improving learning in mathematics and evaluating its relevance for young people's success now and in the future. Schools need to plan opportunities for children and young people to experience a range of personal achievements in the classroom and beyond, in partnership with the community. Within mathematics, there are rich opportunities for links with a range of partners. This can include local employers, health services and community groups such as adult literacy and numeracy tutors.

Eco-house of the future

The eco-house project challenges young people to design and build a house of the future which is environmentally friendly. Working with partners from construction, architectural services and environmental groups, they develop a range of skills for life and skills for work. This context helps young people to see the purpose and relevance of developing and applying a range of mathematical skills, including estimating, measuring, budgeting and problem solving.





3.2 Using learning visits to share good practice

Sharing and evaluating learning and teaching of mathematics can be a dynamic process which opens up discussion on effective learning, involving teachers, senior leaders and young people. The focus should be on both the subject content, including the skills and concepts which underpin the learning of mathematics, and the pedagogy, that is the range of approaches to learning and teaching mathematics. We can also open up the effectiveness of the mathematics curriculum through sharing classroom practice in action. This can include learning, teaching, assessment and teachers' mathematical knowledge.

Direct observation and working together in classrooms can help to determine how well the curriculum for mathematics is designed, reflecting on the principles of *Curriculum for Excellence. Building the Curriculum 3* provides more detailed exemplification of the principles of curriculum design, as well as reflective questions for professional dialogue and debate.

Learning visits to lessons are an important part of opening up learning in mathematics. Many teachers work with each other in classrooms and observe learning, looking at ways to share ideas and challenge colleagues to improve further. This is part of an on-going approach to improvement through self-evaluation. Developing a shared understanding of what high-quality learning in mathematics would look like in a particular school is an important part of this process.

Agreeing the focus for sharing classroom practice

As part of a planned programme, learners' experiences in mathematics were observed and evaluated across the school. Prior to this, staff had agreed the measures of success that would help to ensure high quality mathematical learning experiences for children. They reflected on quality indicators and recent CPD in mathematics and agreed the criteria for success. These included references to active learning, time and space for children to explore and discuss mathematical concepts, opportunities for mental calculation and high quality formative feedback to help young people to reflect and improve. Members of the Pupil Council were asked 'What makes a very good mathematics lesson' at our school? They drew up and agreed criteria which were used by senior leaders when working in classrooms.

Professional dialogue following direct observation is used to identify and share good practice and also look for ways of improving further. This should focus both on the content and pedagogy of the lesson. This direct observation and professional dialogue is not an end in itself. Schools need to develop their own systems for following through on learning visits to ensure a lasting impact of observation on improving learning and teaching and attainment in mathematics.



Opening up learning: using evidence to make overall judgements

After observing learning in mathematics throughout the school, senior leaders worked together to write a whole school summary of strengths and areas for development in mathematics/numeracy. This provided a useful whole school account and challenged teachers to reflect on their own practice and individual discussions after observation.

Extracts from one summary included:

Strengths:

- Teachers used a variety of questioning techniques very well to probe learners' understanding, enabling them to communicate mathematical ideas and develop higher-order thinking skills, including reasoning.
- In most lessons, teachers used young people's answers skilfully to illustrate the type of thinking which leads to the successful solving of a mathematical problem.
- Overall, there is increasing use of well-paced interactive learning activities in mathematics, particularly in aspects of numeracy and mental calculation.
- Learning is enhanced by the well-targeted use of tasks and activities which challenge learners to use their skills flexibly and make connections between prior and current learning.

Areas for development:

- Increase the use of information and communications technology (ICT) in class to promote understanding and to enliven teaching and learning.
- Build on the success of courses and programmes of study which are used well to sequence young people's learning, and provide more open-ended tasks where young people can investigate, explore and debate mathematical concepts.
- Increase the use of formative assessment to help young people to identify next steps and improve their learning.

These high level messages were then shared with young people at a learning assembly. This helped everyone to understand how they could work together to bring about further improvement. A range of approaches to monitoring the long-term impact of this exercise had been developed. This includes visits to parts of or whole lessons; discussions at staff meetings; meetings of young people; sampling young people's work; senior leaders, staff and young people taking learning walks to look for evidence of improvement in relation to the original criteria; and whole school action points.

Increasingly, teachers are involved in these evaluation activities as an individual, with peers and at a strategic level as part of their core duties, working together to develop and improve the curriculum.

The Leuven Scale of active learner engagement¹⁰ is a useful tool to help evaluate how engaged and involved in learning young people are at different points in a mathematics lesson. This scale helps the observer to focus on what the learner does within a lesson and the extent to which they are actually engaged in their own learning. This can be a powerful tool for gathering evidence on challenge, motivation and effectiveness of mathematical learning and teaching. It can be used by teachers, senior leaders and young people.

Direct observation of learning across discrete subjects and within interdisciplinary projects can help schools to determine how well children and young people apply and develop numeracy skills. Observation of specific phases within lessons can also help to determine how well connections are made for children and young people to use their numeracy skills outwith mathematics lessons.

Observing the connections

Teachers wanted to make better use of the ends of lessons to help children to see connections and think about how they could apply their learning in mathematics. They observed lesson endings and discussed the following key features of a lesson plenary within mathematics.

- Children had opportunities to demonstrate their learning and new mathematical ideas.
- Through discussion, young people developed knowledge and awareness of conceptual connections between the new learning and their own existing knowledge.
- The learning intentions were reviewed and next steps were agreed.
- A range of formative feedback was facilitated between individuals.
- Children led the review and discussion, with the teacher acting as a facilitator and coach.
- Success and new learning was celebrated.

Examples of good practice were also shared across the staff as part of this collective focus.

Direct observation through learning visits can also help teachers to determine how effectively they are meeting learning needs in mathematics. Observing particular individuals and groups can help to assess progress and how well tasks and activities are matched to learning needs. National initiatives such as *Assessment is for*

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¹⁰ The 'Leuven Scale of Active Engagement' by Professor Ferre Lavers of Leuven University can be found within the dynamic learning conference report at www.hmie.gov.uk



Learning have helped children and young people to improve their skills as learners. Direct observation of lessons can help teachers to review how well formative assessment is used in mathematics. Discussing classwork with young people can also help to evaluate if they are aware of their strengths and areas for improvement within mathematics and if they have been given time and space to act on feedback.

3.3 Using information about achievement

This aspect of the self-evaluation triangulation process involves the use of a range of information and data. Evidence about learners' progress and their success in undertaking the experiences and achieving the mathematics outcomes within *Curriculum for Excellence* will be gathered from various sources. Such information is particularly powerful when used to improve learning for all children and young people.

Much of the information and data that schools have will be generated from assessment practice. Assessment is an integral part of learning and teaching and, as such, practice in schools has to be well planned and implemented to improve learning. Clearly, quality assessment practice seeks to result in improvement.

Assessment processes need to ensure they provide opportunities for children and young people to show what they can do and address shared learning goals. Evidence about achievement in mathematics will come from what children and young people write, say and do. In order to gather such evidence and evaluations, senior managers and staff need to engage in learning visits to class, talk with learners, and examine wall displays and written work.

Teachers need to share how to apply effective assessment practice and develop a shared understanding of expectations of performance. Moderation of teachers' judgements, with reference to external sources of advice, is required to ensure common and sound interpretation of assessment criteria and practices across the school.

'Teachers judgements of pupils' levels in mathematics were consistently greater than the results based on the main *Scottish Survey of Achievement* assessment'. *Scottish Survey of Achievement*, 2008.

Staff may use baseline or entry achievement information to identify learners' strengths, needs and next steps towards learning goals. This information is used to benchmark learners' progress as they move from stage to stage, track achievement of all learners, as well as against national standards. Senior managers need to work with staff to predict and identify when learners demonstrate secure skills and knowledge. Children and young people should be aware of how well they are progressing. Staff should ensure that when children and young people achieve a national level they have experienced depth of coverage and can demonstrate breadth of learning. Teachers should have evidence of coverage, confidence and consistency across all strands of mathematics to judge whether learners have achieved a specific level.



Data from internal and external examinations can be used to determine how well a school is improving young people's achievement in mathematics. This data can be analysed and broken down in a variety of ways to reflect on the achievement of specific groups of learners and plan for further improvement.





4 Reflective questions

The following appendix provides, for reflection, examples of key questions, issues and features of opening up learning in mathematics. They cover some of the issues and ideas raised in section 3. These questions are intended to be used as a starting point to stimulate and inform discussions and raise debate, with a view to increasing shared understanding and promoting improvement. Some of the questions may also help to inform discussions with learners, their parents and other members of the community.



Examples of reflective questions for staff

4.1 Using people's views

- How do we gather and use the views of stakeholders using a variety of approaches, for example, focus groups, questionnaires, on-line surveys?
- How often do we seek the views of our stakeholders and for what purposes? For example, to explore the challenge and enjoyment within homework tasks.
- How do we build on the views of young people, staff, parents and partners to improve learning in mathematics? How confidently can we show the impact of using people's views on improving outcomes?
- How do we help to ensure positive attitudes and dispositions towards mathematics across the learning community?
- To what extent do the views of all stakeholders match the views of staff in terms of the quality of learning and teaching in mathematics?

4.2 Using learning visits to share practice

The following reflective questions might prompt professional dialogue about your use of direct observation and the impact on improving outcomes and experiences for young people.

- How can we use direct observation of mathematics and numeracy to open up learning and so improve outcomes for our learners?
- How do we create conditions for learning which are supportive and challenging?
- Do we share consistently and fully with young people the purpose of their learning? In particular, can every mathematics lesson be explained on the basis that it has real-life applications, used in other subjects, extends personal interests or is a necessary skill for future learning in mathematics? Are plenaries used well to reflect on learning, discuss transfer of skills and agree next steps?
- When organising direct observations do we include other curriculum areas and record mathematics in the learning of other subjects?
- How well do we interact with young people, for example, to ask more searching
 questions which probe their deeper understanding? How well do we use 'wrong'
 answers to promote learning? Do we praise 'smart mistakes' and encourage
 children to explain the thinking which led to the answer?



- Do we ensure that the teaching of mathematics always challenges young people sufficiently, for example higher achieving children?
- Do we challenge young people enough, for example by providing non-routine tasks and activities of increasing levels of complexity which require them to analyse and make decisions, and synthesise aspects of their existing skills. Do we challenge them to challenge themselves?
- Do we continually develop, reinforce and extend young people's understanding?
- Do young people have the opportunity to work effectively with, and relate appropriately to, others through collaborative activities and accept responsibility for their contribution?
- Do tasks and activities motivate young people and are they actively involved in their own learning?
- Do our teaching approaches enable young people to develop the attitudes and skills of enterprise, including motivation, ambition, self-reliance and team working?

Examples of reflection questions to evaluate the rationale and design of the curriculum for mathematics.

Progression – are curricular pathways ensuring appropriate progression in skills and in understanding for groups and individuals, building effectively on prior learning?

Coherence – do teachers use their mathematical knowledge to ensure that concepts, skills and understanding are developed in a coherent and logical way? Is learning connected to important mathematical concepts prior to and beyond the level currently being taught?

Breadth – is there time and space within the curriculum for young people to experience breadth of learning?

Depth – are there opportunities to explore concepts in depth, to develop and refine understanding and explore learning through a variety of tasks and activities?

Relevance – do young people understand the purpose of their learning? Do they see real life contexts where they would apply this mathematics? Do young people learn to apply their practice problems and exercises in mathematics to develop skills, make connections, reflect and explain their reasoning?

Challenge and Enjoyment – are young people challenged through increasingly complex mathematical questions and problems? Are young people actively involved in learning mathematics, or do they spend long periods completing routine exercises from textbooks which do not challenge their thinking?



Personalisation and Choice - are there appropriate pathways so that young people's progression is personalised to meet their individual needs? Is the mathematics curriculum flexible enough to respond to children and young people's interests and preferences? Do young people have a choice in the way that they demonstrate new learning? Do they make good use of the choices?

4.3 Using information about achievement

- Do our procedures and practices reflect the strategic vision and key principles within the 'Assessment for *Curriculum for Excellence*' framework? Do we reflect on how our practice addresses the ten principles of assessment¹¹ for learning?
- Are all of our assessment processes rigorous and sufficiently robust to monitor progress at individual, class and stage levels?
- Do our assessment policies and practices ensure that those who require additional support with their mathematics are identified at an early stage?
- How effective are our assessment processes in identifying those who require support with their learning or those who are mathematically gifted?
- Are all of our staff regularly assessing learner's work to evaluate their progress?
 How well, and how often, do we ensure all staff understand how to interpret and use assessment information and data?
- Do we have clear evidence that children and young people have made significant progress from prior achievement? Do we know the causes or reasons for improvements or lack of progress?
- Where children and young people self-assess, is this closely monitored by staff and followed up by individual, group or whole class feedback and action?
- Do our comments to learners provide sufficient guidance to enable them to improve?
- Does the data and information from assessment processes demonstrate learners' security and confidence across a range of mathematical concepts and skills? How are results from summative assessments, including preliminary examinations, used? How do they compare with learner's performance in other curriculum areas?
- Do our young people have an appropriate understanding of examination and assessment mark criteria and structures?

¹¹ The 10 principles of assessment as stated in *Changing Assessment Practice* by Gardner, Harlen, Hayward and Stobart. Assessment Reform Group, 2008.



- At points of transition, do we use assessment information to ensure continuity of learning?
- Is homework used as formative assessment to check for understanding before practice is assigned, to determine how much practice is needed to perfect a skill, and to judge the learner's depth of understanding and ability to apply learning?





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Currie High School

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Grange Academy
Invergordon Academy
Killermont Primary School
Kilmartin Primary School
Mid Calder Primary School

Prestwick Academy

St Albert's Primary School St Francis' Primary School

Western Isles Council Shetland Islands Council

Falkirk Council

Perth and Kinross Council Edinburgh City Council Edinburgh City Council East Ayrshire Council Highland Council

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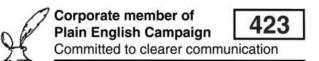
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